Improved Modified Class of Estimators in Estimating the Population Mean

Napattchan Dansawad

Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi, Bangkok 10140, Thailand

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Abstract

In this present paper, following the work of Yadav et al. [1], the author creates alternative class of estimators for estimating the population mean in situation of positive and negative correlations between supplementary and interested variables under simple random sampling without replacement (SRSWOR) scheme. Furthermore, this paper also introduces a modified class of estimators based on combination of alternative class of estimators and unbiased estimator. The expressions for the bias, Mean square error (MSE), and Minimum mean square error (MMSE) of all introduced estimators were considered. Both theoretical and numerical analysis were encouraging and supporting the performance of all introduced estimators for the mean estimation. The MSE and Percent relative efficiencies (PREs) were used as criteria for efficiency comparison between the new estimators and other existing estimators.

Keywords: ratio type estimation, product type estimation, supplementary variable, bias, population mean

Corresponding author: napattchan.dan@kmutt.ac.th

Introduction

In many works related to survey sampling, the use of suitable supplementary information has been permeated the important role to enhance higher precision in the estimates of some population parameters under the SRSWOR scheme such as population total, population mean, population proportion, population variance, etc. One of the popular estimators of population parameters at present is the population mean. In creating any estimators for the population mean, if the relation between supplementary and interested variables is positive, the ratio type estimation first initiated proposed by Cochran [2] is a suitable estimator for the mean of population. On the other hand, if the relationship is negative, the product type estimation as envisaged by Robson [3] and Murthy [4] is appropriate.

In addition, several authors have applied the concept of Cochran [2], Robson [3], and Murthy [4] estimators with the purpose of producing a more efficient population mean estimation. For instance, Sisodia and Dwivedi [5] was first who used a coefficient of variation (C_x) of supplementary variable to improve the efficiency of the Cochran [2]'s estimator. Motivated by Sisodia and Dwivedi [5], Upadhyaya and Singh [6] proposed another ratio type estimator of population mean using the supplementary variable in the form of coefficient of variation (C_x) and coefficient of kurtosis $(\beta_2(x))$, whereas Singh et al. [7] established two ratio and product type estimators using transformation based on known value of coefficient of kurtosis $(\beta_2(x))$.

Kadilar and Cingi [8] suggested a class of ratio estimators for the estimation of population mean by adapting the estimators in Upadhyaya and Singh [6] to the estimator in Singh and Tailor [9]. Previously, Kadilar and Cingi [10] also suggested a chain ratio estimator for estimating the population mean by adapting the estimators' type of Ray and Singh [11]. Al-Omari et al. [12] worked out two new estimators of the population mean using two modified simple random sampling (SRS) and ranked set sampling (RSS) methods when the first or third quartiles of the supplementary variable are available. Koyuncu and Kadilar [13] proposed a general family of combined ratio estimators for population mean using various known population parameters in simple random sampling.

In the next year, Yan and Tian [14] proposed two modified ratio type estimators based on the values of coefficient of skewness $(\beta_1(x))$ and kurtosis $(\beta_2(x))$ of supplementary variable. Subramani and Kumarpandiyan [15] introduced some new modified ratio type estimators using the linear combination of coefficient of variation (C_x) and median (M_d) of the supplementary variable for estimating the population mean. Jeelani and Maqbool [16] used the linear combination of coefficient of skewness $(\beta_1(x))$ and quartile deviation for proposing two modified ratio type estimators of population mean. Also, Jerajuddin and Kishun [17] introduced the use of sample size (n) to increase the performance of modify ratio type estimator. They showed that their estimator is more efficient in the term of the performance than other existing estimators of the population mean.

Pandey and Dubey [18] proposed product estimator for the population mean based on the known of coefficient of variation (C_x) of supplementary variable which turns out to be an alternative estimator in the situation of the relation between supplementary and interested variables is negative. Singh [19] proposed new product estimators under the SRSWOR scheme using known parameters of the supplementary variable such as standard deviation (S_x), coefficient of skewness $(\beta_1(x))$ and coefficient of kurtosis $(\beta_2(x))$ of supplementary variable. By applying the estimator in Koyuncu and Kadilar [13], Yadav and Kadilar [20] also proposed an improved ratio and product family of estimators for the population mean using the information on the coefficient of variation (C_x) and the correlation coefficient (ρ) of the supplementary variable.

Previously, Khoshnevisan et al. [21] suggested using two supplementary information to create a general family of estimators of population mean that cover the other existing estimators. However, the estimator of Khoshnevisan et al. [21] was a quite difficult form to use in practice. Therefore, Yadav et al. [1] suggested adjusting the estimator of Khoshnevisan et al. [21] by removing and adding some values of constants. The estimator of Yadav et al. [1] is given as follows:

$$t_Y = \bar{y}\left(\frac{ab\bar{X}+cd}{ab\bar{x}+cd}\right), \tag{1}$$

where $\bar{y}$ and $\bar{x}$ denotes the sample mean of interested and supplementary variables respectively. $\bar{X}$ denote the population mean of supplementary variables. a, b, c and d are real constants or the functions of known parameters of supplementary variable such as the inter-quartile range (Q_r), skewness $(\beta_1(x))$, kurtosis $(\beta_2(x))$, coefficient of variation (C_x), correlation coefficient (ρ), standard deviation (S_x), median (M_d), and so on.

The bias and the MSE of Yadav et al. [1] estimator, respectively, are

$$Bias(t_Y) = \lambda\bar{Y}(\theta^2 C_x^2 - \theta C_{yx}), \tag{2}$$

$$MSE(t_Y) = \lambda\bar{Y}^2(C_y^2 + \theta^2 C_x^2 - 2\theta C_{yx}), \tag{3}$$

where $\lambda = (N-n)/Nn$, $\theta = ab\bar{X}/(ab\bar{X}+cd)$, $C_x^2 = S_x^2/\bar{X}^2$, $C_y^2 = S_y^2/\bar{Y}^2$, $C_{yx} = S_{yx}/\bar{Y}\bar{X}$, $S_y^2 = \sum_{i=1}^{N}(y_i-\bar{Y})^2/(N-1)$, $S_x^2 = \sum_{i=1}^{N}(x_i-\bar{X})^2/(N-1)$, and $S_{yx} = \sum_{i=1}^{N}(y_i-\bar{Y})(x_i-\bar{X})/(N-1)$.

Likewise, the MMSE of this estimator for the $\theta_{opt} = C_{yx}/C_x^2$ is equal to

$$M\dot{S}E_{\min}(t_Y) = \lambda\bar{Y}^2\left[C_y^2 - \frac{C_x^2}{C_{yx}^2}\right] \tag{4}$$

To improve the efficiency of the Yadav et al. [1] estimator, the author attempts to present two new classes of estimators for estimating the population mean by using the exponentiation method under SRSWOR. The bias, MSE, and MMSE of the new estimators have been obtained. In addition, comparative studies of the new estimators with other existing estimators have been assessed through theoretical and numerical analysis.

Methods

In this paper, the author is interested in improving the efficiency of Yadav et al. [1]'s estimator as a predictor of the mean of the population using the information supplementary when the correlation between interested and supplementary variables is positive in the sample. Therefore, in this section, we introduced an alternative class of estimators in both situations of positive and negative correlation patterns. Furthermore, we also introduce a modified class of estimators based on a combination of alternative class of estimators and unbiased estimator. The details of the methods are as follows:

1. Proposed Alternative Class of Estimators

By adjusting the estimator of Yadav et al. [1], the author introduced an alternative class of estimators for estimating the population mean by using three supplementary information. The alternative class of estimators is given as follows:

$$t_N = \bar{y}\left(\frac{ab\bar{X}+cd}{ab\bar{x}+cd}\right)^g, \qquad (5)$$

where g is constant for creating the different estimators.

For obtaining the expression of bias and MSE of these estimators, we consider the following relative of error terms, as $e_0 = (\bar{y}-\bar{Y})/\bar{Y}$ and $e_1 = (\bar{x}-\bar{X})/\bar{X}$.

Under SRSWOR scheme, we have the following expectations, $E(e_i)=0\,;\, i=0,1,\quad E(e_0^2)=\lambda C_y^2,\quad E(e_1^2)=\lambda C_x^2,$ and $E(e_0e_1)=\lambda C_{yx}=\lambda\rho C_yC_x.$

Rewriting the introduced estimators from equation (5) in term of e_i's, we obtain

$$t_N = \bar{Y}(1+e_0)(1-\theta e_1)^{-g} \qquad (6)$$

Equation (7) can be found by expanding the right-hand side of (6), multiplying out and neglecting any terms that containing powers of higher than two, including subtracting $\bar{Y}$ on both sides, we have

$$t_N - \bar{Y} \approx \bar{Y}\left(e_0 - g\theta e_1 + \frac{g(g+1)}{2}\theta^2 e_1^2 - g\theta e_0 e_1\right) \qquad (7)$$

After that, we just taking expectation on both sides of (7), so the bias of estimators as

$$Bias(t_N) = \lambda\bar{Y}\left[\frac{g(g+1)}{2}\theta^2 C_x^2 - g\theta\rho C_y C_x\right] \qquad (8)$$

Following from equation (7), the MSE of these estimators is achieved by squaring and taking expectation on both sides of its, turns into the below equation

$$MSE(t_N) = \lambda \bar{Y}^2 \left[C_y^2 + g^2\theta^2 C_x^2 - 2g\theta\rho C_y C_x \right] \quad (9)$$

The MSE of alternative class of estimators t_N from equation (9) is minimized for

$$\theta = C_{yx} / gC_x^2 \quad (10)$$

Substituting the value of θ from equation (10) into equation (9), thus the MMSE of alternative class of estimators t_N, as given by

$$MSE_{\min}(t_N) = \lambda \bar{Y}^2 C_y^2 (1-\rho^2) \quad (11)$$

From the alternative class of estimators that showed in equation (5), we can generate a few members of Yadav et al. [1] estimator with its MSE by substituting the values for a, b, c, d and g as shown in the Table 1:

Table 1. A few members of Yadav et al. [1] estimator

Estimator	Values of constants/ functions					MSE
	a	b	c	d	g	
$t_{Y1} = \bar{y}\left(\frac{\beta_{2(x)} M_d \bar{X} + \rho}{\beta_{2(x)} M_d \bar{x} + \rho}\right)$	$\beta_{2(x)}$	M_d	ρ	1	1	$\lambda \bar{Y}^2 (C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 C_{yx})$
$t_{Y2} = \bar{y}\left(\frac{\beta_{2(x)} M_d \bar{X} + \rho C_x}{\beta_{2(x)} M_d \bar{x} + \rho C_x}\right)$	$\beta_{2(x)}$	M_d	ρ	C_x	1	$\lambda \bar{Y}^2 (C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_{yx})$
$t_{Y3} = \bar{y}\left(\frac{\beta_{1(x)} M_d \bar{X} + \rho}{\beta_{1(x)} M_d \bar{x} + \rho}\right)$	$\beta_{1(x)}$	M_d	ρ	1	1	$\lambda \bar{Y}^2 (C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_{yx})$
$t_{Y4} = \bar{y}\left(\frac{\beta_{1(x)} M_d \bar{X} + \rho C_x}{\beta_{1(x)} M_d \bar{x} + \rho C_x}\right)$	$\beta_{1(x)}$	M_d	ρ	C_x	1	$\lambda \bar{Y}^2 (C_y^2 + \theta_4^2 C_x^2 - 2\theta_4 C_{yx})$
$t_{Y5} = \bar{y}\left(\frac{n\bar{X} + \rho}{n\bar{x} + \rho}\right)$	1	n	ρ	1	1	$\lambda \bar{Y}^2 (C_y^2 + \theta_5^2 C_x^2 - 2\theta_5 C_{yx})$
$t_{Y6} = \bar{y}\left(\frac{n\bar{X} + C_x}{n\bar{x} + C_x}\right)$	1	n	C_x	1	1	$\lambda \bar{Y}^2 (C_y^2 + \theta_6^2 C_x^2 - 2\theta_6 C_{yx})$
$t_{Y7} = \bar{y}\left(\frac{n\bar{X} + \rho C_x}{n\bar{x} + \rho C_x}\right)$	1	n	ρ	C_x	1	$\lambda \bar{Y}^2 (C_y^2 + \theta_7^2 C_x^2 - 2\theta_7 C_{yx})$
$t_{Y8} = \bar{y}\left(\frac{n\rho\bar{X} + C_x}{n\rho\bar{x} + C_x}\right)$	n	ρ	C_x	1	1	$\lambda \bar{Y}^2 (C_y^2 + \theta_8^2 C_x^2 - 2\theta_8 C_{yx})$
$t_{Y9} = \bar{y}\left(\frac{nC_x\bar{X} + \rho}{nC_x\bar{x} + \rho}\right)$	n	C_x	ρ	1	1	$\lambda \bar{Y}^2 (C_y^2 + \theta_9^2 C_x^2 - 2\theta_9 C_{yx})$

Similarly, a few members of alternative class of estimators t_N along with its MSE as given in Table 2:

Table 2. A few members of alternative class of estimators t_N

Estimator	Values of constants/ functions					MSE
	a	b	c	d	g	
$t_{N1}=\bar{y}\left(\frac{\beta_{2(x)}M_d\bar{x}+\rho}{\beta_{2(x)}M_d\bar{X}+\rho}\right)$	$\beta_{2(x)}$	M_d	ρ	1	-1	$\lambda\bar{Y}^2(C_y^2+\theta_1^2C_x^2+2\theta_1C_{yx})$
$t_{N2}=\bar{y}\left(\frac{\beta_{2(x)}M_d\bar{x}+\rho C_x}{\beta_{2(x)}M_d\bar{X}+\rho C_x}\right)$	$\beta_{2(x)}$	M_d	ρ	C_x	-1	$\lambda\bar{Y}^2(C_y^2+\theta_2^2C_x^2+2\theta_2C_{yx})$
$t_{N3}=\bar{y}\left(\frac{\beta_{1(x)}M_d\bar{x}+\rho}{\beta_{1(x)}M_d\bar{X}+\rho}\right)$	$\beta_{1(x)}$	M_d	ρ	1	-1	$\lambda\bar{Y}^2(C_y^2+\theta_3^2C_x^2+2\theta_3C_{yx})$
$t_{N4}=\bar{y}\left(\frac{\beta_{1(x)}M_d\bar{x}+\rho C_x}{\beta_{1(x)}M_d\bar{X}+\rho C_x}\right)$	$\beta_{1(x)}$	M_d	ρ	C_x	-1	$\lambda\bar{Y}^2(C_y^2+\theta_4^2C_x^2+2\theta_4C_{yx})$
$t_{N5}=\bar{y}\left(\frac{n\bar{x}+\rho}{n\bar{X}+\rho}\right)$	1	n	ρ	1	-1	$\lambda\bar{Y}^2(C_y^2+\theta_5^2C_x^2+2\theta_5C_{yx})$
$t_{N6}=\bar{y}\left(\frac{n\bar{x}+C_x}{n\bar{X}+C_x}\right)$	1	n	C_x	1	-1	$\lambda\bar{Y}^2(C_y^2+\theta_6^2C_x^2+2\theta_6C_{yx})$
$t_{N7}=\bar{y}\left(\frac{n\bar{x}+\rho C_x}{n\bar{X}+\rho C_x}\right)$	1	n	ρ	C_x	-1	$\lambda\bar{Y}^2(C_y^2+\theta_7^2C_x^2+2\theta_7C_{yx})$
$t_{N8}=\bar{y}\left(\frac{n\rho\bar{x}+C_x}{n\rho\bar{X}+C_x}\right)$	n	ρ	C_x	1	-1	$\lambda\bar{Y}^2(C_y^2+\theta_8^2C_x^2+2\theta_8C_{yx})$
$t_{N9}=\bar{y}\left(\frac{nC_x\bar{x}+\rho}{nC_x\bar{X}+\rho}\right)$	n	C_x	ρ	1	-1	$\lambda\bar{Y}^2(C_y^2+\theta_9^2C_x^2+2\theta_9C_{yx})$

where $\theta_1=\frac{\beta_{2(x)}M_d\bar{X}}{\beta_{2(x)}M_d\bar{X}+\rho}$, $\theta_2=\frac{\beta_{2(x)}M_d\bar{X}}{\beta_{2(x)}M_d\bar{X}+\rho C_x}$, $\theta_3=\frac{\beta_{1(x)}M_d\bar{X}}{\beta_{1(x)}M_d\bar{X}+\rho}$, $\theta_4=\frac{\beta_{1(x)}M_d\bar{X}}{\beta_{1(x)}M_d\bar{X}+\rho C_x}$, $\theta_5=\frac{n\bar{X}}{n\bar{X}+\rho}$, $\theta_6=\frac{n\bar{X}}{n\bar{X}+C_x}$, $\theta_7=\frac{n\bar{X}}{n\bar{X}+\rho C_x}$, $\theta_8=\frac{n\rho\bar{X}}{n\rho\bar{X}+C_x}$, $\theta_9=\frac{nC_x\bar{X}}{nC_x\bar{X}+\rho}$.

2. Modified Class of Estimators

In this section, the author suggests a modified class of estimators by combining the estimators $\bar{y}$ and t_N in order to find the MMSE of the suggest estimators. This modified class of estimators is given as follows:

$$t_N^* = \alpha\bar{y}+(1-\alpha)\,t_N \qquad (12)$$

where α is any chosen constant which makes the MSE of modified class of estimators t_N^* minimum. Now consider adopting the same procedure in previous section, we can obtain the bias and MSE of t_N^*, as

$$Bias(t_N^*) = (1-\alpha)\lambda\overline{Y}\left[\frac{g(g+1)}{2}\theta^2 C_x^2 - g\theta\rho C_y C_x\right] \qquad (13)$$

$$MSE(t_N^*) = \lambda\overline{Y}^2\left[C_y^2 + (1-\alpha)^2 g^2\theta^2 C_x^2 - 2(1-\alpha)g\theta\rho C_y C_x\right] \qquad (14)$$

The MSE of modified class of estimators t_N^* from equation (14) is minimized for

$$\alpha = 1 - \frac{C_{yx}}{g\theta C_x} = \alpha_{opt.} \qquad (15)$$

Substituting the value of α from equation (15) into equation (14), thus the MMSE of modified class of estimators t_N^*, as given by

$$MSE_{\min}(t_N^*) = \lambda\overline{Y}^2 C_y^2(1-\rho^2) = MSE_{\min}(t_N) \qquad (16)$$

It is observed from (16) that the MMSE of modified estimators t_N^* is equal to the MSE of the usual regression estimator.

3. Performance Comparisons

Under SRSWOR, it is generally known that the unbiased estimator $\overline{y}$ has the value of MSE as

$$MSE(\overline{y}) = \lambda\overline{Y}^2 C_y^2 \qquad (17)$$

Therefore, in comparing the performance of an alternative class of estimators t_N and modified estimators t_N^* with other existing estimators, we can show from expression (18) to (21), as

$$MSE(\overline{y}) - MSE_{\min}(t_N, t_N^*) = \lambda\rho^2\overline{Y}^2 C_y^2 > 0 \qquad (18)$$

$$MSE(t_{Y(i)}) - MSE_{\min}(t_N, t_N^*) = (\theta_i C_x - \rho C_y)^2 > 0;\ \ i = 1,\ldots,9 \qquad (19)$$

$$MSE(T_{N(i)}) - MSE_{\min}(t_N, t_N^*) = (\theta_i C_x + \rho C_y)^2 > 0;\ \ i = 1,\ldots,9 \qquad (20)$$

$$MSE_{\min}(t_Y) - MSE_{\min}(t_N, t_N^*) = \rho^4 C_y^4 - 1 > 0 \qquad (21)$$

When these conditions are true, we can extrapolate that the estimators t_N at its minimum condition and the estimators t_N^* will be more efficient than unbiased estimator $\overline{y}$ and other existing estimators listed in Table 1 and 2.

4. Numerical Analysis

To investigate the performance of the introduced estimators, the author considered the data provided in Yadav et al. [1] and Dobson [22] for comparing the performance of their competing estimators, which details are as follows:

Population 1:

Yadav et al. [1] have used the data of peppermint oil production data in Barabanki district of Uttar Pradesh State of India, assumed that the yield of peppermint oil (in kilogram) and the area of the field (in Bigha) were taken as the interested and supplementary variables respectively. The details of any values that calculate from this group are as follows:

$N=150, n=40, \bar{X}=4.20, \bar{Y}=33.46, S_x=3.08, S_y=25.50, C_x=0.73, C_y=0.76, M_d=3, \rho=0.91, \beta_{1(x)}=2.80,$

$\beta_{2(x)}=16.44, \lambda=0.02.$

Population 2:

Dobson [22] presented data about times to death of patients suffering from leukemia. The data consists of a list of 17 leukemia patients, assumed that diagnosis time (weeks) was taken as interested variable and a list of seventeen leukemia patients as taken as supplementary variable. The descriptions of the parameters related to this population have been given as follows:

$N=17, n=4, \bar{X}=4.10, \bar{Y}=62.47, S_x=2.50, S_y=54.35, C_x=0.61, C_y=0.87, M_d=4, \rho=-0.68, \beta_{1(x)}=-0.48,$

$\beta_{2(x)}=-0.68, \lambda=0.19.$

In comparing the performance of the introduced estimators over other existing estimators in this present study, we have used the percent relative efficiencies (PREs) as criteria for comparison with the unbiased estimator, which details are presented in the following tables.

Table 3. MSE and PREs of different estimators for Population 1

Estimator	MSE	PRE
$\bar{y}$	12.93329	100.00
t_{Y1}	2.25099	574.56
t_{Y2}	2.25236	574.21
t_{Y3}	2.23223	579.39
t_{Y4}	2.23713	578.12
t_{Y5}	2.24983	574.86
t_{Y6}	2.25104	574.55

Table 3. (cont.)

Estimator	MSE	PRE
t_{Y7}	2.25149	574.43
t_{Y8}	2.25055	574.67
t_{Y9}	2.24765	575.41
t_{N1}	47.27208	27.36
t_{N2}	47.32668	27.33
t_{N3}	46.31407	27.93
t_{N4}	46.62029	27.74
t_{N5}	47.22509	27.39
t_{N6}	47.27428	27.36
t_{N7}	47.29226	27.35
t_{N8}	47.25453	27.37
t_{N9}	47.13346	27.44
t_N, t_N^*	2.22323	581.73

Table 4. MSE and PREs of different estimators for Population 2

Estimator	MSE	PRE
$\bar{y}$	564.78590	100.00
t_{Y1}	1,323.68508	42.67
t_{Y2}	1,346.98356	41.93
t_{Y3}	1,300.38542	43.43
t_{Y4}	1,331.75378	42.41
t_{Y5}	1,434.91063	39.36
t_{Y6}	1,347.20768	41.92
t_{Y7}	1,415.40958	39.90
t_{Y8}	1,450.65380	38.93
t_{Y9}	1,468.69476	38.45
t_{N1}	300.18729	188.14

Table 4. (cont.)

Estimator	MSE	PRE
t_{N2}	299.91655	188.31
t_{N3}	300.72781	187.81
t_{N4}	300.06346	188.22
t_{N5}	301.17512	187.53
t_{N6}	299.91523	188.32
t_{N7}	300.59785	187.89
t_{N8}	301.75943	187.16
t_{N9}	302.55597	186.67
t_N, t_N^*	299.87934	188.34

From the numerical analysis in Table 3 and 4, it is well established that the introduced estimators t_N, t_N^* are more desirable over all the considered estimators under optimum condition for both population data sets. Because they give the largest and smallest values of PREs and MSE, respectively, as compared to other estimators within the same populations. Besides the introduced estimators t_N, t_N^*, we also consider the efficiency comparison between the other existing by using criterion of MSE and PREs under the same population groups, which details are as follows:

For the data of population 1, when the correlation between supplementary and interested variables is positive, the MSE and PRES of Yadav et al. [1] ($t_{Y1},...,t_{Y9}$) estimators is very close to the introduced estimators t_N, t_N^* .Especially, the estimator t_{Y3} seems to be more suitable estimator in comparison to other estimators in this group, because it gives the closest values of MSE and PREs to the introduced estimators t_N, t_N^* . Therefore, besides the estimators t_N, t_N^* , the estimators t_{Y3} can be used in this group. Also, when comparing the performance between the estimators of population 2 in the case of negative correlation, it has been seen that the MSE and PRES of the estimators $T_{N1},...,T_{N9}$ is very close to the introduced estimators t_N, t_N^* . Among all of estimators $T_{N1},...,T_{N9}$, it is envisaged that the estimator T_{N2} is more appropriate than other estimators, because the MSE and PREs of its is very nearest to the introduced estimators t_N, t_N^* Therefore, besides the estimators t_N, t_N^*, the estimators T_{N2} can be used in situation of negative correlation between supplementary and interested variables.

Finally, from the numerical analysis results, it is well established that the estimators t_N, t_N^* are more desirable over all the considered estimators and should be put into practice.

Results

It is well known in sample surveys that when the correlation between the interested and supplementary variables is positive, the performance of the ratio estimator is generally more efficient than the product estimator, which corresponds to the result of Table 3 and 4. Table 3 and 4 shows that the MSE of the introduced estimators t_N, t_N^* (at its optimum) are persistently less than over all other existing estimators in both situations of positive and negative correlation patterns. While the value of PRE of its are increasingly higher than other ones. Therefore, we can conclude that the introduced estimators t_N, t_N^* have better performance as compared to other estimators in the terms of MSE and PRE. However, besides the introduced estimators t_N, t_N^* , among all estimators, we have also suggested the estimator T_{N2} for the next choice because it gives bigger available.

Conclusions

In this present study, the author introduced an alternative class of estimators for population mean under SRSWOR by utilizing the information from supplementary variables, listed in the section of the proposed alternative class of estimators. In addition, a modified class of estimators is presented by combining the unbiased estimator and alternative class of estimators introduced in the previous section. The author clearly proved through theoretical and numerical analysis that the introduced estimators are always more efficient than other existing estimators in terms of PREs and MSE. Therefore, the introduced estimators are recommended to use in practice for estimating the population mean when the supplementary information is available.

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