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Research Article

วิธีกำลังสองน้อยที่สุดสำหรับแบบจำลองทางคณิตศาสตร์ของกลไกหน้าต่างรถยนต์

Least Squares Method for The Mathematical Model of an Automotive Window Mechanism

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บทคัดย่อ

งานวิจัยนี้นำเสนอแนวทางการหาเอกลักษณ์ของระบบและแบบจำลองทางคณิตศาสตร์สำหรับกลไกหน้าต่างรถยนต์ โดยใช้เทคนิคการประมาณค่าตัวแปร Least Squares (LS) และ Recursive Least Squares (RLS) เนื่องด้วยความเป็นเชิงเส้นของระบบกลไกหน้าต่างรถยนต์ วัตถุประสงค์ของการวิจัย คือการเลือกแบบจำลองทางคณิตศาสตร์ที่เหมาะสมที่สุดสำหรับการทำนายตำแหน่งของกลไกหน้าต่างรถยนต์ และเปรียบเทียบวิธีประมาณค่าตัวแปรระหว่าง Least Squares (LS) และ Recursive Least Squares (RLS) ซึ่งการทดลองของงานวิจัยจะเกี่ยวข้องกับการเก็บสัญญาญขาเข้าและสัญญาญขาออก โดยใช้ตัวตรวจวัดอัลตราโซนิคและระบบเก็บข้อมูลที่ชนิด Arduino โดยข้อมูลที่รวบรวมได้จะถูกนำมาใช้ในการประมาณค่าตัวแปรของระบบสำหรับโครงสร้างแบบจำลองที่แตกต่างกัน ได้แก่ แบบจำลอง Autoregressive Exogenous (ARX) และ Autoregressive Moving Average Exogenous Inputs (ARMAX) ที่ลำดับของแบบจำลองที่แตกต่างกัน โดยที่ประสิทธิภาพของแต่ละแบบจำลองจะได้รับการประเมินโดยการเปรียบเทียบผลลัพธ์จากการจำลองกับข้อมูลจากการทดลอง จากผลการทดลองของงานวิจัยแสดงให้เห็นว่า แบบจำลอง ARMAX ลำดับที่ 4 ที่ใช้วิธี Recursive Least Squares ให้ค่าความแม่นยำสูงสุดที่ 95.563% ขณะที่แบบจำลอง ARMAX ลำดับที่ 4 ที่ใช้วิธี Least Squares ให้ค่าความแม่นยำที่ 94.862% ผลการวิจัยนี้แสดงให้เห็นว่า การเลือกโครงสร้างแบบจำลองและวิธีประมาณค่าตัวแปรที่เหมาะสมมีผลกระทบอย่างมีนัยสำคัญต่อความแม่นยำของกระบวนการหาเอกลักษณ์ของระบบสำหรับกลไกหน้าต่างรถยนต์

คำสำคัญ: แบบจำลองทางคณิตศาสตร์; กลไกหน้าต่างรถยนต์; เอกลักษณ์ของระบบ; ตัวตรวจวัดอัลตราโซนิค

Abstract

This research presents a system identification and mathematical modeling approach for an automotive window mechanism using Least Squares (LS) and Recursive Least Squares (RLS) estimation techniques due to the linearity of the automotive window mechanism system. The objective is to select the suitable dynamic model that can predict the position of the window mechanism and compare two parameter estimation methods between Least Squares (LS) and Recursive Least Squares (RLS) method. The experiment involves collecting input - output data using an ultrasonic sensor and Arduino-based data acquisition system. The collected data is used to estimate the system parameters for different model structures, including Autoregressive exogenous (ARX) and Autoregressive moving average exogenous inputs (ARMAX) models, at various model orders. The performance of each model is evaluated by comparing the simulated outputs with experimental data. The results indicate that the 4^{th} order ARMAX model with Recursive Least Square achieves the highest accuracy of 95.563%. When the 4^{th} order ARMAX model with Least Square achieves the accuracy of 94.862%. The findings of this research demonstrate that the selection of an appropriate model structure and estimation method significantly impacts the accuracy of system identification for an automotive window mechanism.

Keywords: mathematical model; window mechanism; system identification; ultrasonic sensor

Introduction

A mathematical model depends on the selected model structure, such as AR (Autoregressive), ARMA (Autoregressive moving-average), ARMAX (Autoregressive moving average exogenous), ARX (Autoregressive exogenous) and FIR (Finite impulse response) models, while the model parameters depend on the order of the model structure. A higher order of the model structure indicates a mathematical model with numerous system parameters. The primary distinction among these models lies in their utilization of past inputs, past outputs, and noise dynamics in system identification and time-series modeling. The Autoregressive (AR) model relies solely on past output values to predict future outputs, making it suitable for time-series forecasting in the absence of external inputs. The Autoregressive Moving-Average (ARMA) model extends AR by incorporating a moving-average (MA) component, which accounts for stochastic noise, thereby enhancing its capability to model time-dependent data. However, both AR and ARMA do not consider external input variables, limiting their applicability in dynamic systems influenced by external factors. To address this limitation, the Autoregressive Moving-Average with Exogenous Inputs (ARMAX) model builds upon ARMA by incorporating past input values, allowing for more accurate modeling of systems where external inputs significantly impact system behavior. In contrast, the Finite Impulse Response (FIR) model differs fundamentally from the aforementioned models as it exclusively considers past input values while disregarding past outputs. The FIR model is preferable when past outputs are not required, while ARX and ARMAX are more suitable for dynamic system identification involving external inputs. Conversely, AR and ARMA models are most effective for time-series forecasting when exogenous inputs are absent. The appropriate choice among these models significantly influences the accuracy and efficiency of system identification and predictive modeling. In system identification process, input data and output data are measured from numerous experiments when $u(t)$ represents time series input signal to the real system and $y(t)$ represents time series output signal from the real system. All collected data sets are utilized to identify the system parameters when model structure has been selected. However, if the model structure does not match the real system, model errors may occur. Therefore, it is imperative to ensure that the selected model structure and model order must align with the characteristics of the real system [1]. The model error between output from the real system and estimated output from the mathematical model can be expressed as $\hat{e}(t) = y(t) - \hat{y}(t)$, when $\hat{y}(t)$ represents time series output from the mathematical model. In general, a higher model order can better predict the time series of the output signal from the mathematical model than a lower model order. However, this leads to the mathematical model having numerous system parameters. Therefore, the optimum model order of the selected model structure is an essential factor in the system identification process, aiming to minimize the model error and the number of system parameters.

The model structure consists of a time series of input and output data at different time steps, characterized by constant parameters. For example, ARMAX (Autoregressive-moving-average model with exogenous input model) is one kind of time series model and can be expressed as shown in equation (1).

$$y(t) = a_1 y(t-1) + \cdots + a_n y(t-n) + b_1 u(t-1) + \cdots + b_n u(t-n) \qquad (1)$$

when $u(t)$ represents time series input signal and $y(t)$ represents time series output signal. The system parameters $a_1, \cdots, a_n, b_1, \cdots, b_n$ represent the dynamic behavior of the system, when n represents the model order of the system and the parameters $t,\ t-1,\ \ t-n$ are current time step, delay 1-time step and delay n-time step, respectively. Equation (1) can be rewritten as equation (2) when vector Y represents the time series of the output signal from timestep t to timestep $t+N$ when N is total number of data points from the experiment. A regression matrix $\emptyset$ can be formed by delayed time series of the input and output data. The vector θ consists with unknown parameters $a_1, \cdots, a_n, b_1, \cdots, b_n$ and is constructed into a column vector. The number of unknown parameters depends on the model order. As shown in equation (2), the third-order model of the model structure can be written with three-time delay steps of the input and output signal, so the six parameters of the model structure are utilized for predicting the dynamic behavior of the system.

$$\underbrace{y(t)}_{Y} = \underbrace{[y(t-1)\ \ y(t-2)\ y(t-3)\ \ u(t-1)\ \ u(t-2)u(t-3)]}_{\phi} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}}_{\theta} \tag{2}$$

The column vector θ can be extracted using the least square method, which is a mathematical technique used to find the minimum sum of the squares of the differences between measured and predicted values. This method is applied to estimate parameters from all collected input and output data of experiments to form the mathematical model that predicts the dynamic behavior of the system [1], as demonstrated in Equation (3).

$$\theta_{LS} = (\phi^T \phi)^{-1} \phi^T Y \tag{3}$$

Furthermore, another model structure is FIR (Finite Impulse Response). This model utilizes only time delay steps of the input signal, with unknown parameters in each input time delay step, as shown in equation (4).

$$\underbrace{y(t)}_{Y} = \underbrace{[u(t-1)\ \ u(t-2)\ \cdots u(t-M)]}_{\phi} \underbrace{\begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_M \end{bmatrix}}_{\theta} \tag{4}$$

The least square method is a type of offline identification, where a batch of input and output data from a single experiment is utilized to identify the system parameters. Another least square method is known as the recursive least squares (RLS) method. Unlike traditional least squares, recursive least squares method is an adaptive filtering algorithm used to estimate the system parameter of a model structure. The recursive least squares method is useful where the data is changing over time or where real-time processing is required. The main objective of this method is to minimize the error between the measured and predicted values generated by the model structure [2]. The algorithm updates the unknown parameters iteratively as new data becomes available as shown in equation (5).

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K_k \hat{e}(t) \tag{5}$$

When K_k represents the Kalman gain at iterative step k, it is utilized to update the unknown parameters $\hat{\theta}$ based on the error between the measured values from the real system and the predicted values from the mathematical model. The Kalman gain can be calculated as shown in equation (6)

$$K_k = \frac{P_{k-1}\phi_k}{\lambda + \phi_k^T P_{k-1}\phi_k} \quad (6)$$

When P_{k-1} represents the covariance matrix at iterative step $k-1$, the least squares method requires the inversion of the entire data matrix. On the other hand, the recursive least squares method eliminates the need for the computationally intensive matrix inversion. The parameter λ is a forgetting factor that controls the trade-off between the influence of past observations and current observations on the parameter estimates. A smaller value of λ gives more weight to recent observations, while a larger value gives more weight to past observations. The covariance matrix P_k at iterative step k updates based on a forgetting factor λ, the Kalman gain K_k, a regression matrix ϕ_k at iterative step k consists of input and output data used to estimate parameters in a linear model and the matrix is continuously updated as new data arrives as shown in equation (7), and the past covariance matrix P_{k-1} at iterative step $k-1$. The updated covariance matrix can be calculated as shown in equation (8)

$$\phi_k = [y(t-1) \cdots y(t-n) \;\; u(t-1) \cdots u(t-n)] \quad (7)$$

$$P_k = \frac{1}{\lambda}\left(P_{k-1} - K_k \phi_k^T P_{k-1}\right) \quad (8)$$

The update algorithm used to converge the value of the unknown parameters $\hat{\theta}$ to the true value as the number of observations increases each iterative step. However, the rate of convergence may vary depending on factors such as the selected forgetting factor and the characteristics of the experimental data. The least squares (LS) and recursive least squares (RLS) methods are suitable for linear time-invariant (LTI) systems that can be modeled using linear equations, such as the AR, ARMA, and ARMAX models. The least squares method requires batch data processing, where all data is collected first and then analyzed through offline computation. In contrast, the recursive least squares method is suitable for real-time system identification, as it updates system parameters continuously as new data arrives. An automotive window mechanism is an example of a linear time-invariant system. When a time-series input signal is applied to the window mechanism motor, the position of the window varies depending on the direction of the input signal. The behavior of an automotive window mechanism can be modeled using linear equations, making it suitable for system identification using the least squares and recursive least squares methods. From literature review in system identification, numerous researchers have applied system identification process in various applications, including mechanical system such as translation and rotation systems, as well as thermal systems. For example, researcher proposed a system identification process for a thermal process using a step input to the system and measured the output respond to derive a first-order transfer function from experiment data [3, 4]. In control of direct current motor with the PID controller, Researchers utilized the second-order transfer function to represent motor model. All system parameters were identified by open-loop step response test method [5]. Another researcher used state space model as a model structure and used the least square method to identify the system parameters of a thermal system [6]. In a rotational mechanical system, the researcher utilized a system identification toolbox in MATLAB program to synthesize a second-order transfer function model of the studied rotational system. The model can predict the output with 96.87% accuracy when compared with experimental output [7]. Some literature has proposed a comparison study of different model structure such as, ARX, SSEST, N4SID, ERA and OKID in rotational single link robot, with the ARX model demonstrating higher accuracy with 95.8% [8]. Another researcher proposed a rotational mathematical model from physical laws using second-order differential equations and defined numerous energy loss terms in the mathematical model. The results showed that the model with all terms of energy loss has higher accuracy compared to the mathematical model with only some terms of energy loss [9]. In a translational mechanical system, literature reviews revealed that a number of the system identification techniques have been proposed. For example, Researchers proposed the transfer function model to represent the translational mechanical system with subspace-based system identification methods [10], Researchers proposed an on-line algebraic parameter identification method in the time domain for multiple linear mass-spring-damper mechanical systems [11], and the identification of the translational mechanical system using the amplitude dependent frequency and damping extracted from a free decay response [12].

The literature reviews also revealed that many more research studies have utilized system identification in numerous linear time-invariant system. The main focus of these studies is the comparison among various differential model structures to select the most suitable model for predicting the output.

This paper presents the implementation of the least square and recursive least square methods from an automotive window mechanism, which is a kind of linear time-invariant system, with differential model structures and varying model orders. Data is collected from experiments utilizing ultrasonic distance-measuring sensor to measure the displacement of the window as an output data and level of voltage signal from microcontroller as input data. To assess model accuracy, this paper compares the accuracy among different model structures and model orders to select the most suitable model for predicting the position output of an automotive window mechanism. The remaining sections of this paper include the Objective of the research, which define the main purpose of the research. The Methods section discusses the implementation of various equipment for the experimental study. This is followed by the Results section, which presents the results of the experimental study. Finally, the last section concludes the paper.

Objectives

1. To determine the most suitable model for predicting the position output of an automotive window mechanism by comparing FIR and ARMAX models of 2^{nd}, 4^{th} and 6^{th} orders.

2. To compare two parameter estimation methods: Least Squares and Recursive Least Squares to identify the most suitable system parameters for the mathematical model.

Methods

Before conducting experiments to record the position of an automotive window mechanism, numerous electronic devices, including Arduino, a joystick, a circuit board, an ultrasonic sensor, a motor control device, a 12-volt DC power supply, and an automotive window mechanism, are utilized to set up the experiment for the system identification process of an automotive window mechanism as shown in Figure 1. The joy stick is utilized to control the position of car window as the time series of input signal, while the ultrasonic sensor measures the position of the car window as the time series of output signal. All input and output data were collected using an Arduino UNO with a sampling time of 0.25 second over a 60 second test period or 244 data pair of input and output signal were collected. The joy stick is utilized to control the direction of the car window via a motor control device. The device is utilized to connect a source of 12-volt DC power supply to power the window motor. when the joy stick moved forward, the car window moves upward, and when it is moved downward, the car window moves downward due to the changing of current flow direction of the source via a motor control device. The experiments are conducted with two batches of testing. The two batches of data sets are defined over different time sequences by varying the duration of the input signal. The first set of experimental tested data is used to train the characteristics of the mathematical model, while the second set of experimental tested data is used to validate the accuracy of the model. The validation dataset consists of data that the model has not been exposed to during the parameter estimation process. This ensures that the evaluation of the model's performance is unbiased and not influenced by the data used for training.

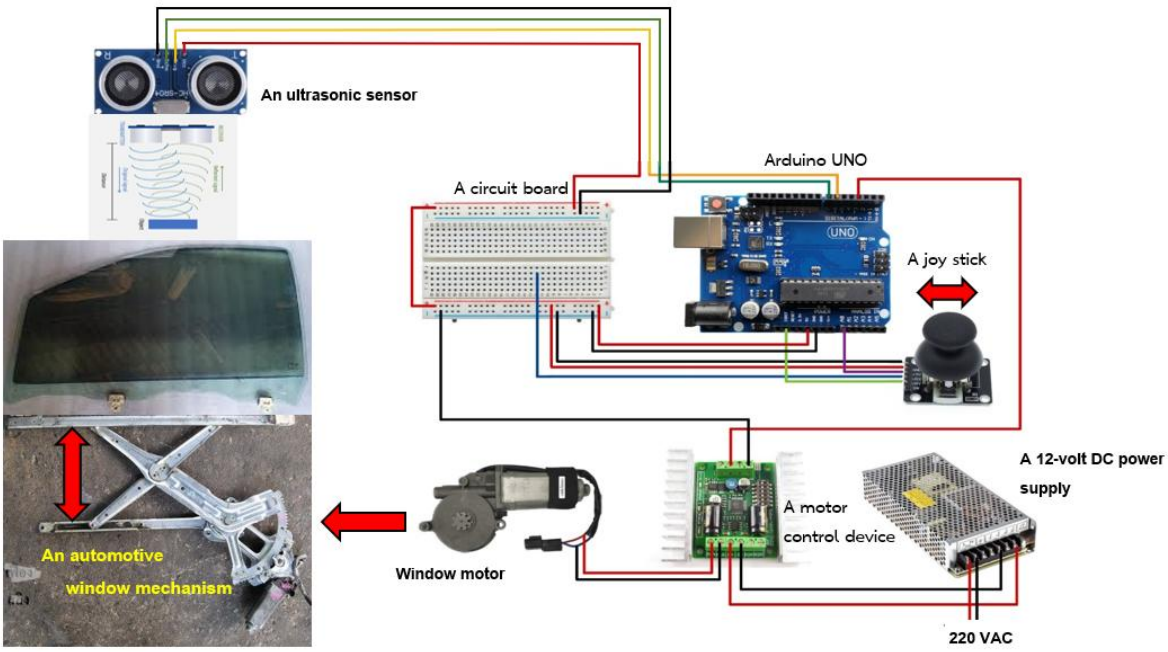

Figure 1 the schematic diagram of the experimental setup.

After collecting the position data of an automotive window mechanism from experiments, the system identification process begins with the selecting the appropriate model structure and model order that suits the dynamic behaviors of the system. To identify the mathematical model from the training dataset, the least squares method is used to find the unknown variables of the mathematical model. Before applying the least squares method, the ARMAX and FIR structures with order of 2nd, 4th and 6th must be rewritten in the from of equation (2) for the ARMAX structure, while the FIR structures must be rewritten as equation (4). Then, the experimental data at each time step must be substituted into the rewritten from of the ARMAX and FIR structures to determine the column vector Y and a regression matrix ϕ, respectively. Consequently, the unknown variables for each model structures and order can be computed using equation (3), and each model structure with its corresponding order can be used to simulate the model output $\hat{y}_i$. After obtaining all the mathematical model outputs, their outputs are compared with the second batch of experiment data to validate the model accuracy, according to equation (9).

$$\%Best\ fit = \left(1 - \frac{\sum_{i=1}^{N} |y_i - \hat{y}_i|}{N}\right) \times 100 \quad (9)$$

When N is the total number of the data points, y_i represents the validated data at data point i and $\hat{y}_i$ represents the model output data at data point i. A higher value of equation (9) indicates higher model accuracy [2]. Another method in the system identification process is the recursive least squares method. This method is useful in situations where data is changing over time, requiring real-time identification, unlike the least square method [1]. The algorithm starts by initializing the estimated system parameters data $\hat{\theta}(t-1)$, the covariance matrix P_k, and the forgetting factor λ, respectively. It then processes these data to estimate the system parameter $\hat{\theta}(t)$ at the current time based on the Kalman gain and the model error $\hat{e}(t)$, as shown in Figure 2. The estimated parameters change over time due to the varying input levels, and all estimated parameters will converge to the true system parameter values in finite time. In the recursive least squares method, the initializations of the system parameters are the identified parameters from the conventional least squares method at the same model order. The true system parameter values obtained from the recursive least squares method are used to calculate the percentage relative error between the parameters estimated by the least squares method and the true parameters from the recursive least squares method, as shown in Equation (10).

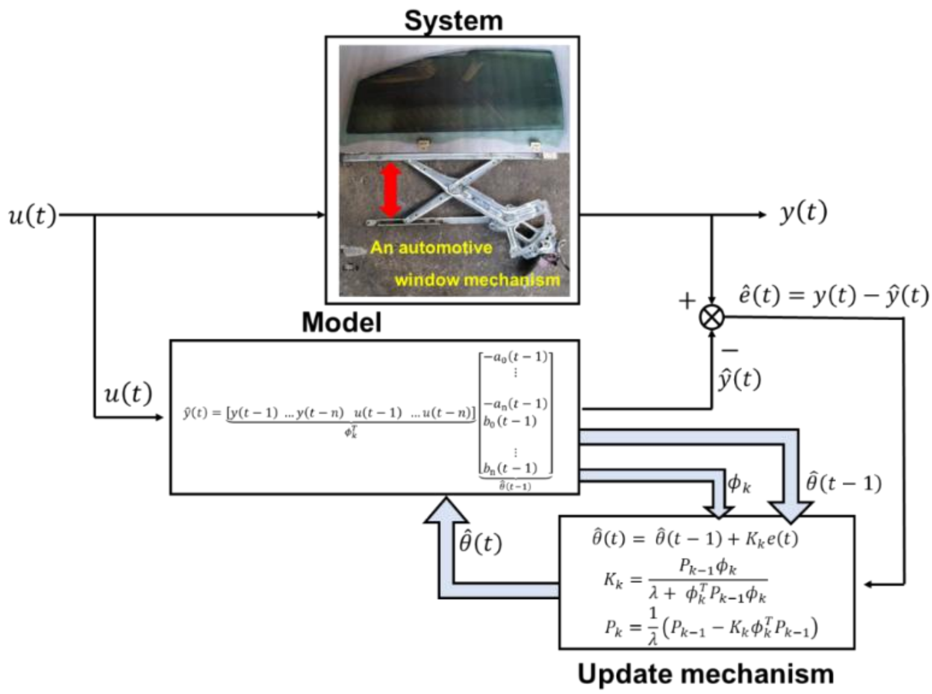

Figure 2 the schematic diagram of the recursive least square method.

$$\%\ Percentage\ relative\ error = \frac{|\theta_{LS} - \theta_{RLS}|}{\theta_{RLS}} \times 100 \quad (10)$$

When θ_{LS} represents a parameter estimated by the Least Squares method, and θ_{RLS} represents a parameter estimated by the Recursive Least Squares method, a lower percentage relative error indicates that the parameter obtained from the Least Squares method is closer to the true parameter from the Recursive Least Squares method. After all experiments, Analysis of Variance (ANOVA) is used to compare the means of model accuracy between two parameter estimation methods. The null hypothesis (H_0) states that the means of model accuracy for the two parameter estimation methods are equal, while the alternative hypothesis (H_1) states that they are not equal. The hypothesis is tested at a 0.05 level of significance.

Results

The validation and training data are shown in Figure 3. To ensure unbiased conditions, the input signal durations in the validation and training datasets were defined differently. The system parameters are identified by the least square method from training data on the ARMAX and FIR model structures with 2nd, 4th and 6th order, as shown in Table 1. The FIR model and ARMAX structures with 2nd, 4th and 6th order have 2, 4 and 6 system parameters, respectively. The parameters in the FIR model focus on the sequence of the input signal, while the parameters in the ARMAX model focus on both of the input and output signal sequence. The model accuracies of the FIR model are 41.23%, 54.88% and 62.55% for 2nd, 4th and 6th order, respectively. A higher order of model structures indicates better model accuracy. In ARMAX model structures, the accuracies are 78.12%, 94.86% and 95.13% for 2nd, 4th and 6th order, respectively. The model accuracy of the 4th model order is close to that of the 6th model order, but the system parameters of the 4th model order are less than those of the 6th model order.

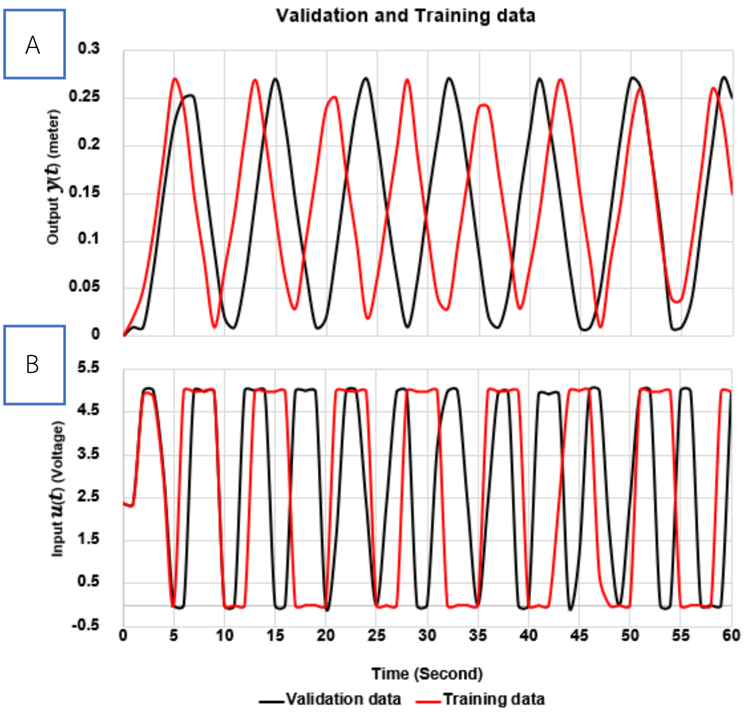

Figure 3 the validation and training data sets: (A) Output of validation and training data; (B) Input of validation and training data.

Table 1 the system parameters of the ARMAX and FIR model structures with 2nd, 4th and 6th order.

System

System parameters	FIR Model			ARMAX Model		
	2nd order	4th order	6th order	2nd order	4th order	6th order
a_0	-	-	-	0.85462	1.54922	1.46522
a_1	-	-	-	-	-0.83173	-0.83432
a_2	-	-	-	-	-	-0.00541
b_0	0.00841	0.01478	0.01863	0.00421	0.00482	0.00863
b_1	0.03793	0.01603	0.01772	-	0.00433	-0.00092
b_2	-	0.01601	0.01352	-	-	0.00782
b_3	-	0.00522	-0.00261	-	-	-
b_4	-	-	0.00481	-	-	-
b_5	-	-	0.00192	-	-	-
$\% Fit$	41.233%	54.884%	62.552%	78.121%	94.862%	95.132%

Experimental model performance of the FIR model in an automotive window mechanism is presented in Figure 4. Solid and dashed lines represent the validation data and the output of the FIR model, respectively. The output of the FIR models can predict only the level of the real system responses. A higher order of the FIR model structures can only raise the level of the output responses of the FIR model, but the responses of all FIR model cannot track the validation data consistently throughout the comparison study of the models.

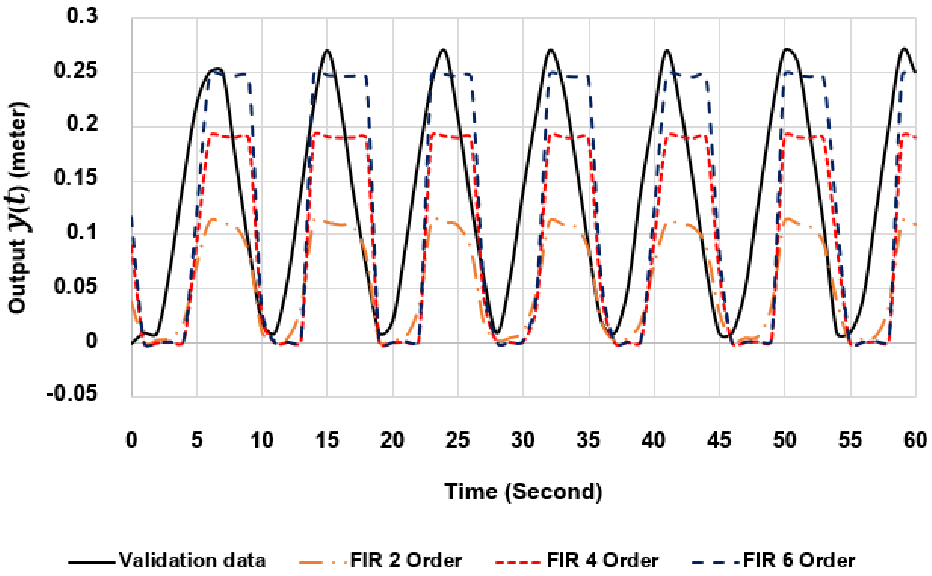

Figure 4 the comparison study of the validation data and the FIR models.

In the comparison study of the ARMAX model, the model performance of the ARMAX model in an automotive window mechanism is presented in Figure 5. Solid and dashed lines represent the validation data and the output of the ARMAX model, respectively. From experimental results, a higher order of the ARMAX model structures can predict the output response better than the lower order when compare between the validation data and the output of the ARMAX models. The patterns of the 4^{th} model order are similar to those of the 6^{th} model order. The 2^{nd} model order can predict only the trend and has a significant gap between the validation data and the model response consistently throughout the comparison study of the models. The system parameters of the 4th order ARMAX model are used as the initial parameters $\hat{\theta}(t-1)$ for the recursive least square method, and the forgetting factor λ is set to 0.97, where a larger value of the forgetting factor gives more weight to the past measured data. The system parameters and model accuracy for both of least square method can be shown in Table 2. The model accuracy of the 4^{th} order ARMAX model by least square method is 94.862%, while the model accuracy of the 4^{th} order ARMAX model by recursive least square method is 95.563%.

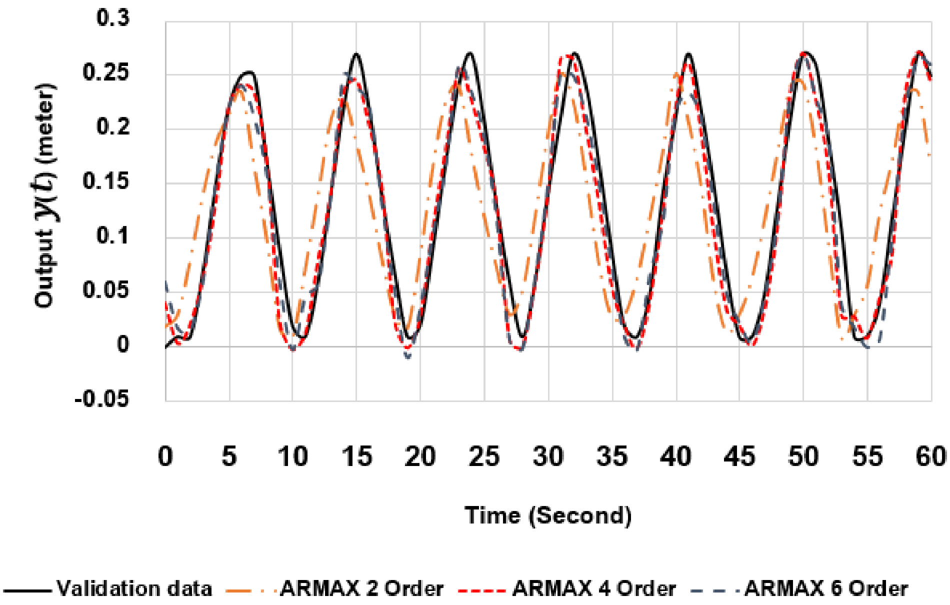

Figure 5 the comparison study of the validation data and the ARMAX models.

The result of the analysis of variance (ANOVA) at the 0.05 level of significance can be shown in Table 3. The P-value is 1.496 x 10^{-11}, which is smaller than the 0.05 level of significance. Therefore, the means of model accuracy between two parameter estimation methods not have the same value. The model accuracy of the recursive least square method is significantly better than that of the conventional least square method. In the recursive least square method, the system parameters a_0 and a_1 change over time from their initial values. After 15 seconds, the system parameters a_0 and a_1 converge to their true values as shown in Figure 6. Figure 7 illustrates the convergence of the system parameters b_0 and b_1. After 12 second, the system parameters b_0 and b_1 converge to their true values.

Table 2 the system parameters of the least square method for 4th ARMAX model and the recursive least square method for 4th ARMAX model.

System parameters	the least square method for 4th ARMAX model	the recursive least square method for 4th ARMAX model	Percentage relative error
	4th order	4th order	% relative error
a_0	1.54921	1.55131	0.13537%
a_1	-0.83172	-0.81922	1.52584%
b_0	0.00481	0.00491	2.03666%
b_1	0.00432	0.00452	4.42477%
$\% \, Fit$	94.862%	95.563%	

The system parameters b_0 and b_1 converge to their true values faster than a_0 and a_1. This is because the initial values of b_0 and b_1 are closer to their true values compared to a_0 and a_1. The percentage relative error of a_0, a_1, b_0 and b_1 are 0.13537%, 1.52584%, 2.03666% and 4.42477%, respectively.

Table 3 Analysis of variance of two parameter estimation methods.

Source of variation	SS	df	MS	F	P-value
Between methods	3.9445	1	3.9445	109.0950	1.496×10^{-11}
Within methods	1.0761	30	0.0359		
Total	6629	55			

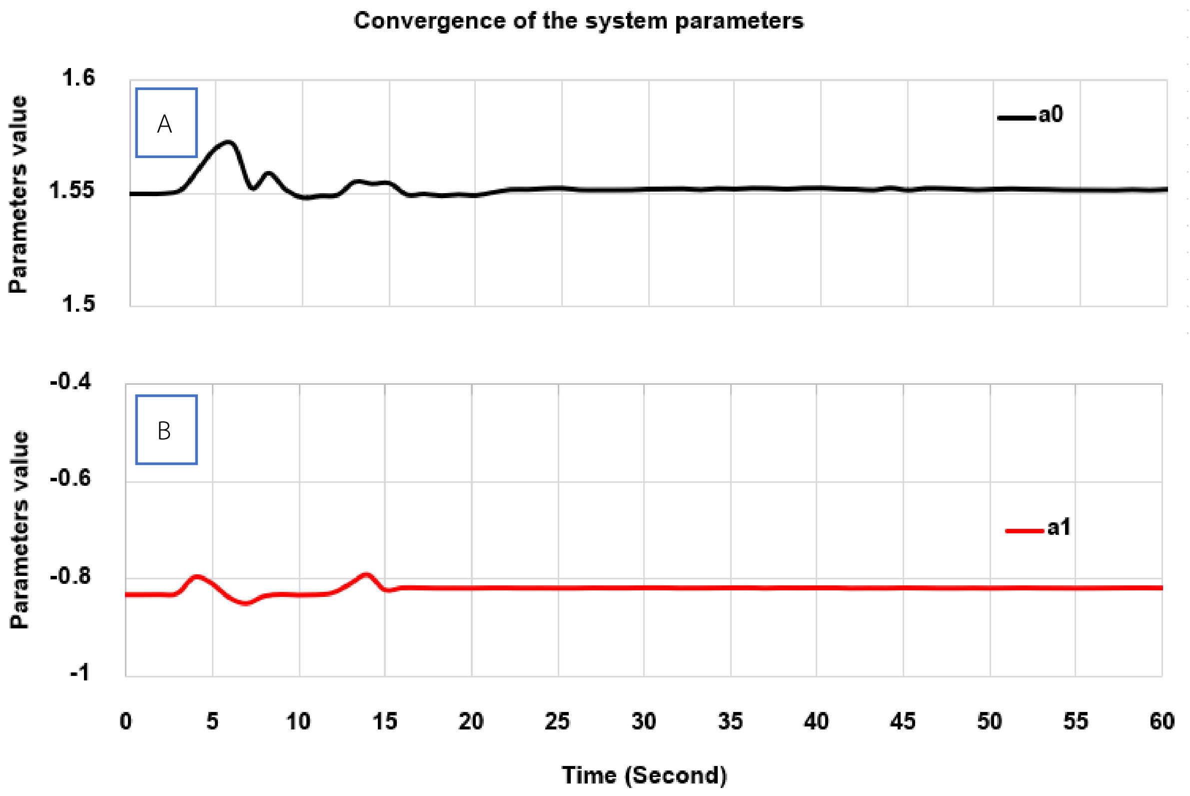

Figure 6 the convergence of the system parameters: (A) the system parameter a_0, and (B) the system parameter a_1.

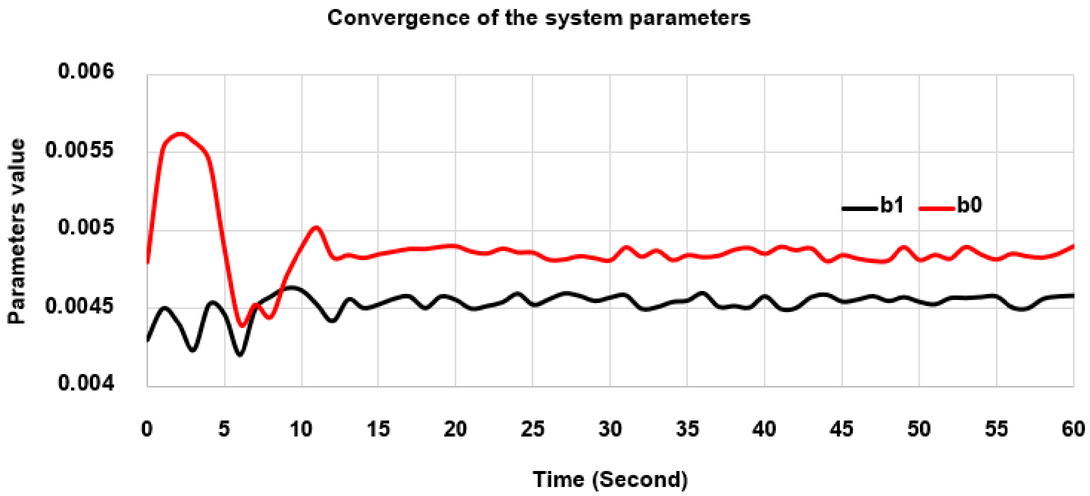

Figure 7 the convergence of the system parameters b_0 and b_1.

The experimental model accuracy of the different least square methods is presented in Figure 8. Solid and dash lines represent the measurement data and the output of the 4th order ARMAX model using both least square methods. The output from the recursive least square method is closer to the measurement data than the output from the conventional least square method. In the first 10 seconds, both outputs exhibit the same trend and have significant gap from the measurement data. After 10 seconds, the output of recursive least square method starts to track the measurement data. Therefore, the model accuracy of the recursive least square method is better than that of the conventional least squares method.

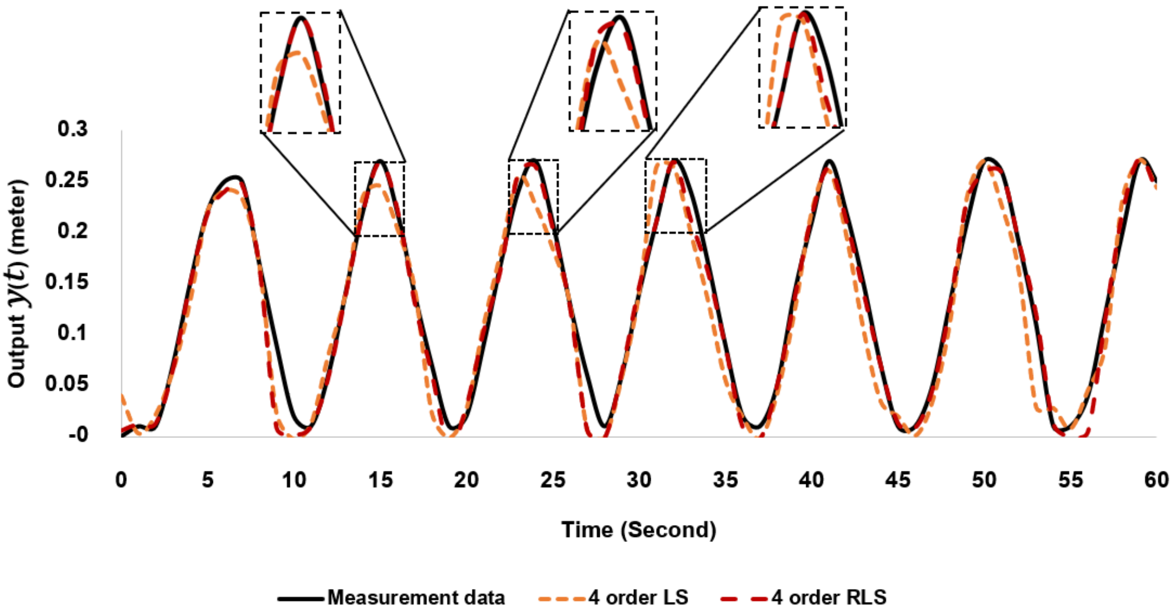

Figure 8 the comparison study of the different least square methods.

Conclusions and Discussion

This paper has applied the least square method to compare different model structures, specifically the FIR model and ARMAX model for 2^{nd}, 4^{th} and 6^{th} model order in an automotive window mechanism. In comparison study, higher-order model structures generally result in better model accuracy for both of the FIR and ARMAX model structures. However, The ARMAX model structures outperforms the FIR model structures. The model accuracy of the 4^{th} order ARMAX model is comparable to that of the 6^{th} order ARMAX model, but the number of system parameters of the 4^{th} order ARMAX model is lower than that of the 6^{th} ARMAX model. Hence, the most suitable model for predicting the output of an automotive window mechanism is the 4^{th} order ARMAX model. This selection is based on both the lower number of system parameters and the comparable model accuracy to that of the 6^{th} order ARMAX model. The model order refers to the number of past input and output terms included in the model equation. In low-order models, the FIR model incorporates only a few past input terms, whereas the ARMAX model includes both past input and output terms. Due to the absence of past output terms, the FIR model is preferred when past system states are not critical or when the system exhibits weak internal dynamics. In contrast, the ARMAX model can capture internal dynamics more effectively due to the inclusion of feedback from past outputs. In high-order models, the FIR model accounts for more past inputs but still lacks feedback from past outputs, whereas the ARMAX model incorporates both past inputs and outputs, enhancing accuracy in complex systems by leveraging feedback mechanisms. Higher order improves accuracy but increases computation time.

To enhance the model accuracy performance, this paper has applied the recursive least square method to the 4^{th} order ARMAX model in an automotive window mechanism. Unlike the conventional least square method, this approach begins by initializing the estimated system parameters data obtained from the conventional least squares method. All estimated parameters then converge to their true system parameter values within 15 seconds. The experimental results showed that the model accuracy achieved using the recursive least square method surpasses that of the conventional least square method in an automotive window mechanism. Future work will focus on deriving the dynamic mathematical model from physical law and will apply system identification methods to determine the values of the system’s physical parameters.

The comparative analysis of various system identification methodologies applied to different mechanical systems highlights the advantages and limitations of each approach in terms of accuracy, complexity, and applicability as shown in table 4. [3] proposed a nonlinear neural network model for a thermal system, achieving a mean squared error of 0.7979. This data-driven approach effectively captures system dynamics; however, the model's complexity poses challenges in implementation.

Table 4 Comparative study and analysis of research findings.

Study	Mechanical system	Methodology	Results	Advantages	Limitations
Arisariyawong, T. et al. (2023) [3]	Thermal system	Nonlinear neural network model	Mean squared error = 0.7979	Model based on Data-driven dynamic modeling	Complicating structure of the model
Pothi, N. et al. (2023) [5]	DC motor	2^{nd} order transfer function model with open-loop step test	Best fit = 96.87%	A few processes for system identification	Highly sensitive to noise
Mahajan, B. D. et al. (2016) [7]	Automotive suspension system	Simulink based modeling	Improved accuracy with sine wave input	Low complexity of model structure	Limited to simulation-based validation
Pappalardo, C. et al. (2023) [8]	Flexible robotic manipulator	ARX with 2^{nd} order mechanical model	Best fit = 95.48%	Easy to implementation	Relies on a lumped parameter
Beltran-Carbajal, F. et al. (2015) [11]	Mass spring damper	2^{nd} order transfer function model with online parameter estimate	Best fit = 97.82%	On line estimation problem under noisy environments	Requires prior knowledge of the system's bandwidth
This study	Automotive window mechanism	4^{th} order ARMAX model with RLS	Best fit = 95.56%	parameters converge to their values	Suitable for online implementation only

[5] identified a DC motor system using a 2^{nd} transfer function model with an open-loop step test. This approach achieved a high accuracy of 96.87% and required only a few processes for system identification. However, the model is highly sensitive to noise, which may affect real-world applications. [7] developed a Simulink-based model for a car suspension system, demonstrating improved results when using a sine wave input. The simplicity of the model structure ensures ease of use, but its validity is limited to simulation studies, lacking experimental verification. [8] applied an ARX-based 2^{nd} mechanical model for a flexible robotic manipulator, achieving a best-fit accuracy of 95.48%. This approach benefits from easy implementation; however, it relies on a lumped parameter model, which may not fully capture complex system dynamics. [11] employed a 2^{nd} transfer function model with online parameter estimation for a mass-spring-damper system, achieving the highest accuracy of 97.82%. The method is particularly useful in handling online estimation problems under noisy environments. Nevertheless, prior knowledge of the

system's bandwidth is required for optimal performance. In this study, the 4th ARMAX model with Recursive Least Squares (RLS) demonstrated a high best-fit accuracy of 95.56% for an automotive window mechanism, effectively converging system parameters. However, its application is constrained to online implementation.

Overall, this study emphasizes the importance of selecting an appropriate system identification technique based on trade-offs between accuracy, computational efficiency, and practical applicability. Future research should focus on hybrid approaches that integrate multiple identification methods to enhance performance while mitigating their individual limitations.

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