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Analyzing Rainfall Condition of Bangladesh: An Application of Markov Chain

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Abstract

This paper aims to study the long term changes of rainfall at two stations in Bangladesh by using Markov Chain Model. For this study daily rainfall data were collected from Bangladesh Meteorological Department (BMD). Stationary distribution test has been employed and to test whether the daily rainfall occurrences are stationary or not. After 10 days and 8 days, stationary probabilities have been observed for Dhaka and Chittagong station, respectively. By analyzing limiting behavior of current day's rainfall, it also has been observed that in Dhaka station 56% status of the day will be rainy and the rest of the condition will be sunny in May to October, whereas in Chittagong it is 58%. Lastly, using Cramer's V^2 and the test of several correlations coefficient, it has found that the association among the daily rainfall occurrences decreases when order of the Markov chain increases.

Keywords: Markov chain, rainfall.

1. Introduction

Bangladesh is an agro-based economic country where 80% of people directly or indirectly depend on agriculture (Rana et al. 2007). In consequence, the prospect of the crops such as rice, jute, wheat etc. would depend upon the sound knowledge of rainfall pattern of Bangladesh. Rainfall governs crop yields in seasonally arid tropics like Bangladesh. Although, Bangladesh is blessed with the largest unbroken sea shore, the largest mangrove forest, rich mountain ranges, vast green scenic serene, ample natural resources, and huge manpower; the country is still on her struggle to advance from an unfavorable economic state. Hence we have got a motivation to analyze the rainfall condition of Bangladesh considering two significant stations of it.

The rainfall in Bangladesh varies and depends on season and space. Only less than 4% annual rainfall and about 10-25% of the total annual rainfall comes from dry winter season (November through February) and pre-monsoon hot season (March-May), respectively. The rainy season accounts for 70-85% of the total annual rainfall. The average annual rainfall in Bangladesh varies from 1,500 mm in the west-central part to over 3,000 mm in the northeast and southeast (Banglapedia 2003).

Moreover, Climate change in Bangladesh is an extremely crucial issue and Bangladesh is the nation most vulnerable to global climate change in the world, according to German Watch's Global climate risk Index (CRI) of 2011. Besides, according to the recent IPCC report (IPCC 2007) it is noteworthy that, Bangladesh will experience 5% to 6% increase of rainfall by 2030. The main important source of water in any area is rain. The amount or availability of water for various purposes is very much depending upon the amount of precipitation in that particular area. Excess or extended absence of rainfall will cause flooding and drought respectively (Adler et al. 2010). The impacts of more variable precipitation and extreme weather events are already felt in Bangladesh. Floods in 1988, 1998, 2004, and 2007; and cyclones and tidal surges in 1991, 1998, 2000, 2004 and 2007 record the increase of extreme events both in frequency and severity (Roy et al. 2015).

Several researches have been carried out so far to study the rainfall related extreme weather events in Bangladesh. Ahmed (1989) studied the probabilistic estimates of rainfall extremes in Bangladesh during the pre-monsoon season. Karmakar and Khatun (1995) repeated similar study for probabilistic estimates of rainfall extremes during the southwest monsoon season. However, both the studies were concentrated only on maximum rainfall events for a limited time period. Shahid (2009) have studied the "Rainfall variability and the trends of wet and dry periods in Bangladesh" he has shown that the trend analysis shows a significant increase of average annual rainfall of Bangladesh at a rate of 5.52 mm/year over the time period 1958-2007. A significant increase of pre-monsoon rainfall by 2.47 mm/year at the 95% level of confidence is noted. The trend analysis of wet and dry months in Bangladesh shows that the number of wet months is increasing and the number of dry months is decreasing both in monsoon and pre-monsoon in most parts of the country. Rahman et al. (2015) have applied Mann-Kendal test to analyze regional variation of temperature and rainfall in Bangladesh and revealed significant increase of maximum temperature has been found in Cox's Bazar and Sylhet, significant decrease of maximum temperature has been found in Dhaka and Bogra. Significant increase of minimum temperature has been found in Dhaka and Cox's Bazar whereas significant decrease has been found in Rajshahi. They also estimated the linear trend and revealed that the maximum temperature increased significantly by 0.021 degree Celsius per year in Cox's Bazar and Sylhet. In case of minimum temperature highest increase was found in Dhaka by 0.049 degree Celsius followed by Cox's Bazar (0.038 degree Celsius per year) whereas significant decrease has been found in Rajshahi by 0.047 degree Celsius per year. The "Trend analysis of Climate Change in Chittagong Station in Bangladesh" has been studied by Roy et al. (2015).

This paper have been made an effort to analyze the daily rainfall of Bangladesh using the latest techniques based on Markov chain developed by, such as, Gabriel and Neumann (1962) and Stern and Coe (1984). The main objective of this paper is to analyze the patterns of rainfall occurrences for Bangladesh and to show the order of Markov model which provides a good fit.

2. Methodology

Dhaka, the capital city of Bangladesh, is also known as the densely populated city in the world with a population over 18 million and located on the east bank of the Buriganga River in the heart of the Bengal delta. It accommodates 40% of the total urban population of the country (Jahan and Moniruzzaman, 2007). The city has a distinct monsoonal season, with an annual average temperature of 26 °C (79 °F) and monthly means varying between 19 °C (66 °F) in January and 29 °C (84 °F) in May. Approximately 87% of the annual average rainfall of 2,123 millimeters (83.6 inches) occurs between May and October (Wikipedia, 2010).

Chittagong, the commercial city of Bangladesh, has a major coastal seaport area in southeastern part of Bangladesh. The city has a population of more than 2.5 million while the metropolitan area has a population of 4,009,423 at the 2011 Census, making it the second largest city in the country. The city is located on the banks of the Karnaphuli River between the Chittagong hill tracts and the Bay of Bengal. Chittagong has a tropical monsoon climate. The average annual rainfall of this city is about 2,700 millimeters. More than 80 percent of the annual rainfall occurs during the monsoon season from May to October.

Daily rainfall data were collected from Bangladesh Meteorological Department for two stations such as Dhaka in period 2008 to 2011 and Chittagong 2008 to 2013. There are 35 meteorological stations of Bangladesh. But the limitations of time and cost we have considered only two stations. Moreover, the data does not represent the long term period because it contains only few years of data.

For analyzing data we have used Microsoft Excel, SPSS and R programming which makes the data analysis quicker, easier and more accurate.

3. Markov Chain Model

The Markov chain model is the chain dependent process (Gabriet and Neumann 1962). The Markov chain model incorporates the current rain depth conditions today in the prediction a significant rain depth tomorrow. The described notation is adapted from Ross (2003).

Consider a stochastic process $X(t)$ ($t = 0,1,2\ldots$) that takes on a finite number of states k. in general, the process $X(t)$ is in state $s = 0,1,2,\ldots, k$ at the time $t = n$ if $X(n) = s$.

Let $s(t)$ denote the observed state of the stochastic process at time t.

For example, the initial state of the stochastic process is represented by $X(t=0) = s(t)$. In this application, the process $X(t)$ has two states, namely $X(t) = 1$ if the observed day is rain, and $X(t) = 0$ if the observed day is sunny. Furthermore, suppose that for all $t = 0$, the Markov property holds, that is,

$$P\left[X(t+1) = st+1 \mid X(t) = st, X(t-1) = st-1, \ldots, X(1) = s1, X(0) = s0\right]$$
$$= P\left[X(t+1) = st+1 \mid X(t) = st\right].$$

In words, this property states that the probability of being in state $st+1$ at time $t+1$ depends only on the current state st. As related to this example, observation of a significant snow depth tomorrow depends only on whether (or not) a significant snow depth is observed today and not on earlier snow depth conditions. This model thus incorporates the dependence of tomorrow's rain depth on today's observed rain depth to inform the probability of observing a significant rain depth tomorrow.

The two state transition probability matrix

		Current Day 0	Current Day 1
Previous Day	0	p_{00}	p_{01}
	1	p_{10}	p_{11}

This chain dependent process is irreducible, since every state communicates to each other. With ergotic states 0,1 the Markov chain (X_n) is said to be of order k (k =1,2,3,4,…) for all n, if and only if,

$$P(X_n = k, X_{n-1} = j, X_{n-2} = i) = p_{ijk} \text{ for all } i, j, k = 0,1.$$

Then p_{ijk} is called a Markov chain of order two, similarly the Markov chain of order three, four and so on can be defined.

4. Maximum Likelihood Estimation of Parameters

Anderson and Goodman (1957) have continued to the development of the statistical inference for Markov chain. Suppose that the states be $j=1,2$ and the time points of observations be $t=0,1,2,...,T.$ Let $p_{jk}(t)$, ($k=1,2$) be the probability of state k at time $t,$ given that the state j at time ($t-1$). If $p_{jk}(t)$ is stationary, then $p_{jk}(t)=p_{jk}$ for all $t,$ otherwise it is non-stationary.

Let us consider a Markov chain with stationary transition probabilities p_{jk}, ($j=1,2$) and finite number of states (1,2). Let n_{jk} be the number of observation in transition from the state j to the state k. The total number of observation is $\sum_{j=0}^{1}\sum_{k=0}^{1} n_{jk}$ where $j,k=0,1,$ $\sum_{k=0}^{1} n_{jk}=n_{j.}$ and $\sum_{j=0}^{1} n_{jk}=n_{.k}$. Whittle (1955) has noted that the observation Markov chain model from a set of independent multinomial trails of exact probability density of the observed n_{jk},

$$f(n_{jk})=T(n_{jk})\frac{\prod_{j}(n_{j.})!}{\prod_{j}\prod_{k}(n_{jk})!}\prod_{j}\prod_{k} p_{jk}^{n_{jk}}.$$

The factor $T(n_{jk})$ is the joint probability density of n_j 's and is independent of p_{jk} 's. If we consider a Markov chain model with non stationary transition probabilities $p_{jk}(t)$ then the probability density of $n_{jk}(t)$ is given by

$$f\{n_{jk}(t)\}=T\{n_{jk}(t)\}\frac{\prod_{j}\{n_{j.}(t)\}!}{\prod_{j}\prod_{k}\{n_{jk}(t)\}!}\prod_{j}\prod_{k} p_{jk}^{n_{jk}}(t).$$

In case of non stationary transition probabilities, the set $\sum_{t=1}^{T} n_{jk}=n_{jk}(t).$

From a set of sufficient statistics and in the case of non stationary transition probabilities $p_{jk}(t).$ The set $n_{jk}(t)$ from a multinational set of sufficient statistics. Medhi (1984) has obtained the logarithm of likelihood function for stationary transition probabilities p_{jk} as,

$$L(p_{jk})=C+\sum_{j=0}^{1}\sum_{k=0}^{1} n_{jk}\log p_{jk}.$$

The factors C contains all terms which are independent of p_{jk}. The maximum likelihood estimates of p_{jk} are found to be

$$\hat{p}_{jk}=\frac{n_{jk}}{n_{j.}}.$$

For non stationary transition probabilities, the maximum likelihood estimates are

$$\hat{p}_{jk}(t)=\frac{n_{jk}(t)}{n_{j.}(t)}=\frac{n_{jk}(t)}{n_{j.}(t-1)}.$$

Thus we may estimate the transition probabilities for second, third and higher order Markov model. The maximum likelihood estimates for the transition probabilities of daily rainfall occurrences for Dhaka and Chittagong stations are shown in following tables.

Table 1 Transition probability matrix for daily rainfall of Dhaka station

			Current Day	
			Sunny	Rainy
			0	1
Previous Day	Sunny	0	0.6338	0.3661
	Rainy	1	0.2916	0.7083

From this matrix 0.6338 indicates that previous day is sunny and the current day is also sunny with probability 0.6338. Similarly, 0.3661 indicates that previous day condition is sunny and current day is rainy with probability 0.3661.Then, 0.2916 indicates that previous day is rainy & the current day is sunny with probability 0.2916 and the probability 0.7083 means that previous day is rainy & the current day is also rainy with probability 0.7083.

Table 2 Transition probability matrix for daily rainfall of Chittagong station

			Current Day	
			Sunny	Rainy
			0	1
Previous Day	Sunny	0	0.5905	0.4095
	Rainy	1	0.2983	0.7017

We can explain just like previous matrix is that the value 0.5905 indicates the probability when the previous condition and the current condition of the day are both sunny, whereas 0.4095 indicates the probability when the previous condition of the day is sunny and the present condition is rainy. Likewise, we can interpret that 0.2983 is the probability of the previous day is rainy and the current day is sunny. Then, 0.7017 means the probability of both the day is rainy.

5. Limiting Behavior of Transition Probabilities

Feller (1957) has developed the unit step, …, m-step transition probabilities by the Chapman-Kolmogorov equation. The Champman-Kolmogrov equation is given

$$p_{jk}^{m+n} = \sum_i p_{ik}^{(n)} p_{ji}^{(m)} = \sum_i p_{ji}^{(n)} p_{ik}^{(m)} \quad \text{for all } i, j, k = 0,1;$$

where m and n denote the m^{th} step and n^{th} step, respectively.

The unit-step transition probabilities denoted by p_{jk} are defined as nth probability of occurrences at the n^{th} given occurrence at the $(n-1)^{th}$ step.

The 2nd -step transition probabilities $p_{jk}^{(2)}$ are defined as

$$p_{jk}^{(2)} = P(x_{n+2} = k \mid x_n = j).$$

So the m^{th} -step transition probabilities are

$$p_{jk}^{(m)} = P(x_{n+m} = k \mid x_n = j)$$

$$p^m = \frac{1}{p_0+p_1}\begin{bmatrix} p_1 & p_0 \\ p_1 & p_0 \end{bmatrix} + \frac{(1-p_0-p_1)^m}{p_0+p_1}\begin{bmatrix} p_0 & -p_0 \\ -p_1 & p_1 \end{bmatrix},$$

where $p_0 = P$ (Rainy day | Previous day Rainy) and $p_1 = P$ (Sunny day | Previous day Sunny).

If m is large, then

$$\lim_{m\to\infty} p^m = \begin{bmatrix} v_0 & v_1 \\ v_0 & v_1 \end{bmatrix}$$

where $v_0 = \frac{p_0}{p_0+p_1}$ and $v_1 = \frac{p_1}{p_0+p_1}$.

In agricultural planning, the limiting behaviors of daily rainfall are an important factor. Hence the Chapman-Kolmogorov equation has been employed to analyze the limiting behavior of daily rainfall occurrences for the selected stations and the results are shown below.

6. Result and Discussion

Using the above transition probability matrix for Dhaka and Chittagong, we can compute the distribution of daily rainfall after n days which are shown in Table 3. Considering the initial probabilities $[0,1]$. Mahanta et al. (2016) showed that initial probabilities not affect for long run distribution.

Table 3 Limiting behavior of daily rainfall of Dhaka and Chittagong station

After n days	Station			
	Dhaka		Chittagong	
	Sunny	Rain	Sunny	Rain
Day 00	0.0000	1.0000	0.0000	1.0000
Day 01	0.2916	0.7083	0.2983	0.7017
Day 02	0.3914	0.6085	0.3854	0.6146
Day 03	0.4256	0.5743	0.4109	0.5891
Day 04	0.4373	0.5626	0.4184	0.5816
Day 05	0.4413	0.5586	0.4205	0.5795
Day 06	0.4426	0.5573	0.4212	0.5788
Day 07	0.4431	0.5568	0.4214	0.5786
Day 08	0.4433	0.5567	0.4214	0.5786
Day 09	0.4433	0.5566		
Day 10	0.4433	0.5566		

It implies for the Dhaka station, after 10 days the distribution becomes stationary i.e. the limiting behavior of current days rainfall will not have any effect over the rainfall after 10 days. Moreover, we may conclude that 56% status will be rainy and rest of the condition will be sunny. That means the status of the rainy is higher than the sunny is May to October.

Likewise, the Chittagong station table represents that, after 08 days the distribution becomes stationary i.e. the limiting behavior of current day rainfall will not have any effect over the rainfall after 08 days. Also we can infer that 58% status will be rainy and rest of the condition will be sunny. That means the status of the rainy is higher than the sunny is May to October.

7. Test for Markov Model

The maximum likelihood estimate of the transition probability has been used to test whether the daily rainfall occurrences are independent or not. To perform this test the null hypothesis that the successive observation for daily rainfall are statistically independent against the alternative that the daily rainfall occurrences follow the Markov chain of order one has been considered, i.e.

$$H_0 : P_{jk} = p_k \text{ against } H_1 : P_{jk} \neq p_k .$$

Using the likelihood ratio criterion Anderson and Goodman (1957) have shown that

$$\chi^2 = 2\sum_{j=0}^{1}\sum_{k=0}^{1} n_{jk} \log\left(\frac{n_{jk} \times N}{n_{j.} \times n_{.k}}\right),$$

where n_{jk} is the transition count for j^{th} and k^{th} cell, $n_{.k}$ is $\sum_{j=0}^{1} n_{jk}$ and $n_{.j}$ is $\sum_{k=0}^{1} n_{jk}$.

Both Chittagong and Dhaka station the above χ^2 statistic has been applied for testing the null hypothesis that daily rainfall occurrence follow Markov model.

7.1. Dhaka Station

Observed value of χ^2 for corresponding null hypothesis have been shown in Table 4.

Table 4 Observed value of χ^2 based on Dhaka station for null hypothesis

Null hypothesis	χ^2 value	p-value
$H_{01} : P_{jk} = p_1$ i.e. Stationary distribution follow 1st order Markov chain.	285.547**	0.000
$H_{02} : P_{jk} = p_2$ i.e. Stationary distribution follow 2nd order Markov chain.	49.757**	0.000
$H_{03} : P_{jk} = p_3$ i.e. Stationary distribution follow 3rd order Markov chain.	39.6809**	0.000
⋮	⋮	⋮
$H_{010} : P_{jk} = p_{10}$ i.e. Stationary distribution follow 10th order Markov chain.	5.942	1.000

The table shows that the observed value of χ^2 is found to be 285.547 for first order Markov chain, and which is significant at 0.01 levels with 1 degree of freedom. Therefore, based on this test we may say that the daily rainfall occurrences are not stationary at 1st order Markov chain for Dhaka.

Then, the observed value of Dhaka station χ^2 is found to be 49.7565 for second order Markov chain, and which is significant at 0.01 levels with 3 degrees of freedom. Therefore, based on this test we may say that the daily rainfall occurrences are not stationary at second order Markov chain for Dhaka.

Also, the observed value of Dhaka station χ^2 is found to be 39.6809 for third order Markov chain, and which is at 0.01 levels of significance with 7 degrees of freedom. Therefore, based on this test we may say that the daily rainfall occurrences are not stationary at third order Markov chain for Dhaka.

Furthermore, the observed value of Dhaka station χ^2 is found to be 5.9421 for 10th order Markov chain, and which is not significant at 0.01 levels with 16383 degrees of freedom and 1.000 of p-value. Therefore, based on this test we may say that the daily rainfall occurrences are stationary at 10th order Markov chain for Dhaka.

From table finally it is notable that, the daily rainfalls of Dhaka station occurrences are not specified by first, second or third orders Markov chain. Here we can observe that the daily rainfall of Dhaka station stationary at 10th order Markov chain. However, test for independence showed that daily rainfall occurrences follow a Markov chain of 10th order. Hence we proceed to measure the strength of association to specify whether the daily rainfall occurrences follow a Markov chain of order 10.

Using the observed value of χ^2 Cramer (1946) has suggested a measurement denoted by V^2 for measuring the strength of association. This measurement V^2 is defined as

$$V^2 = \chi^2 / [n \min(r-1, c-1)],$$

where r is the number of rows and c is the number of columns. This measurement is called Cramer's V^2, falls between 0 and 1 with large values representing stronger association and it is identical to the square of Pearsonian correlation coefficient .The measurement has been employed for measuring the strength of association among the daily rainfall occurrences. The values of V^2 and corresponding correlation coefficient are shown in Table 5.

Table 5 Observed values of V^2 and coefficient of correlation for different order Markov chain (Dhaka station)

Markov chain	Observed values of V^2	Coefficient of correlation r
1st order	0.1413	0.3759
2nd order	0.0227	0.1505
3rd order	0.0136	0.1165
⋮	⋮	⋮
10th order	0.0006	0.0235

The table depicts that the association decreases when the order increases for Dhaka station. In 1st order the co-efficient of correlation is 0.3759 and in 10th order which is 0.0235. That is impact of rainfall decreases when order increases.

7.2. Chittagong Station

Observed value of χ^2 for corresponding null hypothesis have been shown in Table 6.

Table 6 Observed value of χ^2 based on Chittagong station for null hypothesis

Null hypothesis	χ^2 value	p-value
$H_{01}: P_{jk} = p_1$ i.e. Stationary distribution follow 1st order Markov chain.	41.125**	0.000
$H_{02}: P_{jk} = p_2$ i.e. Stationary distribution follow 2nd order Markov chain.	20.541**	0.000
$H_{03}: P_{jk} = p_3$ i.e. Stationary distribution follow 3rd order Markov chain.	6.749	0.455
⋮	⋮	⋮
$H_{08}: P_{jk} = p_8$ i.e. Stationary distribution follow 8th order Markov chain.	6.925	1.000

The observed value of χ^2 for Chittagong station is found to be 41.125, 20.5412, 6.7494 and 6.9245 for 1st order, 2nd order, 3rd order and 8th order Markov chain respectively, and in case of 1st order and 2nd order Markov chain the value of χ^2 is significant at 1% level of significance with their respective degrees of freedom. Therefore, based on this test we can draw a conclusion that the daily rainfall occurrences are not stationary at 1st order and 2nd order Markov chain for Chittagong. But the 3rd order and 8th order Markov chain is not significant at 1% level of significance with 7 and 255 degrees of freedom respectively. Thus we may conclude that the daily rainfall of Chittagong station is stationary at 3rd and 8th order Markov chain.

Hence we proceed to measure the strength of association to specify whether the daily rainfall occurrences of Chittagong station.

Table 7 Observed values of V^2 and coefficient of correlation for different order Markov chain (Chittagong station)

Markov chain	Observed values of V^2	Coefficient of correlation r
1st order	0.0374	0.1933
2nd order	0.0062	0.0789
3rd order	0.0015	0.0392
⋮	⋮	⋮
8th order	0.0007	0.0265

Table 7 showed that the association decreases when the order increases and the coefficient of correlation 0.1933 with observed values of V^2 is 0.0374 for 1st order. The above results show that strength of association decreases with respect to its order.

8. Conclusion

The statistical analysis was carried out based on the daily rainfall of Bangladesh, which is the only source of precipitation in Bangladesh. By analyzing limiting behavior of daily rainfall occurrences current day's rainfall will not have any effect over the rainfall after 10 days in case of Dhaka station and after 8 days in case of Chittagong station. We may also conclude that in Dhaka station 56% status will be rainy and rest of the condition will be sunny and in Chittagong 58% status will be rainy and rest of the condition will be sunny. That means the status of the rainy is higher than the sunny is May to October in Dhaka and Chittagong.

Employing the test of Markov chain, Cramer's V^2 for different order Markov model, it has been found that the daily rainfall of Dhaka station occurrences are not specified by first, second or third orders Markov chain. The daily rainfalls of Dhaka station stationary at 10^{th} order Markov chain. However, test for independence showed that daily rainfall occurrences follow a Markov chain of 10th order with observed value of χ^2 is found to be 5.9421 at 1% level of significance with 16383 degrees of freedom. On the other hand, the daily rainfalls occurrences are not specified by first, second order Markov chain for Chittagong station but it is stationary at 3^{rd} order. Again, as the rainfall is an important factor for agriculture, water resources, public health and economy of Bangladesh, it is hoped that the study in general will be beneficial to a number of stakeholders of Bangladesh particularly disaster management, development and planning organizations.

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