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Modified EWMA Control Chart for Skewed Distributions and Contaminated Processes

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Abstract

In this paper, we proposed the variable control chart for monitoring process mean by modified the traditional Exponentially Weighted Moving Average (EWMA) control chart for skewed and contaminated processes. The proposed EWMA control charts are modified both of mean and standard deviation in the control limits of the traditional EWMA control chart based on three different heuristic methods, namely, the Modified Exponentially Weighted Moving Average (MEWMA) control chart, the Modified Weighted Standard Deviation Exponentially Weighted Moving Average (MWSD-EWMA) control chart, and the Modified Weighted Variance Exponentially Weighted Moving Average (MWV-EWMA) control chart. A comparison of the performance measures is the average run length (*ARL*) were compared with the traditional EWMA control chart in various situations via simulation. The results show that for Gamma distribution, the MWSD-EWMA control chart is performs well for small sample size ($n = 5$), low levels of κ_3, ($\kappa_3 = 1$) and δ are low ($\delta = 0.10$, 0.25) in all values of λ. The MEWMA control chart is the better than other charts for moderate levels of δ ($\delta = 0.50$) in all levels of κ_3, λ, n and relatively small differences with the MWV-EWMA control chart. When the process distribution is Weibull, the MWSD-EWMA control chart is performs well for small sample size ($n = 5$) and δ are low ($\delta = 0.10, 0.25$) in all values of κ_3 and λ. If the sample sizes are moderate and large ($n = 10, 20$), the MWSD-EWMA control chart is better than other charts for very low level of δ ($\delta = 0.10$) in all values of κ_3 and λ. For lognormal distribution, we found that the results as well as the Weibull distribution in all levels of κ_3, λ and n. Finally, all charts are similar performances for all values of κ_3, λ and n when high levels of δ in all skewed distribution processes.

Keywords: EWMA control chart, modified EWMA control chart, skewed distribution, contaminated data, average run length.

1. Introduction

Control charts are widely used and powerful tools for monitor and detect the variation in the process. Quality control of the industrial process is used to ensure that the products are standard and quality. In 1924, Shewhart (Montgomery, 2005) was first introduced control charts for variable,

called the Shewhart $\bar{X}$ and *R* control charts. The $\bar{X}$ and *R* control charts are tools for monitoring change in the process mean and variability. After the $\bar{X}$ and *R* control charts were described by Shewhart, further investigations of the control chart were carried out by many researchers such as Page (1955), Roberts (1958), Champ and Woodall (1987), and others. The researchers were studied and discussed about the performance of the $\bar{X}$ control chart in many cases and found that the $\bar{X}$ control chart is quick for detect in large process mean shifts. In 1959, Roberts (Montgomery 2005) introduced the Exponentially Weighted Moving Average (EWMA) control chart. The properties of the EWMA control chart was studied and disscussed by Crowder (1987) and Lucas and Saccucci (1990). The EWMA control chart is effective for detecting small shifts. The most commonly control charts for variable that used to monitor process mean are the $\bar{X}$, Cumulative Sum (CUSUM), and Exponentially Weighted Moving Average (EWMA). However, these control charts depend on the assumption that the process distribution is normal. In many situations, the process distribution is non-normality. For instance, the characteristics of measurements in chemical, cut tool wear processes, and lifetimes test are often skewed.

Choobinech and Ballard (1987) proposed the weighted variance method (WV method) for constructing the $\bar{X}$ and *R* control charts for skewed distribution processes. The results show that the $\bar{X}$ chart by using the WV method is performing better than other charts in all situations. The *R* control chart by using WV method is performing better than other control charts. Chang and Bai (2001) proposed the weighted standard deviation method (WSD method) for constructing the $\bar{X}$, the CUSUM, and the EWMA control charts for positively skewed distribution processes. Chan and Cui (2003) proposed the skewness correction (SC method) for constructing the $\bar{X}$ and *R* control charts for skewed distribution processes. Khoo and Atta (2008) developed the EWMA control chart by using the WV method for skewed distribution processes. Khoo et al. (2009) developed $\bar{X}$ and *R* control charts by using the WV method and the mean of standard deviations $(\bar{S})$ to construct the control limits. Atta and Ramli (2011) studied the performance of the $\bar{X}$, CUSUM, and EWMA control charts for skewed distribution processes by using the WSD method. Derya and Canan (2012) studied the performance of the $\bar{X}$ and *R* control charts based on the Shewhart, WV, WSD, and SC methods for skewed distribution processes. But in some manufacturing process might have contaminated data, such as the process that consists of two production lines or the machine was broken down. Derya (2016) proposed the robust $\bar{X}$ control chart based on the Shewhart, the WV method and the SC method by using trimmed mean and interquartile range estimators to construct the control limits of the $\bar{X}$ control chart for monitoring skewed and contaminated processes.

Consequently, the aims of this paper are to modify the EWMA, the WSD-EWMA, and the WV-EWMA control charts and study the performance of these control charts when the process distribution is skewed and contaminated in other situations. We modify the control limits of the EWMA, the WSD-EWMA and the WV-EWMA control charts by using the mean of the sample trimmed means $(\overline{TM})$ for estimated the process mean and using the mean of sample interquartile ranges $(\overline{IQR})$ for estimated the process standard deviation. These modified control charts are namely the Modified Exponentially Weighted Moving Average (MEWMA) control chart, the Modified Weighted Standard Deviation Exponentially Weighted Moving Average (MWSD-EWMA) control chart, and the Modified Weighted Variance Exponentially Weighted Moving Average (MWV-EWMA) control chart. The performance was compared in terms of the Average Run Length (*ARL*) for all situations.

2. Materials and Methods

2.1. Control chart

1) EWMA control chart

The EWMA control chart was introduced by Roberts (1959) and developed by Crowder (1989). This control chart is effective for detect small process mean shifts. The EWMA control chart statistics are given as

$$Z_i = \lambda \bar{X}_i + (1-\lambda) Z_{i-1} \ ; i = 1, 2, \ldots \tag{1}$$

where λ is the smoothing constant; $0 < \lambda \leq 1$.

The control limits of the EWMA control chart are given as

$$UCL = \mu_0 + L \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}}, \tag{2}$$

$$LCL = \mu_0 - L \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}}, \tag{3}$$

where L is the control limit coefficient. When the parameters of the process are unknown, the control limits of EWMA chart are given as

$$UCL = \bar{\bar{X}} + L \frac{\bar{R}}{d_2 \sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}}, \tag{4}$$

$$LCL = \bar{\bar{X}} - L \frac{\bar{R}}{d_2 \sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}}, \tag{5}$$

where $\bar{\bar{X}}$ is the mean of sample means, $\bar{R}$ is the mean of sample ranges and d_2 is the factor for control limits.

2) WSD-EWMA control chart

The WSD-EWMA control chart porposed by Chang and Bai (2001) and used to detect the process mean shifts when the process is positively skewed distribution. The WSD-EWMA control chart statistics are given as

$$E_i = \lambda \bar{X}_i + (1-\lambda) Z_{i-1} \ ; i = 1, 2, \ldots . \tag{6}$$

The control limits of the WSD-EWMA control chart are given as

$$UCL_{WSD-EWMA} = \mu + K_1 \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}} 2P_X, \tag{7}$$

$$LCL_{WSD-EWMA} = \mu - K_1 \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}} 2(1-P_X), \tag{8}$$

where K_1 is the control limit coefficient and P_X is the probability of X is less than or equal to μ.

When the parameters of the process are unknown, the control limits of the WSD-EWMA control chart are given as

$$UCL_{WSD-EWMA} = \bar{\bar{X}} + K_1 \frac{\bar{R}}{d_2^{WSD} \sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}} 2\hat{P}_X, \tag{9}$$

$$LCL_{WSD-EWMA} = \bar{\bar{X}} - K_1 \frac{\bar{R}}{d_2^{WSD} \sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}} 2(1-\hat{P}_X), \tag{10}$$

where d_2^{WSD} is the factor for control limits.

3) WV-EWMA control chart

The WV-EWMA control chart proposed by Khoo and Atta (2008) and used to detect the process mean shifts when the process is skewed distribution. The WV-EWMA control chart statistics are given as

$$E_i = \lambda \overline{X}_i + (1-\lambda) Z_{i-1} \ ; i = 1, 2, \ldots . \tag{11}$$

The control limits of the WV-EWMA control chart are given as

$$UCL_{WV-EWMA} = \mu + K_2 \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}} \sqrt{2P_X}, \tag{12}$$

$$LCL_{WV-EWMA} = \mu - K_2 \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}} \sqrt{2(1-P_X)}. \tag{13}$$

When the parameters of the process are unknown, the control limits of the WV-EWMA control chart are given as

$$UCL_{WV-EWMA} = \overline{\overline{X}} + K_2 \frac{\overline{R}}{d_2^{'}\sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}} \sqrt{2P_X}, \tag{14}$$

$$LCL_{WV-EWMA} = \overline{\overline{X}} - K_2 \frac{\overline{R}}{d_2^{'}\sqrt{n}} \sqrt{\frac{\lambda}{2-\lambda}} \sqrt{2(1-\hat{P}_X)}, \tag{15}$$

where K_2 is the control limit coefficient.

2.2. Modified control charts methods

In this study, the control charts are modified both of mean and standard deviation in the control limits. The mean in the control limit was modified by using the mean of sample trimmed means $(\overline{TM})$. The mean of the sample trimmed means is

$$\overline{TM} = \frac{1}{k} \sum_{i=1}^{k} TM_i, \tag{16}$$

where $TM_i = \frac{1}{n-np} \left[\sum_{j=\frac{np}{2}+1}^{n-\frac{np}{2}} X_{ij} \right]$, and p is the percent of the samples to be trimmed. The standard deviation in the control limit was modified by using the mean of the sample interquartile ranges $(\overline{IQR})$. The mean of the sample interquartile ranges is

$$\overline{IQR} = \frac{1}{k} \sum_{i=1}^{k} IQR_i, \tag{17}$$

where $IQR_i = Q_{75,i} - Q_{25,i}$.

1) MEWMA control chart

The MEWMA control chart statistics are defined as

$$M_i = \lambda TM_i + (1-\lambda) M_{i-1} \ ; i = 1, 2, \ldots . \tag{18}$$

The control limits of the MEWMA control chart are defined as

$$UCL_{MEWMA} = \overline{TM} + L_1 \frac{\overline{IQR}}{d_*^Q \sqrt{n-np}} \sqrt{\frac{\lambda}{2-\lambda}}, \tag{19}$$

$$LCL_{MEWMA} = \overline{TM} - L_1 \frac{\overline{IQR}}{d_*^Q \sqrt{n-np}} \sqrt{\frac{\lambda}{2-\lambda}}, \tag{20}$$

where L_1 is the control limit coefficient and d_*^Q is the factor for control limits that calculated when the distribution is skewed. The factor for control limits is

$$d_*^Q = E\left(\frac{IQR}{\sigma}\right). \tag{21}$$

Then, d_*^Q is changed by process distribution, where d_G^Q, d_W^Q, d_L^Q are the factor for control limits for gamma, Weibull, and lognormal distributions, respectively.

2) MWSD-EWMA control chart

The MWSD-EWMA control chart statistics are defined as

$$M_i = \lambda TM_i + (1-\lambda)M_{i-1} \; ; i = 1, 2, \ldots . \tag{22}$$

The control limits of the MWSD-EWMA control chart are defined as

$$UCL_{MWSD-EWMA} = \overline{TM} + L_2 \frac{\overline{IQR}}{d_*^Q \sqrt{n-np}} \sqrt{\frac{\lambda}{2-\lambda}} (2\hat{P}_X), \tag{23}$$

$$LCL_{MWSD-EWMA} = \overline{TM} - L_2 \frac{\overline{IQR}}{d_*^Q \sqrt{n-np}} \sqrt{\frac{\lambda}{2-\lambda}} 2(1-\hat{P}_X), \tag{24}$$

where L_2 is the control limit coefficient and d_*^Q is the factor for control limits that calculated when the distribution is skewed and calculated from (21).

3) MWV-EWMA control chart

The MWV-EWMA control chart statistics are defined as

$$M_i = \lambda TM_i + (1-\lambda)M_{i-1} \; ; i = 1, 2, \ldots . \tag{25}$$

The control limits of the MWV-EWMA control chart are defined as

$$UCL_{MWV-EWMA} = \overline{TM} + L_3 \frac{\overline{IQR}}{d_*^Q \sqrt{n-np}} \sqrt{\frac{\lambda}{2-\lambda}} \sqrt{2\hat{P}_X}, \tag{26}$$

$$LCL_{MWV-EWMA} = \overline{TM} - L_3 \frac{\overline{IQR}}{d_*^Q \sqrt{n-np}} \sqrt{\frac{\lambda}{2-\lambda}} \sqrt{2(1-\hat{P}_X)}, \tag{27}$$

where L_3 is the control limit coefficient and d_*^Q is the factor for control limits that calculated when the distribution is skewed and calculated from (21).

2.3. Experimental design and methodology

This study compared the performance of these modified control charts that the MEWMA, MWSD-EWMA, and MWV-EWMA with the traditional control charts that the WSD-EWMA and WV-EWMA for skwed and contaminated processes in terms of the average run length (*ARL*). The results show the *ARL* values for various levels of the skewness coefficients (κ_3) the smoothing

constants (λ), and the mean shift rates (δ). All the charts considered in this study are designed based on the in-control *ARL* of 370 when the process follows the gamma, Weibull, and lognormal distributions.

To evaluate the performance of five control charts that the WSD-EWMA, WV-EWMA, MEWMA, MWSD-EWMA, and MWV-EWMA control charts. This study defined the process distributions in three type of skewed distributions that is Weibull, gamma, and lognormal. In the application, the scale parameter of gamma distribution is $\beta = 1$, the scale parameter of Weibull distribution is $\eta = 1$, and the location parameter of lognormal distribution is $\mu = 1$. The contaminated processes are gamma distribution with shape parameter 2α and scale parameter $\beta = 0.2$, Weibull distribution with shape parameter 2β and scale parameter $\eta = 0.2$, lognormal distribution with scale parameter $1/2\sigma$ and location parameter $\mu = -1$. The percent of contaminated data is 10%. The skewness coefficients (κ_3) considered for all distributions are 1.0, 2.0, and 3.0. The sample sizes (n) considered for all distributions are 5, 10 and 20. The process mean shift rates (δ) are 0.10, 0.25, and 0.50. The smoothing constants (λ) are 0.1, 0.2, 0.3, and 0.4. The percent of samples to be trimmed (p) is 20%. The in-control average run length (ARL_0) is equal to 370. The average run length (ARL) is a criterion for measure control chart performances. The simulation using R program is repeated for 10,000 simulation trials.

3. Results

3.1. Simulation study

In general, Tables 1-3 show the *ARL* values for gamma distribution when $\kappa_3 =$ 1.0, 2.0, 3.0, $\lambda = 0.1$, 0.3, $\delta = 0.10$, 0.25, 0.50, and $n = 5$, 10, 20, respectively. In this case, the MWSD-EWMA control chart is preferred for small sample size ($n =$ 5), low level of κ_3 ($\kappa_3 = 1.00$) and δ are low ($\delta =$ 0.10, 0.25) in all values of λ. When the sample sizes are moderate and large ($n =$ 10, 20), the MEWMA control chart is the best for all levels of κ_3 and λ for $\delta =$ 0.50 and relatively small differences with the MWV-EWMA control chart.

Tables 4-6 show the *ARL* values for Weibull distribution when $\kappa_3 =$ 1.0, 2.0, 3.0, $\lambda =$ 0.1, 0.3, $\delta = 0.10$, 0.25, 0.50, and $n =$ 5, 10, 20, respectively. In this case, for $n =$ 5, the MWSD-EWMA control chart is performs better than other control charts for all values of κ_3 and λ when δ are low ($\delta =$ 0.10, 0.25) because this control chart can detect the process mean shifts to faster. When the sample sizes are moderate and large ($n =$ 10, 20), the MWSD-EWMA control chart is performs well for very low δ ($\delta = 0.10$) in all values of κ_3 and λ.

Table 1 The Average Run Length (ARL) values for the gamma distribution with contaminated data 10% when $ARL_0 = 370$ and $n = 5$

		$\lambda = 0.1$					$\lambda = 0.3$				
κ_3	δ	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA
1.00	0.00	369.46	371.11	369.49	369.16	369.75	370.99	369.48	371.71	370.26	371.32
	0.10	29.52	35.89	49.90	27.32	35.48	80.30	122.17	276.85	76.70	121.76
	0.25	238.59	294.94	319.65	236.65	264.60	354.81	433.50	381.29	278.64	329.80
	0.50	73.27	51.44	22.51	41.85	29.28	151.67	93.82	47.01	81.14	56.16
2.00	0.00	370.89	371.86	369.45	371.31	371.98	371.23	369.78	369.78	370.26	371.84
	0.10	216.95	271.13	529.58	178.79	238.81	330.39	486.35	669.40	366.36	617.19
	0.25	522.93	216.67	162.64	555.50	261.32	437.17	217.48	232.38	408.47	246.18
	0.50	29.10	10.16	6.50	29.25	10.66	69.69	34.65	32.41	59.84	34.06
3.00	0.00	370.18	370.85	371.77	369.54	369.03	371.50	371.26	370.82	371.10	369.14
	0.10	452.97	419.30	496.05	532.26	493.90	480.42	417.50	477.95	548.98	474.68
	0.25	646.16	122.21	64.82	504.02	104.78	448.36	135.76	146.29	295.05	144.87
	0.50	51.58	8.42	1.67	30.40	4.94	83.52	27.52	25.30	46.35	26.06

Table 2 The Average Run Length (ARL) values for the gamma distribution with contaminated data 10% when $ARL_0 = 370$ and $n = 10$

		$\lambda = 0.1$					$\lambda = 0.3$				
κ_3	δ	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA
1.00	0.00	369.46	369.92	371.27	369.06	369.06	369.49	369.75	371.69	369.91	371.11
	0.10	223.99	258.73	293.78	203.87	228.00	284.00	346.21	348.12	265.04	327.40
	0.25	93.18	59.17	41.19	84.40	52.97	208.91	117.67	72.26	168.39	100.83
	0.50	0.00	0.00	0.00	0.00	0.00	10.08	6.77	4.50	8.69	5.89
2.00	0.00	370.91	371.87	371.17	371.76	369.31	369.17	369.85	371.46	371.42	369.11
	0.10	622.33	376.23	251.38	621.44	391.42	544.78	339.69	284.54	549.60	355.59
	0.25	68.73	22.52	10.69	63.05	20.37	175.84	52.77	35.96	149.40	47.06
	0.50	0.00	0.00	0.00	0.00	0.00	12.24	4.79	2.88	9.64	3.94
3.00	0.00	369.28	369.00	370.54	371.66	369.26	371.06	369.64	369.77	369.99	369.00
	0.10	812.31	343.60	129.39	833.64	296.43	690.97	267.74	187.07	681.28	228.16
	0.25	166.38	21.91	3.86	89.03	12.60	354.49	47.98	28.04	191.24	33.50
	0.50	0.59	0.00	0.00	0.03	0.00	23.62	4.65	2.16	11.56	2.74

Table 3 The Average Run Length (ARL) values for the gamma distribution with contaminated data 10% when $ARL_0 = 370$ and $n = 20$

		$\lambda = 0.1$					$\lambda = 0.3$				
κ_3	δ	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA
1.00	0.00	371.59	369.12	370.28	371.65	371.49	371.70	369.96	369.02	371.12	369.01
	0.10	152.79	209.09	206.41	129.08	149.73	231.94	290.54	367.81	200.72	251.73
	0.25	39.07	29.67	16.88	37.39	25.02	122.43	67.00	45.39	109.54	62.67
	0.50	0.00	0.00	0.00	0.00	0.00	2.03	1.03	0.63	1.73	0.97
2.00	0.00	369.29	370.89	370.31	369.43	369.58	371.05	371.16	369.95	371.36	370.29
	0.10	680.84	612.57	209.68	689.65	375.46	638.39	349.37	242.43	649.78	360.16
	0.25	20.17	9.99	0.56	15.62	3.06	113.64	27.58	14.80	92.68	24.44
	0.50	0.00	0.00	0.00	0.00	0.00	2.48	0.45	0.08	1.61	0.23
3.00	0.00	369.36	371.22	371.36	369.58	369.89	369.56	369.12	371.26	371.53	370.46
	0.10	908.40	731.22	71.18	883.89	215.59	833.49	417.60	339.94	989.60	690.26
	0.25	50.75	17.13	0.00	22.30	0.48	300.76	26.00	23.01	534.75	42.01
	0.50	0.00	0.00	0.00	0.00	0.00	8.01	0.63	0.17	8.83	0.57

Table 4 The Average Run Length (ARL) values for the Weibull distribution with contaminated data 10% when $ARL_0 = 370$ and $n = 5$

		$\lambda = 0.1$					$\lambda = 0.3$				
κ_3	δ	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA
1.00	0.00	369.57	369.54	369.22	369.17	371.80	369.54	371.69	370.53	371.46	370.61
	0.10	15.45	17.93	2.35	0.97	1.31	20.77	26.26	22.36	9.40	12.80
	0.25	23.40	23.82	2.22	1.06	1.43	15.79	17.12	14.29	6.31	8.24
	0.50	13.47	11.87	2.46	1.40	1.76	8.28	8.02	9.55	5.05	6.31
2.00	0.00	369.19	371.32	371.29	371.88	369.61	371.18	371.47	369.85	371.19	369.89
	0.10	46.91	53.98	29.81	9.11	12.42	49.55	80.65	553.61	25.49	70.97
	0.25	54.19	41.99	28.81	10.88	14.42	31.86	31.22	213.44	17.88	39.03
	0.50	17.33	11.11	23.66	12.96	14.26	13.13	10.61	70.21	14.35	23.82
3.00	0.00	371.48	371.92	369.62	369.36	369.31	370.94	369.81	371.66	371.07	371.21
	0.10	74.48	78.25	103.17	18.13	26.45	72.21	141.81	717.54	43.72	328.38
	0.25	78.90	49.05	82.36	22.39	28.35	40.37	37.84	318.89	31.15	125.99
	0.50	22.83	11.41	58.69	27.21	29.21	15.89	13.25	105.27	20.84	56.86

Table 5 The Average Run Length (ARL) values for the Weibull distribution with contaminated data 10% when $ARL_0 = 370$ and $n = 10$

		$\lambda = 0.1$					$\lambda = 0.3$				
κ_3	δ	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA
1.00	0.00	369.87	370.38	370.37	369.34	371.16	369.34	369.32	369.52	369.67	369.83
	0.10	53.29	54.66	18.72	12.47	14.39	37.09	39.18	34.50	20.10	23.87
	0.25	21.72	17.83	19.52	14.91	16.37	16.18	14.98	19.73	13.18	15.69
	0.50	0.57	0.25	17.04	16.70	16.78	4.49	3.83	11.54	10.11	10.35
2.00	0.00	371.70	371.59	371.09	371.62	370.51	369.40	371.50	371.11	370.35	371.43
	0.10	141.44	105.05	81.16	43.22	49.64	93.91	79.71	218.09	49.06	71.33
	0.25	33.30	17.50	62.89	57.91	53.27	27.12	17.23	58.00	39.00	38.40
	0.50	0.81	0.03	26.89	41.59	29.48	6.37	4.27	19.32	20.07	16.99
3.00	0.00	371.03	370.57	369.58	371.22	369.85	371.66	370.30	371.56	369.88	369.09
	0.10	237.02	126.52	172.22	77.43	81.32	130.24	87.94	251.93	75.78	110.40
	0.25	49.30	17.87	75.77	110.66	73.22	39.52	19.89	66.43	59.86	51.61
	0.50	2.10	0.00	21.46	58.42	28.32	8.91	4.99	21.45	30.13	20.38

Table 6 The Average Run Length (ARL) values for the Weibull distribution with contaminated data 10% when $ARL_0 = 370$ and $n = 20$

		$\lambda = 0.1$					$\lambda = 0.3$				
κ_3	δ	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA
1.00	0.00	369.00	370.02	371.97	370.95	370.00	371.00	370.11	371.37	369.54	371.56
	0.10	45.52	47.80	3.15	1.47	2.00	34.84	37.20	15.13	9.07	10.77
	0.25	16.10	12.51	5.14	3.33	3.81	15.96	12.69	11.79	8.39	9.48
	0.50	0.00	0.00	11.72	9.48	9.39	2.33	1.81	9.77	8.40	8.52
2.00	0.00	369.56	369.42	371.89	370.22	369.75	369.25	370.16	370.47	371.56	369.13
	0.10	162.52	113.32	35.03	18.62	21.08	101.73	80.42	99.10	27.64	35.52
	0.25	18.62	8.26	47.35	35.21	34.81	25.85	13.48	53.33	27.87	30.62
	0.50	0.00	0.00	31.43	47.36	33.36	3.28	1.63	22.00	24.17	19.26
3.00	0.00	370.25	369.56	370.67	370.00	371.23	369.99	371.27	371.11	371.26	370.55
	0.10	296.26	133.49	82.51	40.77	43.61	168.34	90.61	275.88	49.28	64.94
	0.25	27.11	6.52	82.28	80.43	68.71	34.61	13.01	80.78	56.11	49.14
	0.50	0.00	0.00	25.66	81.97	36.02	4.64	1.67	21.32	41.18	22.00

Tables 7-9 show the ARL values for lognormal distribution when $\kappa_3 =$ 1.0, 2.0, 3.0, $\lambda =$ 0.1, 0.3, $\delta =$ 0.10, 0.25, 0.50, and $n =$ 5, 10, 20, respectively. In this case, for $n =$ 5, the MWSD-EWMA control chart is performed better than other control charts when δ are low ($\delta =$ 0.10, 0.25) for all values of κ_3, λ. When the sample sizes are moderate and large ($n =$ 10, 20), the MWSD-EWMA control chart is performed well for very low δ ($\delta =$ 0.10) in all values of κ_3 and λ. We found that this result as well as the Weibull distribution in all levels of κ_3, λ and n.

Table 7 The Average Run Length (ARL) values for the lognormal distribution with contaminated data 10% when $ARL_0 =$ 370 and $n =$ 5

		$\lambda = 0.1$					$\lambda = 0.3$				
κ_3	δ	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA
1.00	0.00	369.18	369.60	371.44	371.29	370.45	370.55	371.88	369.05	369.55	369.63
	0.10	1.50	1.68	2.71	1.12	1.51	8.08	10.33	17.63	7.32	12.06
	0.25	6.70	7.97	2.26	1.49	1.76	10.57	12.30	10.08	6.33	7.50
	0.50	15.07	14.76	2.86	1.93	2.21	8.51	8.66	6.72	4.67	5.31
2.00	0.00	371.74	370.37	370.23	371.20	371.14	370.14	370.52	371.05	370.33	371.90
	0.10	20.16	26.19	15.00	5.14	7.26	34.38	65.64	145.46	18.61	31.79
	0.25	43.29	44.10	17.84	8.00	10.43	28.98	36.63	69.08	14.41	21.70
	0.50	31.34	16.45	21.57	13.28	15.30	14.49	13.75	32.14	11.60	16.07
3.00	0.00	369.81	369.39	369.55	371.49	369.71	369.80	371.36	370.82	371.38	370.47
	0.10	43.57	55.75	35.77	10.64	14.78	59.25	159.06	554.64	28.08	74.04
	0.25	62.82	52.43	39.77	17.05	20.81	40.78	50.67	209.54	22.72	49.15
	0.50	22.28	15.05	40.53	23.78	25.09	16.46	14.84	64.63	18.71	28.27

3.2. Empirical real data

To illustrate the proposed control charts from data investigated, we applied to monitoring of Urinary Tract Infections (UTIs) data. The UTIs data was considered by Muhammad et al. (2017). This data is followed gamma distribution with shape parameter (α) equals to 2. In this study, we replaced 10% of contaminated data from gamma distribution with 2α and $\beta = 0.2$ in each of samples. The UTIs data with contamination is given in Table 10.

Table 8 The Average Run Length (ARL) values for the lognormal distribution with contaminated data 10% when $ARL_0 = 370$ and $n = 10$

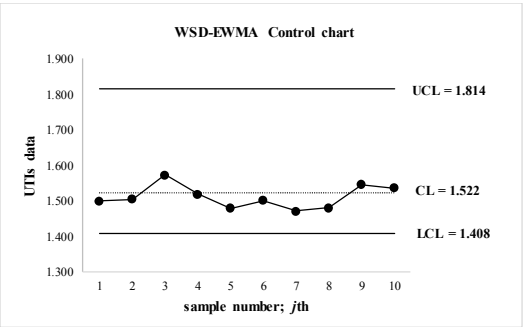
		$\lambda = 0.1$					$\lambda = 0.3$				
κ_3	δ	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA
1.00	0.00	371.44	371.09	371.13	371.86	369.30	371.49	370.30	371.67	369.12	371.33
	0.10	19.96	22.69	20.16	14.00	16.39	25.86	30.68	36.67	22.40	27.21
	0.25	33.50	31.85	25.09	21.53	22.64	19.29	18.74	19.93	15.67	16.66
	0.50	3.05	2.25	18.12	18.57	17.83	6.00	5.30	10.03	9.53	9.46
2.00	0.00	371.33	371.68	370.29	371.42	370.40	369.50	369.90	369.87	369.67	369.10
	0.10	90.81	92.67	62.68	38.39	43.31	74.50	84.43	143.35	43.50	61.38
	0.25	42.43	27.21	61.13	56.60	54.06	31.42	23.59	50.49	38.38	37.79
	0.50	0.82	0.07	19.71	31.82	22.11	6.44	4.17	16.06	18.41	15.35
3.00	0.00	371.04	369.78	369.58	369.52	369.70	369.65	371.31	370.72	369.71	369.03
	0.10	157.55	124.58	113.92	61.29	72.24	109.64	103.41	259.34	64.54	96.26
	0.25	42.17	22.29	71.13	96.55	71.82	34.12	23.47	62.51	55.41	48.82
	0.50	0.72	0.01	16.29	39.63	21.22	8.02	5.23	17.27	26.31	17.63

Table 9 The Average Run Length (ARL) values for the lognormal distribution with contaminated data10% when $ARL_0 = 370$ and $n = 20$

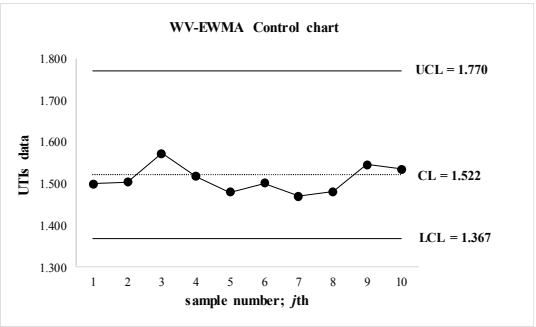
		$\lambda = 0.1$					$\lambda = 0.3$				
κ_3	δ	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA	WSD-EWMA	WV-EWMA	Modified EWMA	Modified WSD-EWMA	Modified WV-EWMA
1.00	0.00	369.63	369.41	371.61	369.01	371.52	371.85	369.20	369.25	371.46	369.72
	0.10	7.41	8.54	3.55	1.77	2.41	16.23	18.93	16.02	10.31	11.81
	0.25	34.22	28.98	10.77	7.69	8.86	21.48	19.28	14.02	11.30	12.16
	0.50	0.21	0.07	17.45	16.88	16.14	3.96	3.30	9.05	8.98	8.64
2.00	0.00	371.00	370.26	369.67	370.26	371.15	371.27	371.20	369.59	369.34	370.25
	0.10	81.14	80.78	24.52	14.28	17.00	66.39	75.01	62.52	25.08	31.11
	0.25	24.29	14.02	51.76	45.61	44.75	27.15	17.01	42.95	33.45	31.09
	0.50	0.00	0.00	20.21	34.55	23.52	3.34	2.07	16.04	22.03	16.17
3.00	0.00	371.02	371.01	371.59	369.66	370.37	369.78	371.24	370.57	371.40	369.31
	0.10	163.64	123.45	52.62	27.88	32.08	109.24	94.56	154.74	39.57	51.26
	0.25	24.50	9.50	68.07	81.85	70.51	30.30	16.41	59.85	52.64	47.87
	0.50	0.00	0.00	14.34	44.02	20.85	3.63	1.79	16.13	30.87	17.92

Table 10 The Urinary Tract Infections (UTIs) data with contaminated data 10%

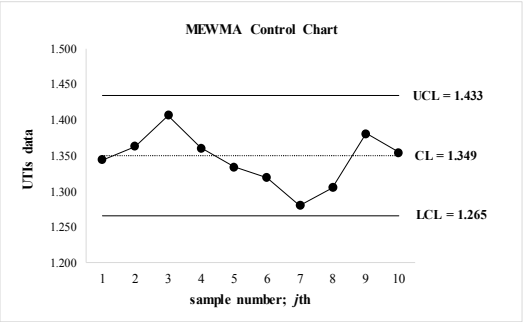
Sample j	Subgroup i				
	1	2	3	4	5
1	3.159024	0.275058	2.227719	0.754739	1.939090
2	1.011845	2.007316	0.644884	2.466912	1.564738
3	5.170865	0.363686	2.026336	2.808165	0.571657
4	0.538155	0.487815	0.716230	1.584113	1.800484
5	1.318261	1.221514	2.091515	0.740039	0.285453
6	0.640254	1.355277	0.318997	1.553403	4.615132
7	1.176958	2.534075	0.662896	0.641414	0.952253
8	1.463346	1.438160	1.561982	1.836224	1.565102
9	0.409005	3.502164	1.734065	0.943032	4.043703
10	1.247893	3.202629	1.295080	0.809534	0.667236



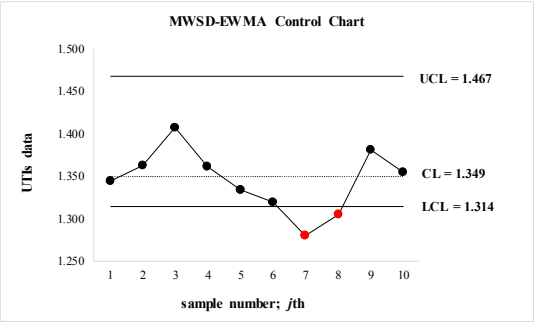
(a) The WSD-EWMA control chart for the UTIs data



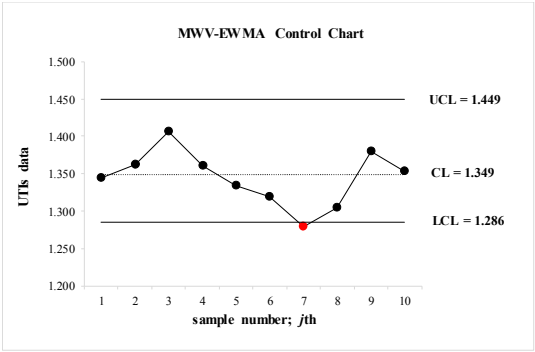
(b) The WV-EWMA control chart for the UTIs data



(c) The MEWMA control chart for the UTIs data



(d) The MWSD-EWMA control chart for the UTIs data



(e) The MWV-EWMA control chart for the UTIs data

Figure 1 Control charts for the UTIs data

From Figure 1, we found that the proposed control charts, the MWSD-EWMA and MWV-EWMA scheme declare the out-of-control at sample 7 from Figure 1(d) and (e), respectively. Which the result consists with the simulation data. Accordingly, it can confirm that the proposed control charts are more effective than the other control charts for skew distributions and contaminated processes.

4. Conclusions

In this paper, we compare the performance of the control charts based on skewed distributions and contaminated processes. These proposed control charts are modified in both of mean and standard deviation based on three different heauristic methods. The performance of the control charts is considered in terms of the average run length (ARL) for various situations that the skewed distributions, the skewness coefficients (κ_3) the smoothing constants (λ), sample sizes (n), and the mean shift rates (δ).

The results show that for the gamma distribution, the MEWMA control chart performs better than the other control charts in many situations that $n =$ 5, 10, 20 and relatively small differences with the MWV-EWMA control chart in all levels of κ_3, λ and δ. For Weibull and Lognormal distributions, the MWSD-EWMA control chart has higher efficiency than the other control charts when $n =$ 5 in all levels of κ_3, λ, and δ. On the other hand, the MWSD-EWMA control chart is performed well for low levels of the skewness coefficient and mean shift rate ($\kappa_3 = 1.0$, $\delta =$ 0.10, 0.25) when $n =$ 10, 20.

In summarize, when the process distribution is gamma and contaminated, the MEWMA control chart is recommend for using with all values of κ_3, λ and n. For Weibull and lognormal distributions, the MWSD-EWMA control chart is recommend for using with small sample size ($n = 5$) in all values of κ_3 and λ. If the smaple sizes are moderate and large ($n =$ 10, 20), the proposed control charts similar perform as the traditional control charts.

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