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Short Communication

A Note on the Transmuted Generalized Inverted Exponential Distribution with Application to Reliability Data

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Abstract

Based on the transmuted generalized inverted exponential (TGIE) distribution (Elbatal 2013), Khan (2018) revisited the TGIE distribution with an illustrative application to a reliability data-set. Here, we revisit the data application and discuss the inadequacy of the TGIE distribution to the applied data-set.

Keywords: Reliability, unimodal, bathtub, maximum likelihood estimation.

1. Introduction

Recent advancements in statistics and probability include the generalization of standard probability distributions through parameter induction and distribution mixing. Generalizations of distributions have received the resounding attention of several scholars. Standard probability distributions are extended with the exclusive objective of improving their fitting tendencies in real-life application situations. In most cases the distributions in their generalized forms demonstrate better flexibility and fitting ability than their standard forms in modelling real-life data; for instance, the exponentiated Kumaraswamy-power function distribution due to Bursa and Ozel (2017), Marshall-Olkin Kappa distribution due to Javed et al. (2018), Burr X Pareto Distribution due to Korkmaz et al (2017), a new extension of Weibull distribution due to Nassar et al. (2017), the modified power function distribution due to Okorie et al. (2017a), extended Erlang-truncated exponential distribution due to Okorie et al. (2017b), and so on.

A couple of years ago Elbatal (2013) introduced the transmuted generalized inverted exponential (TGIE) distribution as an extension of the generalized inverted exponential (GIE) distribution due to Abouammoh and Alshingiti (2009). The TGIE distribution follows from the pioneering work of Shaw and Buckley (2007) - quadratic rank transmutation map (QRTM). The cumulative distribution function (CDF) of the TGIE distribution is given by

$$F_X(x)=\left\{1-\left(1-e^{-\frac{\theta}{x}}\right)^{\alpha}\right\}\left[1+\lambda\left(1-e^{-\frac{\theta}{x}}\right)^{\alpha}\right];\theta,\alpha>0,|\lambda|\leq 1,$$

and the probability density function (PDF) is given by

$$f_X(x)=\frac{\alpha\theta}{x^2}e^{-\frac{\theta}{x}}\left(1-e^{-\frac{\theta}{x}}\right)^{\alpha-1}\left[1-\lambda+2\lambda\left(1-e^{-\frac{\theta}{x}}\right)^{\alpha}\right];\theta,\alpha>0,|\lambda|\leq 1.$$

Elbatal (2013) studied various statistical properties of the TGIE distribution such as the qth quantile, moments, and order statistics. The maximum likelihood estimates and the corresponding information matrix of the parameters were derived and discussed. However, Elbatal (2013) failed to illustrate the potential usefulness of the TGIE distribution with real-life data.

More recently, Khan (2018) revisited the TGIE distribution with mainly diagrammatic contributions (plots of the PDF, reliability function, hazard rate function, median, coefficient of quartile deviation, skewness, and kurtosis) and numerical contributions (computation of some quartile and related measures and a Monte-Carlo simulation study of the parameter estimates based on the method of maximum likelihood estimation (MLE)) and a particular focus on the real-data application of the distribution.

Credit should be given to Khan (2018) for making the first attempt to illustrate the utility and flexibility of the TGIE distribution by fitting the distribution to a real data-set (survival times of 50 devices put on life test at time zero (see; Aarset 1987) and comparing its fit with those of known special cases of the TGIE distribution namely, the transmuted inverted exponential (TIE) due to Oguntunde and Adejumo (2014), generalized inverted exponential (GIE) due to Abouammoh and Alshingiti (2009), and the inverted exponential (IE) distributions. Based on the smallest AIC (Akaike 1974) and AICc (Hurvich and Tsai 1989) values of the TGIE distribution, Khan (2018) concluded that the TGIE distribution provides an adequate fit to the survival times' data-set. The TGIE distribution may provide fantastic fits to many real data-sets, particularly the unimodal data-sets. For the data set on the survival times of 50 devices put on life test at time zero (Aarset 1987) considered by Khan (2018); however, several one and two-parameter distributions can perform better than the TGIE distribution. This is illustrated in Section 2.

The aim of this note is to point out that the TGIE distribution is inadequate for modeling the survival times' data of 50 devices put on life test at time zero (Aarset 1987) because the distribution of the data has a bathtub or bimodal characteristics. But the TGIE distribution with unimodal density function could provide excellent fits to some other real-life unimodal distributed data-sets but, definitely not the survival times' data of 50 devices put on life test at time zero in Aarset (1987). The four-parameter Weibull-power function distribution with more flexible shape characteristics (including bathtub shape) due to Tahir et al. (2014) is known to provide better fit to the survival times' data of 50 devices put on life test at time zero (Aarset 1987).

2. Data Application

In this section, we compare the fit of the three-parameter TGIE distribution to some standard and well-known one and two-parameter unimodal distributions in modeling the survival times' data of 50 devices put on life test at time zero. We give evidence that some of the one and two-parameter distributions perform better than the TGIE distribution; although their fits are not adequate for the survival times' data. The TGIE distribution of Elbatal (2013) may give adequate fits to many unimodal data-sets but, certainly not the survival times' data. The intention of this note is not to vilify the phenomenal contribution by Khan (2018); but generally, in trying to access the fit of a generalized distribution to a real data-set it is often a good practice to compare the fits of some simple, standard, and well-known distributions with that of the generalized one which in most cases have complicated analytical expression.

The fitted distributions and their corresponding CDF's are:

Weibull: $F_X(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^{\alpha}}; x > 0, \alpha, \beta > 0,$

Rayleigh: $F_X(x) = 1 - e^{-\frac{x^2}{2\sigma^2}}; x \ge 0, \sigma > 0,$

gamma: $F_X(x) = \dfrac{\Upsilon\left(\alpha, \frac{x}{\beta}\right)}{\Gamma(\alpha)}; x > 0, \alpha, \beta > 0,$

generalized exponential (EE) due to Gupta and Kundu (2001): $F_X(x) = \left(1 - e^{-\lambda x}\right)^{\alpha}; x, \alpha, \lambda > 0,$

Erlang: $F_X(x) = \dfrac{\Upsilon(k, \lambda x)}{\Gamma(k)}; x \ge 0, k \in N, \lambda > 0,$

Lindley: $F_X(x) = 1 - \left(1 + \dfrac{\theta x}{\theta + 1}\right) e^{-\theta x}; x, \theta > 0,$

and Lindley-geometric (LP) due to Zakerzadeh and Mahmoudi (2012):

$$F_X(x) = \frac{1 - \left(1 + \frac{\theta x}{\theta + 1}\right) e^{-\theta x}}{1 - p\left(1 + \frac{\theta x}{\theta + 1}\right) e^{-\theta x}}; x > 0, \theta > 0, p \in (0,1).$$

All the distributions are fitted to the survival times data of 50 devices put on life test at time zero by the method of maximum likelihood estimation and the goodness-of-fit of the fitted distributions are compared by their AIC, BIC (Schwarz 1978), AICc, and Kolmogorov-Smirnov (K-S) (see; Kolmogorov 1933, Smirnoff 1939, Scheffé 1943, and Wolfowitz 1949) values. In comparison, the distribution with the smallest values of all these goodness-of-fit statistics is said to offer a better fit to the data-set than the others.

The analytical forms of the goodness-of-fit measures are:

$$AIC = 2k - 2\log L(\theta),$$

$$BIC = k\log n - 2\log L(\theta),$$

and

$$K - S = \sup_x | F_n(x) - F_X(x) |,$$

where k is the number of parameters in the distribution, n is the total number of observations in the data, $-\log(\theta)$ is the estimated value of the minimized log-likelihood function of the PDF, $F_n(x)$ is the empirical CDF, and $F_X(x)$ is the fitted CDF.

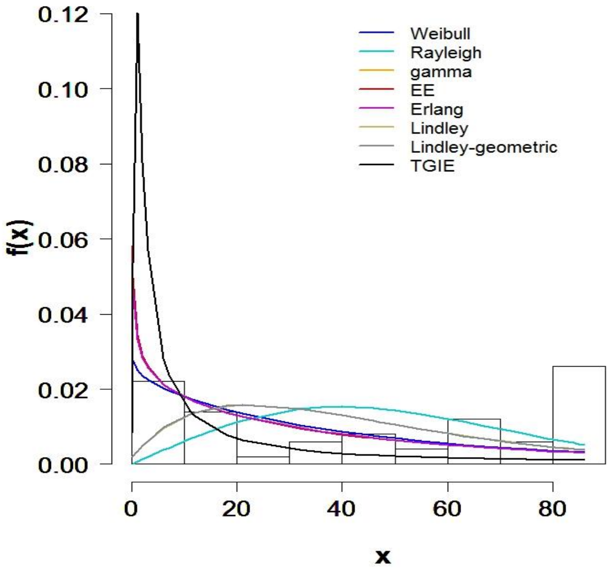

Figure 1 Histogram of the data and fitted densities of the Weibull, Rayleigh, gamma, EE, Erlang, Lindley, Lindley-geometric and TGIE distributions

The results in Table 1 indicate that the Weibull, Rayleigh, gamma, EE, Erlang, Lindley, and Lindley-geometric, distributions with comparatively smaller AIC, BIC, AICc, K-S values, and larger K-S p-values performs better than the TGIE distribution which gave the largest AIC, BIC, AICc, K-S values, and the smallest K-S p-value. Figure 1 indicates that none of the fitted unimodal distributions provides a good fit to the survival times' data of 50 devices put on life test at time zero (Aarset 1987). For now, we are not aware of any probability distribution providing a better fit than the Weibull-power function distribution for the survival times' data of 50 devices put on life test at time zero hence; we recommend the Weibull-power function distribution for modeling the survival times' data of 50 devices put on life test at time zero (Aarset 1987); for detail see, Tahir et al. (2014).

Table 1 Parameter estimates, log-likelihood values, AIC values, BIC values, AICc values and Kolmogorov-Smirnov (K-S) statistic and K-S p-values for the fitted distributions

Distributions	MLEs [Standard errors]			$-\log L(\theta)$
Weibull	$\hat{\alpha}: 0.9490425$ [0.1195723]	$\hat{\beta}: 44.9125137$ [6.9465188]		241.0018
Rayleigh	$\hat{\sigma}: 39.64718$ [2.804178]			264.0528
Gamma	$\hat{\alpha}: 0.79911358$ [0.13837959]	$\hat{\beta}: 0.01749159$ [0.004132499]		240.1902
EE	$\hat{\alpha}: 0.77983525$ [0.13520767]	$\hat{\lambda}: 0.01870096$ [0.003647848]		239.9951
Erlang	$\hat{k}: 0.79907439$ [0.13837218]	$\hat{\lambda}: 0.01749007$ [0.00413218]		240.1902
Lindley	$\hat{\theta}: 0.04287723$ [0.004299543]			251.4303
LG	$\hat{p}: 2.049295\times10^{-6}$ [0.5213013]	$\hat{\theta}: 4.314914\times10^{-2}$ [0.008239275]		251.4323
TGIE	$\hat{\alpha}: 0.3763702$ [0.05068110]	$\hat{\theta}: 0.6032954$ [0.20582068]	$\hat{\lambda}: -0.8447299$ [0.12497786]	270.4390

Table 1 (Continued)

Distributions	Goodness-of-fit measures				
	AIC	BIC	AICc	K-S	K-S p-values
Weibull	486.0036	489.8277	486.2590	486.2590	0.048600
Rayleigh	530.1057	532.0177	530.1890	0.2621	0.002081
Gamma	484.3804	488.2045	484.6358	0.2022	0.033490
EE	483.9903	487.8143	484.2456	0.2042	0.030950
Erlang	484.3804	488.2045	484.6358	0.2022	0.033500
Lindley	504.8606	506.7726	504.9439	0.1990	0.038090
LG	506.8646	510.6886	507.1199	0.2021	0.033700
TGIE	546.8780	552.6140	547.3997	0.8668	$<2.2\times10^{-16}$

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