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Short Communication

A Note on the Transmuted Inverse Weibull Distribution

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Abstract

Khan, King and Hudson [ANZIAM Journal, 55, 2014, C197-C217] introduced a novel three-parameter distribution, studied its mathematical properties and illustrated a data application. We show that a one-parameter distribution can provide a better fit to the data.

Keywords: Maximum likelihood, probability plots, quantile plots.

1. Introduction

Khan et al. (2014) introduced a novel three-parameter distribution referred to as the transmuted inverse Weibull distribution. It has the probability density and cumulative distribution functions specified by

$$f(x) = \alpha \theta^\alpha x^{-\alpha-1} \left\{ 1 + \lambda - 2\lambda \exp \left[- \left(\frac{\theta}{x} \right)^\alpha \right] \right\} \exp \left[- \left(\frac{\theta}{x} \right)^\alpha \right] \quad (1)$$

and

$$F(x) = \left\{ 1 + \lambda - \lambda \exp \left[- \left(\frac{\theta}{x} \right)^\alpha \right] \right\} \exp \left[- \left(\frac{\theta}{x} \right)^\alpha \right], \quad (2)$$

respectively, for $x > 0$, $\alpha > 0$, $\theta > 0$ and $-1 \leq \lambda < +1$. Khan et al. (2014) derived various mathematical properties of the distribution and illustrated a data application. The derived properties included moment generating function, moments, quantiles, order statistic properties, entropies, mean deviation and maximum likelihood estimation.

The aim of this note is to show that the data illustration is inappropriate. The transmuted inverse Weibull distribution may provide adequate fits for many data sets. For the data set considered by Khan et al. (2014), however, a one-parameter distribution can provide a better fit. This is illustrated in Section 2.

2. Two Data Applications

In this section, we compare the fits the one-parameter exponential and three-parameter transmuted inverse Weibull distributions. We consider two data sets. First are the data considered in Section 7 of Khan et al. (2014). The data are survival times of 128 bladder cancer patients:

4.50, 32.15, 3.88, 13.80, 19.13, 4.87, 5.85, 14.24, 5.71, 7.09, 7.87, 7.59, 20.28, 5.32, 5.49, 3.02, 46.12, 2.02, 4.51, 5.17, 2.83, 9.22, 1.05, 0.20, 8.37, 3.82, 9.47, 36.66, 14.77, 26.31, 79.05, 10.06,

8.53, 2.02, 4.98, 11.98, 2.62, 4.26, 5.06, 1.76, 0.90, 11.25, 16.62, 4.40, 21.73, 10.34, 12.07, 34.26, 10.66, 6.97, 2.07, 0.51, 12.03, 0.08, 17.12, 3.36, 2.64, 1.40, 12.63, 43.01, 14.76, 2.75, 7.66, 0.81, 1.19, 7.32, 4.18, 3.36, 8.66, 1.26, 13.29, 1.46, 14.83, 6.76, 23.63, 5.62, 3.25, 18.10, 7.62, 7.63, 17.14, 25.74, 3.52, 2.87, 15.96, 17.36, 9.74, 3.31, 7.28, 1.35, 0.40, 2.26, 4.33, 9.02, 5.41, 2.69, 22.69, 6.94, 2.54, 11.79, 2.46, 7.26, 2.69, 5.34, 3.48, 8.26, 6.93, 4.23, 3.70, 0.50, 10.75, 6.54, 3.64, 5.32, 13.11, 8.65, 3.57, 5.09, 7.39, 5.41, 11.64, 2.09, 2.23, 6.25, 7.93, 4.34, 25.82, 12.02.

The second data set contains the number of operating hours between successive failure times of the air conditioning systems in Boeing airplanes (Proschan, 1963):

1, 1, 2, 3, 3, 3, 3, 4, 5, 5, 5, 5, 5, 7, 7, 7, 9, 9, 10, 11, 11, 11, 11, 12, 12, 12, 12, 13, 14, 14, 14, 14, 14, 14, 15, 15, 15, 16, 16, 16, 18, 18, 18, 18, 18, 18, 20, 20, 21, 21, 22, 22, 22, 23, 23, 23, 24, 24, 25, 26, 26, 27, 27, 29, 29, 29, 29, 30, 31, 31, 32, 33, 33, 34, 34, 34, 35, 35, 36, 36, 37, 39, 39, 41, 42, 43, 44, 44, 44, 46, 46, 47, 47, 48, 49, 50, 50, 51, 52, 54, 54, 55, 56, 56, 57, 57, 57, 58, 59, 59, 59, 60, 61, 61, 62, 62, 62, 63, 65, 66, 67, 67, 68, 70, 70, 71, 71, 72, 74, 76, 77, 79, 79, 80, 82, 84, 85, 87, 88, 90, 90, 91, 95, 97, 97, 98, 100, 100, 101, 102, 102, 104, 104, 104, 106, 111, 118, 118, 120, 120, 130, 130, 130, 134, 139, 141, 142, 152, 153, 156, 163, 169, 176, 181, 182, 184, 186, 188, 191, 194, 197, 201, 206, 208, 208, 209, 210, 216, 220, 225, 230, 230, 239, 246, 246, 254, 261, 270, 283, 310, 320, 326, 359, 386, 413, 438, 447, 487, 493, 502, 603.

The values of Akaike information criterion (Akaike, 1974), Bayesian information criterion (Schwarz, 1978) and p -value of the Kolmogorov-Smirnov test are given in Table 1 for both data sets. Also given in Table 1 are values of AIC and BIC for the transmuted inverse Weibull distribution. The p -value reported in Khan et al. (2014) for the transmuted inverse Weibull distribution does not appear correct. The recomputed p -value is given in Table 1.

Table 1 Comparison of the values of AIC, BIC and p -value of the Kolmogorov-Smirnov test.

	Cancer data		Airconditioning data	
	TIW	Exponential	TIW	Exponential
AIC	879.4	830.7	-160.7	-365.6
BIC	879.7	833.5	-150.7	-362.2
KS p -value	0.049	0.318	0.107	0.211

Table 1 shows that the one-parameter exponential distribution provides the better fit with respect to AIC, BIC and the p -value for both data sets. The p -value of the transmuted inverse Weibull distribution for the first data set suggests that its fit is not adequate at the five percent level of significance. The better fits of the one-parameter exponential distribution for both data sets is confirmed by the probability and quantile plots shown in Figures 1 to 4.

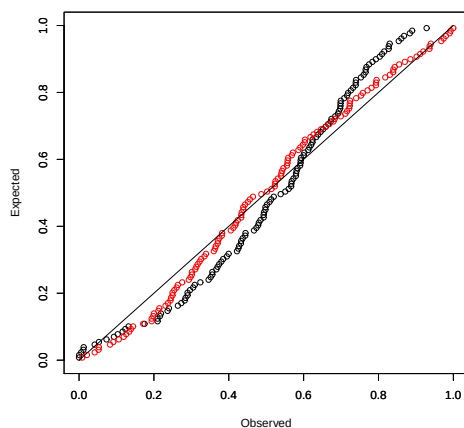


Figure 1 Probability plots of the fits of transmuted inverse Weibull (black) and exponential (red) distributions for the cancer data.

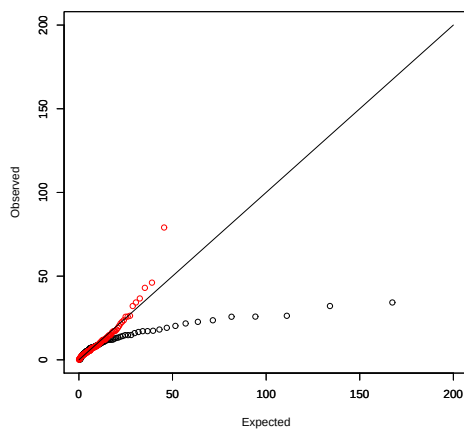


Figure 2 Quantile plots of the fits of transmuted inverse Weibull (black) and exponential (red) distributions for the cancer data.

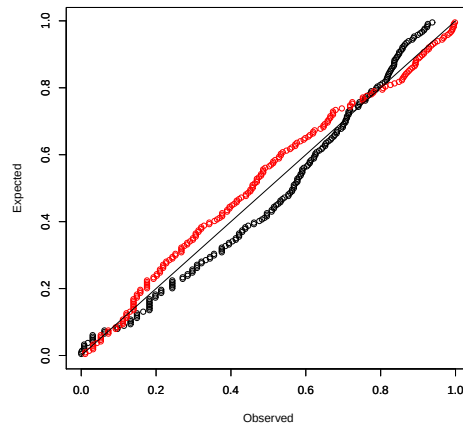


Figure 3 Probability plots of the fits of transmuted inverse Weibull (black) and exponential (red) distributions for the airconditioning data.

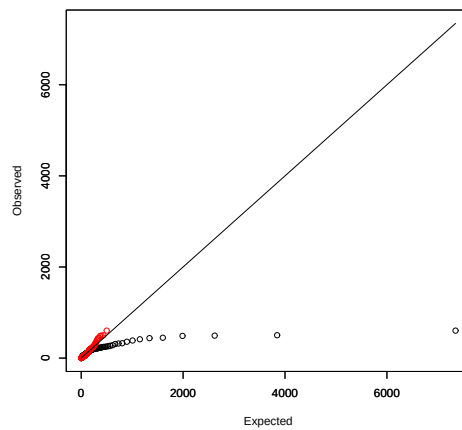


Figure 4 Quantile plots of the fits of transmuted inverse Weibull (black) and exponential (red) distributions for the airconditioning data.

This is not a criticism of Khan et al.'s novel contribution. The transmuted inverse Weibull distribution of Khan et al. (2014) may give adequate fits to many data sets. But in general it is a good idea to try out simpler distributions than more complicated ones.

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