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A Periodic Review Inventory Model for Weibull Deteriorating Items under Trade Credit Offer Using Discounted Cash-Flow Approach

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Abstract

The paper considers a periodic review inventory model for items having deterioration time distributed as two-parameter Weibull. The inventory manager is offered progressive trade credit whereby if the manager pays his dues within the first credit period, he does not have to pay any interest. However, he is charged an interest on the unpaid balance if he pays in the second credit period and the interest is increased if he pays beyond that period. The model is analyzed using the discounted-cash-flow approach, and the optimal stock height and cycle length are determined so as to minimize the present value of all future cash out-flows. Numerical examples are cited, and a sensitivity analysis is carried out to study how robust the model is to change in the parameter values.

Keywords: Periodic review model, Weibull deterioration, progressive payment scheme, time value of money.

1. Introduction

Trade credit offer is an effective tool to encourage the inventory manager to buy more. It also enables the manager to sell his goods and invest his revenue to earn interest before he proceeds to settle his account with the supplier. Goyal (1985) first studied an economic order quantity (EOQ) model where the supplier allows the retailer a fixed permissible delay in paying his dues. Shah (1993) extended his study to the case where the items in inventory are subject to a constant rate of deterioration. Liao et al. (2000) formulated an inventory model with stock dependent demand under permissible delay in payments. Pal and Ghosh (2007) discussed an inventory model with stock dependent demand and general rate of deterioration when the inventory manager is offered trade credit. Teng (2002) indicated that the inventory manager should buy in small quantities and take advantage of the trade credit repeatedly. Huang (2003) investigated an inventory model in which a supplier offers the inventory manager a permissible delay period M and the manager also provides a trade credit period $N(N \leq M)$ to his customers. Some models have also been developed where the credit period is dependent on the order quantity, see, for example Shinn and Hwang (2003), Pal and Ghosh (2006, 2007).

In classical inventory models, it is tacitly assumed that all costs associated with the inventory system remain constant over time. Much research has been carried out under this assumption. However, from financial point of view, inventory can be considered to be a capital investment, and should, therefore, compete with other assets for an organization's limited capital fund. It is, thus, important to investigate how time-value of money influences an inventory policy. The discounted-cash-flow (DCF) approach, which is widely used in business decisions, takes into account the time value of money. Hadley (1964) compared the optimal policy under the classical EOQ approach and that considering the net present value approach, and showed that for some extreme cases the two policies are widely different. Kim et al. (1986) used the DCF approach to analyse various inventory systems. The DCF approach was also employed by Rachamadugu (1989) to study the EOQ model with trade credit period. Chao (1992) investigated the EOQ model using net present value approach for both deterministic and stochastic demands. Sun and Queyranne (2002) discussed production and inventory model through net present value.

Progressive credit period is a generalization of the one period credit that has been studied by most authors. The notion of progressive credit period is as follows: If the inventory manager settles his outstanding due by time M, the supplier does not charge him any interest. If, however, he pays after M but before the time point $N(M < N)$ specified by the supplier, the supplier charges him any interest on the unpaid balance. If the manager settles his account after N, he has to pay a higher interest on the unpaid balance. Soni et al. (2006) studied an ordering policy for stock-dependent demand under progressive payment scheme. Teng et al. (2011) extended the work to allow for non-zero ending-inventory and limited inventory space. Jaggi et al. (2012) considered credit policy in EOQ model with two-levels of trade credit policy when demand is influenced by credit period. Glocket al. (2015) modified some assumptions made by Soni et al. (2006) to increase the model's applicability. Ries et al. (2016) generalized trade credit inventory models with progressive interest schemes by considering the impact of varying financial environments and time-value of money. Widyadana et al. (2017) developed single vendor-single buyer non-cooperative models with permissible delay in payments using Stackelberg equilibrium) and Nash equilibrium. Of the above studies in which deteriorating items have been considered, a constant rate of deterioration has been assumed. However, in real life, the deterioration rate tends to increase with time.

In this paper we discuss a periodic review model for items having Weibull failure rate under progressive payment scheme. The Weibull distribution is a very popular failure time distribution, and is found to fit many failure time data. The discounted cash flow approach is used to analyse the model. The paper is organized as follows. Section 2 defines the notations used and the assumptions made in the model. Section 3 analyses the model. In Section 4, the model is illustrated through examples, and a study of the sensitivity of the model to change in the model parameters is carried out. Finally, in Section 5, some concluding remarks have been made.

2. Notations and Assumptions

The paper considers a periodic review inventory model. Orders are placed at the beginning of each reorder cycle, and the inventory manager is given a credit period such that if he pays all his dues within that period, he is not charged any interest. However, if he fails to do so, the supplier allows him a second credit period, but charges an interest on the unpaid balance. If the manager has unpaid balance even after the second credit period, the supplier charges him a higher interest than that in the second credit period. During any credit period, the manager can earn interest on his revenue, which, if necessary, goes towards paying his debt.

The following notations have been used in the paper:

A	:	Ordering cost/order
C	:	Unit purchase cost/unit
R	:	Demand rate per year
h	:	Inventory holding cost per unit per year
r	:	Discount rate per year
M	:	Permissible credit period without any interest charged
N	:	Second permissible credit period in settling the account with an interest charged $(N > M)$
I_{C_1}	:	Interest charged per unit currency in stock per year by the supplier when the retailer pays after M but before N
I_{C_2}	:	Interest charged per unit currency in stock per year by the supplier when the retailer pays after $N (I_{C_2} > I_{C_1})$
p	:	Selling price per unit
I_e	:	Interest earned per year
T	:	Length of a replenishment cycle
Q	:	Quantity procured at each reorder point
$I(t)$	:	Inventory level at time t, $(0 \le t \le T)$.

The assumptions made in the model are as follows:

1. The inventory system deals with a single item only.
2. The planning horizon is of infinite length.
3. The demand rate R remains constant over the planning horizon.
4. Orders are placed at the beginning of each replenishment cycle.
5. Replenishment is instantaneous on ordering.
6. Shortages and excess stock are not allowed during a replenishment cycle.
7. The units in inventory deteriorate following a Weibull (α, β) distribution, so that the deterioration rate is given by $r(t) = \alpha\beta t^{\beta-1}$, $t \ge 0$.
8. There is no repair or replacement of deteriorated items.

3. Analysis of the Model

Noting that depletion from stock occurs due to demand and deterioration, the inventory level $I(t)$ at t satisfies the differential equation

$$\frac{d}{dt} I(t) + r(t) I(t) = -R, \qquad 0 \le t \le T,$$

i.e., $$\frac{d}{dt}I(t)+\alpha\beta t^{\beta-1}I(t)=-R, \qquad 0\le t\le T.$$

The boundary conditions are $I(0)=Q$ and $I(T)=0.$

We, therefore get $I(t)=R\int_t^T e^{\alpha(t'^{\beta}-t^{\beta})}dt',\ 0\le t\le T,$ and

$$Q=R\int_0^T e^{\alpha t^{\beta}}\,dt. \tag{1}$$

The inventory policy is determined by (Q,T). Since these are related by (1), we take T as our independent decision variable. We find T so as to minimize the present value of all future cash out-flows.

The components of total inventory cost per replenishment cycle are as follows:

1. Ordering cost: $OC=A.$
2. Procurement cost: $PC=C\times Q=CR\int_0^T e^{\alpha t^{\beta}}\,dt.$
3. Holding cost: $IHC=h\int_0^T I(t)e^{-rt}dt=hR\int_0^T\left(\int_t^T e^{\alpha(t'^{\beta}-t^{\beta})}dt'\right)e^{-rt}dt.$
4. Interest earned and interest charged are given by the following.

Case 1: $T\le M$

In this case, the inventory manager sells the whole stock before the end of the first credit period and hence repays his dues within that period. He, therefore is not charged any interest, but earns interest on the revenue collected from selling his stock. Thus,

Interest charged: $IC_1 = 0.$

Interest earned: $IE_1=pI_eR\left[\frac{1}{r^2}\{1-e^{-rT}(rT+1)\}+T(M-T)e^{-r(M-T)}\right].$

The present value of the cash out-flow in a reorder cycle is, therefore, given by

$$PV_1(T)=\left[OC+PC+IHC+IC_1-IE\right]_1.$$

Hence, the present value of all future cash out-flows is given by

$$PV_{1\infty}(T)=\sum_{n=0}^{\infty}PV_1(T)e^{-nrT}=\frac{PV_1(T)}{1-e^{-rT}}=\left(\frac{1}{rT}+\frac{1}{2}+\frac{rT}{4}\right)PV_1(T).$$

Case 2: $M<T\le N$

The interest earned by the inventory manager on the items sold during $(0,M)$ is given by

$$IE_2=pI_e\int_0^T Rte^{-rt}dt=\frac{pI_eR}{r^2}\left[1-(1+rM)e^{-rM}\right].$$

The interest paid, however, depends on whether the total amount of money through selling of items and interest earned on revenue available to the inventory manager at M is sufficient to meet the procurement cost or not.

Case 2.1: $pRMe^{-rM} + IE_2 \geq CQ$

Since the revenue from selling stock during $(0, M)$ and the interest earned $(= pRMe^{-rM} + IE_2)$ is at least as much as the cost CQ of units procured from the supplier, the inventory manager is able to repay his dues by the end of the first credit period. Hence, the interest paid is $IC_{2.1} = 0.$

Then, the present value of the cash out-flow in a reorder cycle

$$PV_{2.1}(T) = [OC + PC + IHC + IC_{2.1} - IE_2].$$

The present value of all future cash out-flows is therefore given by

$$PV_{2.1,\infty}(T) = \sum_{n=0}^{\infty} PV_{2.1}(T)e^{-nrT} = \frac{PV_{2.1}(T)}{1-e^{-rT}} = \left(\frac{1}{rT} + \frac{1}{2} + \frac{rT}{4}\right) PV_{2.1}(T).$$

Case 2.2: $pRMe^{-rM} + IE_2 < CQ$

Here the revenue from selling stock during $(0, M)$ and the interest earned is less than the total procurement cost, and the unpaid balance at M is given by

$$U_1 = CQ - \left[pRMe^{-rM} + IE_2\right].$$

Hence, the supplier will charge the inventory manager an interest on the unpaid balance U_1 at the rate I_{C_1} after M. Noting that the time by which the unpaid balance is paid back is $\frac{U_1}{pR}$, and the interest charged on the unpaid balance per year is $\int_M^{\infty} U_1 I_{C_1} I(t) e^{-rt} dt,$ the total interest payable is

$$IC_{2.2} = \frac{U_1^2}{pR} I_{C_1} \int_M^T I(t) e^{-rt} dt.$$

On the other hand, the interest earned is

$$IE_{2.2} = pI_e \int_0^T Rte^{-rt} dt = pI_e R\left[\frac{1}{r^2}\{1 - e^{-rT}(rT+1)\}\right].$$

The cash outflow in a replenishment cycle is, therefore,

$$PV_{2.2}(T) = \left[OC + PC + IHC + IC_{2.2} - IE_{2.2}\right].$$

The present value of all future cash out- flows then comes out to be

$$PV_{2.2,\infty}(T) = \sum_{n=0}^{\infty} PV_{2.2}(T)e^{-nrT} = \frac{PV_{2.2}(T)}{1-e^{-rT}} = \left(\frac{1}{rT} + \frac{1}{2} + \frac{rT}{4}\right) PV_{2.2}(T).$$

Case 3: $T > N$

As noted earlier, the total amount available to the inventory manager at M is $pRMe^{-rM} + IE_2$, where

$$IE_2 = pI_e \int_0^T Rte^{-rt} dt = \frac{pI_e R}{r^2}\left[1 - (1 + rM)e^{-rM}\right].$$

For $T > N,$ we can have three possibilities as given below:

Case 3.1: $pRMe^{-rM} + IE_2 \geq CQ$

Here the present value of all future cash-out flows is

$$PV_{3.1,\infty}(T) = (1-e^{-rT})^{-1} PV_{2.1}(T).$$

Case 3.2: $pRMe^{-rM} + IE_2 < CQ$ but $pR(N-M)e^{-r(N-M)} + pI_e \int_M^N Rte^{-rt}dt \geq U_1$

In this case, the total amount available to the inventory manager at M is less than the total procurement cost, but the amount accumulated through sale and interest earned in the period $(M, N]$, namely $pR(N-M)e^{-r(N-M)} + pI_e \int_M^N Rte^{-rt}dt,$ is sufficient to meet the unpaid balance $U_1.$ Hence, the present value of all future cash out-flows is

$$PV_{3.2,\infty}(T) = (1-e^{-rT})^{-1} PV_{2.2}(T).$$

Case 3.3: $pRMe^{-rM} + IE_2 < CQ$ and $pR(N-M)e^{-r(N-M)} + pI_e \int_M^N Rte^{-rt}dt < U_1$

In this case the inventory manager is not in a position to pay off the total purchase cost at $N.$ He pays $pRMe^{-rM} + IE_2$ at M and $pR(N-M)e^{-r(N-M)} + pI_e \int_M^N Rte^{-rt}dt$ at $N,$ and the remaining after $N.$

Hence, the inventory manager is charged an interest at the rate I_{C_1} during $(M, N]$ on the unpaid balance

$$U_1 = CQ - [pRMe^{-rM} + IE_2],$$

and at the rate I_{C_2} during $(N, T]$ on the unpaid balance

$$U_2 = U_1 - \left[pR(N-M)e^{-r(N-M)} + pI_e \int_M^N Rte^{-rt}dt \right].$$

The total interest charged is, therefore,

$$IC_{3.3} = U_1 I_{C_1}(N-M) + \frac{U_2^2}{pR} I_{C_2} \int_N^T I(t)e^{-rt}dt,$$

and the total interest earned is

$$IE_{3.3} = pI_e \int_0^T Rte^{-rt}dt = pI_e R\left[\frac{1}{r^2}\{1-e^{-rT}(rT+1)\} \right].$$

Hence, the cash out-flow in a replenishment cycle is

$$PV_{3.3}(T) = [OC + PC + IHC + IC_{3.3} - IE_{3.3}],$$

and the present value of all future cash out-flows is given by

$$PV_{3.3,\infty}(T) = \sum_{n=0}^{\infty} PV_{3.3}(T)e^{-nrT} = \frac{PV_{3.3}(T)}{1-e^{-rT}} = \left(\frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) PV_{3.3}(T).$$

In the i^{th} case, the optimal value of T that minimizes $PV_{i\infty}$ is obtained by solving $\frac{d}{dT}PV_{i\infty} = 0, i = 1,\ 2.1,\ 2.2,\ 3.1,\ 3.2,\ 3.3$. Because of the complexity in the expression of the present value of future cash out-flows in each case, MATLAB software (R2010) has been used to find optimal T.

To find the optimum value of T that minimizes the present value of all future cash out-flows, we compare the minimum costs obtained in the different situations. The optimal value of T in the situation which gives the minimum cost among all situations is the required optimal T. Optimal value of Q is then obtained from (1) by replacing T by its optimal value.

4. Numerical Illustrations and Sensitivity Analysis

1. Consider the parameters of the model to be as follows in some suitable units: $A = 400; C = 20; h = 5; p = 30; I_e = 0.05; I_{C_1} = 0.07; I_{C_2} = 0.08; R = 4000; M = 30/365$ year; $N = 60/365$ year; $\alpha = 1; \beta = 3; r = 0.1$.

The optimal policy and the minimum PV_∞ (in '000) corresponding to the different cases discussed in Section 3 are given in Table 1.

Table 1 The optimal policies and the minimum present value of future cash out-flows under different cases

Case	T (in Year)	Q	PV_∞
1	29/365	318	859.242
2.1	43/365	471	848.976
2.2	51/365	559	845.341
3.2	61/365	669	850.023
3.3	90/365	990	849.253

Note: Case 3.1 does not occur for the given set of parameter values.

The optimal policy, therefore, corresponds to Case 2.2, which gives the minimum present value of future cash out-flows. Hence, the optimal values of T and Q are $T^* = 51$ days and $Q^* = 559$ units, respectively and the corresponding present value of all future cash-out-flows is 845.341.

2. Consider the parameter set $A = 400; C = 150; h = 2; p = 400; I_e = 0.05; I_{C_1} = 0.07;$ $I_{C_2} = 0.08;\ R = 2000; M = 30/365$ year; $N = 60/365$ year; $\alpha = 0.1; \beta = 3; r = 0.01$.

Table 2 gives the optimal policy and the minimum PV_∞ (in '000) corresponding to the different cases discussed in Section 3.

Table 2 The optimal policies and the minimum present value of future cash out-flows under different cases

Case	T (in Year)	Q	PV_∞
1	29/365	318	855.851
2.1	55/365	603	845.462
3.1	61/365	669	845.736
3.2	75/365	824	836.022
3.3	148/365	1,649	856.187

Note: Case 2.2 does not occur for the given set of parameter values.

Here the optimal policy corresponds to Case 3.2. The optimal values of T and Q are $T^* = 75$ days and $Q^* = 824$ units, respectively and the corresponding present value of all future cash-out-flows is 836.022.

Sensitivity Analysis

Here we examine how sensitive the model is to change in the model parameters. The change is studied with respect to the set of parameter values given in Example 1 above. Tables 3 and 4 show the change in the optimal policy with change in a model parameter when the other parameters remain the same (PV_∞ is in '000).

The following observations are made from Tables 3 and 4:

1. Q and T are highly sensitive to change in the in the demand rate $R,$ the procurement cost C and the discount rate r, while these are fairly robust to changes in the other parameters.

2. The future cash out-flows change rapidly with change in R, C and $r,$ while the change is sufficiently small for change in other parameters.

5. Conclusions

In this paper, we take into account the present values of all future cash-out-flows. We develop a model for deteriorating items where the deterioration rate is time dependent, and the supplier offers the inventory manager two progressive credit periods to settle his account for the procured items. The present value of all future costs is observed to decrease with increase in the first allowable credit period while it increases with increase in extended credit period. Further, the present value of all future costs decreases with increase in inflation rate.

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Table 3 Change in optimal policy and future cash out-flows with change in model parameters

R	T	Q	PV_∞
10,000	45/365	1,233	2,067,623
8,000	45/365	987	1,660,626
6,000	48/365	789	1,253,341
4,000	51/365	559	845,345
2,000	90/365	495	432,832
A	T	Q	PV_∞
1,000	90/365	990	873,884
800	90/365	990	865,672
600	90/365	990	857,461
400	51/365	559	845,342
200	46/365	504	830,303
C	T	Q	PV_∞
28	64/365	702	1,170,020
25	45/365	493	1,048,985
20	51/365	559	845,342
15	62/365	680	641,665
10	90/365	990	440,415
h	T	Q	PV_∞
2	90/365	990	834,327
5	51/365	559	845,342
7	50/365	548	850,892
10	47/365	515	858,876
15	45/365	493	871,357
r	T	Q	PV_∞
0.2	44/365	482	173,502
0.1	51/365	559	845,342
0.08	52/365	570	1,055,194
0.05	52/365	570	1,684,687
0.02	52/365	570	4,202,674
I_e	T	Q	PV_∞
0.03	51/365	559	847,021
0.05	51/365	559	845,342
0.07	52/365	570	843,651
α	T	Q	PV_∞
0.1	52/365	570	844,821
0.2	52/365	570	844,881
0.5	51/365	559	845,058
1.0	51/365	559	845,342
β	T	Q	PV_∞
3.0	51/365	559	845,342
2.5	50/365	549	846,509
2.0	49/365	540	849,997
1.0	30/365	343	892,093

Table 4 Change in optimal policy with change in M and N

$N \downarrow$	$M \rightarrow$	10	20	30
40	PV_∞	847,094	845,165	844,000
	T	61/365	61/365	61/365
	Q	669	669	669
50	PV_∞	849,182	847,012	845,342
	T	75/365	75/365	51/365
	Q	824	824	559
60	PV_∞	853,137	850,538	845,342
	T	89/365	41/365	51/365
	Q	979	449	559

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