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On Characterizations of the Continuous Distributions via Identically Distributed Functions of Upper Record Values

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Abstract

In this paper, we obtain two characterizations of a family of continuous probability distribution via identically distributed functions of upper record values. Examples of special cases of general classes as Pareto, power, beta, Lomax, Singh-Maddala, Gompertz, Rayleigh, Weibull, Kumaraswamy, rectangular, extream value, Burr type and Lindley distributions are discussed.

Keywords: Identical distribution, hazard rate, functional equation.

1. Introduction

Record values are found in many situations of daily life as well as in many statistical applications. Example of application areas include industrial stress testing, meteorological analysis, sporting and athletic events and oil and mining surveys. Many researchers have studied the characterizations of probability distributions via identically distributed functions of upper and lower record values. The starting point for many characterizations of probability distributions via identically distributed functions of record values is the well-known result of Ahsanullah (1978) when he characterized the exponential distribution. Several characterization results involving identical distributed can be found in the literature. For example Ahsanullah (1979) studied the characterizations based on identical distributed. Also, Azedine (2013) presented characteristic properties of lower records associated with this distribution. For further various characterizations of probability distributions based on record values and order statistics the interested readers are referred to Arslan et al. (2005) Arslan (2014), Nadarajah et al. (2014), Lee (2015) and Skřivánková and Juhás (2011) among others.

Now some notations and definitions. Suppose that $\{X_n, n \geq 1\}$ is a sequence of independent and identically distributed(i.i.d.) random variables with cumulative distribution function (cdf) $F(x)$ and probability density function (pdf) $f(x)$. Let $Y_n = max(min)\{X_1, X_2, \cdots, X_n\}$ for $n \geq 1$. We say X_j is an upper(lower) record value of $\{X_n, n \geq 1\}$ if $Y_j > (<) Y_{j-1}$ for $j > 1$. By definition, X_1 is an upper as well as a lower record value. One can transform the upper records to lower records by replacing the original sequence of $\{X_j\}$ by $\{-X_j, j \geq 1\}$ or (if $P(X_j > 0) = 1$ for all j) by $\{1/X_j, j \geq 1\}$. We defined the upper record times $\{U(n), n \geq 2\}$ where $U(1) = 1$, and $U(n) = \min\{j \mid j > U(n-1), X_j > X_{U(n-1)}\}$. Similarly, the lower record times $\{L(n), n \geq 2\}$ where $L(1) = 1$, and $L(n) = \min\{j \mid j > L(n-1), X_j < X_{L(n-1)}\}$.

If $F(x)$ is the distribution function of a nonnegative random variable, we say that $F(x)$ is "new better than used" (NBU), if $\bar{F}(xy) \leq \bar{F}(x)\bar{F}(y)$, for $x, y > 1$ and $F(x)$ is "new worse than used"

(NWU), if $\bar{F}(xy) \geq \bar{F}(x)\bar{F}(y)$, for $x > 1$ and $y > 1$. We say the $F(x)$ belongs to the class C_1 if $F(x)$ is either NBU or NWU(see Ahsanullah (2004)).

If $F(x)$ has density $f(x)$ and $R(x) = -ln\bar{F}(x)$, $r(x) = \frac{f(x)}{\bar{F}(x)}$, $\bar{F}(x) = 1 - F(x)$, $0 < \bar{F}(x) < 1$. We say the random value X_n belongs to the class C^* if $\frac{d}{dx}R(h(xy)) \geq \frac{d}{dx}R(h(x))$ or $\frac{d}{dx}R(h(xy)) \leq \frac{d}{dx}R(h(x))$ where $h(x)$ is an increasing and differentiable function, for all $x > 1$ and $y > 1$.

The continuous distributions have been studied by many researchers. Juhás and Skřivánková (2014) showed characterization of general classes of distributions with the independent property that the random variables $g(L_n)$ and $g(L_{n+1}) - g(L_n)$ are independent where $g(\cdot)$ is an increasing and differentiable function, $n \geq 1$ if and only if X has general classes of distributions. Recently, Khan et al. (2016) characterized a general class of distributions $\bar{F}(x) = [ah(x) + b]^c$ by the conditional expectation of $X_{U(s)}$ and $X_{U(r)}$ conditioned on $X_{U(m)}$ where $h(x)$ is a continuous, differentiable, and non-decreasing function of x and for all $s > r \geq m$.

The current investigation was induced by characterizations based on identical distribution(see Ahsanullah et al. (2012) and Lee (2015)). Namely, we write $V_{n+1,n} = \frac{g(X_{U(n+1)})}{g(X_{U(n)})}$, one can ask whether the identical distribution of $V_{n+1,n}$ and $g(X_k)$ or $V_{n+1,n}$ and $V_{n,n-1}$ characterizes general classes of distributions.

In this paper we investigate new characterizations of various continuous distributions via identical distribution of quotient function forms of upper record values.

2. Main Results

Theorem 1 *Let $\{X_n, n \geq 1\}$ be a sequence of independent and identically distributed random variables with absolutely continuous cdf $F(x)$ and the pdf $f(x)$ on the support (α, β), where α and β may be finite or infinite. Assume that F belongs to the class C_1. Let $g(x)$ be an increasing and differentiable function with $g(x) \to 1$ as $x \to \alpha^+$ and $g(x) \to \infty$ as $x \to \beta^-$ for all $x \in (\alpha, \beta)$. Then $F(x) = 1 - (g(x))^{-\lambda}$, for $\lambda > 0$, if and only if $\frac{g(X_{U(n+1)})}{g(X_{U(n)})}$ has an identical distribution with $g(X_k)$, for some k between 1 and n.*

Proof: For necessary part, let the joint pdf of $X = g(X_{U(n)})$ and $Y = g(X_{U(n+1)})$ be $f_{X,Y}(x,y)$. Then

$$f_{X,Y}(x,y) = \frac{1}{\Gamma(n)}(R(x))^{n-1}r(x)f(y) \tag{1}$$

for $1 \leq x < y < \infty$.

Consider the transformations $U = g(X_{U(n)})$ and $V = \frac{g(X_{U(n+1)})}{g(X_{U(n)})}$. Then the Jacobian of the transformation is $J = \frac{\partial}{\partial u}(g^{-1}(u)) \times \frac{\partial}{\partial v}(g^{-1}(uv))$. Since $g(x)$ is an increasing and differentiable function, both $\frac{\partial}{\partial u}(g^{-1}(u))$ and $\frac{\partial}{\partial v}(g^{-1}(uv))$ are positive. Thus we can write the joint pdf $f_{U,V}(u,v)$ of U and V as

$$f_{U,V}(u,v) = \frac{1}{\Gamma(n)}(R(g^{-1}(u)))^{n-1}r(g^{-1}(u))f(g^{-1}(uv))\frac{\partial}{\partial u}(g^{-1}(u))\frac{\partial}{\partial v}(g^{-1}(uv))$$

for $u > 1$ and $v > 1$.

The marginal pdf of $f_V(v)$ of V is given by

$$f_V(v) = \frac{1}{\Gamma(n)}\int_1^{\infty}(R(g^{-1}(u)))^{n-1}r(g^{-1}(u))f(g^{-1}(uv))\frac{\partial}{\partial u}(g^{-1}(u))\frac{\partial}{\partial v}(g^{-1}(uv))du$$

for $u > 1$ and $v > 1$.

Substituting

$$\Gamma(n)=\int_1^{\infty}(R(g^{-1}(u)))^{n-1}f(g^{-1}(u))\frac{\partial}{\partial u}(g^{-1}(u))du,\ u>1,$$

we have

$$f_V(v)=\frac{\int_1^{\infty}(R(g^{-1}(u)))^{n-1}r(g^{-1}(u))f(g^{-1}(uv))\frac{\partial}{\partial u}(g^{-1}(u))\frac{\partial}{\partial v}(g^{-1}(uv))du}{\int_1^{\infty}(R(g^{-1}(u)))^{n-1}f(g^{-1}(u))\frac{\partial}{\partial u}(g^{-1}(u))du} \tag{2}$$

for $u>1$ and $v>1$.

If $F(x)=1-(g(x))^{-\lambda}$, for all $g(x)>1$ and $\lambda>0$, then we obtain

$$f_V(v)=\frac{\int_1^{\infty}\lambda^{n+1}\big(ln(u)\big)^{n-1}(uv)^{-\lambda-1}du}{\int_1^{\infty}\lambda^{n}\big(ln(u)\big)^{n-1}u^{-\lambda-1}du} \tag{3}$$

for $u>1$ and $v>1$.

From (3), we have on simplification

$$f_V(v)=\lambda v^{-\lambda-1},\ 1\le v<\infty.$$

Hence $\frac{g(X_{U(n+1)})}{g(X_{U(n)})}$ and $g(X_k)$ identically distributed.

Now we prove the sufficient part of the Theorem 1. Let us use the transformation $U=g(X_{U(n)})$ and $V=\frac{g(X_{U(n+1)})}{g(X_{U(n)})}$. The Jacobian of the transformation is $J=\frac{\partial}{\partial u}(g^{-1}(u))\times\frac{\partial}{\partial v}(g^{-1}(uv))$. Since $g(x)$ is an increasing and differentiable function, both $\frac{\partial}{\partial u}(g^{-1}(u))$ and $\frac{\partial}{\partial v}(g^{-1}(uv))$ are positive. Thus we can write the joint pdf $f_{U,V}(u,v)$ of U and V as

$$f_{U,V}(u,v)=\frac{1}{\Gamma(n)}(R(g^{-1}(u)))^{n-1}r(g^{-1}(u))f(g^{-1}(uv))\frac{\partial}{\partial u}(g^{-1}(u))\frac{\partial}{\partial v}(g^{-1}(uv)) \tag{4}$$

for $u>1$ and $v>1$.

The marginal pdf of $f_V(v)$ of V is given by

$$\frac{1}{\Gamma(n)}\int_1^{\infty}(R(g^{-1}(u)))^{n-1}r(g^{-1}(u))f(g^{-1}(uv))\frac{\partial}{\partial u}(g^{-1}(u))\frac{\partial}{\partial v}(g^{-1}(uv))du$$

for $u>1$ and $v>1$.

By the assumption of the identical distribution of V and $g(X_k)$, we have

$$\frac{1}{\Gamma(n)}\int_1^{\infty}(R(g^{-1}(u)))^{n-1}r(g^{-1}(u))f(g^{-1}(uv))\frac{\partial}{\partial u}(g^{-1}(u))\frac{\partial}{\partial v}(g^{-1}(uv))du$$
$$=f(g^{-1}(v))\frac{\partial}{\partial v}(g^{-1}(v)),\ u>1,\ v>1.$$

Substituting

$$\Gamma(n)=\int_1^{\infty}(R(g^{-1}(u)))^{n-1}f(g^{-1}(u))\frac{\partial}{\partial u}(g^{-1}(u))du,\ u>1,$$

we obtain

$$\begin{aligned}&\int_1^{\infty}(R(g^{-1}(u)))^{n-1}r(g^{-1}(u))f(g^{-1}(uv))\frac{\partial}{\partial u}(g^{-1}(u))\frac{\partial}{\partial v}(g^{-1}(uv))du\\&=\int_1^{\infty}(R(g^{-1}(u)))^{n-1}f(g^{-1}(u))\frac{\partial}{\partial u}(g^{-1}(u))f(g^{-1}(v))\frac{\partial}{\partial v}(g^{-1}(v))du\end{aligned} \tag{5}$$

for $u > 1$ and $v > 1$.

Now equating (5), we have on simplification

$$\int_1^{\infty} (R(g^{-1}(u)))^{n-1} \frac{\partial}{\partial u}(g^{-1}(u)) f(g^{-1}(u)) \times \left[f(g^{-1}(v)) \frac{\partial}{\partial v}(g^{-1}(v)) - \frac{f(g^{-1}(uv))}{\bar{F}(g^{-1}(u))} \frac{\partial}{\partial v}(g^{-1}(uv)) \right] du = 0 \tag{6}$$

for $u > 1$ and $v > 1$.

Integration (6) with respect to v from 1 to v_1, we obtain

$$\int_1^{\infty} (R(g^{-1}(u)))^{n-1} \frac{\partial}{\partial u}(g^{-1}(u)) f(g^{-1}(u)) \left[\frac{\bar{F}(g^{-1}(uv_1))}{\bar{F}(g^{-1}(u))} - \bar{F}(g^{-1}(v_1)) \right] du = 0$$

for $u > 1$ and $v_1 > 1$.

Since $F(x)$ belongs to the class C_1, we must have

$$\bar{F}(g^{-1}(v_1)) \bar{F}(g^{-1}(u)) = \bar{F}(g^{-1}(uv_1)) \tag{7}$$

for almost all u, $u > 1$ and any fixed v_1, $v_1 > 1$

By the theory of functional equation (see Aczel 1966) the only continuous solution of (7) with boundary conditions $F(\alpha) = 0$ and $F(\beta) = 1$ is

$$\bar{F}(x) = (g(x))^{-\lambda}$$

for all $g(x) > 1$ and $\lambda > 0$. This completes the proof.

Example 1 If we set $g(x) = x$, $x > 1$, then identical distribution of $\frac{X_{U(n+1)}}{X_{U(n)}}$ and X_k characterizes pareto distribution in Lee (2015).

Remark 1 Let $\{X_n, n \geq 1\}$ be a sequence of independent and identically distributed random variables with absolutely continuous cdf $F(x)$ and the pdf $f(x)$ on the support (α, β), where α and β may be finite or infinite. Assume that F belongs to the class C_1. Let $g(x)$ be an increasing and differentiable function with $g(x) \to 0$ as $x \to \alpha^+$ and $g(x) \to \infty$ as $x \to \beta^-$ for all $x \in (\alpha, \beta)$. Then $F(x) = 1 - e^{-g(x)}$, if and only if $g(X_{U(n+1)}) - g(X_{U(n)})$ has an identical distribution with $g(X_k)$, for some k between 1 and n.

Proof: The proof is similar to the proof of Theorem 1.

Example 2 If we set $g(x) = x$, $x > 0$, then identical distribution of $X_{U(n+1)} - X_{U(n)}$ and X_k characterizes exponential distribution in Ahsanullah (2004).

Theorem 2 *Let $\{X_n, n \geq 1\}$ be a sequence of independent and identically distributed random variables with absolutely continuous cdf $F(x)$ and the pdf $f(x)$ on the support (α, β), where α and β may be finite or infinite. Assume that F belongs to the class C^*. Let $g(x)$ be an increasing and differentiable function with $g(x) \to 1$ as $x \to \alpha^+$ and $g(x) \to \infty$ as $x \to \beta^-$ for all $x \in (\alpha, \beta)$. Then $F(x) = 1 - (g(x))^{-\lambda}$, for $\lambda > 0$, if and only if $\frac{g(X_{U(n+1)})}{g(X_{U(n)})}$ has an identical distribution with $\frac{g(X_{U(n)})}{g(X_{U(n-1)})}$.*

Proof: For necessary part, the joint pdf of $X = g(X_{U(n)})$ and $Y = g(X_{U(n+1)})$ is given by (1). Consider the transformations $U = g(X_{U(n)})$ and $V = \frac{g(X_{U(n+1)})}{g(X_{U(n)})}$. Then the Jacobian of the transformation is $J = \frac{\partial}{\partial u}(g^{-1}(u)) \times \frac{\partial}{\partial v}(g^{-1}(uv))$. Since $g(x)$ is an increasing and differentiable function, both $\frac{\partial}{\partial u}(g^{-1}(u))$ and $\frac{\partial}{\partial v}(g^{-1}(uv))$ are positive. Then for the joint pdf of transformed U and V we get

$$f_{U,V}(u,v) = \frac{1}{\Gamma(n)}(R(g^{-1}(u)))^{n-1}r(g^{-1}(u))f(g^{-1}(uv))\frac{\partial}{\partial u}(g^{-1}(u))\frac{\partial}{\partial v}(g^{-1}(uv))$$

for $u > 1$ and $v > 1$.

The marginal pdf of $f_V(v)$ of V is given by

$$f_V(v) = \frac{1}{\Gamma(n)}\int_1^\infty (R(g^{-1}(u)))^{n-1}r(g^{-1}(u))f(g^{-1}(uv))\frac{\partial}{\partial u}(g^{-1}(u))\frac{\partial}{\partial v}(g^{-1}(uv))du \tag{8}$$

for $u > 1$ and $v > 1$.

From (8), we have

$$\begin{aligned} P(V_{n+1,n} > v_1) &= \frac{1}{\Gamma(n)}\int_1^\infty (R(g^{-1}(u)))^{n-1}r(g^{-1}(u)) \\ &\quad \times\frac{\partial}{\partial u}(g^{-1}(u))\bar{F}(g^{-1}(uv_1))du,\ u>1,\ v_1>1, \\ \text{and}\quad P(V_{n,n-1} > v_1) &= \frac{1}{\Gamma(n-1)}\int_1^\infty (R(g^{-1}(u)))^{n-2}r(g^{-1}(u)) \\ &\quad \times\frac{\partial}{\partial u}(g^{-1}(u))\bar{F}(g^{-1}(uv_1))du,\ u>1,\ v_1>1, \end{aligned}$$

where $V_{n+1,n} = \frac{g(X_{U(n+1)})}{g(X_{U(n)})}$ and $V_{n,n-1} = \frac{g(X_{U(n)})}{g(X_{U(n-1)})}$.

For the continuous distribution $F(x) = 1-(g(x))^{-\lambda}$, $g(x) > 1$, $\lambda > 0$, we obtain

$$P(V_{n+1,n} > v_1) = \frac{\lambda^n}{\Gamma(n)}\int_1^\infty \big(ln(u)\big)^{n-1}u^{-\lambda-1}v_1^{-\lambda}du,\ u>1,\ v_1>1,$$

and

$$P(V_{n+1,n} > v_1) = \frac{\lambda^{n-1}}{\Gamma(n-1)}\int_1^\infty \big(ln(u)\big)^{n-2}u^{-\lambda-1}v_1^{-\lambda}du,\ u>1,\ v_1>1. \tag{9}$$

Integrating (9) by parts, we get

$$P(V_{n,n-1} > v_1) = \frac{\lambda^n}{\Gamma(n)}\int_1^\infty \big(ln(u)\big)^{n-1}u^{-\lambda-1}v_1^{-\lambda}du,\ u>1,\ v_1>1.$$

Hence $\frac{g(X_{U(n+1)})}{g(X_{U(n)})}$ and $\frac{g(X_{U(n)})}{g(X_{U(n-1)})}$ are identically distributed.

Now we prove the sufficient part of the Theorem 2. Let us use the transformation $U = g(X_{U(n)})$ and $V = \frac{g(X_{U(n+1)})}{g(X_{U(n)})}$. The Jacobian of the transformation is $J = \frac{\partial}{\partial u}(g^{-1}(u)) \times \frac{\partial}{\partial v}(g^{-1}(uv))$. Since $g(x)$ is an increasing and differentiable function, both $\frac{\partial}{\partial u}(g^{-1}(u))$ and $\frac{\partial}{\partial v}(g^{-1}(uv))$ are positive. Thus we can write the joint pdf $f_{U,V}(u,v)$ of U and V as

$$f_{U,V}(u,v) = \frac{1}{\Gamma(n)}(R(g^{-1}(u)))^{n-1}r(g^{-1}(u))f(g^{-1}(uv))\frac{\partial}{\partial u}(g^{-1}(u))\frac{\partial}{\partial v}(g^{-1}(uv))$$

for $u > 1$ and $v > 1$.

The marginal pdf of $f_V(v)$ of V is given by

$$f_V(v) = \frac{1}{\Gamma(n)}\int_1^\infty (R(g^{-1}(u)))^{n-1}r(g^{-1}(u))f(g^{-1}(uv))\frac{\partial}{\partial u}(g^{-1}(u))\frac{\partial}{\partial v}(g^{-1}(uv))du \tag{10}$$

for $u > 1$ and $v > 1$.

From (10), we have

$$P(V_{n+1,n} > v_1) = \frac{1}{\Gamma(n)} \int_1^{\infty} (R(g^{-1}(u)))^{n-1} r(g^{-1}(u)) \times \frac{\partial}{\partial u}(g^{-1}(u)) \bar{F}(g^{-1}(uv_1)) du,$$

for $u > 1$ and $v_1 > 1$.

Since $V_{n+1,n}$ and $V_{n,n-1}$ are identically distributed, we get

$$\begin{aligned} &\int_1^{\infty} (R(g^{-1}(u)))^{n-1} r(g^{-1}(u)) \frac{\partial}{\partial u}(g^{-1}(u)) \bar{F}(g^{-1}(uv_1)) du \\ &= (n-1) \int_1^{\infty} (R(g^{-1}(u)))^{n-2} r(g^{-1}(u)) \frac{\partial}{\partial u}(g^{-1}(u)) \bar{F}(g^{-1}(uv_1)) du \end{aligned} \tag{11}$$

for all $u > 1$ and $v_1 > 1$.

But we know that

$$\begin{aligned} &(n-1) \int_1^{\infty} (R(g^{-1}(u)))^{n-2} r(g^{-1}(u)) \frac{\partial}{\partial u}(g^{-1}(u)) \bar{F}(g^{-1}(uv_1)) du \\ &= \int_1^{\infty} (R(g^{-1}(u)))^{n-1} f(g^{-1}(uv_1)) \frac{\partial}{\partial u}(g^{-1}(uv_1)) du, \ u > 1, \ v_1 > 1. \end{aligned} \tag{12}$$

Substituting (12) in (11) and on further simplification, we get

$$\int_1^{\infty} (R(g^{-1}(u)))^{n-1} \bar{F}(g^{-1}(uv_1)) r(g^{-1}(u)) \frac{\partial}{\partial u}(g^{-1}(u)) \times \left[1 - \frac{r(g^{-1}(uv_1)) \frac{\partial}{\partial u}(g^{-1}(uv_1))}{r(g^{-1}(u)) \frac{\partial}{\partial u}(g^{-1}(u))}\right] du = 0.$$

Since $F(x)$ belongs to the class C^*, we must have

$$r(g^{-1}(uv_1)) \frac{\partial}{\partial u}(g^{-1}(uv_1)) = r(g^{-1}(u)) \frac{\partial}{\partial u}(g^{-1}(u)) \tag{13}$$

for almost all $v_1 > 1$ and $u > 1$.

Integrating (13) with respect to u from 1 to u_1 and simplifying, we get

$$\bar{F}(g^{-1}(u_1 v_1)) = \bar{F}(g^{-1}(v_1)) \bar{F}(g^{-1}(u_1)) \tag{14}$$

for all $u_1 > 1$ and $v_1 > 1$.

By the theory of functional equation(see Aczel (1966)) the only continuous solution of (14) with boundary conditions $F(\alpha) = 0$ and $F(\beta) = 1$ is

$$\bar{F}(x) = (g(x))^{-\lambda}$$

for all $g(x) > 1$ and $\lambda > 0$.

This completes the proof.

Example 3 If we set $g(x) = x$, $x > 1$, then identical distribution of $\frac{X_{U(n+1)}}{X_{U(n)}}$ and $\frac{X_{U(n)}}{X_{U(n-1)}}$ characterizes Pareto distribution in Ahsanullah et al. (2012).

Remark 2 Let $\{X_n, n \geq 1\}$ be a sequence of independent and identically distributed random variables with absolutely continuous cdf $F(x)$ and the pdf $f(x)$ on the support (α, β), where α and β may be finite or infinite. Let $g(x)$ be an increasing and differentiable function with $g(x) \to 0$ as $x \to \alpha^+$ and $g(x) \to \infty$ as $x \to \beta^-$ for all $x \in (\alpha, \beta)$. Assume that F belongs to the class C_2 if the hazard rate $r(x) = \frac{f(x)}{1-F(x)}$ increases monotonically increases or decreases for all x. Then $F(x) = 1 - e^{-g(x)}$, if and only if $g(X_{U(n+1)}) - g(X_{U(n)})$ has an identical distribution with $g(X_{U(n)}) - g(X_{U(n-1)})$.

Proof: The proof is similar to the proof of Theorem 2.

Example 4 If we set $g(x) = x$, $x > 0$, then identical distribution of $X_{U(n+1)} - X_{U(n)}$ and $X_{U(n)} - X_{U(n-1)}$ characterizes exponential distribution in Ahsanullah (2004).

Remark 3 A list of continuous distributions with cdf and the corresponding forms of $g(x)$ are given in Table 1.

Table 1 Examples based on the distribution function $F(x) = 1 - (g(x))^{-\lambda}$

Distribution	$g(x)$	$F(x)$
Pareto	x	$1 - x^{-\lambda},\ x > 1$
Power	$(1 - (\frac{x}{\beta})^p)^{-\frac{1}{\lambda}}$	$(\frac{x}{\beta})^p,\ 0 < x < \beta$
Beta 2nd kind	$(1 + x)$	$1 - (1 + x)^{-\lambda},\ x > 0$
Lomax	$(1 + \frac{x}{c})^{\frac{1}{\lambda}}$	$1 - (1 + \frac{x}{c})^{-1},\ x > 0$
Singh-Maddala	$(1 + \beta x^p)$	$1 - (1 + \beta x^p)^{-\lambda},\ x > 0$
Gompertz	$\left(exp[\frac{c}{\alpha}(e^{\alpha x} - 1)]\right)^{\frac{1}{\lambda}}$	$1 - exp[-\frac{c}{\alpha}(e^{\alpha x} - 1)], x > 0$
Rayleigh	$\left(exp[2^{-1}\beta^{-2}x^2]\right)^{\frac{1}{\lambda}}$	$1 - exp[-2^{-1}\beta^{-2}x^2], x > 0$
Weibull	e^{x^p}	$1 - e^{-\lambda x^p},\ x > 0$
Kumaraswamy	$(1 - x^p)^{-1}$	$1 - (1 - x^p)^{\lambda},\ 0 < x < 1$
Rectangular	$(1 - (\frac{x-\alpha}{\beta-\alpha}))^{-\frac{1}{\lambda}}$	$(\frac{x-\alpha}{\beta-\alpha}),\ \alpha < x < \beta$
Extream value I	$\left(exp[e^x]\right)^{\frac{1}{\lambda}}$	$1 - exp[-e^x],\ -\infty < x < \infty$
Burr type III	$(1 - (1 + x^{-p})^{-\beta})^{-\frac{1}{\lambda}}$	$(1 + x^{-p})^{-\beta},\ x > 0$
Burr type IX	$\left(\frac{c((1+e^x)^p-1)}{2} + 1\right)^{\frac{1}{\lambda}}$	$1 - \left(\frac{c((1+e^x)^p-1)}{2} + 1\right)^{-1},\ -\infty < x < \infty$
Burr type X	$\left(1 - (1 - e^{-\beta x^2})^{\alpha}\right)^{-\frac{1}{\lambda}}$	$(1 - e^{-\beta x^2})^{\alpha},\ x > 0$
Lindley	$\left(\frac{c+1+cx}{c+1}e^{-cx}\right)^{-\frac{1}{\lambda}}$	$1 - \frac{c+1+cx}{c+1}e^{-cx},\ x > 0$

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