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Measures of Efficiency of Nearest Neighbour Balanced Block Designs for First Order Correlated Models

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Abstract

The comparison of efficiency of complete and incomplete nearest neighbour balanced block designs (NNBD) over regular block design using average variance, generalized variance and min-max variance with the error term ε given in the NNBD model follows using first order correlated models. It is observed that, R_H and R_D show increasing efficiency values for direct and neighbour effects (left and right) for MA(1) models. The R_A and R_G show neither increasing nor decreasing efficiency values are observed for direct and neighbouring effects for AR(1) and MA(1) models. In the case of ARMA(1,1) model, neither increasing nor decreasing efficiency values have observed for average variance and generalized variance. The R_E shows decreasing efficiency values with ρ in the interval 0.1 to 0.4 for direct and neighbouring effects for AR(1), MA(1) and ARMA(1,1) models.

Keywords: Autoregressive, moving average, autoregressive moving average, information matrix, efficiency, regular block design, average variance, generalized variance, min-max variance.

1. Introduction

The assumptions in the classical (Fisherian) block model are that the response on a plot to a particular treatment does not affect the response on the neighbouring plots and the fertility associated with plots in a block is constant. However, in many fields of agricultural research, like horticultural and agro-forestry experiments, the treatment applied to one experimental plot in a block may affect the response on the neighbouring plots if the blocks are linear with no guard areas between the plots. The treatments are varieties, neighbour effects may be caused by differences in height, root vigor, or germination date especially on small plots, which are used in plant breeding experiments. Treatments such as fertilizer, irrigation, or pesticide may spread to adjacent plots causing neighbour effects. Such experiments exhibit neighbour effects, because the effect of having no treatment as a neighbour is different from the neighbour effects of any treatment. Competition or interference between neighbouring units in field experiments can contribute to variability in experimental results and lead to substantial losses in efficiency. In case of block design setup, if each block is a single line of plots and blocks are well separated, extra parameters are needed for the effect of left and right neighbours. An alternative is to have border plots on both ends of every block. Each border plot receives an experimental treatment, but it is not used for measuring the response variable. These border plots do not add too much to the cost of one-dimensional experiments. The estimates of treatment differences may therefore deviate because of interference from neighbouring units. Neighbour balanced block designs, where in the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbours, are used for modeling and controlling interference effects between neighbouring plots. Azais et al. (1993) obtained a series of efficient neighbour designs with border plots that are balanced in $v-1$ blocks of size v and v blocks of size $v-1$, where v is the number of treatments. Santharam and Ponnuswamy (1997a) observed that the performance of NNBD is quite satisfactory for the remaining models. Druilhet (1999) studied optimality of circular neighbour balanced block designs obtained by Azais et al. (1993). Bailey (2003) had given some designs for studying one-sided neighbour effects. These neighbour balanced block designs have been developed under the assumption that the observations within a block are uncorrelated. In situations where the correlation structure among the observations within a block is known, may be from the data of past similar experiments, it may be advantageous to use this information in designing an experiment and analyzing the data so as to make more precise inference about treatment effects (Gill and Shukla 1985). Kunert et al. (2003) considered two related models for interference and have shown that optimal designs for one model can be obtained from optimal designs for the other model. Martin and Eccelston (2004) has given variance balanced designs under interference and dependent observations. Tomar and Jaggi (2007) observed that efficiency is quite high, in case of complete block designs for both AR(1) and Nearest Neighbour (NN) correlation structures. In case of incomplete block designs, designs with AR(1) structure turns out to be more efficient. However, the efficiency of direct effects of treatments is more as compared to neighbour effects under both the structures. Mingyao et al. (2007) studied the optimality of circular neighbour balanced designs for total effects when the one-sided or two-sided neighbour effects are present in the model and the observation errors are correlated according to a first-order circular autoregressive (AR(1,C)) process.

In this article, we have compared the efficiencies of nearest neighbour balanced block design (NNBD) and nearest neighbour balanced incomplete block design (NNBIBD) over regular block design using average variance, generalized variance and min-max variance with the error term ε given in the NNBD model follows AR(1), MA(1) and ARMA(1,1) models. We have investigated the various measures of efficiencies $(R_A, R_H, R_G, R_D$ and $R_E)$ of NNBD over regular block design using first order correlated models. We have also investigated the various measures of efficiencies $(R_A, R_H, R_G, R_D$ and $R_E)$ of NNBIBD over regular block design using first order correlated models.

2. Model Structures

The designs considered here are assumed to be in linear blocks, with neighbour effects only in the direction of the blocks (say left-neighbour or right-neighbour or both). Because the effect of having no treatment differs from the neighbour effects of any treatment, designs with border plots have been considered, which is, designs with one point added at each end of each block. The border plots receive treatments but are not used for measuring the response variables. The plots, which are not on the borders, are inner plots. The length of a block is the number of its inner plots. It is further assumed that all the designs are circular, that is the treatment on border plots is same as the treatment on the inner plot at the other end of the block.

Let Δ be a class of binary neighbour balanced block designs with $n = bk$ units that form b blocks each containing k units. Y_{ij} be the response from the i^{th} plot in the j^{th} block

$(i = 1, 2, ..., k; j = 1, 2, ..., b)$. The layout includes border plots at both ends of every block, i.e., at 0^{th} and $(k+1)^{\text{th}}$ position and observations for these units are not modelled. It is assumed that the design is circular, that is the treatment on border plots is same as the treatment on the inner plot at the other end of the block.

The following fixed effects additive model is considered for analyzing a neighbour balanced block design under correlated observations:

$$Y_{ij} = \mu + \tau_{(i,j)} + l_{(i-1,j)} + \gamma_{(i+1,j)} + \beta_j + e_{ij}, \tag{1}$$

where μ is the general mean, $\tau_{(i,j)}$ is the direct effect of the treatment in the i^{th} plot of j^{th} block, β_j is the effect of the j^{th} block, $l_{(i-1,j)}$ is the left neighbour effect due to the treatment in the $(i-1)^{\text{th}}$ plot of j^{th} block, $\gamma_{(i+1,j)}$ is the right neighbour effect due to the treatment in the $(i+1)^{\text{th}}$ plot in j^{th} block, e_{ij} are error terms distributed with mean zero and a variance-covariance structure $\Omega = I_b \otimes \Lambda$ (I_b is an identity matrix of order b and $\otimes$ denotes the Kronecker product). The ARMA(1,1) model along with AR(1) and MA(1) and explored the performance of NNBD range from ρ = −0.4 to 0.4. If the errors within a block follow a AR(1) structure, then Λ is a $k \times k$ matrix with $(i,i')^{\text{th}}$ entry $(i,i' = 1, 2, \ldots, k)$ as $\rho^{|i-i'|}$, $|\rho| < 1$. The MA(1) structure, then Λ is a matrix with diagonal entries as 1 and $(i,i')^{\text{th}}$ entry $(i,i' = 1, 2, \ldots, k)$ as ρ, when $|i - i'| = 1$, otherwise zero (Gill and Shukla 1985). If the errors within a block follow an ARMA(1,1) model then $\Omega = I_b \otimes \Lambda$. I_b is an identity matrix of order b and $\Lambda = \begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{k-1} \\ r_1 & r_0 & r_1 & \cdots & r_{k-2} \\ r_2 & r_1 & r_0 & \cdots & r_{k-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{k-1} & r_{k-2} & r_{k-3} & \cdots & r_0 \end{bmatrix}$, $r_0 = \dfrac{1 + 2\rho_1\rho_2 + \rho_2^2}{1 - \rho_1^2}$, $r_1 = \dfrac{\rho_1(1+\rho_2^2) + \rho_2(1+\rho_1^2)}{1-\rho_1^2}$, $r_k = \rho_1^r(k-1)$ for $k \geq 2$ (Santharam and Ponnuswamy 1997b). The Nearest Neighbour (NN) correlation structure, the Λ is a matrix with diagonal entries as 1 and off-diagonal entries as ρ. Model (1) can be rewritten in the matrix notation as follows

$$Y = \mu 1 + \Delta'\tau + \Delta_1' l + \Delta_2'\gamma + D'\beta + e, \tag{2}$$

where Y is $n \times 1$ vector of observations, 1 is $n \times 1$ vector of ones, Δ' is an $n \times v$ incidence matrix of observations versus direct treatments, τ is $v \times 1$ vector of direct treatment effects, Δ_1' is a $n \times v$ matrix of observations versus left neighbour treatment, Δ_2' is a $n \times v$ matrix of observations versus right neighbour treatment, l is $v \times 1$ vector of left neighbour effects, γ is $v \times 1$ vector of right neighbour effects, D' is an $n \times b$ incidence matrix of observations versus blocks, β is $b \times 1$ vector of block effects and e is $n \times 1$ vector of errors. The joint information matrix for estimating the direct and neighbour (left and right) effects under correlated observations estimated by generalized least squares is obtained as follows:

$$C = \begin{bmatrix} \Delta\left(I_b \otimes \Lambda^*\right)\Delta' & \Delta\left(I_b \otimes \Lambda^*\right)\Delta_1' & \Delta\left(I_b \otimes \Lambda^*\right)\Delta_2' \\ \Delta_1\left(I_b \otimes \Lambda^*\right)\Delta' & \Delta_1\left(I_b \otimes \Lambda^*\right)\Delta_1' & \Delta_1\left(I_b \otimes \Lambda^*\right)\Delta_2' \\ \Delta_2\left(I_b \otimes \Lambda^*\right)\Delta' & \Delta_2\left(I_b \otimes \Lambda^*\right)\Delta_1' & \Delta_2\left(I_b \otimes \Lambda^*\right)\Delta_2' \end{bmatrix} \tag{3}$$

with

$$\Lambda^* = \Lambda^{-1} - \left(1_k' \Lambda^{-1} 1_k\right)^{-1} \Lambda^{-1} 1_k 1_k' \Lambda^{-1}.$$

The above $3v \times 3v$ information matrix (C) for estimating the direct effects and neighbour effects of treatments in a block design setting is symmetric, non-negative definite with row and column sums equal to zero. The information matrix for estimating the direct effects of treatments from (3) is as follows

$$C_\tau = C_{11} - C_{12} C_{22}^{-1} C_{21}, \tag{4}$$

where $C_{11} = \Delta\left(I_b \otimes \Lambda^*\right)\Lambda'$, $C_{12} = \left[\Delta\left(I_b \otimes \Lambda^*\right)\Lambda_1' \quad \Delta\left(I_b \otimes \Lambda^*\right)\Lambda_2'\right]$ and

$$C_{22} = \begin{bmatrix} \Delta_1\left(I_b \otimes \Lambda^*\right)\Delta_1' & \Delta_1\left(I_b \otimes \Lambda^*\right)\Delta_2' \\ \Delta_2\left(I_b \otimes \Lambda^*\right)\Delta_1' & \Delta_2\left(I_b \otimes \Lambda^*\right)\Delta_2' \end{bmatrix}.$$

Similarly, the information matrix for estimating the left neighbour effect of treatments (C_l) and right neighbour effect of treatments (C_γ) can be obtained.

Definition 1 *A block design is neighbour balanced if every treatment has every treatment appearing as a neighbour (left and right) constant number of times (say, λ).*

Definition 2 *A neighbour balanced block design is called pair-wise uniform on the plots if each treatment $s\,(=1,2,\ldots,v)$ occurs equally often in each plot position i $(=1,2,\ldots,k)$ and each pair of treatments s and s', $s \neq s'\,(=1,2,\ldots,v)$ occurs equally often $(\alpha$ time$)$ within the same block in each unordered pair of plot positions i and i', $i \neq i'$ $(=1,2,\ldots,k)$.*

Definition 3 *A neighbour balanced block design with correlated observations permitting the estimation of direct and neighbour (left and right) effects, is called variance balanced if the variance of any estimated elementary contrast among the direct effects is constant, say V_1, the variance of any estimated elementary contrast among the left neighbour effect is constant, say V_2, and the variance of any estimated elementary contrast among the right neighbour effects is constant, say V_3. The constants V_1, V_2 and V_3 may not be equal. A block design is totally balanced if $V_1 = V_2 = V_3$.*

3. Construction of Designs

Tomer et al. (2005) has constructed neighbour balanced block design with parameters v (prime or prime power), $b = v(v-1)$, $r = (v-1)(v-m)$, $k = (v-m)$, $m = 1,2,\ldots,v-4$ and $\lambda = (v-m)$ using mutually orthogonal Latin squares (MOLS) of order v. This series of design has been investigated under the correlated error structure. It is seen that the design turns out to be pair-wise uniform with $\alpha = 1$ and also variance balanced for estimating direct (V_1) and neighbour effects $(V_2 = V_3)$.

Example 1 Let $v = 6$ and $m = 0$. The following is a neighbour balanced pair-wise uniform complete block design with parameters $v = 6, b = 30, r = 30, k = 6,$ $\lambda = 6$ and $\alpha = 1$:

4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3.

The information matrices for estimating the direct and neighbour effects (left and right) of treatments for AR(1) structure with $\rho = 0.1$ was obtained as given below

$$C_\tau = 14.620\left[I - \frac{J}{5}\right] \text{ and } C_l = C_\gamma = 15.893\left[I - \frac{J}{5}\right].$$

Similarly for MA(1), ARMA(1,1) and NN structures

$$C_\tau = 15.561\left[I - \frac{J}{5}\right] \text{ and } C_l = C_\gamma = 16.886\left[I - \frac{J}{5}\right],$$

$$C_\tau = 14.687\left[I - \frac{J}{5}\right] \text{ and } C_l = C_\gamma = 13.678\left[I - \frac{J}{5}\right],$$

$$C_\tau = 16.572\left[I - \frac{J}{5}\right] \text{ and } C_l = C_\gamma = 17.391\left[I - \frac{J}{5}\right].$$

These matrices have been worked out using in R (R Core Team 2018).

Similarly, we have worked a neighbour balanced pair-wise uniform complete block design with parameter $v = 5$ and $m = 0$.

Example 2 Let $v = 5$ and $m = 1$. The following is a neighbour balanced pair-wise uniform incomplete block design with parameters $v = 5, b = 20, r = 16, k = 4, \lambda = 4$ and $\alpha = 1$:

5	2	3	4	5	2
1	3	4	5	1	3
2	4	5	1	2	4
3	5	1	2	3	5
4	1	2	3	4	1
1	3	4	2	1	3
2	4	1	3	2	4
3	5	2	4	3	5
4	1	3	5	4	1
5	2	4	1	5	2
3	4	2	5	3	4
4	5	3	1	4	5
5	1	4	2	5	1
1	2	5	3	1	2
2	3	1	4	2	3
2	5	4	3	2	5
3	1	5	4	3	1
4	2	1	5	4	2
5	3	2	1	5	3
1	4	3	2	1	4.

The information matrices for estimating the direct and neighbour effects (left and right) of treatments for AR(1) structure with $\rho = 0.1$ is obtained as given below

$$C_\tau = 12.02696\left[I - \frac{J}{5}\right] \text{ and } C_l = C_\gamma = 12.20456\left[I - \frac{J}{5}\right].$$

Similarly for MA(1), ARMA(1,1) and NN structures,

$$C_\tau = 11.18748\left[I - \frac{J}{5}\right] \text{ and } C_l = C_\gamma = 13.68067\left[I - \frac{J}{5}\right],$$

$$C_\tau = 11.18748\left[I - \frac{J}{5}\right] \text{ and } C_l = C_\gamma = 12.62843\left[I - \frac{J}{5}\right],$$

$$C_\tau = 13.17057\left[I - \frac{J}{5}\right] \text{ and } C_l = C_\gamma = 12.60091\left[I - \frac{J}{5}\right].$$

These matrices have been worked out using in R (R Core Team 2018).

Similarly, we have worked a neighbour balanced pair-wise uniform incomplete block design with parameters (i) $v=6$ and $m=1,2$ and (ii) $v=7$ and $m=3$.

4. Comparison of Measures of Efficiency of NNBD

In this section, we study the behaviour of some estimators of ρ and σ_ε^2. The nearest neighbour balanced block design (NNBD) and regular block design data sets were generated with the following true parameters: $\rho=-0.4$ to 0.4, $\sigma_\varepsilon^2=1$, $t=5, r=20$ and $t=6, r=30$.

The estimation of σ_ε^2 based on NNBD and regular block design were compared using the following three measures.

4.1. Average variance comparison

Consider the measure

$$R_A=\frac{\sigma^2_{\varepsilon(RBD)}\sum_{i=1}^{t-1}\gamma_{RBD}^{-1}(i)}{\sigma^2_{\varepsilon(NNBD)}\sum_{i=1}^{t-1}\gamma_{NNBD}^{-1}(i)},$$

where $\sigma^2_{\varepsilon(RBD)}$ denotes the estimate of σ_ε^2 based on regular block design $\sigma^2_{\varepsilon(NNBD)}$ denotes the estimate of σ_ε^2 based on NNBD $\gamma_{d(i)}$'s and are nonzero eigen values of the information matrix.

The above measure R_A compares the average variance of elementary treatment contrast when the same data are analysed by regular block design and nearest neighbour balanced block design. It may be noted that the estimates of σ_ε^2 and ρ can be different in case of regular block design and nearest neighbour balanced block design. The ratio $\sigma^2_{\varepsilon RBD}/\sigma^2_{\varepsilon NNBD}$ could mask the genuine efficiency of NNBD. Therefore, the ratio

$$R_H=\frac{\sum_{i=1}^{t-1}\gamma_{RBD}^{-1}(i)}{\sum_{i=1}^{t-1}\gamma_{NNBD}^{-1}(i)}$$

of harmonic means will also be considered as an index of efficiency.

4.2. Generalized variance comparison

Another way to compare regular block design and nearest neighbour balanced block design is the ratio

$$R_G=\left[\sigma^2_{RBD}/\sigma^2_{NNBD}\right]^{t-1}\prod_{i=1}^{t-1}\gamma_{NNBD(i)}\gamma^{-1}_{RBD(i)}$$

of generalized variances of $t-1$ orthonormal treatment contrasts estimated under regular block design and nearest neighbour balanced block design. It may be noted that R_G is very sensitive to the ratio $\sigma^2_{RBD}/\sigma^2_{NNBD}$. We therefore, consider the ratio

$$R_D=\prod_{i=1}^{t-1}\gamma_{NNBD(i)}\gamma^{-1}_{RBD(i)}.$$

This gives a better comparison of regular block design and nearest neighbour balanced block design.

4.3. Min-max variance comparison

This closeness is measured by the ratio of the smallest nonzero eigen-value to the largest eigen value of the information matrix. Note that this ratio independent of σ_ε^2. For comparing nearest neighbour balanced block design (NNBD) and regular block design, we take the ratio

$$R_E = \frac{\gamma_{NNBD(1)}}{\gamma_{NNBD(t-1)}} \times \frac{\gamma_{RBD(t-1)}}{\gamma_{RBD(1)}}.$$

Tables 1, 2 and 3 show the efficiencies of AR(1), MA(1) and ARMA(1,1) models with $t = 5, r = 20$ and $\alpha = 1,$ there is considerable advantage in using NNBD as far as average variance $(R_A$ and $R_G)$, generalized variance $(R_H$ and $R_D)$ and min-max variance (R_E) are concerned. The efficiency factor (E_τ) for direct effects of the neighbour, left and right (E_l) and (E_γ) neighbour effects of treatments is obtained by Tomer et al. (2007). The R_H and R_D show increasing efficiency values, R_A and E_γ show decreasing efficiency values for direct effects of treatments for both AR(1) and MA(1) models. In the case of ARMA(1,1) model, neither increasing nor devcreasing efficiency values are observed for average variance and generalized variance. The R_E show decreasing efficiency values with ρ in the interval 0.1 to 0.4 for direct and neighbouring effects for AR(1), MA(1) and ARMA(1,1) models. We have concluded that, the higher efficiency values are observed for direct effects of treatments for both MA(1) and ARMA(1,1) models for average variance. The lower efficiency values are observed for direct, left and right neighbour effects of treatments for AR(1), MA(1) and ARMA(1,1) models for min-max variance.

Tables 4, 5 and 6 show the efficiencies of AR(1), MA(1) and ARMA(1,1) models with $t = 6, r = 30$ and $\alpha = 1,$ there is considerable advantage in using NNBD as far as average variance $(R_A$ and $R_G)$, generalized variance $(R_H$ and $R_D)$ and min-max variance (R_E) are concerned. The efficiency factor (E_τ) for direct effects of the neighbour, left and right (E_l) and (E_γ) neighbour effects of treatments is obtained by Tomer et al. (2007). The R_H and R_D show increasing efficiency values for direct, left and right neighbour effects for MA(1) models. Whereas neither increasing nor decreasing efficiency values are observed for R_A and R_G for AR(1), MA(1) and ARMA(1,1) models. The R_E show decreasing efficiency values with ρ in the interval 0.1 to 0.4 for direct and neighbouring effects for AR(1), MA(1) and ARMA(1,1) models. We have concluded that, the higher efficiency values are observed for direct for MA(1) and ARMA(1,1) models for average variance. The lower efficiency values are observed for direct, left and right neighbour effects of treatments for ARMA(1,1) model for min-max variance.

Table 1 AR(1) - R_H, R_A, R_D, R_G and R_E values for NNBD $t = 5, r = 20$ and $\alpha = 1$

AR(1)		$\rho = -0.4$	$\rho = -0.3$	$\rho = -0.2$	$\rho = -0.1$	$\rho = 0$	$\rho = 0.1$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$
R_H	E_τ	0.82206	0.84056	0.84849	0.92776	0.99731	1.07535	1.11937	1.19711	1.33701
	E_l	0.88057	0.88385	0.87055	0.93609	0.99731	1.07443	1.14159	1.19428	1.38385
	E_γ	0.87535	0.87293	0.90202	0.93467	0.99731	1.07842	1.14588	1.17232	1.44921
R_A	E_τ	1.71980	1.62361	1.43725	1.16062	1.03300	0.88094	0.70476	0.67051	0.58753
	E_l	1.04258	1.23553	1.35437	1.05331	1.03300	1.02668	0.95449	0.93141	0.90494
	E_γ	0.84175	0.83609	0.77479	0.78136	1.03300	1.02670	0.94460	0.94350	0.72666
R_D	E_τ	0.65693	0.76922	0.80208	0.91668	0.99742	1.06470	1.07019	1.10153	1.15849
	E_l	0.75975	0.80215	0.79492	0.93214	0.99742	1.13410	1.10125	1.13779	1.18341
	E_γ	0.75451	0.73194	0.86888	0.93505	0.99742	1.08569	1.11999	1.07228	1.21074
R_G	E_τ	1.37434	1.68040	1.35863	1.14676	1.03311	0.87221	0.67380	0.61698	0.50908
	E_l	1.09700	1.02896	1.28008	1.13454	1.03311	0.98796	0.87841	0.84006	0.83029
	E_γ	0.87294	0.74411	0.72419	0.86790	1.03311	1.02708	1.02430	0.83693	0.73724
R_E	E_τ	0.34195	0.41605	0.58207	0.80513	1.02397	0.80117	0.61089	0.51894	0.38149
	E_l	0.50890	0.50607	0.49748	0.90509	1.02397	0.90788	0.71547	0.51062	0.39041
	E_γ	0.40937	0.50946	0.57910	0.79090	1.0297	0.77891	0.51938	0.44272	0.35021

Table 2 MA(1) - R_H, R_A, R_D, R_G and R_E values for NNBD $t = 5, r = 20$ and $\alpha = 1$

MA(1)		$\rho = -0.4$	$\rho = -0.3$	$\rho = -0.2$	$\rho = -0.1$	$\rho = 0$	$\rho = 0.1$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$
R_H	E_τ	0.84249	0.86817	0.86119	0.93935	0.99731	1.08974	1.14109	1.24952	1.46584
	E_l	0.84164	0.86526	0.86778	0.92234	0.99731	1.11430	1.14735	1.26867	1.54839
	E_γ	0.85300	0.86664	0.93185	0.96478	0.99731	1.08162	1.18884	1.30402	1.50626
R_A	E_τ	1.74409	1.71577	1.38649	1.13382	1.03300	0.85413	0.73994	0.59917	0.45844
	E_l	1.44720	1.36032	1.25010	1.22789	1.03300	0.99040	0.85540	0.81432	0.79324
	E_γ	0.97037	0.89610	0.86960	0.83836	1.03300	0.79912	0.72593	0.76606	0.68569
R_D	E_τ	0.72640	0.89669	0.89790	0.92919	0.99742	1.06906	1.08207	1.07493	1.08956
	E_l	0.78649	0.81766	0.89177	0.91675	0.99742	1.10748	1.09527	1.13450	1.14402
	E_γ	0.77146	0.76634	0.89765	0.94456	0.99742	1.06893	1.12330	1.12611	1.19308
R_G	E_τ	1.50378	1.47508	1.31983	1.12156	1.03311	0.83792	0.70167	0.51545	0.34076
	E_l	1.35236	1.29648	1.20373	1.12044	1.03311	0.98434	0.81658	0.81763	0.73881
	E_γ	0.98717	0.83761	0.83769	0.82070	1.03311	0.86120	0.81950	0.81146	0.71732
R_E	E_τ	0.42834	0.53675	0.58745	0.81568	1.02397	0.74110	0.56530	0.40386	0.24452
	E_l	0.63471	0.56779	0.65418	0.90441	1.02397	0.93008	0.66682	0.42463	0.25251
	E_γ	0.51057	0.52467	0.59003	0.70308	1.02397	0.77840	0.55752	0.36290	0.22587

Table 3 ARMA(1) - R_H, R_A, R_D, R_G and R_E values for NNBD $t=5, r=20$ and $\alpha=1$

ARMA (1,1)		$\rho_1=-0.4$ $\rho_2=-0.4$	$\rho_1=-0.3$ $\rho_2=-0.3$	$\rho_1=-0.2$ $\rho_2=-0.2$	$\rho_1=-0.1$ $\rho_2=-0.1$	$\rho_1=0$ $\rho_2=0$	$\rho_1=0.1$ $\rho_2=0.1$	$\rho_1=0.2$ $\rho_2=0.2$	$\rho_1=0.3$ $\rho_2=0.3$	$\rho_1=0.4$ $\rho_2=0.4$
R_H	E_τ	1.73929	1.40782	1.13165	1.03135	0.99731	1.07079	1.25615	1.80200	1.73929
	E_l	1.27542	1.24219	1.11838	1.00941	0.99731	1.14190	1.33971	1.97894	1.27542
	E_γ	1.64768	1.14631	1.19344	1.01525	0.99731	1.10410	1.36069	1.86532	1.64768
R_A	E_τ	2.38589	2.29292	1.66082	1.25045	1.03300	0.81184	0.49451	0.39819	2.38589
	E_l	1.38934	1.28048	1.20192	1.28074	1.03300	0.96485	0.41140	0.32256	1.38934
	E_γ	1.11417	1.14094	1.12845	1.08510	1.03300	0.93729	0.41176	0.31165	1.11417
R_D	E_τ	1.24259	1.11109	0.99616	1.00093	0.99742	1.02257	0.99765	1.04287	1.24259
	E_l	2.21619	1.65773	1.03140	0.97924	0.99742	1.08982	1.05636	1.02048	2.21619
	E_γ	1.64380	1.62707	1.06986	0.98238	0.99742	1.05094	1.07005	1.07228	1.64380
R_G	E_τ	1.70454	1.70964	1.46198	1.21357	1.03311	0.77528	0.39275	0.23044	1.70454
	E_l	2.41413	1.70565	1.41117	1.24246	1.03311	0.92084	0.82094	0.82197	2.41413
	E_γ	1.78736	1.76492	1.70332	1.28235	1.03311	0.97291	0.78334	0.81147	1.78736
R_E	E_τ	0.25425	0.30840	0.40455	0.71882	1.02397	0.58190	0.29767	0.16850	0.25425
	E_l	0.49622	0.19640	0.52302	0.70143	1.02397	0.64260	0.34204	0.25408	0.49622
	E_γ	0.26500	0.29737	0.40302	0.62134	1.02397	0.57200	0.27076	0.21844	0.26500

Table 4 AR(1) - R_H, R_A, R_D, R_G and R_E values for NNBD $t=6, r=30$ and $\alpha=1$

AR(1)		$\rho=-0.4$	$\rho=-0.3$	$\rho=-0.2$	$\rho=-0.1$	$\rho=0$	$\rho=0.1$	$\rho=0.2$	$\rho=0.3$	$\rho=0.4$
R_H	E_τ	0.77832	0.83364	1.18472	0.94421	1.00000	1.07791	1.16081	1.25413	1.36637
	E_l	1.01559	1.19869	0.88125	0.94508	1.00000	1.05268	1.30270	1.28229	1.41284
	E_γ	0.84044	0.88062	0.85890	0.96807	1.00000	1.08877	1.16076	1.26551	1.38382
R_A	E_τ	2.40263	1.61135	1.84549	1.03443	1.00000	1.00032	0.76598	0.63805	0.57940
	E_l	1.26510	1.51088	0.98075	0.99408	1.00000	1.12142	1.27964	1.25184	1.36862
	E_γ	0.86283	0.81914	0.91018	0.87069	1.00000	1.06665	1.21837	1.33880	1.55820
R_D	E_τ	0.56238	0.72942	0.83232	0.93186	1.00000	1.06837	1.10932	1.14256	1.16253
	E_l	0.98933	0.77681	0.84631	0.93420	1.00000	1.04625	1.12387	1.16745	1.21139
	E_γ	0.66824	0.72392	0.82160	0.95574	1.00000	1.08248	1.10874	1.14809	1.16696
R_G	E_τ	1.73602	1.40989	1.29655	1.02091	1.00000	0.87209	0.73199	0.58129	0.49297
	E_l	1.23239	0.97912	0.94187	0.98263	1.00000	1.11458	1.10398	1.13974	1.17347
	E_γ	0.68604	0.77824	0.87065	0.85955	1.00000	1.06049	1.16377	1.21458	1.31401
R_E	E_τ	0.20321	0.38802	0.22031	0.74089	1.00000	0.81287	0.56315	0.43954	0.31437
	E_l	0.37858	0.18556	0.59450	0.78300	1.00000	0.88736	0.49702	0.44176	0.34283
	E_γ	0.28271	0.37227	0.55862	0.79059	1.00000	0.83293	0.55672	0.42128	0.31865

Table 5 MA(1) - R_H, R_A, R_D, R_G and R_E values for NNBD $t = 6, r = 30$ and $\alpha = 1$

MA(1)		$\rho=-0.4$	$\rho=-0.3$	$\rho=-0.2$	$\rho=-0.1$	$\rho=0$	$\rho=0.1$	$\rho=0.2$	$\rho=0.3$	$\rho=0.4$
R_H	E_τ	0.81306	0.87290	0.88637	0.95041	1.00000	1.06427	1.17653	1.30554	1.48406
	E_l	0.83938	0.86753	0.88976	0.94990	1.00000	1.07059	1.17567	1.31853	1.61137
	E_γ	0.83063	0.83380	0.87932	0.95711	1.00000	1.08149	1.18381	1.32796	1.52774
R_A	E_τ	1.86872	1.56309	1.35701	1.13493	1.00000	0.86109	0.75763	0.61426	0.45750
	E_l	0.89214	0.91607	0.95486	1.00367	1.00000	1.10168	1.10623	1.21367	1.48584
	E_γ	0.94657	0.92795	0.87022	1.05528	1.00000	1.16852	1.27713	1.59889	1.62079
R_D	E_τ	0.66748	0.79552	0.84534	0.93460	1.00000	1.04624	1.10775	1.11528	1.12876
	E_l	0.74005	0.80019	0.86031	0.93618	1.00000	1.05731	1.11667	1.16043	1.25151
	E_γ	0.70136	0.75171	0.84213	0.94158	1.00000	1.06839	1.11933	1.15073	1.17200
R_G	E_τ	1.53412	1.42454	1.29418	1.11604	1.00000	0.84623	0.71333	0.52474	0.31714
	E_l	0.78657	0.84496	0.92326	0.98917	1.00000	1.08803	1.05072	1.06815	1.13281
	E_γ	0.79927	0.83659	0.83342	1.03816	1.00000	1.15436	1.20756	1.38551	1.69864
R_E	E_τ	0.29554	0.46831	0.57965	0.76171	1.00000	0.71332	0.51385	0.33502	0.18494
	E_l	0.39038	0.49585	0.62406	0.74412	1.00000	0.81210	0.54731	0.38071	0.25370
	E_γ	0.33012	0.41470	0.57104	0.73282	1.00000	0.75395	0.51760	0.34924	0.18346

Table 6 ARMA(1,1) - R_H, R_A, R_D, R_G and R_E values for NNBD $t = 6, r = 30$ and $\alpha = 1$

ARMA (1,1)		$\rho_1=-0.4$ $\rho_2=-0.4$	$\rho_1=-0.3$ $\rho_2=-0.3$	$\rho_1=-0.2$ $\rho_2=-0.2$	$\rho_1=-0.1$ $\rho_2=-0.1$	$\rho_1=0$ $\rho_2=0$	$\rho_1=0.1$ $\rho_2=0.1$	$\rho_1=0.2$ $\rho_2=0.2$	$\rho_1=0.3$ $\rho_2=0.3$	$\rho_1=0.4$ $\rho_2=0.4$
R_H	E_τ	1.61665	1.29861	1.09650	0.90622	1.00000	1.09023	1.22625	1.83774	2.15711
	E_l	1.51890	1.77775	1.14375	1.01721	1.00000	1.07553	1.39113	1.70890	1.98760
	E_γ	0.94790	1.39883	1.11723	1.04661	1.00000	1.10188	1.39116	1.99808	1.99897
R_A	E_τ	2.86234	2.71432	1.82587	1.28585	1.00000	0.73195	0.45467	0.25593	0.23836
	E_l	1.35838	1.28848	0.92658	0.92199	1.00000	1.24563	1.27625	1.47355	1.60108
	E_γ	0.97760	0.83002	0.92133	0.88979	1.00000	1.24768	1.40949	1.67064	1.72301
R_D	E_τ	0.99559	0.89055	0.93523	0.84926	1.00000	1.03193	0.92058	0.90077	0.90214
	E_l	0.88983	1.43282	1.04002	0.98194	1.00000	1.02545	1.19841	1.19478	1.38984
	E_γ	1.91865	1.14741	0.96832	0.95574	1.00000	1.04569	1.07585	1.18192	1.27654
R_G	E_τ	1.76278	1.86140	1.55732	1.20503	1.00000	0.84234	0.34133	0.12544	0.22336
	E_l	0.79579	1.03848	0.84255	0.89002	1.00000	1.18763	1.17720	1.32107	1.31403
	E_γ	0.19787	0.68081	0.79854	0.81254	1.00000	1.18405	1.32204	1.44180	1.53403
R_E	E_τ	0.14236	0.19702	0.35713	0.48975	1.00000	0.53588	0.23090	0.09060	0.00867
	E_l	0.13541	0.27493	0.45441	0.62825	1.00000	0.62983	0.36037	0.29192	0.18219
	E_γ	0.41038	0.31932	0.35525	0.61146	1.00000	0.54712	0.24009	0.21890	0.17624

5. Comparison of Measures of Efficiency of NNBIBD

In this section, we study the behavior of some estimators of ρ and σ_ε^2. The nearest neighbour balanced incomplete block design (NNBIBD) data sets were generated with the following true parameters: $\rho = -0.4$ to 0.4, $\sigma_\varepsilon^2 = 1, t = 5, r = 16$ and $t = 6, r = 25$. Tables 7, 8 and 9 show the efficiencies of AR(1), MA(1) and ARMA(1,1) models with $t = 5$, $r = 16$ and $\alpha = 1$, there is considerable advantage in using NNBIBD as far as average variance (R_A and R_G), generalized variance (R_H and R_D) and min-max variance (R_E) are concerned. The efficiency factor (E_τ) for direct effects of the neighbour, left and right (E_l) and (E_γ) neighbour effects of treatments is obtained by Tomer et al. (2005). The R_H and R_D show increasing efficiency values with ρ in the interval 0.1 to 0.4 for direct, left and right neighbour effects for AR(1) and MA(1) models. Whereas neither increasing nor decreasing efficiency values are observed for R_A and R_G for both AR(1) and MA(1) models. In the case of ARMA(1,1) model, neither increasing nor decreasing efficiency values are observed for average variance and generalized variance. The R_E shows decreasing efficiency values with ρ in the interval 0.1 to 0.4 for direct, left and right neighbour effects for AR(1), MA(1) and ARMA(1,1) models. We have concluded that, the higher efficiency values are observed for direct, left and right neighbour effects of treatments for AR(1) and MA(1) models for average variance. The lower efficiency values are observed for direct, left and right neighbour effects of treatments for AR(1), MA(1) and ARMA(1,1) models for min-max variance.

Table 7 AR(1) - R_H, R_A, R_D, R_G and R_E values for NNBIBD $t = 5, r = 16$ and $\alpha = 1$

AR(1)		$\rho=-0.4$	$\rho=-0.3$	$\rho=-0.2$	$\rho=-0.1$	$\rho=0$	$\rho=0.1$	$\rho=0.2$	$\rho=0.3$	$\rho=0.4$
R_H	E_τ	0.77786	0.83529	0.88792	0.99261	1.00000	1.01727	1.09963	1.15674	1.19348
	E_l	0.89672	0.83497	0.96549	0.95583	1.00000	1.09572	1.11314	1.14588	1.25147
	E_γ	0.88430	0.89980	0.94050	0.93840	1.00000	1.05232	1.07227	1.15538	1.24844
R_A	E_τ	1.27944	1.28998	1.13598	1.08621	1.00000	0.88803	0.94038	0.90180	0.86450
	E_l	1.01177	1.24454	1.34190	0.97799	1.00000	0.76243	0.93141	0.91137	1.06250
	E_γ	1.05091	0.65846	0.96372	0.84897	1.00000	1.07127	1.06240	1.19190	1.19601
R_D	E_τ	0.69488	0.78461	0.86158	0.98803	1.00000	1.01377	1.07613	1.10434	1.11442
	E_l	0.81570	0.80860	0.93505	0.95152	1.00000	1.08822	1.09513	1.10064	1.18071
	E_γ	0.79788	0.81662	0.90576	0.93376	1.00000	1.04854	1.04827	1.11896	1.16330
R_G	E_τ	1.14295	1.21171	1.10228	1.08120	1.00000	0.88498	0.92028	0.86094	0.80724
	E_l	0.92036	1.20524	1.29959	0.97358	1.00000	0.75721	0.91634	0.87539	1.00242
	E_γ	0.94821	0.59759	0.92812	0.84477	1.00000	1.06743	1.03862	1.15433	1.11445
R_E	E_τ	0.53018	0.57122	0.69325	0.93115	1.00000	0.92787	0.71842	0.65256	0.58474
	E_l	0.45317	0.75637	0.68621	0.90216	1.00000	0.83254	0.73473	0.64376	0.51054
	E_γ	0.44395	0.46696	0.65170	0.89680	1.00000	0.92690	0.78959	0.69013	0.54842

Tables 10, 11 and 12 show the efficiencies of AR(1), MA(1) and ARMA(1,1) models with $t = 6$, $r = 25$ and $\alpha = 1$, there is considerable advantage in using NNBIBD as far as average variance (R_A and R_G), generalized variance (R_H and R_D) and min-max variance (R_E) are concerned. The efficiency factor (E_τ) for direct effects of the neighbour, left and right (E_l) and (E_γ) neighbour effects of treatments is obtained by Tomer et al. (2005). The R_H and R_D show increasing efficiency values for direct, left and right neighbour effects whereas neither increasing nor decreasing efficiency values are observed for R_A and R_G for both AR(1) and MA(1) models. In the case of ARMA(1,1) model, neither increasing nor decreasing efficiency values are observed for average variance and generalized variance. The R_E show decreasing efficiency values with ρ in the interval 0.1 to 0.4 for direct, left and right neighbour effects for AR(1), MA(1) and ARMA(1,1) models. We have concluded that, the higher efficiency values are observed for direct effects of treatments for AR(1), MA(1) and ARMA(1,1) models for average variance. The lower efficiency values are observed for direct, left and right neighbour effects of treatments for AR(1), MA(1) and ARMA(1,1) models for min-max variance.

Table 8 MA(1) - R_H, R_A, R_D, R_G and R_E values for NNBIBD $t=5, r=16$ and $\alpha=1$

MA(1)		$\rho=-0.4$	$\rho=-0.3$	$\rho=-0.2$	$\rho=-0.1$	$\rho=0$	$\rho=0.1$	$\rho=0.2$	$\rho=0.3$	$\rho=0.4$
R_H	E_τ	0.86083	0.88038	0.89597	0.96944	1.00000	1.06681	1.11907	1.18888	1.26490
	E_l	0.88099	0.83125	0.93124	1.00291	1.00000	1.03877	1.09574	1.23675	1.35791
	E_γ	0.84355	0.86391	0.91231	0.94774	1.00000	1.02830	1.08648	1.23418	1.35574
R_A	E_τ	1.13284	1.40250	1.19237	1.07885	1.00000	0.97629	0.87999	0.97257	0.83735
	E_l	1.10314	1.10937	1.08170	0.75168	1.00000	0.96844	0.94299	0.92745	0.95085
	E_γ	0.79164	0.71453	0.92204	0.87113	1.00000	1.07506	1.15091	1.47112	1.55587
R_D	E_τ	0.80087	0.83408	0.87537	0.96886	1.00000	1.05467	1.07995	1.09565	1.09872
	E_l	0.70347	0.82745	0.91711	0.99903	1.00000	1.01769	1.05690	1.13925	1.14744
	E_γ	0.79398	0.83028	0.89981	0.94494	1.00000	1.02287	1.06245	1.15902	1.17491
R_G	E_τ	1.05393	1.32875	1.16495	1.07821	1.00000	0.96518	0.84922	0.89630	0.68176
	E_l	0.88086	1.10430	1.06528	0.74877	1.00000	0.94880	0.90957	0.85430	0.80347
	E_γ	0.74512	0.68672	0.90942	0.86856	1.00000	1.06938	1.12545	1.38153	1.34835
R_E	E_τ	0.63365	0.57701	0.73024	0.99507	1.00000	0.92787	0.71842	0.65256	0.58474
	E_l	0.55367	0.90369	0.77949	0.89515	1.00000	0.68152	0.63403	0.51547	0.36212
	E_γ	0.52558	0.66838	0.79267	0.91481	1.00000	0.86625	0.77192	0.56403	0.39041

Table 9 ARMA(1,1) - R_H, R_A, R_D, R_G and R_E values for NNBIBD $t=5, r=16$ and $\alpha=1$

ARMA (1,1)		$\rho_1=-0.4$ $\rho_2=-0.4$	$\rho_1=-0.3$ $\rho_2=-0.3$	$\rho_1=-0.2$ $\rho_2=-0.2$	$\rho_1=-0.1$ $\rho_2=-0.1$	$\rho_1=0$ $\rho_2=0$	$\rho_1=0.1$ $\rho_2=0.1$	$\rho_1=0.2$ $\rho_2=0.2$	$\rho_1=0.3$ $\rho_2=0.3$	$\rho_1=0.4$ $\rho_2=0.4$
R_H	E_τ	1.71592	1.26114	1.29357	1.01964	1.00000	1.03598	1.14253	1.37358	1.81046
	E_l	2.59153	1.49485	1.12954	1.05501	1.00000	1.02735	1.22523	2.01832	2.09753
	E_γ	1.72077	1.38987	1.15548	1.04394	1.00000	1.03976	1.15657	1.67583	2.14000
R_A	E_τ	1.47312	1.55894	1.86328	1.15464	1.00000	0.76940	0.84150	0.88997	1.27818
	E_l	0.72351	1.01397	0.89693	1.05316	1.00.68	0.84368	1.01321	2.24453	2.23429
	E_γ	1.27917	0.95827	0.88167	0.99717	1.00000	1.15908	1.50302	2.32502	2.45010
R_D	E_τ	1.40212	1.05921	1.17711	1.01432	1.00000	0.99693	0.97581	0.85306	0.50820
	E_l	1.87295	1.31061	1.08719	1.04027	1.00000	0.99888	1.08788	1.36470	1.46250
	E_γ	1.56621	1.26519	1.10650	1.03511	1.00000	1.01353	1.04043	1.23131	1.35643
R_G	E_τ	1.20373	1.30932	1.69553	1.14862	1.00000	0.74039	0.71870	0.55271	0.35879
	E_l	0.52290	0.88900	0.86330	1.03844	1.00000	0.82030	0.89962	1.21632	0.96622
	E_γ	1.16427	0.84279	0.84430	0.98873	1.00000	1.12984	1.35209	1.70830	1.52313
R_E	E_τ	0.39495	0.42941	0.47639	0.90605	1.00000	0.92787	0.44354	0.23878	0.08087
	E_l	0.21028	0.45689	0.62885	0.80132	1.00000	0.67754	0.43285	0.10564	0.09841
	E_γ	0.42202	0.47483	0.63525	0.85213	1.00000	0.73622	0.49250	0.26196	0.09723

Table 10 AR(1) - R_H, R_A, R_D, R_G and R_E values for NNBIBD $t=6, r=25$ and $\alpha=1$

AR(1)		$\rho=-0.4$	$\rho=-0.3$	$\rho=-0.2$	$\rho=-0.1$	$\rho=0$	$\rho=0.1$	$\rho=0.2$	$\rho=0.3$	$\rho=0.4$
R_H	E_τ	0.75715	0.81741	0.87770	0.94752	1.00000	1.07554	1.13153	1.18912	1.25529
	E_l	0.78991	0.82209	0.87333	0.98171	1.00000	1.07280	1.13023	1.21005	1.28822
	E_γ	0.76911	0.79349	0.89559	0.92212	1.00000	1.05652	1.14089	1.19583	1.27012
R_A	E_τ	1.54026	1.31995	1.20874	1.07690	1.00000	0.96936	0.90038	0.85262	0.84582
	E_l	1.03521	0.93301	0.97859	1.18969	1.00000	1.01818	1.11520	1.16400	1.20850
	E_γ	0.77878	1.16049	1.19915	0.86793	1.00000	1.04292	1.16891	1.16952	1.28236
R_D	E_τ	0.63806	0.748000	0.84232	0.93948	1.00000	1.06938	1.09691	1.11265	1.12397
	E_l	0.70426	0.76875	0.84913	0.95630	1.00000	1.06987	1.10394	1.15359	1.18623
	E_γ	0.67859	0.71308	0.86226	0.91027	1.00000	1.05014	1.11379	1.12729	1.14820
R_G	E_τ	1.29801	1.20786	1.16001	1.06776	1.00000	0.96381	0.87283	0.79778	0.80211
	E_l	0.92296	0.87248	0.95148	1.15890	1.00000	1.01540	1.08926	1.10968	1.11283
	E_γ	0.68709	1.04288	1.15452	0.85677	1.00000	1.03662	1.14114	1.10249	1.15926
R_E	E_τ	0.33526	0.46349	0.58736	0.85204	1.00000	0.83486	0.64981	0.54126	0.47121
	E_l	0.36148	0.52131	0.66916	0.65587	1.00000	0.89743	0.69088	0.58654	0.52776
	E_γ	0.38828	0.38737	0.64417	0.76051	1.00000	0.85220	0.67219	0.55838	0.47193

Table 11 MA(1) - R_H, R_A, R_D, R_G and R_E values for NNBIBD $t=6, r=25$ and $\alpha=1$

MA(1)		$\rho=-0.4$	$\rho=-0.3$	$\rho=-0.2$	$\rho=-0.1$	$\rho=0$	$\rho=0.1$	$\rho=0.2$	$\rho=0.3$	$\rho=0.4$
R_H	E_τ	0.79925	0.84719	0.89198	0.93825	1.00000	1.06842	1.14266	1.22395	1.32585
	E_l	0.82072	0.84036	0.88665	0.92991	1.00000	1.06064	1.15554	1.24376	1.38881
	E_γ	0.78236	0.78786	0.88938	0.94766	1.00000	1.05418	1.13472	1.23931	1.34378
R_A	E_τ	1.43244	1.27030	1.15913	1.10366	1.00000	0.95051	0.85093	0.83533	0.82640
	E_l	0.80049	0.91451	0.96212	0.91096	1.00000	1.11134	1.08876	1.16081	1.27921
	E_γ	0.96247	0.89751	0.95033	0.95515	1.00000	1.05072	1.16246	1.33932	1.71044
R_D	E_τ	0.70495	0.78941	0.86471	0.93623	1.00000	1.05878	1.09525	1.09093	1.12666
	E_l	0.73048	0.79861	0.86714	0.92454	1.00000	1.05143	1.11810	1.14792	1.15737
	E_γ	0.62016	0.74205	0.86729	0.94049	1.00000	1.04549	1.09393	1.11950	1.15923
R_G	E_τ	1.26342	1.18365	1.12369	1.10129	1.00000	0.94193	0.81563	0.74454	0.63992
	E_l	0.71248	0.86909	0.94095	0.90569	1.00000	1.10170	1.05348	1.07136	0.97393
	E_γ	0.76294	0.84532	0.92673	0.94792	1.00000	1.04206	1.12066	1.20984	1.34825
R_E	E_τ	0.42503	0.52744	0.63889	0.92392	1.00000	0.79331	0.58556	0.41827	0.27592
	E_l	0.42951	0.58203	0.72463	0.86101	1.00000	0.80398	0.62561	0.48249	0.36456
	E_γ	0.52545	0.55748	0.77617	0.80675	1.00000	0.82280	0.60038	0.43560	0.28557

Table 12 ARMA(1,1) - R_H, R_A, R_D, R_G and R_E values for NNBIBD $t = 6, r = 25$ and $\alpha = 1$

ARMA (1,1)		$\rho_1 = -0.4$ $\rho_2 = -0.4$	$\rho_1 = -0.3$ $\rho_2 = -0.3$	$\rho_1 = -0.2$ $\rho_2 = -0.2$	$\rho_1 = -0.1$ $\rho_2 = -0.1$	$\rho_1 = 0$ $\rho_2 = 0$	$\rho_1 = 0.1$ $\rho_2 = 0.1$	$\rho_1 = 0.2$ $\rho_2 = 0.2$	$\rho_1 = 0.3$ $\rho_2 = 0.3$	$\rho_1 = 0.4$ $\rho_2 = 0.4$
R_H	E_τ	1.52232	1.25590	1.09540	1.01367	1.00000	1.05858	1.19610	1.39511	1.34311
	E_l	1.84867	1.31759	1.29502	1.04689	1.00000	1.05439	1.24908	1.74777	1.22016
	E_γ	1.76384	1.79428	1.11238	1.01631	1.00000	1.09183	1.23092	1.57102	1.16281
R_A	E_τ	1.96092	1.65797	1.38649	1.17350	1.00000	0.89106	0.81524	0.83425	1.46755
	E_l	0.99938	0.89095	1.08132	0.94590	1.00000	1.11011	1.19732	1.36251	1.41134
	E_γ	0.95179	0.97118	1.00614	0.90295	1.00000	1.18075	1.57642	1.30807	1.49198
R_D	E_τ	1.11823	1.02162	0.98559	0.98581	1.00000	1.01768	0.98571	0.79179	0.86027
	E_l	1.53911	1.16715	1.16601	1.02631	1.00000	1.02261	1.11055	1.51474	1.42191
	E_γ	1.44149	1.24356	1.02314	0.98868	1.00000	1.05391	1.03416	0.88280	1.13944
R_G	E_τ	1.44042	1.34870	1.24750	1.14125	1.00000	0.85664	0.67185	0.47347	0.64453
	E_l	0.83203	0.78923	0.97360	0.92730	1.00000	1.07665	1.06453	1.91419	1.56911
	E_γ	0.77785	0.67309	0.92543	0.87841	1.00000	1.13975	1.32443	1.73118	1.63531
R_E	E_τ	0.26028	0.33942	0.44471	0.70276	1.00000	0.60797	0.31515	0.13047	0.18893
	E_l	0.28127	0.41457	0.37418	0.74233	1.00000	0.63558	0.41272	0.36627	0.10550
	E_γ	0.36160	0.17001	0.56562	0.65460	1.00000	0.62718	0.33230	0.12040	0.14824

6. Results and Conclusions

We have compared the efficiencies of NNBD using average variance, generalized variance and min-max variance when the errors follow first order correlated models. The R_H and R_D show increasing efficiency values for direct, left and right neighbour effects for MA(1) models. The R_A and R_G show neither increasing nor decreasing efficiency values are observed for AR(1), MA(1) and ARMA(1,1) models. The R_E show decreasing efficiency values with ρ in the interval 0.1 to 0.4 for direct and neighbouring effects for AR(1), MA(1) and ARMA(1,1) models. Finally, we have concluded that, the efficiencies of NNBD using the three measures when the errors follow the first order correlated models. The higher efficiency values are observed for direct effects of treatments for MA(1) and ARMA(1,1) models for average variance. The lower efficiency values are observed for direct, left and right neighbour effects of treatments for ARMA(1,1) model for min-max variance.

We have compared the efficiencies of NNBIBD using average variance, generalized variance and min-max variance when the errors follow the first order correlated models. The R_H and R_D show increasing efficiency values with ρ in the interval 0.1 to 0.4 for direct, left and right neighbour effects for AR(1) and MA(1) models. Whereas neither increasing nor decreasing efficiency values are observed for R_A and R_G for both AR(1) and MA(1) models. In the case of ARMA(1,1) model, neither increasing nor decreasing efficiency values are observed for average variance and generalized variance. The R_E show decreasing efficiency values with ρ in the interval 0.1 to 0.4 for direct, left and right neighbour effects for AR(1), MA(1) and ARMA(1,1) models. Finally, we have concluded that, the efficiencies of NNBIBD using the three measures when the errors follow the first order correlated models. The higher efficiency values are observed for direct, left and right neighbour effects of treatments for AR(1) and MA(1) models for average variance. The lower efficiency values are observed for direct, left and right neighbour effects of treatments for AR(1), MA(1) and ARMA(1,1) models for min-max variance.

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