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Gompertz-Alpha Power Inverted Exponential Distribution: Properties and Applications

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Abstract

This article proposes the Gompertz-Alpha power inverted exponential distribution for lifetime processes. The statistical properties of the distribution such as survival, hazard, reversed hazard, cumulative, odd functions, quantiles, order statistics and entropies were derived. The parameters of the new distribution were obtained by maximum likelihood method. A simulation study was performed to test the flexibility of the propose model. However, the flexibility of the new distribution was also examined using two real life data. The goodness-of-fit of the proposed model indicates that the new distribution perform favourably when compare with existing distributions.

Keywords: Alpha power transformation, Gompertz distribution, maximum likelihood estimation, odd functions.

1. Introduction

Lifetime processes has received several attentions recently through modeling the manner in which they are distributed. Thus, developing compound but flexible distributions depends on the researcher's ability to compound one or more distributions to form a better or a comparable distribution in Lee et al. (2013). The exponential distribution is widely used to describing the time between events with a Poisson process. However, the exponential distribution is used to model processes with continuous memoryless random processes with constant failure rates. The occurrence of these constant failure rates is almost impossible in real life. However, to account for this disadvantage, Keller et al. (1982) introduced the inverted exponential (IE) distribution with inverted bathtub hazard rate. The inverted exponential distribution was further studied extensively in Lin et al. (1989) and applied in engineering and medicine in Oguntunde (2019), Oguntunde (2017), Oguntunde et al. (2017a) and Oguntunde et al. (2017b). Oguntunde et al. (2015) proposed the transmuted inverse exponential distribution. The statistical properties of the exponentiated generalized inverted exponential distribution was examined in Oguntunde et al. (2014a). Oguntunde et al. (2014b) proposed the Kumaraswamy inverse exponential distribution. Yousof et al. (2015) proposed the transmuted exponentiated generalized-G family. Afify et al. (2017) proposed the odd exponentiated half-logistic-G family and tangent function was proposed in Al-Moflel (2018). Anake et al. (2015) proposed a fractional beta exponential distribution. Aryal and Yousof (2017) proposed the exponentiated generalized-G Poisson distribution.

Cordeiro et al. (2013) proposed the exponentiated generalized class. Oguntunde et al. (2013) proposed the sum of the exponential distributed. Pinho et al. (2015) proposed the Harris extended exponential distribution. Abouammoh and Alshingiti (2009) proposed the generalized inverted exponential (GIE) distribution method whose parameters were estimated in Dey et al. (2017). Mahdavi and Kundu (2017) proposed a for generating distributions with an application to exponential distribution. Eghwerido et al. (2019) proposed extended new generalized exponential distribution. Nadarajah and Okorie (2017) extended the work by introducing a third parameter.

Also, the Gompertz distribution is a continuous distribution used to describe the lifespan of stochastic processes. Hence, there is a relationship exit between the inverted exponential and the Gompertz distributions.

However, many Gompertz distributions have been developed, but little knowledge have been developed for alpha power inverted exponential distribution. Hence, this article seeks to develop a distribution that has the characterization of the Gompertz distribution and the alpha power inverted exponential called Gompertz alpha power inverted exponential (GAPIE) distribution. This distribution is further applied to glass fibre and carbon dataset to examine its efficiency.

The Gompertz-G family of distribution was proposed by Alizadeh et al. (2017) with a cumulative distribution function given as

$$F(x)=\int_0^{B[G(x)]}u(t)dt, \tag{1}$$

where $u(t)$ is the probability density function (pdf) of the Gompertz distribution and the link function

$$B\left[G(x)=-\log\left[1-G(x)\right]\right].$$

The cumulative distribution function (cdf) of (1) is explicitly expressed in Alizadeh et al. (2017) as

$$F(x)=\int_0^{=\log[1-G(x)]}a\exp\left(bt-\frac{a}{b}\left(\exp(bt)-1\right)\right)dt=1-\exp\left(\frac{a}{b}\left(1-\left(1-G(x)\right)^{-b}\right)\right),\ \ a>0, b>0, \tag{2}$$

where b and a are the shape and scale parameters, respectively. The corresponding pdf of the G-family of distribution is given as

$$f(x)=\left[\frac{d}{dx}B\left[G(x)\right]\right]u\left[B\left[G(x)\right]\right] \tag{3}$$

which implies that

$$f(x)=ag(x)\left[1-G(x)\right]^{-b-1}\exp\left(\frac{a}{b}\left(1-\left(1-G(x)\right)^{-b}\right)\right). \tag{4}$$

Also, the inverted exponential (IE) distribution has an inverted bathtub hazard rate function with the pdf given in Unal et al. (2018) as

$$f_{IE}(x)=\frac{c}{x^2}\exp\left(-\frac{c}{x}\right),\quad x>0, c>0, \tag{5}$$

where c is an additional parameter. Also, the cdf of the IE distribution is expressed as

$$F_{IE}(x)=\exp\left(-\frac{c}{x}\right),\quad x>0, c>0. \tag{6}$$

Mahdavi and Kundu (2017) proposed a G family of distributions called the alpha power (AP) with cdf given by

$$F_{AP}(x)=\frac{\alpha^{G(x)}-1}{\alpha-1},\quad \alpha>0, \alpha\neq 1, \tag{7}$$

with its pdf expressed as

$$f_{AP}(x) = \frac{\log\alpha}{(\alpha-1)} g(x)\alpha^{G(x)}, \qquad \alpha > 0, \quad \alpha \neq 1, \tag{8}$$

where $g(x)$ and $G(x)$ are the baseline pdf and cdf, respectively. This family of distribution has been studied by many authors. Nassar et al. (2017) studied the situation when $g(x)$ and $G(x)$ correspond to the Weibull distribution. Dey et al. (2017) represented $g(x)$ and $G(x)$ as the generalized exponential distribution of the Gupta and Kundu (2001). Unal et al. (2018) studied the case when $g(x)$ and $G(x)$ correspond to the inverted exponential distribution and gave the cdf of the alpha power inverted exponential (APIE) distribution as

$$F_{APIE}(x) = \frac{\alpha^{\exp\left(-\frac{c}{x}\right)} - 1}{\alpha - 1}, \quad \alpha > 0, \alpha \neq 1, \tag{9}$$

with its pdf given as

$$f_{APIE}(x) = \frac{c\log\alpha}{x^2(\alpha-1)} \exp\left(-\frac{c}{x}\right) \alpha^{\exp\left(-\frac{c}{x}\right)}, \quad \alpha > 0, \alpha \neq 1. \tag{10}$$

In this article, the GAPIE distribution with its statistical properties will be studied intensely. This article is organized as follows: The introduction was given in Section 1; Section 2 is the formulation of the GAPIE distribution together with the maximum likelihood of its parameters. In Section 3, we discuss some properties of the GAPIE distribution. Section 4 is the application to examine the proposed distribution. The results obtained are compared with existing distributions in this section also. Section 5 is the conclusion.

2. The Gompertz-Alpha Power Inverted Exponential Distribution

In this article, we shall propose a four parameter class of distribution called the GAPIE distribution.

Let $x_1, x_2, \ldots, x_n$ be a random sample of the GAPIE distribution. Now, suppose the link function is $B[D(x)] = -\log[1 - G(x)]$. Then, the pdf of the GAPIE that corresponds to (4) is given as

$$f_{GAPIE}(x) = \frac{ac\log\alpha}{x^2(\alpha-1)} \exp\left(-\frac{c}{x}\right) \alpha^{\exp\left(-\frac{c}{x}\right)} \left(\frac{\alpha-1}{\alpha - \alpha^{\exp\left(-\frac{c}{x}\right)}}\right)^{b+1} \exp\left(\frac{a}{b}\left(1 - \left[\frac{\alpha-1}{\alpha - \alpha^{\exp\left(-\frac{c}{x}\right)}}\right]^b\right)\right), \tag{11}$$

where c and α additional parameters. Figure 1 shows the pdf of the GAPIE distribution for various values of parameters.

From Figure 1, the shape of the GAPIE distribution could be inverted bathtub pdf depending on the value of the parameters. Also, it could be negatively skewed or positively skewed.

The cdf of the GAPIE is given as

$$F_{GAPIE}(x) = 1 - \exp\left(\frac{a}{b}\left(1 - \left[\frac{\alpha-1}{\alpha - \alpha^{\exp\left(-\frac{c}{x}\right)}}\right]^b\right)\right), \quad \alpha > 0, \alpha \neq 1, c, b > 0. \tag{12}$$

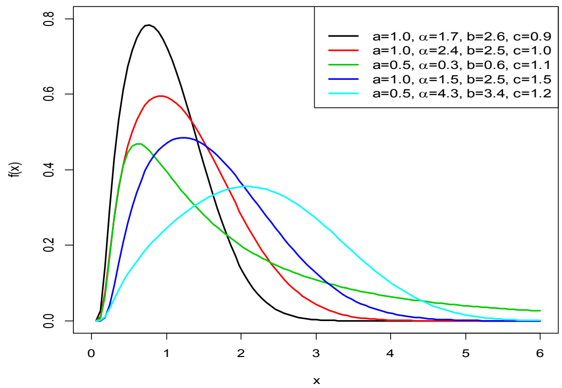

Figure 1 The pdf of the GAPIE distribution for different parameter values

Figure 2 is the shape of the cdf of the GAPIE distribution. It shows that it is increasing depending on the value of the parameters.

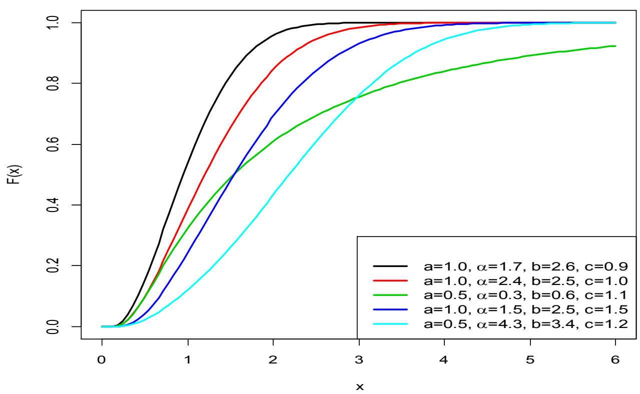

Figure 2 The cdf of the GAPIE distribution for different parameter values

2.1. Parameter estimation for the Gompertz-Alpha power inverted exponential distribution formulation

The parameters of the GAPIE distribution can be obtained by maximum likelihood (MLE) as follows. Let $x_1, x_2, \ldots, x_n$ be a random sample from an infinite population with a pdf $f_{GAPIE}(x)$ at the point x with distribution of vector of parameter $\Theta = (a, b, c, \alpha)^u$, then the likelihood function is given by

$$\prod_{i=1}^{n} f(x_1, x_2 \ldots, x_n; a, b, c, \alpha). \tag{13}$$

Let ℓ be the log-likelihood function, then

$$\ell = \log \prod_{i=1}^{n} f(x_1, x_2 \ldots, x_n; a, b, c, \alpha) = n \log a + \sum_{i=1}^{n} \log g(x)$$
$$-(b+1)\sum_{i=1}^{n} \log(1 - G(x)) + \frac{an}{b} - \frac{a}{b}\sum_{i=1}^{n}(1 - G(x))^{-b}. \tag{14}$$

Now, differentiating the nonlinear equation with respect to the parameters, noting that

$$z_i = \frac{a}{b}\sum_{i=1}^{n}\left(1-\left(1-G(x)\right)^{-b}\right); \quad \frac{\partial z_i}{\partial b} = z_i'; \quad \frac{\partial g(x_i)}{\partial c} = c_i'; \quad \frac{\partial r_i}{\partial c} = m_i'; \quad \frac{\partial g(x_i)}{\partial \alpha} = k_i'; \quad \frac{\partial r_i}{\partial \alpha} = p_i';$$

$$\frac{\partial g(x_i)}{\partial \alpha} = \frac{\partial g(x_i)}{\partial b} = 0; \text{ and } r_i = \left(1-G(x)\right).$$

Thus,

$$U_a = nb\sum_{i=1}^{n}\left(1-\left(1-G(x)\right)^{b}\right), \tag{15}$$

$$U_b = \sum_{i=1}^{n}\log\left(1-G(x)\right) - z_i', \tag{16}$$

$$U_c = \sum_{i=1}^{n}\frac{c_i'}{g(x)} - (b+1)\sum_{i=1}^{n}\frac{m_i'}{r_i} + a\sum_{i=1}^{n}m_i', \tag{17}$$

$$U_\alpha = \sum_{i=1}^{n}\frac{k_i'}{g(x)} - \left(b+1\right)\sum_{i=1}^{n}\frac{p_i'}{r_i} + a\sum_{i=1}^{n}p_i'. \tag{18}$$

The solution might not be gotten analytically except, when data sets are available. Thus, software like MATLAB, R, MAPLE, and so on could be used to obtain the estimates. Thus, setting $U_a = U_b = U_c = U_\alpha = 0$ and solving the nonlinear equations yield the MLEs $\hat{\theta} = (\hat{a}, \hat{b}, \hat{c}, \hat{\alpha})$.

3. Some Statistical Properties of the Gompertz-Alpha Power Inverted Exponential Distribution Formulation

3.1. Reliability analysis

We shall provide the survival and hazard rate functions of the GAPIE distribution. The survival function is given as

$$S_{GAPIE}(x) = 1 - P_{(GAPIE)}(X \le x),$$

$$S_{GAPIE}(x) = \exp\left(\frac{a}{b}\left(1-\left[\frac{\alpha-1}{\alpha-\alpha^{\exp\left(-\frac{c}{x}\right)}}\right]^{b}\right)\right), \quad \alpha > 0, \alpha \ne 1, c, b > 0. \tag{19}$$

Figure 3 shows the plot for the survival function of the GAPIE distribution.

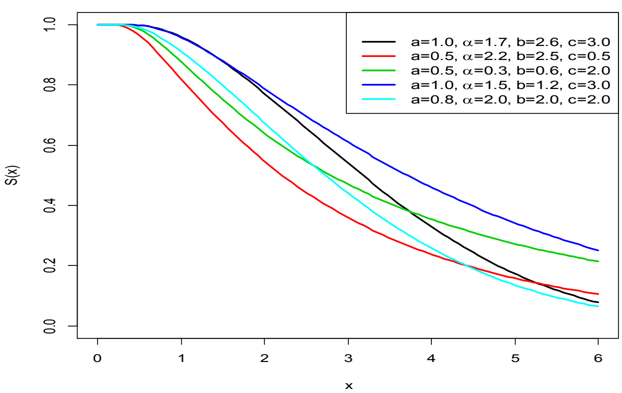

Figure 3 The survival function of the GAPIE distribution for different parameter values

3.2. Hazard rate function

The hazard rate function or failure rate for the GAPIE distribution is the conditional density given as

$$h(x)=\frac{ac\log\alpha}{x^2(\alpha-1)}\exp\left(-\frac{c}{x}\right)\alpha^{\exp\left(-\frac{c}{x}\right)}\left[\frac{\alpha-1}{\alpha-\alpha^{\exp\left(-\frac{c}{x}\right)}}\right]^{b+1}. \tag{20}$$

Plots for the hazard function of the GAPIE distribution at various selected values are displayed in Figure 4.

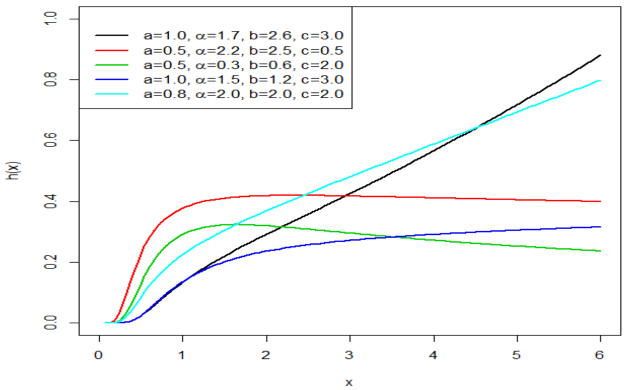

Figure 4 The hazard rate function of the GAPIE distribution for different parameter values

It can be deduced from Figure 4 that the shape of the hazard function of the GAPIE distribution could be constant, increasing, or decreasing (depending on the value of the parameters).

3.3. Cumulative hazard rate

The cumulative hazard rate function of the GAPIE distribution is given by

$$H_{(GAPIE)}(x)=-\frac{a}{b}\left(1-\left[\frac{\alpha-1}{\alpha-\alpha^{\exp\left(-\frac{c}{x}\right)}}\right]^{b}\right). \tag{21}$$

3.4. Quantile function and median

The Quantile function is derived from

$$Q(x)=F^{-1}(x). \tag{22}$$

Thus, the quantile function of the GAPIE distribution is given by

$$Q(u)=-c\left[\log\left[(\log\ \alpha)^{-1}\log\left[\alpha-(\alpha-1)\left[1-\frac{b}{a}\log(1-u)\right]^{\frac{1}{b}}\right]\right]\right]^{-1}, \tag{23}$$

where $u\sim\text{Uniform}[0,1]$. GAPIE random variate can be generated from the GAPIE distribution using

$$x=-c\left[\log\left[(\log\alpha)^{-1}\log\left[\alpha-(\alpha-1)\left[1-\frac{b}{a}\log(1-u)\right]^{\frac{1}{b}}\right]\right]\right]^{-1}. \tag{24}$$

The median of the GAPIE distribution can be obtained when $u = 0.5$ is substituting into Equation (23) as

$$\text{Median} = -c\left[\log\left[\left(\log\alpha\right)^{-1}\log\left[\alpha-(\alpha-1)\left[1-\frac{b}{a}\log(0.5)\right]^{\frac{1}{b}}\right]\right]\right]^{-1}. \tag{25}$$

Hence, the 25^{th} percentile and the 75^{th} percentile are given as

$$Q_1 = -c\left[\log\left[\left(\log\alpha\right)^{-1}\log\left[\alpha-(\alpha-1)\left[1-\frac{b}{a}\log(0.75)\right]^{\frac{1}{b}}\right]\right]\right]^{-1}, \tag{26}$$

$$Q_3 = -c\left[\log\left[\left(\log\alpha\right)^{-1}\log\left[\alpha-(\alpha-1)\left[1-\frac{b}{a}\log(0.25)\right]^{\frac{1}{b}}\right]\right]\right]^{-1}. \tag{27}$$

3.5. Distribution of order statistics

Let $x_1, x_2, \ldots, x_n$ be a random sample from an infinite population with a pdf $f_{GAPIE}(x)$ at the point x then the pdf of the k^{th} order statistics is given as

$$g_k(y_k)_{GAPIE} = \frac{n\;!}{(k-1)!(n-k)!}\left[1-\exp\left(\frac{a}{b}\left(1-\left[\frac{\alpha-1}{\alpha-\alpha^{\exp\left(-\frac{c}{x}\right)}}\right]^{b}\right)\right)\right]^{k-1}\frac{ac\log\alpha}{x^2(\alpha-1)}\exp\left(-\frac{c}{x}\right)$$

$$\times\alpha^{\exp\left(-\frac{c}{x}\right)\left[\frac{\alpha-1}{\alpha-\alpha^{\exp\left(-\frac{c}{x}\right)}}\right]^{b+1}}\left[\exp\left(\frac{a}{b}\left(1-\left[\frac{\alpha-1}{\alpha-\alpha^{\exp\left(-\frac{c}{x}\right)}}\right]^{b}\right)\right)\right]^{n-k-1}, \tag{28}$$

for $k = 1$ we have the minimum order statistics and for $k = n$ we obtain the maximum order statistics.

3.6. Reversed hazard function

The reversed hazard function of the GAPIE distribution is given by

$$r_{(GAPIE)}(x) = \frac{\frac{ac\log\alpha}{x^2(\alpha-1)}\exp\left(-\frac{c}{x}\right)\alpha^{\exp\left(-\frac{c}{x}\right)\left[\frac{\alpha-1}{\alpha-\alpha^{\exp\left(-\frac{c}{x}\right)}}\right]^{b+1}}\exp\left(\frac{a}{b}\left(1-\left[\frac{\alpha-1}{\alpha-\alpha^{\exp\left(-\frac{c}{x}\right)}}\right]^{b}\right)\right)}{1-\exp\left(\frac{a}{b}\left(1-\left[\frac{\alpha-1}{\alpha-\alpha^{\exp\left(-\frac{c}{x}\right)}}\right]^{b}\right)\right)}. \tag{29}$$

3.7. Odds function

The odds function of the GAPIE distribution is given by

$$O_{(GAPIE)}(x)=\exp\left(-\frac{a}{b}\left(1-\left[\frac{\alpha-1}{\alpha-\alpha^{\exp\left(-\frac{c}{x}\right)}}\right]^b\right)\right)-1 \text{ for } \alpha>0,\alpha\neq 1,c,b>0. \quad (30)$$

3.8. The entropies

The measure of the uncertainty of the random variable X is either measure as Renyi entropy which is defined as

$$I_R(\theta)=\frac{1}{1-\theta}\log\left[\int_0^\infty f_{GAPIE}^{\theta}(x)dx\right], \text{ for } \theta>0 \text{ and } \theta\neq 1. \quad (31)$$

Also, the Shannon entropy of the random variable X is defined as

$$E\left[-\log\left[f(x)\right]\right]=-\log a-E\left[\log g(x)\right]+(b+1)E\left[\log\left(1-G(x)\right)\right]-\frac{a}{b}\left[1-E\left(1-G(x)\right)^b\right]. \quad (32)$$

4. Applications

A gas fiber and carbon data real life datasets are applied to the proposed model to examine the performance of the model based on its statistic. Several criteria were used to determine the distribution for the best fit: Akaike Information Criteria (AIC), Consistent Akaike Information Criteria (CAIC), Bayesian Information Criteria (BIC), and Hannan and Quinn Information Criteria (HQIC). The Anderson Darling (A) statistic, Cramér-von Mises statistic (W), Kolmogorov-Smirnov (KS) statistic, and the p-value were also provided.

4.1. Glass fibres data

The data on 1.5 cm strengths of glass fibres were obtained by workers at the UK National Physical Laboratory was also used to compare the performance of the GAPIE distribution as used by Smith and Naylor (1987), Haq et al. (2016), Bourguinon et al. (2014), Merovci et al. (2016), Rastogi and Oguntunde (2019), and Oguntunde et al. (2017a). The parameters were obtained using R (R Core Team 2013). The observations are as follows:

0.55, 0.74, 0.77, 0.81, 0.84, 1.24, 0.93, 1.04, 1.11, 1.13, 1.30, 1.25, 1.27, 1.28, 1.29, 1.48, 1.36, 1.39, 1.42, 1.48, 1.51, 1.49, 1.49, 1.50, 1.50, 1.55, 1.52, 1.53, 1.54, 1.55, 1.61, 1.58, 1.59, 1.60, 1.61, 1.63, 1.61, 1.61, 1.62, 1.62, 1.67, 1.64, 1.66, 1.66, 1.66, 1.70, 1.68, 1.68, 1.69, 1.70, 1.78, 1.73, 1.76, 1.76, 1.77, 1.89, 1.81, 1.82, 1.84, 1.84, 2.00, 2.01, 2.24.

The descriptive statistics of the glass fibres dataset are shown in Table 1. Table 2 shows the measure of comparison for the various distributions under consideration.

Table 1 Descriptive statistics for the glass fibres dataset to 2 decimal points

Mean	Median	Mode	St.D.	IQR	Variance	Skewness	Kurtosis	25^{th} percent	75^{th} percent
1.51	1.59	1.61	0.32	0.31	0.11	−0.81	0.80	1.38	1.69

A plot of some distributions against the empirical histogram of the glass fibres data is as shown in Figure 5. This is to demonstrate the performance of the GAPIE distribution. Also, a plot for the empirical cdf of the competing GAPIE distribution of the glass fibres data is shown in Figure 6.

Table 3 shows the Kolmogorov-Smirnov test, p-value and the log-likelihood test of the various distributions under consideration for glass fibres dataset.

Table 2 Performance rating of the GAPIE distribution with glass fibres dataset

Distributions	Parameter MLEs	AIC	CAIC	BIC	HQIC	W	A
GAPIE	$\hat{a} = 0.1459$	35.9175	36.6072	44.4901	39.2891	0.1708	0.9538
	$\hat{b} = 6.2954$						
	$\hat{c} = 2.4575$						
	$\hat{\alpha} = -2.5792$						
Gompertz Weibull	$\hat{\alpha} = 0.2245$	38.3769	39.0666	46.9495	41.7486	0.2330	1.2832
	$\hat{\beta} = 0.0092$						
	$\hat{a} = 0.7973$						
	$\hat{b} = 5.6176$						
Gompertz Lomax	$\hat{\alpha} = 0.0046$	37.0055	37.6951	45.5780	40.3771	0.1685	0.9462
	$\hat{\beta} = 8.1791$						
	$\hat{a} = 0.5070$						
	$\hat{b} = 1.5158$						
Gompertz exponential	$\hat{\alpha} = -0.0048$	35.6353	36.0421	42.0647	38.1640	0.1445	0.8425
	$\hat{\beta} = -1.8200$						
	$\hat{a} = -1.9877$						
Kumaraswamy Lomax	$\hat{\alpha} = 9.8352$	44.2055	44.8951	52.7779	47.5771	1.6446	1.9915
	$\hat{\beta} = 45.3107$						
	$\hat{a} = 15.1182$						
	$\hat{b} = 0.0483$						
Beta Lomax	$\hat{\alpha} = 18.1737$	56.8268	57.4964	65.3793	60.1784	2.5426	3.1986
	$\hat{\beta} = 26.7645$						
	$\hat{a} = 10.8769$						
	$\hat{b} = 0.0329$						
Alpha power inverted exponential	$\hat{\alpha} = 53.5634$	196.3253	196.5253	200.6110	198.0111	0.7775	4.2384
	$\hat{\lambda} = 0.3509$						

Table 3 Test statistic for the GAPIE distribution with glass fibres dataset

Distributions	KS	p-value	Log-likelihood
GAPIE	0.1338923	0.2087089	13.95875
Gompertz Weibull	0.1520247	0.1087110	15.18846
Gompertz Lomax	0.1542000	0.0998000	14.50270
Gompertz exponential	0.1313551	0.2271038	14.81765
Kumaraswamy Lomax	0.1854000	0.0263000	18.10270
Beta Lomax	0.2182000	0.0049000	24.40340
Alpha power inverted exponential	0.4645605	3.099632e-12	96.16265

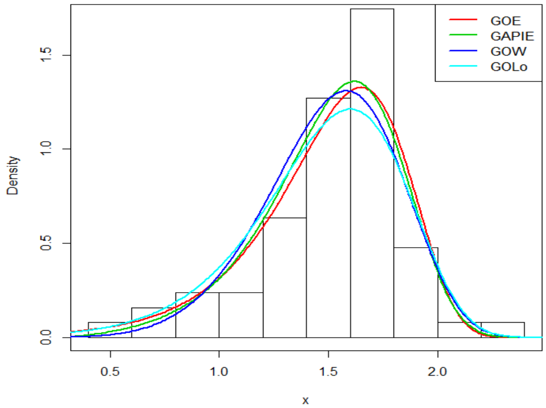

Figure 5 A plot of APEGE distributions with the empirical histogram of the glass fibres data

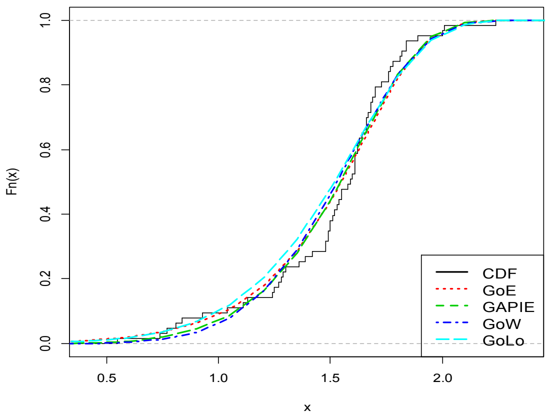

Figure 6 A plot of empirical cdf of the distributions of the glass fibers data

The plots in Figure 5 and Figure 6 show the GAPIE distribution is more suitable for the glass fibres data than the other competing distributions.

4.2. Carbon data

Our second set of data is from Nichols and Padgett (2006). It consists of 100 observations taken on breaking stress of carbon fibres. The dataset are as follow:

3.7, 2.74, 2.73, 2.5, 3.6, 3.11, 3.27, 2.87, 1.47, 3.11,4.42, 2.41, 3.19, 3.22, 1.69, 3.28, 3.09, 1.87, 3.15, 4.9, 3.75, 2.43,2.95, 2.97, 3.39, 2.96, 2.53,2.67, 2.93, 3.22, 3.39, 2.81, 4.2, 3.33, 2.55, 3.31, 3.31, 2.85,2.56, 3.56, 3.15, 2.35, 2.55, 2.59,2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36, 0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19,1.57, 0.81, 5.56, 1.73, 1.59, 2, 1.22, 1.12, 1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69,1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.7, 2.03, 1.8, 1.57, 1.08, 2.03, 1.61, 2.12,1.89, 2.88, 2.82, 2.05, 3.65.

Table 4 shows the goodness of fit and the performance rating of the GAPIE distribution for several test statistics with the carbon dataset, while Table 5 is the test statistic.

4.3. Discussion

The performance of a model is determined by the value that corresponds to the lowest Akaike Information Criteria (AIC) or the highest log-likelihood value is regarded as the best model. In the two real life cases considered, the GAPIE distribution has the lowest AIC value with 35.9175 in glass fibres data and 290.6222 in carbon data, respectively. Also, the GAPIE has the value of log-likelihood as 13.9588 and 141.3111 for glass fibres and carbon data, respectively. Hence, it competes favourably with other existing model for the data used.

Table 4 Performance rating of the GAPIE distribution with carbon dataset

Distributions	Parameter MLEs	AIC	CAIC	BIC	HQIC	W	A
GAPIE	$\hat{a}=0.0277$ $\hat{b}=2.1443$ $\hat{c}=1.4190$ $\hat{\alpha}=0.0549$	290.6222	291.0433	301.0429	294.8397	0.0675	0.4091
Gompertz Weibull	$\hat{\alpha}=2.2594$ $\hat{\beta}=-0.2017$ $\hat{a}=0.2650$ $\hat{b}=2.9808$	290.5644	290.9854	300.9850	294.7818	0.0648	0.3834
Gompertz Lomax	$\hat{\alpha}=0.0091$ $\hat{\beta}=5.0656$ $\hat{a}=1.9848$ $\hat{b}=0.6471$	292.8646	293.2857	303.2853	297.0821	0.0611	0.4763
Gompertz exponential	$\hat{\alpha}=0.0842$ $\hat{\beta}=0.8660$ $\hat{a}=0.9134$	304.2500	304.5000	312.0656	307.4131	0.1597	1.2608
Kumaraswamy Lomax	$\hat{\alpha}=3.7970$ $\hat{\beta}=24.3670$ $\hat{a}=0.0334$ $\hat{b}=6.0885$	290.9681	291.3891	301.3888	295.1855	0.0842	0.4532
Beta Lomax	$\hat{\alpha}=18.1737$ $\hat{\beta}=26.7645$ $\hat{a}=10.8769$ $\hat{b}=0.0329$	315.0974	317.4653	320.1753	317.4653	1.0896	2.0088
Alpha power inverted exponential	$\hat{\alpha}=11.0029$ $\hat{\lambda}=0.8694$	422.3312	422.4550	427.5416	424.4400	0.3726	2.0427

Table 5 Test statistic for the GAPIE distribution with Carbon Dataset

Distributions	KS	p-value	Log-likelihood
GAPIE	0.06185131	0.8388617	141.3111
Gompertz Weibull	0.06325020	0.8185524	141.2822
Gompertz Lomax	0.06365319	0.8125448	142.4323
Gompertz Exponential	0.09621573	0.3128060	149.1250
Kumaraswamy Lomax	0.07543761	0.6198049	141.4840
Beta Lomax	0.17654926	0.0045972	156.7625
Alpha power inverted exponential	0.35031040	4.384659e-11	209.1656

5. Conclusions

The GAPIE distribution has been derived. The statistical properties which include the order statistics distribution, cumulative hazard function, reversed hazard function, quantile, median, hazard function, odds function have been established. The shape of the distribution could be inverted bathtub or decreasing (depending on the value of the parameters). Applications to a two real life data show that the GAPIE distribution competes favourably with the Gompertz Weibull and exponential, and better than the Kumaraswamy Lomax distribution, beta Lomax distribution and some other families of distributions.

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