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A Three Parameter Shifted Exponential Distribution: Properties and Applications

Agu Friday Ikechukwu*[a], Okoi Emmanuel Jaspa [b], Runyi Francis Emmanuel [c] and Ogunsanya Adeyinka [d]

[a] Department of Statistics, University of Calabar, Calabar, Nigeria.
[b] Department of Statistics, Cross River University of Technology, Calabar, Nigeria.
[c] Department of Mathematics, Arthur Jarvis University, Akpabuyo, Cross River State, Nigeria.
[d] Department of Statistics, University of Ilorin, Kwara State, Nigeria.
*Corresponding author; e-mail: epsilomanalyst@gmail.com

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Abstract

This paper proposed a three parameter exponentiated shifted exponential distribution and derived some of its statistical properties including the order statistics and discussed in brief details. Method of maximum likelihood was used to estimate the parameters of the proposed distribution. The proposed distribution was applied on two real life positively skewed data sets with different level of kurtosis and simulation was done. The results obtained indicate that the proposed distribution with unimodal, positively skewed and decreasing shapes property fits better on the data set with higher kurtosis than the data set with lower kurtosis when compared. The simulation results showed that as the sample size increases the biasedness and the mean square error (MSE) of the proposed distribution decreases showing its flexibility property. In both real life applications, the proposed distribution was compared with the three parameter generalized inverted generalized exponential distribution, a three parameter generalized Lindley distribution and the two parameter shifted exponential distribution based on their Alkaike Information Criteria (AIC), Bayesian Information Criteria (BIC), Negative Log-likelihood (NLL) and Hanniquin Information Criteria (HQIC) values and it indicated that the proposed distribution can be used to model real life situations of positively skewed data with high kurtosis.

Keywords: Quantile function, probability density function, cumulative density function, order statistics, reliability function.

1. Introduction

Over the decades, researchers have introduced in literature several continuous univariate distributions particularly on lifetime distributions that is rich and still growing rapidly. Various distributions have been proposed by the extension of the existing distributions to serve as useful models for a wide applications on data arising from different real life situations. This has been done through different approaches. Few recent studies in this line of research involve extending probability distributions with the aim of increasing their modeling capability include the works of Nekoukhou et al. (2015), Al-Babtain et al. (2015), Pogány and Saboor (2016), Rodrigues et al. (2016), Natalie et al. (2016), Elgarhy and Shawki (2017) and Agu and Onwukwe (2019), Eghwerido et al. (2020a), Egwherido et al. (2020b). All this researchers used the exponentiation method proposed by Gupta et al. (1998) to extend the baseline distributions by the injection of a shape parameter which helps to control the skewness and peakness inherent nature in the data sets. This allow a more realistic modeling of data arising from different real life situations. Most of these distributions proposed by researchers have been extended or applied in different areas of real life problems by other researchers. Agu and Francis (2018) compared goodness of fit test for normal distribution to determine the normality of a given data.

Obubu et al. (2019) introduced a four parameter odd generalized exponentiated inverse Lomax distribution for modeling lifetime data and derived some of its basic statistical properties.

The interest of this article is on the extension of the two parameters shifted exponential distribution to three parameters distribution. The shifted exponential distribution is simply the distribution of where X is exponentially distributed and T is a parameter. In exponential distribution, the distribution begins at $x = 0$ but when the distribution begin at any positive value of x the resulting distribution is the shifted exponential distribution. This distribution is a life time distribution appropriate in modeling life or time failure of system under chance or constant failure rate condition.

Numerous researchers have studied Bayesian prediction problems for the shifted exponential distribution. Evans and Nigm (1980) discussed Bayesian prediction of future observations based on type II censored sample. Madi and Tsui (1990) derived a class of smooth estimators Madi and Leonard (1996) addressed the problem of estimating the scale parameter and proposed Bayesian estimators. Madi and Raqab (2003) had discussed on the basis of a doubly censored random sample of failure times drawn from a shifted exponential distribution.

Chang et al. (2013) dealt with hypothesis testing on the common location parameter of shifted exponential distribution with unknown and possibly unequal scale parameters. Sánchez et al. (2014) represented shifted exponential as likelihood function and conjugate inverted gamma prior for making Bayesian inference comparatively robust against a prior density poorly specified.

Ahuja and Stanley (1967) introduced a two parameter exponentiated exponential distribution with the distribution begin at $x = 0$ which was further studied in detail by Gupta and Kundu (1999). Gupta and Kundu (1999) introduced a two parameter generalized exponential distribution using the exponentiation method proposed by Gupta et al. (1998).

This article is interested in extending the shifted exponential distribution by the addition of a shape parameter to make it a three parameter distribution (where $x \geq \theta$) with the aim to increase its flexibility to real life situations using the exponentiation method proposed by Gupta et al. (1998).

The basic motivations for obtaining this exponentiated shifted exponential distribution is to make the kurtosis more flexible as compared to the parent distribution (shifted exponential distribution), to generate a distribution with right skewed, unimodal and to provide consistently better fits on higher kurtosis data than other generated distributions with less or equal number of parameter.

2. Shifted Exponential Distribution

A random variable X is said to have a shifted exponential distribution if the corresponding probability density function (pdf) and cumulative density function (cdf) begin at θ are given by

$$f(x)=\begin{cases}\lambda e^{-\lambda(x-\theta)}, & x\geq\theta,\\ 0, & x<\theta,\end{cases} \tag{1}$$

$$F(x)=\begin{cases}1-e^{-\lambda(x-\theta)}, & x\geq\theta,\\ 0, & x<\theta,\end{cases} \tag{2}$$

where θ is the location parameter interpreted as unknown point at which life begins while λ represents the scale parameter.

A random variable X is said to have an exponentiated distribution if the corresponding pdf and cdf proposed by Gupta et al. (1998) are given by

$$h(x)=\beta[F(x)]^{\beta-1}f(x), \tag{3}$$

$$H(x)=[F(x)]^{\beta}, \beta>0. \tag{4}$$

3. Exponentiated Shifted Exponential (ESE) Distribution

The pdf of the proposed ESE distribution is derived by substituting (1) and (2) into (3) to have

$$h(x)=\lambda\beta e^{-\lambda(x-\theta)}\left(1-e^{-\lambda(x-\theta)}\right)^{\beta-1}, x\geq\theta, \beta>0, \lambda>0. \tag{5}$$

This is represented graphically in Figure 1. At $\beta=1,$ (5) becomes the shifted exponential distribution, at $\theta=0$ and $\beta=1,$ (5) becomes the exponential distribution. Also the corresponding cdf of the ESE distribution is derived by substituting (2) into (4) to have

$$H(x)=\left(1-e^{-\lambda(x-\theta)}\right)^{\beta}, x\geq\theta, \beta>0, \lambda>0. \tag{6}$$

This is represented graphically in Figure 2. Where λ is the scale parameter, θ is the location parameter and β is the shape parameter. (6) becomes the cdf of the ESE distribution.

3.1. Some basic properties of the ESE distribution

Some of the basic properties of ESE distribution are obtained as follows. The reliability function is obtained using the relation

$$R_f=1-H(x),$$

where $H(x)$ is the cdf of the proposed ESE distribution. Therefore, the reliability function of ESE distribution is

$$R_f=1-[1-e^{-\lambda(x-\theta)}]^{\beta}, \quad x\geq\theta, \beta>0, \lambda>0.$$

The corresponding failure rate of ESE distribution is obtained using the relation $F_{rf} = \frac{h(x)}{R_f}$ and is given by $F_{rf} = \frac{\lambda\beta e^{-\lambda(x-\theta)}\left[1-e^{-\lambda(x-\theta)}\right]^{\beta-1}}{1-\left[1-e^{-\lambda(x-\theta)}\right]^{\beta}}, \quad x \geq \theta, \beta > 0, \lambda > 0.$

3.2. Quantile function and median

The quantile function $Q(u)$ is derived from the relation

$$Q(u) = H^{-1}(u).$$

Hence, the quantile function of the ESE distribution is derived as

$$Q(u) = \theta - \frac{\ln\left(1-u^{\frac{1}{\beta}}\right)}{\lambda}, \tag{7}$$

where $u \sim \text{uniform}(0,1)$. This simply mean that random samples from the proposed ESE distribution can be generated using

$$x = \theta - \frac{\ln\left(1-u^{\frac{1}{\beta}}\right)}{\lambda}.$$

The median of the ESE distribution can be obtained by making the substitution of $u = 0.5$ in (7) to have

$$median = \theta - \frac{\ln\left(1-0.5^{\frac{1}{\beta}}\right)}{\lambda}.$$

3.3. Order statistics

The pdf of the j^{th} order statistic for a random sample of size n from a distribution function $H(x)$ and an associated $h(x)$ is given by

$$h_{jm}(x) = \frac{n!}{(j-1)!(n-j)!} h(x)\left[H(x)\right]^{j-1}\left[1-H(x)\right]^{n-j},$$

where $h(x)$ and $H(x)$ are respectively the pdf and the cdf of the ESE distribution.

The pdf of the j^{th} order statistic for a random sample of size n from the ESE distribution is given by

$$h_{jm}(x) = \frac{n!}{(j-1)!(n-j)!}\left(\frac{\lambda\beta e^{-\lambda(x-\theta)}(1-e^{-\lambda(x-\theta)})^{\beta}}{(1-e^{-\lambda(x-\theta)})}\right)\left[(1-e^{-\lambda(x-\theta)})^{\beta}\right]^{j-1}\left[1-(1-e^{-\lambda(x-\theta)})^{\beta}\right]^{n-j}. \tag{8}$$

The pdf of the minimum order statistics is obtained by setting $j = 1$ in (8) to have

$$h_{1m}(x) = \frac{n\lambda\beta e^{-\lambda(x-\theta)}(1-e^{-\lambda(x-\theta)})^{\beta}}{(1-e^{-\lambda(x-\theta)})}\left[1-(1-e^{-\lambda(x-\theta)})^{\beta}\right]^{n-1}$$

and the corresponding pdf of the maximum order statistics is obtained by setting $j=n$ in (8) to have

$$h_{nm}(x)=\frac{n\lambda\beta e^{-\lambda(x-\theta)}(1-e^{-\lambda(x-\theta)})^{\beta}}{(1-e^{-\lambda(x-\theta)})}\left[(1-e^{-\lambda(x-\theta)})^{\beta}\right]^{n-1}.$$

3.4. Parameter estimation

The method of maximum likelihood estimation is used to estimate the parameters of the ESE distribution.

Let $x_1,...,x_n$ be a random sample distributed according to the pdf of ESE distribution, the log-likelihood function is obtained as

$$L\left(x_1,...,x_n;\lambda,\beta,\theta\right)=\prod_{i=1}^{n}h\left(x_1,...,x_n;\lambda,\beta,\theta\right)$$

$$L\left(x_1,...,x_n;\lambda,\beta,\theta\right)=\prod_{i=1}^{n}\left[\frac{\lambda\beta e^{-\lambda(x-\theta)}(1-e^{-\lambda(x-\theta)})^{\beta}}{(1-e^{-\lambda(x-\theta)})}\right].$$

The log-likelihood function L is derived as

$$L=n\ln\lambda+n\ln\beta+n\lambda\theta-\lambda\sum_{i=1}^{n}x_i+(\beta-1)\sum_{i=1}^{n}\ln(1-e^{-\lambda(x_i-\theta)}). \tag{9}$$

The solution to (9) may not be obtained in closed form, hence, R package maxLik function (Henningsen and Toomet 2010) can be used to obtain the estimates numerically.

Figure 1 shows that the distribution is unimodal, positively skewed, heavy tail, and kurtosis increases as the shape parameter value increases. Figure 2 shows that the distribution function of the proposed model has an S-shape and is a monotonically non-decreasing function.

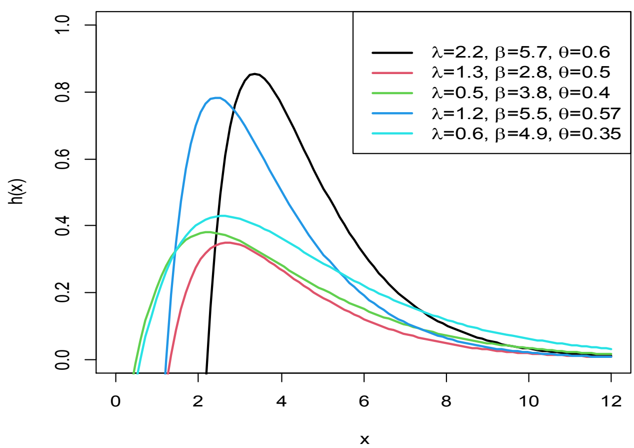

Figure 1 the pdf plot of the ESE distribution

4. Numerical Applications

The ESE distribution is applied to two data sets and comparisons are made with the three parameter generalized inverted generalized exponential distribution (Oguntunde and Adejumo

2015), a three parameter generalized Lindley distribution (Nosakhare and Festus 2018) and the two parameter shifted exponential distribution. R package maxLik function was used to enhance data analysis and the criteria used for model selection are Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), Hanniquin Information Criteria (HQIC), Negative Log-likelihood (NLL). The criteria for selecting the distribution with the best fit depends on the values of the AIC, BIC, NLL and HQIC, and small values of this criteria indicate a better fit.

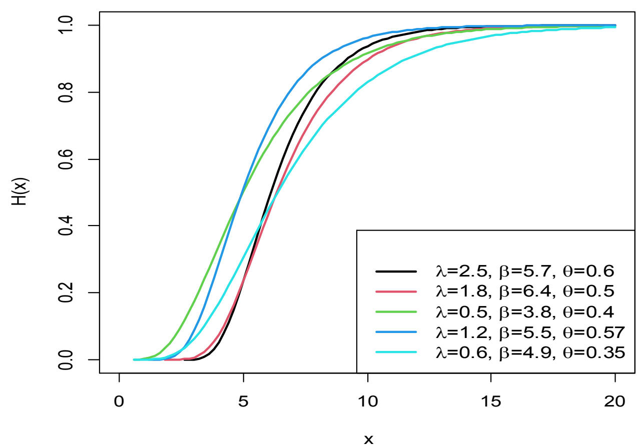

Figure 2 the cdf plot of the ESE distribution

Data set I: This data on Nigerian inflation rate was downloaded from the CBN portal through the link https://www.cbn.gov.ng/rates/inflrates.asp.

Data set II: The data consist of 72 exceedances of flood peaks (in m^3/s) of the Wheaton River near Carcross in Yukon Territory, Canada for the years 1958-1984. This data was first used by Choulakian and Stephens (2001) to examine the applicability of the generalized Pareto distribution and also was reported in Akinsete et al. (2008). Ekhosuehi and Opone (2018) used this data to model a three parameter generalized Lindley distribution. The data set is given below.

Table 1 Exceedances of Wheaton river flood data

1.7, 2.2, 14.4, 1.1, 0.4, 20.6, 5.3, 0.7, 1.9, 13.0, 12.0, 9.3, 1.4, 18.7, 8.5, 25.5, 11.6, 14.1, 22.1, 1.1, 2.5, 27.0, 14.4, 1.7, 37.6, 0.6, 2.2, 39.0, 0.3, 15.0, 11.0, 7.3, 22.9, 1.7, 0.1, 1.1, 0.6, 9.0, 1.7, 7.0, 20.1, 0.4, 2.8, 14.1, 9.9, 10.4, 10.7, 30.0, 3.6, 5.6, 30.8, 13.3, 4.2, 25.5, 3.4, 11.9, 21.5, 27.6, 36.4, 2.7, 64.0, 1.5, 2.5, 27.4, 1.0, 27.1, 20.2, 16.8, 5.3, 9.7, 27.5, 2.5

Table 2 Summary of the Nigerian monthly inflation rate data from January 2003 to December 2018 for data set I

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Skewness	Kurtosis
0.100	8.600	11.400	11.740	14.000	41.2.00	1.278	7.718

Table 2 displays the summary of the Nigerian monthly inflation rate data from January 2003 to December 2018. It shows that the data is positively skewed and have highest kurtosis among the sets of data used in this study.

Table 3 Summary of the exceedances of Wheaton river flood data for data set II

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Skewness	Kurtosis
0.100	2.125	9.500	12.204	20.125	64.000	1.473	5.889

Table 3 displays the summary of the exceedances of Wheaton river flood data. It shows that the data is positively skewed and have lower kurtosis among the sets of data used in this study.

Table 4 Parameter estimates and fitness of generalized Lindley distribution (GLD), generalized inverted generalized exponential distribution (GIGED), ESE distribution and the shifted exponential distribution (SED) based on data set I

Models	Estimates	NLL	AIC	BIC	HQIC	Rank
GIGED	$\hat{\lambda}$ = 2.71768 $\hat{\beta}$ = 4.14376 $\hat{\theta}$ = 1.07146	−6,661.317	13,328.630	13,344.630	13,334.590	3
GLD	$\hat{\lambda}$ = 0.02244 $\hat{\beta}$ = 5.02501 $\hat{\theta}$ = 1.75395	−4,614.239	9,234.478	9,250.473	9,240.431**	2**
ESE	$\hat{\lambda}$ = 0.22500 $\hat{\beta}$ =149.00390 $\hat{\theta}$ = −12.87110	−4,611.697	9,229.394	9,245.389	9,235.347	1*
SED	$\hat{\lambda}$ = 0.80230 $\hat{\theta}$ = 4,001.1970	−17,090,651	34,181,322	34,181,322	34,181,312	4

Note: * Best fit model. ** Competing model with the best fit model.

5. Simulation Study

This section presents simulation study to examine the efficiency and flexibility of the proposed ESE distribution. The simulation is performed as follows.

1. Data are generated using the quantile function of the ESE distribution defined in (7).
2. The samples sizes are taken as n = 100, 250 and 350.
3. The parameters values are set as λ = 2.5, β = 5.7, θ = 0.6, and λ = 1.8, β = 6.4 and θ = 0.5 and are presented in Tables 6 and 7, respectively.
4. Each sample size is replicated 1,500 times.

The simulation results are shown in Tables 6 and 7.

6. Discussion

Table 1 presented the exceedances of the Wheaton river flood data. Tables 2 and 3 show the summary of the two sets of data used for numerical analysis in the study and it revealed that data set I have a higher kurtosis than data set II. Table 4 shows the parameter estimates and fitness of GLD, GIGED, ESE distribution, and SED based on the Nigerian monthly inflation rate data from January 2003 to December 2018. The result obtained in Table 4 shows that the proposed ESE distribution has the lowest values for all the criteria used. So, the ESE distribution is considered best to fit the data with higher kurtosis than other distributions used for comparison. The competing distribution with the ESE model is the GLD.

Table 5 presents the parameter estimates and fitness of GLD, GIGED, ESE distributions, and SED based on the exceedances of Wheaton river flood data. The results obtained in Table 5 show that the GLD has the lowest values for all the criteria used than the proposed ESE distribution. This result is in line with our basic motivation to develop a distribution to model high kurtosis data. The data set II used to obtain the result in Table 5 has lower kurtosis as compared to the data set I. So, the GLD, in this case, is considered best to fit the data with lower kurtosis.

Table 5 Parameter estimates and fitness of GLD, GIGED, ESE distribution and SED based on the exceedances of Wheaton river flood data

Models	Estimates	NLL	AIC	BIC	HQIC	Rank
GIGED	$\hat{\lambda}$ = 1.55244 $\hat{\beta}$ = 0.76105 $\hat{\theta}$ = 0.26808	−324.254	654.508	661.338	657.227	3
GLD	$\hat{\lambda}$ = 0.15367 $\hat{\beta}$ = 16.47772 $\hat{\theta}$ = 0.87045	−251.365	508.730	515.560	511.449	1*
ESE	$\hat{\lambda}$ = 0.12808 $\hat{\beta}$ = 0.99309 $\hat{\theta}$ = 0.09968	−259.606	525.213	532.043	527.932	2**
SED	$\hat{\lambda}$ = 0.87041 $\hat{\theta}$ = 0.55325	−55,043.0	110,089.9	110,096.8	110,092.6	4

Note: * Best fit model. ** Competing model with the best fit model.

Tables 6 and 7 show the simulation illustration of the proposed ESE distribution. The results revealed that as the sample size increases the biasedness and the root mean square error (RMSE) values also reduce and approaches zero which is an indication that the ESE distribution fits better as the sample size increases. This result corresponds to the first-order asymptotic theory.

7. Conclusions

The ESE distribution has been successfully introduced in this paper and some of its basic statistical properties have been derived. The pdf of the ESE distribution has a unimodal, positively skewed, high kurtosis, and decreasing shapes. This means that ESE distribution would be very useful to model real-life events with unimodal, positively skewed, high kurtosis, and decreasing shapes. The model is tractable and flexible and shows high modeling capacity on positively skewed and high kurtosis data as it performs better than the GLD, GIGED, and the SED on positively skewed and high kurtosis data. The performance of the ESE distribution was judged based on the AIC, BIC, NLL, and HQIC values of these distributions on the positively skewed data, high kurtosis data, and the simulation results also proved its fitness. The ESE distribution has been seen to fit better to positively skewed data with high kurtosis is of no doubt a competitive model for positively skewed, high kurtosis data and it is hoped that it will be used in fields like finance, engineering, physics, geology biology, and medicine. The study, therefore, recommends the ESE distribution to be used to model the real-life situation of positively skewed data with high kurtosis. The study suggests further research on some other statistical properties of the ESE distribution not considered in this paper.

Table 6 Simulation results for mean estimates, biases and root mean squared errors (RMSE) of $\hat{\lambda}$, $\hat{\beta}$ and $\hat{\theta}$ for the ESE distribution

Sample size	Parameter values	Mean	Bias	RMSE
100	$\lambda = 2.5$	2.5321	0.0321	0.2204
	$\beta = 5.7$	5.6134	0.0866	1.7936
	$\theta = 0.6$	0.6254	0.0254	0.0961
250	$\lambda = 2.5$	2.5054	0.0054	0.1507
	$\beta = 5.7$	5.7080	0.0080	1.4395
	$\theta = 0.6$	0.6103	0.0103	0.0699
350	$\lambda = 2.5$	2.4991	−0.0009	0.1296
	$\beta = 5.7$	5.7068	0.0068	1.3906
	$\theta = 0.6$	0.6078	0.0078	0.0656

Table 7 Simulation results for mean estimates, biases and root mean squared errors (RMSE) of $\hat{\lambda}$, $\hat{\beta}$ and $\hat{\theta}$ for the ESE distribution

Sample size	Parameter values	Mean	Bias	RMSE
100	$\lambda = 1.8$	1.8248	0.0248	0.1599
	$\beta = 6.4$	6.3156	0.0844	1.9100
	$\theta = 0.5$	0.5359	0.0359	0.1359
250	$\lambda = 1.8$	1.8063	0.0063	0.1035
	$\beta = 6.4$	6.4231	0.0231	1.5053
	$\theta = 0.5$	0.5131	0.0131	0.0981
350	$\lambda = 1.8$	1.8021	0.0021	0.0870
	$\beta = 6.4$	6.4226	0.0226	1.4658
	$\theta = 0.5$	0.5095	0.0095	0.0917

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