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Short Communication

A Note on Second Order Rotatable Designs under Intra-Class Correlated Structure of Errors Using a Pair of Partially Balanced Incomplete Block Type Designs

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Abstract

In this paper, following the works of Das (1997, 1999 and 2003), Rajyalakshmi (2014), Ravindrababu and Victorbabu (2018), Victorbabu and Ravindrababu (2019), second order rotatable designs (SORD) under intra-class correlated structure of errors using a pair of partially balanced incomplete block type designs is suggested. Here we study the variance function of estimated response for different values of intra-class correlated coefficient (ρ) and distance from the center (d) for various factors. This method gives designs with less number of design points than the corresponding SORD under intra-class correlated structure of errors using central composite designs, pairwise balanced designs, balanced incomplete block designs and symmetrical unequal block arrangements with two unequal block sizes in some cases.

Keywords: Response surface designs, rotatability, robustness, incomplete block designs.

1. Introduction

The concept of rotatability was proposed by Box and Hunter (1957). Das and Narasimham (1962) constructed second order rotatable designs through balanced incomplete block designs. Narasimham et al. (1983) constructed second order rotatable designs (SORD) through a pair of balanced incomplete block designs (BIBD). Chowdhury and Gupta (1985) constructed SORD associated with partially balanced incomplete block designs (PBIBD). Victorbabu (2004) suggested SORD using a pair of PBIBD. Panda and Das (1994) studied first order rotatable designs with correlated errors. Das (1997, 1999, 2003, 2014) introduced and studied robust SORD. Rajyalakshmi (2014) suggested some contributions to second order rotatable and slope rotatable designs under different correlated structures. Ravindrababu and Victorbabu (2018), Victorbabu and Ravindrababu (2019) suggested an empirical study of SORD under intra-class correlated structures of errors using a pair of BIBD and pair of symmetrical unequal block arrangements (SUBA) with two unequal block size, respectively.

In this paper, following the works of Das (1997, 1999, 2003), Rajyalakshmi (2014), Ravindrababu and Victorbabu (2018), Victorbabu and Ravindrababu (2019), second order rotatable designs under intra-class correlated structure of errors using a pair of partially balanced incomplete block designs is suggested. Here we study the variance function of estimated response for the different values of intra-class correlated coefficient (ρ) and distance from the center (d) for various factors. This method give designs with less number of design points than the corresponding SORD under intra-class correlated structure of errors using central composite designs (CCD), BIBD, PBD and symmetrical unequal block arrangements (SUBA) with two unequal block sizes in some cases.

2. Conditions for SORD under Intra-Class Correlated Structure of Errors (cf. Das 1997, 2003)

A second order response surface design $D=((x_{iu}))$ for fitting,

$$Y_u(x)=b_0+\sum_{i=1}^{v}b_ix_{iu}+\sum_{i=1}^{v}b_{ii}x_{iu}^2+\sum_{i=1}^{v}\sum_{j=1}^{v}b_{ij}x_{iu}x_{ju}+e_u,\quad (i\le j), \tag{1}$$

where x_{iu} denotes the level of the i^{th} factor $(i=1,2,...,v)$ in the u^{th} run $(u=1,2,...,N)$ of the experiment, e_u 's are correlated random errors. The design D is said to be a SORD under intra-class correlated error of structure, if the variance of the estimated response of $\hat{Y}_u$ from the fitted surface is only a function of the distance, $\left(d^2=\sum_{i=1}^{v}x_i^2\right)$ of the point from the origin (center) of the design. Such a spherical variance function for estimation of responses in the second order response surface is achieved if the design points satisfy the following conditions (cf. Das 1997, 2003).

Let ρ be the correlation between errors of any two observations, each has the same variance σ^2 then the necessary and sufficient conditions for second order rotatability under the intra-class variance covariance structure are

$$\sum_{u=1}^{N}\prod_{i=1}^{v}x_{iu}^{\alpha_i}=0,\ \text{if any } \alpha_i \text{ is odd, for } \sum_{i=1}^{4}\alpha_i\le 4, \tag{2}$$

$$\sum_{u=1}^{N}x_{iu}^2=\text{constant}=N\lambda_2,\quad 1\le i\le v, \tag{3}$$

$$\sum_{u=1}^{N}x_{iu}^4=\text{constant}=3N\lambda_2,\quad 1\le i\le v, \tag{4}$$

$$\sum_{u=1}^{N}x_{iu}^2x_{ju}^2=\text{constant}=N\lambda_4,\quad 1\le i,j\le v, i\ne j, \tag{5}$$

$$\sum_{u=1}^{N}x_{iu}^4=3\sum_{u=1}^{N}x_{iu}^2x_{ju}^2. \tag{6}$$

Conditions (2) to (6) are independent of intra-class correlated coefficient ρ and are the same as the necessary and sufficient conditions of usual (i.e., when errors are uncorrelated and homoscedastic) SORD. The variances and covariances of the estimated parameters under the intra-class model are as follows:

$$V(\widehat{b}_0)=\frac{\left[\lambda_4(v+2)\{1+(N-1)\rho\}-v\rho N\lambda_2^2\right]\sigma^2\{1+(N-1)\rho\}}{N\Delta},$$

$$V(\widehat{b}_i)=\frac{\sigma^2(1-\rho)}{N\lambda_2},$$

$$V(\widehat{b}_{ij})=\frac{\sigma^2(1-\rho)}{N\lambda_4},$$

$$V(\widehat{b}_{ii})=\frac{\sigma^2(1-\rho)\left[\lambda_4(v+1)\{1+N-1)\rho\}-(v-1)\rho N\lambda_2^2-(v-1)\lambda_2^2(1-\rho)\right]}{2N\lambda_4\Delta},$$

$$Cov(\widehat{b}_0,\widehat{b}_{ii})=\frac{-\lambda_2\sigma^2(1-\rho)\{1+(N-1)\rho\}}{N\Delta},$$

$$Cov(\widehat{b}_{ii},\widehat{b}_{ij})=\frac{\sigma^2(1-\rho)\left[\lambda_2^2(1-\rho)-\lambda_4\{1+(N-1)\rho\}+N\rho\lambda_2^2\right]}{2N\lambda_4\Delta}. \quad (7)$$

Here, $\Delta=\left[\lambda_4(v+2)\{1+(N-1)\rho\}-v\rho N\lambda_2^2-v\lambda_2^2(1-\rho)\right]$ and other covariances are zero. An inspection of the variance of $\hat{b}_0$ shows that a necessary condition for the existence of a non-singular second order design is $\Delta>0,$ which leads to

$$\frac{\lambda_4}{\lambda_2^2}>\frac{v}{(v+2)} \text{ (non-singularity condition).} \quad (8)$$

The variance of the response $\hat{Y}_u$ at any point estimated through the surface comes out as

$$V(\widehat{Y}_u)=V(\widehat{b}_0)+d^2[(V(\widehat{b}_i)+2Cov(\widehat{b}_0,\widehat{b}_{ii})]+d^4V(\widehat{b}_{ii}). \quad (9)$$

Hence, the variance of estimate of $\hat{Y}_u$ becomes

$$V(\widehat{Y}_u)=\frac{\left[\lambda_4(v+2)\{1+(N-1)\rho\}-v\rho N\lambda_2^2\right]\sigma^2\{1+(N-1)\rho\}}{N\Delta}$$

$$+\left[\frac{\sigma^2(1-\rho)}{N\lambda_2}+2\left(\frac{\lambda_2\sigma^2(1-\rho)\{1+(N-1)\rho\}}{N\Delta}\right)\right]d^2$$

$$+\left[\frac{\sigma^2(1-\rho)[\lambda_4(v+1)\{1+(N-1)\rho\}-(v-1)\rho N\lambda_2^2-(v-1)\lambda_2^2(1-\rho)]}{2N\lambda_4\Delta}\right]d^4. \quad (10)$$

Here ρ is the correlation between errors of any two observations each has the same variance σ^2 and N-denotes number of design points.

3. An Empirical Study on SORD under Intra-Class Correlated Structure of Errors Using a Pair of Partially Balanced Incomplete Block Type Designs

Consider an incomplete block arrangement with constant block size and replication in which some pair of treatments occur λ_{11} times each $(\lambda_{11}\neq 0)$ and some other pairs do not occur at all $(\lambda_{12}=0)$ (the design need not be PBIBD). Take this as the first design. For the second design take the incomplete block design with all missing pairs (in the first design) once each with $k_2=2,\lambda_{21}=0,\lambda_{22}=1.$ Such pairs of designs can be construct in a straight forward manner using existing two-associate PBIB designs with one of the λ 's equal to zero. Let $D_1=(v,b_1,r_1,k_1,\lambda_{11}\neq 0,\lambda_{12}=0)$ be an incomplete block design with constant replication in which only some pair of treatments occur a constant number of times $\lambda_{11}\neq 0$ $(\lambda_{12}=0)$, and let $2^{t(k_1)}$ denote a fractional replicate of 2^{k_1} in +1 and −1 levels, in which no interaction with less than five factors is confounded. Let $[1-(v,b_1,r_1,k_1,\lambda_{11},\lambda_{12}=0)]$ denote the design points generated from the transpose of the incidence matrix of incomplete block design D_1. $[1-(v,b_1,r_1,k_1,\lambda_{11},\lambda_{12}=0)]2^{(k_1)}$ are the $b_1 2^{t(k_1)}$ design points generate from D_1 by "multiplication" (see Raghavarao 1971, pp.298-300). Let $D_2=(v,b_2,r_2,k_2=2,\lambda_{21}=0,\lambda_{22}=1)$ be the associated second design containing only the missing pairs of treatments of above design D_1. Let $[a-(v,b_2,r_2,k_2=2,\lambda_{21}=0,\lambda_{22}=1)]2^2$ be the $b_2 2^2$ design points generated from D_2 by "multiplication", with levels $+a$ and $-a$. The number of design points of SORD under intra-class correlated structure of errors using a pair of PBIBD is $N=b_1 2^{t(k_1)}+b_2 2^2$.

For the design points generated from the pair of PBIBD, simple symmetry conditions (2) to (5) are true. Further, from (3) to (5), we have

$$\sum_{u=1}^{N} x_{iu}^2 = r_1 2^{t(k_1)} + r_2 2^2 a^2 = \text{constant} = N\lambda_2, \text{ for all } i, \tag{11}$$

$$\sum_{u=1}^{N} x_{iu}^4 = r_1 2^{t(k_1)} + r_2 2^2 a^4 = \text{constant} = 3N\lambda_4, \text{ for all } i, \tag{12}$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = \lambda_{11} 2^{t(k_1)} + \lambda_{21} 2^2 a^4 = \text{constant} = N\lambda_4, \text{ for all } i \neq j. \tag{13}$$

From (12) and (13), we get $a^4 = \dfrac{3(\lambda_{11}-r_1)2^{t(k_1)}}{r_2 2^2}$. Substituting a^4 in (12) and (13) also a^2 in (11), we get λ_2 and λ_4, respectively. The variance of estimated response for a various factors is tabulated for $(0 \le \rho \le 0.9)$. The numerical calculations are appended in the Appendix in Table 1.

Example: An empirical study on SORD under intra-class correlated structure of errors using a pair of partially balanced incomplete block designs with $D_1=(v=6,b_1=4,r_1=2,k_1=3,\lambda_{11}=1,\lambda_{12}=0)$, $D_2=(v=6,b_2=3,r_2=1,k_2=2,\lambda_{21}=0,\lambda_{22}=1)$ is given below with $N=44$ design points.

From (11), (12) and (13), we have

$$\sum_{u=1}^{N} x_{iu}^2 = 16+4a^2 = \text{constant} = N\lambda_2, \tag{14}$$

$$\sum_{u=1}^{N} x_{iu}^4 = 16+4a^4 = \text{constant} = 3N\lambda_4, \tag{15}$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = 8 = \text{constant} = N\lambda_4, \tag{16}$$

From (15) and (16), we get $a^4=2$. From (16), we get $\lambda_4=0.18181818$. Substituting $a^2=1.414213562$ in (14) and on simplification, we obtain $\lambda_2=0.49220123$. Here $\dfrac{\lambda_4}{\lambda_2^2}=0.75050212$ and $\dfrac{v}{(v+2)}=0.75$. Hence $0.75050212>0.75$, and the non-singularity condition (8) is also satisfied.

4. A Study of Dependence of the Variance Function of the Response at Different Design Points

Here, the dependence of variance function of response at different design points for SORD under intra-class correlated structure of errors using a pair of PBIBD; $D_1 = (v = 6, b_1 = 4, r_1 = 2, k_1 = 3, \lambda_{11} = 1, \lambda_{12} = 0)$, $D_2 = (v = 6, b_2 = 3, r_2 = 1, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)$ with intra-class correlation coefficient 'ρ' and distance from the center 'd' is studied. The variance function is given by

$$V(\hat{Y}) = V(\hat{b}_0) + \left(V(\hat{b}_i) + 2Cov(\hat{b}_0, \hat{b}_{ii})\right)d^2 + V(\hat{b}_{ii})d^4,$$

$$V(\hat{y}) = 24.0794\sigma^2 + 16.1257\sigma^2 d^2 + 2.7562\sigma^2 d^4 = 30.2418 \text{ (Taking } \rho = 0.3,\ d = 0.6 \text{ and } \sigma^2 = 1).$$

The study of variance function of response at different design points for SORD under intra-class correlated structure of errors using a pair of PBIBD and distance from center d for $0.1 \le d \le 1$ are tabulated. The numerical calculations are appended in the Appendix in Table 2.

5. Conclusions

We may point out here that the SORD under intra-class correlated structure of errors using a pair of PBIBD has 44 design points for 6-factors whereas the corresponding SORD under intra-class correlated structure of errors by Rajyalakshmi (2014) using CCD for $v = 6$ factors, PBD with parameters $(v = 6, b = 7, r = 3, k_1 = 3, k_2 = 2, \lambda = 1)$, BIBD with parameters $(v = 6, b = 10, r = 5, k = 3, \lambda = 2)$ and SUBA with two unequal block sizes with parameters $(v = 6, b = 7, r = 3, k_1 = 2, k_2 = 3, b_1 = 3, b_2 = 4\lambda = 1)$, needs 45, 57, 93 and 57 design points, respectively. Therefore this method gives less number of design points than the corresponding SORD under intra-class correlated structure of errors using CCD, PBD, BIBD and SUBA with two unequal block sizes.

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Appendix

Table 1 Variance of the estimated response under intra-class correlated structure of errors using a pair of PBIBD

ρ	$V(\widehat{Y})$
	$D_1 = (v = 6, b_1 = 4, r_1 = 2, k_1 = 3, \lambda_{11} = 1, \lambda_{12} = 0)$, $D_2 = (v = 6, b_2 = 3, r_2 = 1, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1); N = 44$
0.0	$33.9706\sigma^2 + 23.0367\sigma^2 d^2 + 3.9374\sigma^2 d^4$
0.1	$30.6735\sigma^2 + 20.7330\sigma^2 d^2 + 3.5437\sigma^2 d^4$
0.2	$27.3765\sigma^2 + 18.4293\sigma^2 d^2 + 3.1499\sigma^2 d^4$
0.3	$24.0794\sigma^2 + 16.1257\sigma^2 d^2 + 2.7562\sigma^2 d^4$
0.4	$20.7823\sigma^2 + 13.8220\sigma^2 d^2 + 2.3625\sigma^2 d^4$
0.5	$17.4853\sigma^2 + 11.5183\sigma^2 d^2 + 1.9687\sigma^2 d^4$
0.6	$14.1882\sigma^2 + 9.2147\sigma^2 d^2 + 1.5750\sigma^2 d^4$
0.7	$10.8912\sigma^2 + 6.9110\sigma^2 d^2 + 1.1812\sigma^2 d^4$
0.8	$7.5941\sigma^2 + 4.6073\sigma^2 d^2 + 0.7875\sigma^2 d^4$
0.9	$4.2971\sigma^2 + 2.3037\sigma^2 d^2 + 0.3937\sigma^2 d^4$
	$D_1 = (v = 10, b_1 = 8, r_1 = 4, k_1 = 5, \lambda_{11} = 2, \lambda_{12} = 0)$, $D_2 = (v = 10, b_2 = 5, r_2 = 1, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1); N = 148$
0.0	$3.4766\sigma^2 + 1.3700\sigma^2 d^2 + 0.1481\sigma^2 d^4$
0.1	$3.2289\sigma^2 + 1.2393\sigma^2 d^2 + 0.1332\sigma^2 d^4$
0.2	$2.9813\sigma^2 + 1.1016\sigma^2 d^2 + 0.1184\sigma^2 d^4$
0.3	$2.7336\sigma^2 + 0.9639\sigma^2 d^2 + 0.1036\sigma^2 d^4$
0.4	$2.4860\sigma^2 + 0.8262\sigma^2 d^2 + 0.0888\sigma^2 d^4$
0.5	$2.2383\sigma^2 + 0.6885\sigma^2 d^2 + 0.0740\sigma^2 d^4$
0.6	$1.9906\sigma^2 + 0.5508\sigma^2 d^2 + 0.0592\sigma^2 d^4$
0.7	$1.7430\sigma^2 + 0.4131\sigma^2 d^2 + 0.0444\sigma^2 d^4$
0.8	$1.4953\sigma^2 + 0.2754\sigma^2 d^2 + 0.0296\sigma^2 d^4$
0.9	$1.2477\sigma^2 + 0.1377\sigma^2 d^2 + 0.0148\sigma^2 d^4$
	$D_1 = (v = 12, b_1 = 8, r_1 = 4, k_1 = 6, \lambda_{11} = 2, \lambda_{12} = 0)$, $D_2 = (v = 12, b_2 = 6, r_2 = 1, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1); N = 280$
0.0	$0.4375\sigma^2 + 0.1476\sigma^2 d^2 + 0.0186\sigma^2 d^4$
0.1	$0.4938\sigma^2 + 0.1328\sigma^2 d^2 + 0.0167\sigma^2 d^4$
0.2	$0.5500\sigma^2 + 0.1181\sigma^2 d^2 + 0.0148\sigma^2 d^4$
0.3	$0.6063\sigma^2 + 0.1033\sigma^2 d^2 + 0.0130\sigma^2 d^4$
0.4	$0.6625\sigma^2 + 0.0885\sigma^2 d^2 + 0.0111\sigma^2 d^4$
0.5	$0.7188\sigma^2 + 0.0738\sigma^2 d^2 + 0.0093\sigma^2 d^4$
0.6	$0.7750\sigma^2 + 0.0590\sigma^2 d^2 + 0.0074\sigma^2 d^4$
0.7	$0.8313\sigma^2 + 0.0443\sigma^2 d^2 + 0.0056\sigma^2 d^4$
0.8	$0.8875\sigma^2 + 0.0295\sigma^2 d^2 + 0.0037\sigma^2 d^4$
0.9	$0.9438\sigma^2 + 0.0148\sigma^2 d^2 + 0.0018\sigma^2 d^4$

Table 2 Study of variance function of SORD under intra-class correlated structure of errors using a pair of PBIBD for different values of 'ρ' and 'd' by taking $\sigma^2 = 1$

$D_1 = (v=6, b_1=4, r_1=2, k_1=3, \lambda_{11}=1, \lambda_{12}=0), D_2 = (v=6, b_2=3, r_2=1, k_2=2, \lambda_{21}=0, \lambda_{22}=1); N=44$

ρ	$d=0.1$	$d=0.2$	$d=0.3$	$d=0.4$	$d=0.5$	$d=0.6$	$d=0.7$	$d=0.8$	$d=0.9$	$d=1.0$
0.0	34.2013	34.8983	36.0758	37.7572	39.9758	42.7741	46.2039	35.5833	55.2136	60.9446
0.1	30.8812	31.5085	32.5682	34.0815	36.0782	38.5966	41.6835	32.1250	49.7922	54.9402
0.2	27.5611	28.1187	29.0606	30.4058	32.1807	34.4192	37.1631	28.6667	44.3709	48.9557
0.3	24.2409	24.7288	25.5530	26.7301	28.2831	30.2418	32.6427	25.2083	38.9495	42.9613
0.4	20.9208	21.3390	22.0455	23.0543	24.3855	26.0644	28.1223	21.7500	33.5282	36.9668
0.5	17.6007	17.9492	18.5379	19.3786	20.4879	21.8870	23.6020	18.2917	28.1068	30.9723
0.6	14.2805	14.5593	15.0303	15.7029	16.5903	17.7096	19.0816	14.8333	22.6854	24.9779
0.7	10.9604	11.1695	11.5227	12.0272	12.6927	13.5322	14.5612	11.3750	17.2641	18.9834
0.8	7.6403	7.7797	8.0152	8.3514	8.7952	9.3548	10.0408	7.9167	11.8427	12.9889
0.9	4.3201	4.3898	4.5076	4.6757	4.8976	5.1774	5.5204	4.4583	6.4214	6.9945

$D_1 = (v=10, b_1=8, r_1=4, k_1=5, \lambda_{11}=2, \lambda_{12}=0), D_2 = (v=10, b_2=5, r_2=1, k_2=2, \lambda_{21}=0, \lambda_{22}=1); N=148$

ρ	$d=0.1$	$d=0.2$	$d=0.3$	$d=0.4$	$d=0.5$	$d=0.6$	$d=0.7$	$d=0.8$	$d=0.9$	$d=1.0$
0.0	3.4904	3.5319	3.6017	3.7007	3.8301	3.9915	4.1869	3.5372	4.6891	5.0016
0.1	3.2413	3.2787	3.3415	3.4306	3.5471	3.6923	3.8682	3.2835	4.3202	4.6015
0.2	2.9923	3.0255	3.0814	3.1606	3.2641	3.3932	3.5495	3.0298	3.9513	4.2013
0.3	2.7433	2.7723	2.8212	2.8905	2.9811	3.0940	3.2308	2.7761	3.5824	3.8011
0.4	2.4942	2.5191	2.5610	2.6204	2.6981	2.7949	2.9121	2.5223	3.2135	3.4010
0.5	2.2452	2.2660	2.3009	2.3503	2.4150	2.4957	2.5934	2.2686	2.8445	3.0008
0.6	1.9961	2.0128	2.0407	2.0803	2.1320	2.1966	2.2747	2.0149	2.4756	2.6007
0.7	1.7471	1.7596	1.7805	1.8102	1.8490	1.8974	1.9561	1.7612	2.1067	2.2005
0.8	1.4981	1.5064	1.5203	1.5401	1.5660	1.5983	1.6374	1.5074	1.7378	1.8003
0.9	1.2490	1.2532	1.2602	1.2701	1.2830	1.2991	1.3187	1.2537	1.3689	1.4002

$D_1 = (v=12, b_1=8, r_1=4, k_1=6, \lambda_{11}=2, \lambda_{12}=0), D_2 = (v=12, b_2=6, r_2=1, k_2=2, \lambda_{21}=0, \lambda_{22}=1); N=280$

ρ	$d=0.1$	$d=0.2$	$d=0.3$	$d=0.4$	$d=0.5$	$d=0.6$	$d=0.7$	$d=0.8$	$d=0.9$	$d=1.0$
0.0	0.4390	0.4434	0.4509	0.4616	0.4756	0.4930	0.5143	0.4451	0.5692	0.6036
0.1	0.4951	0.4991	0.5058	0.5154	0.5280	0.5437	0.5628	0.5006	0.6123	0.6433
0.2	0.5512	0.5547	0.5607	0.5693	0.5804	0.5944	0.6114	0.5561	0.6554	0.6829
0.3	0.6073	0.6104	0.6157	0.6231	0.6329	0.6451	0.6600	0.6116	0.6984	0.7225
0.4	0.6634	0.6661	0.6706	0.6770	0.6853	0.6958	0.7086	0.6671	0.7415	0.7622
0.5	0.7195	0.7217	0.7255	0.7308	0.7378	0.7465	0.7571	0.7225	0.7846	0.8018
0.6	0.7756	0.7774	0.7804	0.7846	0.7902	0.7972	0.8057	0.7780	0.8277	0.8414
0.7	0.8317	0.8330	0.8353	0.8385	0.8427	0.8479	0.8543	0.8335	0.8708	0.8811
0.8	0.8878	0.8887	0.8902	0.8923	0.8951	0.8986	0.9029	0.8890	0.9138	0.9207
0.9	0.9439	0.9443	0.9451	0.9462	0.9476	0.9493	0.9514	0.9445	0.9569	0.9604