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Detection Capability of the Modified EWMA Chart for the Trend Stationary AR(1) Model

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Abstract

A performance of control chart is frequently evaluated by using the average run length (ARL) which is calculated with several methods. Regarding this research paper, an explicit formula is a presented technique for computing the ARL of the modified EWMA chart with the trend AR(1) model in the scope of exponential white noise. The numerical integral equation (NIE) method is compared to the performance with the explicit formula. Accordingly, the results show that the explicit formula can find the solutions correctly and quickly. Moreover, the modified EWMA chart is used to compare a capability with the EWMA scheme. As a result, the performance of the modified EWMA control chart is better for small and moderated shifts. In addition, this explicit formula of the ARL is applied by using the real data of a health field.

Keywords: Average run length (ARL), exponential white noise, autoregressive model, explicit formula.

1. Introduction

A statistician used the statistical process control (SPC) as a quality control for an autoregressive model of time series. In addition, a control chart was a well-known measurement. For instance, Jiang (2001) proposed to analyze the average run length of the autoregressive moving average (ARMA) chart applied to an ARMA(1,1) model. Furthermore, Lu and Reynolds (2001) investigated the cumulative sum (CUSUM) chart on the first autoregressive (AR) process. Costa and Castagliola (2011) also showed a performance of $\bar{x}$ chart on an AR(1) model. Additionally, Suriyakat et al. (2012) presented the AR(1) process on the exponentially weighted moving average (EWMA) chart. According to a statistical analysis of time series, a real trend may exist on numerous models. However, one of the most popular models is the first autoregressive model with trend or the trend AR(1) model which can be applied to many fields. Regarding the previous research, Yue and Pilon (2003) studied the annual mean daily streamflow data of 15 watersheds with a linear trend and an AR(1) process. In the study of Hamed (2009), the first order autocorrelated series with a linear trend were used for hydrologic data. Moreover, Karaoglan and Bayhan (2012) presented a case study on the peroxide values of stored vegetable oil for the trend stationary first order autoregressive model. Regarding to this reason, the trend AR(1) model will be analyzed on a control chart.

This research studies the first autoregressive model with trend, thus, the researcher has looked for a suitable control chart with autocorrelated processes. The modified exponentially weighted moving average or modified EWMA chart were presented by Patel and Divecha (2011) and developed from the EWMA chart which was a primal control chart created by Roberts in 1959. Additionally, both charts were appropriate for a small shift detection. However, the modified EWMA chart was invented for highly autocorrelated observations. Recently, a modified EWMA statistic is brought to study in the new research; see Khan et al. (2017), Herdiani et al. (2018). In addition, the stationary error term of this model is studied on an exponential distribution called an exponential white noise which is non-Gaussian white noise. An exponential white noise was studied by Andel (1988), Ibazizen and Fellag (2003), Pereira and Antonia Amaral-Turkman (2004). Therefore, the modified EWMA chart is chosen to study on the trend AR(1) model with a condition of an exponential white noise.

The average run length (ARL) is the usual performance measure of control charts. Besides, the ARL is the expected number of observations that must be plotted for a specified control chart until an out-of-control signal is obtained. Regarding a control chart, the ARL should be large before a process started and when a mean shift occurs, the ARL should be small. For example, The ARL was used to compute the efficacy of exponential EWMA charts by Gan and Chang (2000). The average run length was used to study small shifts of the process mean on a synthetic control chart from the work of Wu and Spedding (2000). Faraz et al. (2017) evaluated the in-control performance of the np control chart with estimated parameters on the ARL measure. According to the previous research, the ARL could be approximated by numerous techniques, such as Monte Carlo simulation, Markov Chain approach, Martingale approach, and the numerical integral equation method. In the study of Areepong and Novikov (2008), the martingale approach was used to approximate ARL of an EWMA control chart. Next, the average run length of the multivariate exponentially weighted moving average (MEWMA) chart and the combined control chart were evaluated with Monte Carlo simulation by Zhang et al. (2009). Moreover, Khoo et al. (2016) presented a Markov chain approach for computing the ARL of EWMA charts. Peerajit et al. (2018) also estimated the average run length of a CUSUM chart for long-memory process by using the numerical integral equation (NIE) method. For other methods, the explicit formula is a method which provides an accurate value of ARL and a fast computation. For example, Suriyakat et al. (2012) proposed an explicit formula of the ARL on an EWMA control chart for AR(1) process with exponential white noise. Meanwhile, Suriyakat et al. (2012) also derived the ARL solution with an explicit formula for the trend exponential AR(1) processes on the an EWMA chart. Besides, Busaba et al. (2012) also solved the explicit formulas of the average run length on CUSUM chart with the trend stationary first order of the autoregressive model by using an integral equation approach. Moreover, Petcharat et al. (2015) also found the average run length of a CUSUM chart on a moving average (MA) process of the order q with exponential white noise by using an explicit formula. Then, Petcharat (2016) analyzed the average run length by using the explicit formula on an EWMA chart for a seasonal moving average model of order q with exponential white noise. Furthermore, Sukparungsee and Areepong (2017) presented the explicit formulas of ARL for an EWMA chart. Moreover, they also observed an autoregressive model. Recently, Sunthornwat et al. (2017) had found an analytical ARL for the long memory autoregressive which fractionally integrated moving average (ARFIMA) process on an EWMA control chart. The latest study of Zhang and Busababodhin (2018) derived the explicit formula for the average run length of a CUSUM control chart on autoregressive integrated moving average (ARIMA) process observations with exponential white noise. According to various past research, the author is interested in developing the explicit formula of the average run length for performance evaluations of the modified EWMA control chart on the trend AR(1) model.

The purpose of this paper is to further investigate the explicit formulas of the average run length on the modified EWMA control chart for the trend stationary AR(1) process with exponential white noise. The explicit formula is compared with the numerical integral equation (NIE) method by using the absolute percentage relative error (APRE) (Sunthornwat et al. 2018) in order to check an infallibility of solutions. Moreover, a performance of the modified EWMA control chart is tested against an EWMA scheme on the same process for both simulated data and real data.

2. A Modified EWMA Control Chart

Definition 1 (Modified EWMA Statistic) *For the modified EWMA control chart, given* $\{X_t, t = 1, 2, \ldots\}$ *as a sequence of the trend AR(1) model with the target mean* μ *and constant variance* σ^2. *A modified EWMA statistic is developed from an EWMA statistic which is* $Z_t = (1-\lambda)Z_{t-1} + \lambda X_t$ *and can be written as follows*

$$Z_t = (1-\lambda)Z_{t-1} + \lambda X_t + (X_t - X_{t-1}), \tag{1}$$

where λ *is an exponential smoothing parameter with* $\lambda \in (0,1]$ *and the first value* Z_0 *is* μ.

Definition 2 (Modified EWMA Control Chart) *The upper and lower control limits of the modified EWMA control chart are*

$$UCL = \mu + B\sigma\sqrt{\frac{\lambda}{2-\lambda} + \frac{2\lambda(1-\lambda)}{2-\lambda}},$$

$$LCL = \mu - B\sigma\sqrt{\frac{\lambda}{2-\lambda} + \frac{2\lambda(1-\lambda)}{2-\lambda}}, \tag{2}$$

where B *is a suitable control width limit and the control bound of the EWMA chart is* $\mu \pm B\sigma\sqrt{\frac{\lambda}{2-\lambda}}$.

3. Stationary AR(1) Model with Trend

Regarding time series, the general autoregressive process is denoted the AR(p) process where a parameter p is an order of the autoregressive process (Brockwell and Davis 2002). Therefore, the trend stationary first order autoregressive model is the AR(1) process with a linear trend and a stationary error term.

Definition 3 (AR(1) Process) *The equation of observations for the first autoregressive process with trend or the trend AR(1) process in the case of exponential white noise is defined as*

$$X_t = \eta + \gamma t + \phi X_{t-1} + \varepsilon_t, \tag{3}$$

where η *is a suitable constant,* γ *is a slope,* ϕ *is an autoregressive coefficient* $(|\phi| < 1)$ *and* ε_t *is exponential white noise sequences.*

Remark 1 From (3), the process X_t is said to be trend stationary if

$$X_t = f(t) + \varepsilon_t, \tag{4}$$

where $f(t)$ is a function $f:\mathbb{R}\to\mathbb{R}$ for any time t, ε_t is a stationary process. Moreover, ε_t is a white noise process if ε_t is uncorrelated sequences with a constant mean, and a finite variance.

4. Explicit Formulas of Average Run Length on the Modified EWMA Chart for the Trend AR(1) Model

Before finding the explicit formula of the ARL, the trend AR(1) process is enforced on the modified EWMA chart by using (3) to put into (1). Thus, Z_t can be rewritten as follows:

$$\begin{aligned} Z_t &= (1-\lambda)Z_{t-1} + \lambda(\eta+\gamma t+\phi X_{t-1}+\varepsilon_t) + (\eta+\gamma t+\phi X_{t-1}+\varepsilon_t - X_{t-1}), \\ &= (1-\lambda)Z_{t-1} + (\lambda\phi+\phi-1)X_{t-1} + (1+\lambda)\eta + (1+\lambda)\gamma t + (1+\lambda)\varepsilon_t. \end{aligned} \tag{5}$$

After the initial values are given to be $Z_0 = u$ and $X_0 = v$, the equation would be

$$Z_1 = (1-\lambda)u + (\lambda\phi+\phi-1)v + (1+\lambda)\eta + (1+\lambda)\gamma + (1+\lambda)\varepsilon_1. \tag{6}$$

For this scheme, the explicit formula of the ARL is presented to change the upper control bound, in which $0 \le Z_1 \le b$ and b is the upper control limit of Z_1 for an in-control process. When (6) is used instead of Z_1 and the interval of Z_1 is formatted to be ε_1, it can be written as

$$\frac{-(1-\lambda)u-(\lambda\phi+\phi-1)v}{(1+\lambda)} - (\eta+\gamma) \le \varepsilon_1 \le \frac{b-(1-\lambda)u-(\lambda\phi+\phi-1)v}{(1+\lambda)} - (\eta+\gamma). \tag{7}$$

When $L(u)$ is set to be the ARL that is derived by the method of Champ and Rigdon (1991) and applies Fredholm integral equation of the second kind (Zemyan 2012), the equation would be

$$L(u) = 1 + \frac{1}{1+\lambda}\int_0^b L(k) f\left(\frac{k-(1-\lambda)u-(\lambda\phi+\phi-1)v}{(1+\lambda)} - (\eta+\gamma)\right)dk. \tag{8}$$

This research studies on the error term $\varepsilon_t \sim Exp(\beta)$, so

$$L(u) = 1 + \frac{e^{\frac{(1-\lambda)u}{\beta(1+\lambda)}} \cdot e^{\frac{(\lambda\phi+\phi-1)v}{\beta(1+\lambda)}} \cdot e^{\frac{\eta+\gamma}{\beta}}}{\beta(1+\lambda)} \int_0^b L(k)\cdot e^{\frac{-k}{\beta(1+\lambda)}} dk. \tag{9}$$

For an in-control process, the average run length is called ARL_0 and the exponential parameter (β) is determined with β_0 in this stage. However, the average run length is called ARL_1 if the exponential parameter (β) is changed to $\beta_1 = (1+\delta)\beta_0$ where $\beta_1 > \beta_0$ and δ is a shift size for $\beta = \beta_1$ an out-of-control process.

For solving the explicit formula of the average run length on this chart, $H = \int_0^b L(k)\cdot e^{\frac{-k}{\beta(1+\lambda)}} dk$ and $G(u) = e^{\frac{(1-\lambda)u}{\beta(1+\lambda)}} \cdot e^{\frac{(\lambda\phi+\phi-1)v}{\beta(1+\lambda)}} \cdot e^{\frac{\eta+\gamma}{\beta}}$, where $0 \le u \le b$ are given and substituted into (9). Therefore, it can be rearranged as follows

$$L(u) = 1 + \frac{G(u)}{\beta(1+\lambda)}\cdot H. \tag{10}$$

From (10), H is separated to analyse as follows

$$H = \int_0^b \left(1 + \frac{G(k)}{\beta(1+\lambda)}\cdot H\right) e^{\frac{-k}{\beta(1+\lambda)}} dk,$$

$$= -\beta(1+\lambda)\left[e^{\frac{-b}{\beta(1+\lambda)}}-1\right]-\frac{H}{\lambda}\cdot e^{\frac{(\lambda\phi+\phi-1)v}{\beta(1+\lambda)}}\cdot e^{\frac{\eta+\gamma}{\beta}}\left[e^{\frac{-\lambda b}{\beta(1+\lambda)}}-1\right],$$

$$= \frac{-\beta(1+\lambda)\left[e^{\frac{-b}{\beta(1+\lambda)}}-1\right]}{1+\frac{1}{\lambda}\cdot e^{\frac{(\lambda\phi+\phi-1)v}{\beta(1+\lambda)}}\cdot e^{\frac{\eta+\gamma}{\beta}}\left[e^{\frac{-\lambda b}{\beta(1+\lambda)}}-1\right]}. \tag{11}$$

For the next step, the solution of H is replaced into (10), and $L(u)$ is obtained as

$$L(u) = 1+\frac{e^{\frac{(1-\lambda)u}{\beta(1+\lambda)}}\cdot e^{\frac{(\lambda\phi+\phi-1)v}{\beta(1+\lambda)}}\cdot e^{\frac{\eta+\gamma}{\beta}}}{\beta(1+\lambda)}\left(\frac{-\beta(1+\lambda)\left[e^{\frac{-b}{\beta(1+\lambda)}}-1\right]}{1+\frac{1}{\lambda}\cdot e^{\frac{(\lambda\phi+\phi-1)v}{\beta(1+\lambda)}}\cdot e^{\frac{\eta+\gamma}{\beta}}\left[e^{\frac{-\lambda b}{\beta(1+\lambda)}}-1\right]}\right),$$

$$= 1-\frac{\lambda e^{\frac{(1-\lambda)u}{\beta(1+\lambda)}}\left(e^{\frac{-b}{\beta(1+\lambda)}}-1\right)}{\lambda e^{-\frac{(\lambda\phi+\phi-1)v+(1+\lambda)(\eta+\gamma)}{\beta(1+\lambda)}}+e^{\frac{-\lambda b}{\beta(1+\lambda)}}-1}. \tag{12}$$

Therefore, this solution of $L(u)$ is the explicit formula of the ARL on the modified EWMA control chart for the trend AR(1) model.

5. Solution Comparison of Average Run Length

The numerical integral equation (NIE) method is one of the techniques that used for approximating the ARL of the modified EWMA control chart on the trend AR(1) model. $\tilde{L}(u)$ is set to be an estimated value of the ARL with the m linear equation systems by using the composite midpoint quadrature rule (Phanthuna et al. 2018) as well as others. Therefore, this solution can be explained as

$$\tilde{L}(u) \approx 1+\frac{1}{1+\lambda}\sum_{k=1}^{m} w_k L(a_k) f\left(\frac{a_k-(1-\lambda)u-(\lambda\phi+\phi-1)v}{(1+\lambda)}-(\eta+\gamma)\right), \tag{13}$$

where a_k is a set of the division points on the interval $[0,b]$ as: $a_k = \left(k-\frac{1}{2}\right)w_k$; $k = 1,2,...,m$, w_k is a weight of the composite midpoint formula as: $w_k = b/m$, and $\varepsilon_t \sim Exp(\beta)$.

According to (13), the average run length of the numerical integral equation method is approximated by partitive points (m) 1,000 nodes. Also, the solutions of the NIE method are compared to the explicit formula for the trend AR(1) model of a modified EWMA chart by using the time of ARL computations and the absolute percentage relative error (APRE), which can be calculated as

$$\text{APRE}\,(\%) = \left|\frac{L(u)-\tilde{L}(u)}{L(u)}\right|\times 100\%. \tag{14}$$

In speed comparison, the time calculations of two methods are presented by the central processing unit (CPU) time (PC System: Windows 10 Education, i7-6500U CPU @ 2.50GHz Processor, 8.00 GB RAM, 64-bit Operating System) in seconds.

In addition, the modified EWMA control chart is compared to performance with the EWMA chart on the trend AR(1) process; therefore, the ARL should be a low value in order to indicate the occurrence of the mean which is shift quickly. For 3σ control limits of a control chart, ARL_0 is 1/0.0027 or 370. In this research, the initial parameter values are studied at $\text{ARL}_0 = 370$ and another one $\text{ARL}_0 = 500$, and given $\phi = 0.5, -0.5$ ($|\phi| < 1$), $\lambda = 0.05, \eta = 2, \gamma = 0.8$ and $\beta_0 = 1$ for the in-control process. For the out-of-control process, a shift size (δ) of β is displayed to be 0.01, 0.03, 0.05, 0.08, 0.10, 0.30, 0.50 and 1.00.

Table 1 shows the ARL on the trend AR(1) process at $\phi = 0.5$ by using the explicit formula and the NIE method for the modified EWMA chart with an initial upper bound b = 0.0999752411 at ARL_0 = 370 and b = 0.1001416741 at $\text{ARL}_0 = 500$ and the explicit formula for the EWMA chart with an initial upper bound $b = 3.812665\times10^{-9}$ at ARL_0 = 370 and $b = 5.15588\times10^{-9}$ at ARL_0 = 500, respectively. The APRE (%) and CPU time are used to compare the performance of the explicit formula and the NIE method for a modified EWMA chart. Therefore, the results present very low APRE (%) showing that both methods can be used to calculate the ARL. However, the CPU time of the explicit formula is much faster than the NIE method for all results. In addition, the ARL of an EWMA chart on the trend AR(1) model is found by using the explicit formula. After that, this result is compared to performance with the modified EWMA control chart. When the mean shift of process occurs, the ARL of the modified EWMA scheme decreases abruptly whereas the ARL of the EWMA chart decreases slowly. A performance of the modified EWMA control chart is better than the EWMA control chart for small and intermediate shifts and worse than the large shift at the same shift size.

According to Table 2, the ARL of explicit formulas and the NIE method on the trend AR(1) process is shown at $\phi = -0.5$ for the modified EWMA chart with an initial upper bound b = 0.273008016 at ARL_0 = 370 and b = 0.273431328 at ARL_0 = 500 and for the EWMA chart with an initial upper bound $b = 1.03639\times10^{-8}$ at ARL_0 = 370 and $b = 1.401513\times10^{-8}$ at $\text{ARL}_0 = 500$. The results are the same direction with in the case of $\phi = 0.5$, as the ARL of the explicit formula and the NIE method is not obviously different. Moreover, the modified EWMA control chart are still shown a good performance for small and intermediate shifts.

6. Application for Real Data

With the use of real data, the trend AR(1) model with exponential white noise is tested with the observations of the annual male melanoma incidence (Andrew and Herzberg 1985) that best fits with this model. The data of the annual male melanoma incidence in Connecticut of the United States in 1936-1972 is recorded, which included 37 observations and can be predicted for the trend AR(1) model by using parameter values $\eta = 0.522004$, $\gamma = 0.121989$, and $\phi = 0.408647$. The performance of control charts that are the modified EWMA chart and the EWMA chart is compared by using the ARL with the initial parameter values as $\lambda = 0.05$, $\beta_0 = 0.4698$, $\text{ARL}_0 = 370$ and 500.

Table 1 Explicit formula against the NIE method on the modified EWMA control chart and performance comparison with the EWMA chart given $\phi = 0.5$ for $\text{ARL}_0 = 370$ and 500

ARL_0	Shift size (δ)	Modified EWMA Explicit (CPU time)	Modified EWMA NIE (CPU time)	APRE (%)	EWMA (Explicit)
370	0.00	370.0000280630 (< 0.001)	370.0000278695 (9.766)	5.23×10^{-8}	370.000
	0.01	59.06981473641 (< 0.001)	59.06981471358 (10.313)	3.86×10^{-8}	293.965
	0.03	21.97309660551 (< 0.001)	21.97309659788 (10.265)	3.47×10^{-8}	188.115
	0.05	13.49104429212 (< 0.001)	13.49104428779 (10.344)	3.21×10^{-8}	122.523
	0.08	8.563447447119 (< 0.001)	8.563447444649 (11.250)	2.88×10^{-8}	66.496
	0.10	6.904054753465 (< 0.001)	6.904054751609 (10.328)	2.69×10^{-8}	45.177
	0.30	2.552727341157 (< 0.001)	2.552727340809 (10.297)	1.36×10^{-8}	2.652
	0.50	1.771631172048 (< 0.001)	1.771631171918 (10.250)	7.34×10^{-9}	1.145
	1.00	1.279347708441 (< 0.001)	1.279347708415 (10.593)	2.03×10^{-9}	1.003
500	0.00	500.0000430153 (< 0.001)	500.0000427270 (10.031)	5.77×10^{-8}	500.000
	0.01	61.65894970748 (< 0.001)	61.65894968348 (10.110)	3.89×10^{-8}	397.178
	0.03	22.328821891155 (< 0.001)	22.32882188336 (11.297)	3.49×10^{-8}	254.036
	0.05	13.626406726523 (< 0.001)	13.62640672212 (12.564)	3.23×10^{-8}	165.336
	0.08	8.6186632603828 (< 0.001)	8.618663257886 (10.391)	2.90×10^{-8}	89.570
	0.10	6.9401235608870 (< 0.001)	6.940123559014 (10.859)	2.70×10^{-8}	60.741
	0.30	2.5572923966644 (< 0.001)	2.557292396314 (10.672)	1.37×10^{-8}	3.235
	0.50	1.7734345206069 (< 0.001)	1.773434520477 (10.735)	7.32×10^{-9}	1.197
	1.00	1.2798863308321 (< 0.001)	1.279886330806 (10.688)	2.04×10^{-9}	1.004

Table 2 Explicit formula against the NIE method on the modified EWMA control chart and performance comparison with the EWMA chart given $\phi = -0.5$ $\text{ARL}_0 = 370$ and 500

ARL_0	Shift size (δ)	Modified EWMA Explicit (CPU time)	Modified EWMA NIE (CPU time)	APRE (%)	EWMA (Explicit)
370	0.00	370.0001962608 (< 0.001)	370.0001947836 (10.766)	3.99×10^{-7}	370.000
	0.01	74.48352656467 (< 0.001)	74.48352634462 (9.750)	2.95×10^{-7}	296.880
	0.03	28.66250251134 (< 0.001)	28.66250243552 (9.797)	2.65×10^{-7}	193.645
	0.05	17.75950997676 (<0.001)	17.75950993309 (10.281)	2.46×10^{-7}	128.450
	0.08	11.338006447277 (< 0.001)	11.33800642200 (10.202)	2.23×10^{-7}	71.531
	0.10	9.1565358395794 (< 0.001)	9.1565358204014 (10.109)	2.09×10^{-7}	49.382
	0.30	3.3360080741079 (< 0.001)	3.3360080702018 (9.734)	1.17×10^{-7}	3.081
	0.50	2.2398739949440 (< 0.001)	2.2398739933889 (10.265)	6.94×10^{-8}	1.203
	1.00	1.5040903621634 (< 0.001)	1.5040903618081 (10.110)	2.36×10^{-8}	1.004
500	0.00	500.0000064256 (< 0.001)	500.0000042123 (10.812)	4.43×10^{-7}	500.000
	0.01	78.62686258312 (< 0.001)	78.62686234889 (13.219)	2.98×10^{-7}	401.120
	0.03	29.26290785241 (< 0.001)	29.26290777465 (10.625)	2.66×10^{-7}	261.515
	0.05	17.99038100311 (< 0.001)	17.99038095869 (11.578)	2.47×10^{-7}	173.351
	0.08	11.43262379381 (< 0.001)	11.43262376822 (10.125)	2.24×10^{-7}	96.380
	0.10	9.218414560763 (< 0.001)	9.218414541378 (10.500)	2.10×10^{-7}	66.427
	0.30	3.343903354657 (< 0.001)	3.343903350725 (10.140)	1.18×10^{-7}	3.815
	0.50	2.243046073638 (< 0.001)	2.243046072074 (10.453)	6.97×10^{-8}	1.274
	1.00	1.505088267025 (< 0.001)	1.505088266668 (10.406)	2.37×10^{-8}	1.006

Table 3 Comparison of the ARL values between the modified EWMA chart and the EWMA chart for the annual male melanoma incidence in Connecticut of the United States in 1936-1972

Shift size (δ)	$ARL_0 = 370$		$ARL_0 = 500$	
	Modified EWMA	EWMA	Modified EWMA	EWMA
0.00	370.000	370.000	500.000	500.000
0.01	45.764	235.311	47.296	317.860
0.03	16.699	100.647	16.901	135.753
0.05	10.285	46.190	10.361	62.110
0.08	6.608	16.333	6.640	21.734
0.10	5.382	8.933	5.402	11.728
0.20	2.973	1.523	2.979	1.707
0.30	2.213	1.068	2.216	1.093
0.50	1.649	1.004	1.651	1.005

Regarding Table 3, the ARL results are shown for the modified EWMA chart with an initial upper bound b = 0.216926813 at ARL_0 = 370 and b = 0.217238851 at ARL_0 = 500 and for the EWMA chart with an initial upper bound $b = 8.19441\times10^{-9}$ at ARL_0 = 370 and $b = 1.108132\times10^{-8}$ at ARL_0 = 500. When the processes start and mean shifts occur, the modified EWMA chart can detect shifts to fasten the EWMA chart at the earliest stage until shifts have intermediate size. However, the ARL of the EWMA chart at large shift is lower than the ARL of the modified EWMA chart. Moreover, this result is similarly shown the simulated data in Tables 1 and 2.

From Figures 1 and 2, two control charts are created by using the observations of the annual male melanoma incidence with the target mean $\mu = 2.7324$, the standard deviation $\sigma = 1.3431$, the exponential smoothing parameter $\lambda = 0.05$ and the control width limit B = 3. The results show that the modified EWMA control chart can detect the abrupt shift at the 7th observation and the EWMA chart detects the process shift at the 29th observation. Accordingly, the changed detection of the modified EWMA chart is faster than the EWMA chart.

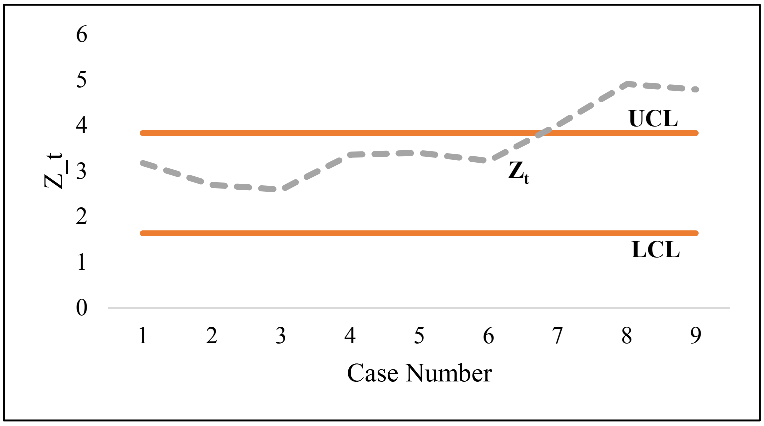

Figure 1 Plot of the modified EWMA control chart with the annual male melanoma incidence

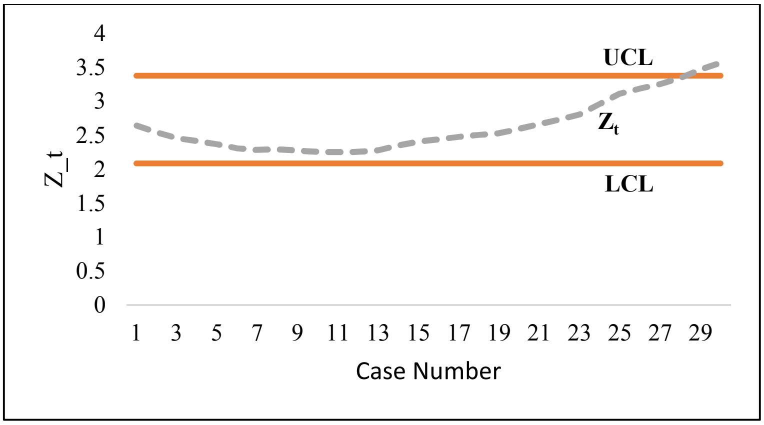

Figure 2 Plot of the EWMA control chart with the annual male melanoma incidence

7. Discussion and Conclusions

In this research, the modified EWMA control chart on the trend AR(1) model with exponential white noise is evaluated based on the performance by finding the explicit formula of the ARL. The explicit formula is a method for finding the exact value of the ARL and decreased process time. The numerical integral equation (NIE) method is used to compare an effect of this explicit formula by measuring the APRE (%) and CPU time. Therefore, both methods are shown to ARL values closely but the explicit formula can be computed with a small amount of time. After that, the modified EWMA chart is tested against the primal EWMA scheme. The results show that the modified EWMA chart can be detected shifts more quickly for small and intermediate levels. This explicit formula of the ARL can be applied with real observations of the trend AR(1) process in various fields, such as economics, environment and health. In addition, the annual male melanoma incidence data is experimented to calculate on this explicit formula of the ARL. Accordingly, the results are similar to simulated data. For the future research, this result can be developed for new control charts or other interesting models.

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