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Measure of Modified Slope Rotatability for Second Order Response Surface Designs

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Abstract

In this paper, measure of modified slope rotatability for second order response surface designs using central composite designs is suggested which enables us to assess the degree of modified slope rotatability for a given second order response surface design and variance of the estimated responses are also obtained.

Keywords: Response surface designs, slope rotatable designs, central composite designs, modified designs, measure.

1. Introduction

The concept of rotatability, which is very important in response surface designs, was proposed by Box and Hunter (1957). Das et al. (1999) suggested modified second order response surface designs. Victorbabu et al. (2008) suggested modified rotatable central composite designs. A design is said to be rotatable if the variance of the response estimate is a function only of the distance of the point from the design centre. The study of rotatable designs is mainly emphasized on the estimation of absolute response. Estimation of differences in response at two different points in the factor space will often be of great importance. If differences at two points close together, estimation of local slope (rate of change) of the response is of interest. Estimation of slopes occurs frequently in practical situations. For instance, there are cases in which we want to estimate rate of reaction in chemical experiment, rate of change in the yield of a crop to various fertilizer doses, rate of disintegration of radioactive material in an animal etc. (cf. Park 1987).

Hader and Park (1978) introduced slope rotatable central composite designs (SRCCD). Victorbabu and Narasimham (1991) studied in detail the conditions to be satisfied by a general second order slope rotatable designs (SOSRD) and constructed SOSRD using balanced incomplete block designs (BIBD). Victorbabu (2005) studied modified SRCCD. Victorbabu (2006) suggested modified SOSRD using BIBD. Victorbabu (2007) suggested a review on SOSRD.

Park and Kim (1992) suggested measure of slope rotatability for second order response surface experimental designs. Jang and Park (1993) suggested measure and a graphical method for evaluating slope rotatability in response surface designs. Victorbabu and Surekha (2011, 2012a, 2012b, 2012c, 2013a, 2013b, 2016) studied different measures of second order response surface designs using different methods. In this paper measure of modified slope rotatability for second order response surface designs using central composite designs (CCD) for $2 \le v \le 16$ factors are studied. These measures are useful to enable us to assess the degree of modified slope rotatability for a given second order response surface designs.

2. Conditions for Second Order Slope Rotatable Designs

Suppose we want to use the second order response surface design $D = ((x_{iu}))$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i=1}^{v} \sum_{j=1}^{v} b_{ij} x_{iu} x_{ju} + e_u,$$

where x_{iu} denotes the level of the i^{th} factor $(i = 1,2,\ldots,v)$ in the u^{th} run $(u = 1,2,\ldots,N)$ of the experiment, e_u's are uncorrelated random errors with mean zero and variance σ^2 is said to be SOSRD if the variance of the estimate of first order partial derivative of $Y_u(x_1, x_2, \ldots, x_v)$ with respect to each of independent variables (x_i) is only a function of the distance $(d^2 = \sum_{i=1}^{v} x_i^2)$ of the point $(x_1, x_2, \ldots, x_v)$ from the origin of the design.

Following Box and Hunter (1957), Hader and Park (1978) and Victorbabu and Narasimham (1991) the general conditions for second order slope rotatability can be obtained as follows. To simplify the fit of the second order polynomial from design points '*D*' through the method of least squares, we impose the following simple symmetry conditions on D to facilitate easy solutions of the normal equations:

1. $\sum x_{iu} = 0, \sum x_{iu} x_{ju} = 0, \sum x_{iu} x_{ju}^2 = 0, \sum x_{iu}\ x_{ju}\ x_{ku} = 0, \sum x_{iu}^3 = 0, \sum x_{iu} x_{ju}^3 = 0,$
$\sum x_{iu} x_{ju}\ x_{ku}^2 = 0, \sum x_{iu} x_{ju} x_{ku} x_{lu} = 0;$ for $i \ne j \ne k \ne l,$
2. (i) $\sum x_{iu}^2 = \text{constant} = N\lambda_2,$
(ii) $\sum x_{iu}^4 = \text{constant} = cN\lambda_4;$ for all i,
3. $\sum x_{iu}^2 x_{ju}^2 = \text{constant} = N\lambda_4;$ for $i \ne j,$ where c, λ_2 and λ_4 are constants. (1)

The variances and covariances of the estimated parameters are

$$V(\hat{b}_0) = \frac{\lambda_4(c+v-1)\sigma^2}{N\left[\lambda_4(c+v-1) - v\lambda_2^2\right]}, \quad V(\hat{b}_i) = \frac{\sigma^2}{N\lambda_2}, \quad V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4},$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2) - (v-1)\lambda_2^2}{\lambda_4(c+v-1) - v\lambda_2^2}\right], \quad Cov(\hat{b}_0, \hat{b}_{ii}) = \frac{-\lambda_2\sigma^2}{N\left[\lambda_4(c+v-1)\right] - v\lambda_2^2},$$

$$Cov(\hat{b}_{ii}, \hat{b}_{jj}) = \frac{(\lambda_2^2 - \lambda_4)\sigma^2}{(c-1)N\lambda_4\left[\lambda_4(c+v-1) - v\lambda_2^2\right]}, \tag{2}$$

and other covariances vanish.

An inspection of the variance of $\hat{b}_0$ shows that a necessary condition for the existence of a non-singular second order design is

4. $\frac{\lambda_4}{\lambda_2^2} > \frac{v}{(c+v-1)}$ (3)

For the second order model

$$\frac{\partial \hat{Y}}{\partial x_i} = \hat{b}_i + 2\hat{b}_{ii}x_{iu} + \sum_{j\neq i}\hat{b}_{ij}x_{ju}, \tag{4}$$

$$V\left(\frac{\partial \hat{Y}}{\partial x_i}\right) = V(\hat{b}_i) + 4x_{iu}^2 V(\hat{b}_{ii}) + \sum_{j\neq i} x_{ju}^2 V(\hat{b}_{ij}). \tag{5}$$

The condition for right hand side of the Equation (5) to be a function of $d^2 = \sum_{i=1}^{v} x_i^2$ alone (for slope rotatability) is

$$4V(\hat{b}_{ii}) = V(\hat{b}_{ij}). \tag{6}$$

On simplification of (6), using (1), (2) and (3), we get

5. $\left[v(5-c)-(c-3)^2\right]\lambda_4 + \left[v(c-5)+4\right]\lambda_2^2 = 0.$ (7)

Therefore, 1, 2 and 3 of (1), (2), (3) and (7) give a set of conditions for slope rotatability in any general second order response surface design (cf. Hader and Park 1978, Victorbabu and Narasimham 1991).

3. Conditions for Modified SOSRD (cf. Victorbabu 2005)

Following Hader and Park (1978), Victorbabu and Narasimham (1991), equations 1, 2 and 3 of (1), (2), (3) and (7) give the necessary and sufficient conditions for modified second order slope rotatable designs.

The usual method of construction of SOSRD is to take combinations with unknown constants, associate a 2^v factorial combinations or a suitable fraction of it with factors each at ± 1 levels to make the level codes equidistant. All such combinations form a design. Generally SOSRD need at least five levels (suitably coded) at $0, \pm 1, \pm a$ for all factors $((0,0,\ldots,0)$ – chosen center of the design, unknown level 'a' to be chosen suitably to satisfy slope rotatability). Generation of design points this way ensures satisfaction of all the conditions even though the design points contain unknown levels.

Alternatively by putting some restrictions indicating some relation among $\sum x_{iu}^2, \sum x_{iu}^4$ and $\sum x_{iu}^2 x_{ju}^2$ some equations involving the unknowns are obtained and their solution gives the unknown levels. In SOSRD the restrictions used is seems, not exploited well. We shall investigate the restriction $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$, i.e., $(N\lambda_2)^2 = N(N\lambda_4)$, $\lambda_2^2 = \lambda_4$ to get modified SOSRD. By applying the new restriction in (7), we get $c = 1$ or $c = 5$. The non-singularity condition (3) leads to $c = 5$. It may be noted $\lambda_2^2 = \lambda_4$ and $c = 5$ are equivalent conditions. The variances and covariances of the estimated parameters are

$$V(\hat{b}_0) = \frac{(v+4)\sigma^2}{4N}, \quad V(\hat{b}_i) = \frac{\sigma^2}{N\sqrt{\lambda_4}}, \quad V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4}, \quad V(\hat{b}_{ii}) = \frac{\sigma^2}{4N\lambda_4}.$$

$Cov(\hat{b}_0, \hat{b}_{ii}) = \dfrac{-\sigma^2}{4N\sqrt{\lambda_4}}$ and other covariances are zero, $V\left(\dfrac{\partial \hat{Y}}{\partial x_i}\right) = \left[\dfrac{\sqrt{\lambda_4} + d^2}{N\lambda_4}\right]\sigma^2.$

4. Conditions of Measure of SOSRD (cf. Park and Kim (1992))

Following Hader and Park (1978), Victorbabu and Narasimham (1991), Park and Kim (1992), (1), (2), (3) and (7) give the necessary and sufficient conditions for a measure of slope rotatability for any general second order response surface designs. Further we have, $V(b_i)$ are the same (identical) for all i, $V(b_{ii})$ are the same (identical) for all i, $V(b_{ij})$ are the same (identical) for all i, j, where $i \neq j$, $Cov(b_i, b_{ii}) = Cov(b_i, b_{ij}) = Cov(b_{ii}, b_{ij}) = Cov(b_{ij}, b_{il})$ for all $i \neq j \neq l$.

The measure of slope rotatability for second order response surface design can be obtained by using the following equation (cf. Park and Kim 1992, p.398).

$$Q_v(D) = \frac{1}{2(v-1)\sigma^4}\left\{(v+2)(v+4)\sum_{i=1}^{v}\left[\left[\left(V(b_i)-\frac{1}{v}\sum_{i=1}^{v}V(b_i)\right)+\frac{\left(4V(b_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}V(b_{ij})\right)-\frac{1}{v}\sum_{i=1}^{v}\left(4V(b_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}V(b_{ij})\right)}{v+2}\right]^2\right]\right.$$

$$+\frac{4}{v(v+2)}\sum_{i=1}^{v}\left(4V(b_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}V(b_{ij})\right)-\frac{1}{v}\sum_{i=1}^{v}\left(4V(b_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}V(b_{ij})\right)$$

$$+2\sum_{i=1}^{v}\left[\left[4V(b_{ii})-\frac{\left(4V(b_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}V(b_{ij})\right)}{v}\right]^2+\sum_{\substack{j=1\\j\neq i}}^{v}\left[V(b_{ij})-\frac{\left(4V(b_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}V(b_{ij})\right)}{v}\right]^2\right]$$

$$\left.+4(v+4)\left[4\operatorname{cov}^2(b_i,b_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}\operatorname{cov}^2(b_i,b_{ij})\right]+4\sum_{i=1}^{v}\left[4\sum_{\substack{j=1\\j\neq i}}^{v}\operatorname{cov}^2(b_i,b_{ij})+\sum_{\substack{j<l\\j,l\neq i}}^{v}\sum\operatorname{cov}^2(b_{ij},b_{il})\right]\right\},$$

where $Q_v(D)$ is the measure of slope-rotatability. It can be verified that $Q_v(D)$ is zero if and only if a design D is slope-rotatable. $Q_v(D)$ becomes larger as D deviates from a slope-rotatable design. Further $Q_v(D)$ is greatly simplified to $Q_v(D) = \frac{1}{\sigma^4}\left[4V(b_{ii}) - V(b_{ij})\right]^2$.

5. Measure of Slope Rotatability for Second Order Response Surface Designs Using CCD

The well-known type of design consists of $2^{t(v)}$ factorial points $(\pm1, \pm1, \ldots, \pm1)$, $2v$ axial points of the form $(\pm a, 0, \ldots, 0)$, etc., and centre point $(0, 0, \ldots, 0)$, may be replicated n_0 times, and the axial points are also allowed to be replicated n_a times. Thus the total number of experimental points (N) can be written as, $N = F + T$, where F is the number of factorial points i.e., $F = 2^{t(v)}$ and $T = 2vn_a + n_0$.

The simple symmetry conditions 1, 2, 3 of (1) are true. Condition 1 of (1) is true obviously. Conditions 2 and 3 of (1) are true as follows:

$$\sum x_{iu}^2 = 2^{t(v)} + 2n_a a^2 = N\lambda_2, \tag{8}$$

$$\sum x_{iu}^4 = 2^{t(v)} + 2n_a a^4 = cN\lambda_4, \tag{9}$$

$$\sum x_{iu}^2 x_{ju}^2 = 2^{t(v)} = N\lambda_4. \tag{10}$$

From (9) and (10), we get $c = \frac{2^{t(v)} + 2n_a a^4}{2^{t(v)}}$. From (8), we get $\lambda_2 = \frac{2^{t(v)} + 2n_a a^2}{N}$ and (10) we get $\lambda_4 = \frac{2^{t(v)}}{N}$. For this design $Q_v(D)$ is found to be

$$Q_v(D) = \left(\frac{F + 2n_a a^2}{N}\right)^4 \left(4e - \frac{1}{F}\right)^2.$$

Substituting c, λ_2 and λ_4 in $V(b_{ii})$ of (2) and on simplification, we get,

$$V(b_{ii}) = e = \frac{(v-1)FT - 4(v-1)Fn_a a^2 + 2n_a\left[N - 2(v-1)n_a\right]a^4}{2n_a a^4\left[vFT - 4vFn_a a^2 + 2n_a[N - 2vn_a]a^4\right]}.$$

If $Q_v(D)$ is zero, if and only if, a design 'D' is slope-rotatable. $Q_v(D)$ becomes larger as 'D' deviates from a Slope Rotatable Design (cf. Park and Kim (1992)).

6. Measure of Modified Slope Rotatability for Second Order Response Surface Designs

In this section, the proposed measure of modified slope rotatability for second order response surface designs using CCD is given below.

Consider the following set of points: (i) $2^{t(v)}$ (where $2^{t(v)}$ is resolution V fraction of 2^v) (cf. Raghavarao 1971) points on cube viz., coordinates $(\pm 1, \pm 1, \ldots, \pm 1)$, (ii) 2^v axial points, viz., $(\pm a, 0, \ldots, 0), (0, \pm a, \ldots, 0), \ldots, (0, 0, \ldots, \pm a)$ − repeated n_a times, (iii) n_0 central points, where n_a is chosen to satisfy the criterion of modified slope rotatability.

The design points, $(\pm 1, \pm 1, \ldots, \pm 1)2^{t(v)} \cup n_a(\pm a,\ 0, \ldots, 0)2^1 \cup (n_0)$ generated from CCD will give a measure of modified slope rotatability for second order response surface designs in $N = \frac{(2^{t(v)} + 2n_a a^2)^2}{2^{t(v)}}$ with

$$a^4 = \frac{2^{t(v)}}{n_a}, \tag{11}$$

$$n_0 = \frac{(2^{t(v)} + 2n_a a^2)^2}{2^{t(v)}} - 2^{t(v)} - 2n_a v. \tag{12}$$

Alternatively N may be obtained directly as $N = 2^{t(v)} + n_a 2v + n_0$ design points, for the above design points the simple symmetry conditions 1, 2, 3 of (1) are satisfied. Condition 1 of (1) is true obviously. Conditions 2 and 3 of (1) are true as follows

$$\sum x_{iu}^2 = 2^{t(v)} + 2n_a a^2 = N\lambda_2, \tag{13}$$

$$\sum x_{iu}^4 = 2^{t(v)} + 2n_a a^4 = cN\lambda_4, \tag{14}$$

$$\sum x_{iu}^2 x_{ju}^2 = 2^{t(v)} = N\lambda_4. \tag{15}$$

From (13) and (15) we get $\lambda_2 = \frac{2^{t(v)} + 2n_a a^2}{N}$ and $\lambda_4 = \frac{2^{t(v)}}{N}$. To obtain measure of modified slope rotatability for second order response surface designs using CCD we investigate the restriction $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ (cf. Victorbabu 2005) then we get,

$$Q_v(D)=\left[\frac{\sum x_{iu}^2}{N}\right]^4\left[4e-v(\hat{b}_{ij})\right]^2,$$

where $e=V(\hat{b}_{ii})=\dfrac{2^{t(v)}+2vn_a+n_0}{4[(2^{t(v)})^2+4[2^{t(v)}n_a a^2+n_a^2a^4]]}$, (since $\lambda_2^2=\lambda_4$),

i.e., $e=V(\hat{b}_{ii})=\dfrac{F+T}{4[F^2+4[Fn_a a^2+n_a^2a^4]]}$, where $F=2^{t(v)}$ and $T=2vn_a+n_0$.

Example: We illustrate the measure of modified slope rotatability for second order response surface designs using central composite design for $v=3$ factors. The design points are $N=32$, modified slope rotatability value is $a^2=4$, $n_0=18$ from (11) and (12) respectively axial points are replicated $n_a=1$ time. At $a=2$, we get $e=0.0313$ then $Q_v(D)$ is zero. Then the design is modified slope rotatable. Variance of the estimated response for measure of modified slope rotatability is $V\left(\dfrac{\partial\hat{Y}}{\partial x_i}\right)=0.0625\sigma^2+0.125\sigma^2d^2.$

Suppose if we take $a=3.5$ instead of taking $a=2$ for $v=3$ factors we get $e=0.0101$ then $Q_v(D)=7.6350\times10^{-3}$. Here $Q_v(D)$ becomes larger it deviates from modified slope rotatability. Variance of the estimated response for measure of modified slope rotatability is $V\left(\dfrac{\partial\hat{Y}}{\partial x_i}\right)=0.0308\sigma^2+0.0403\sigma^2d^2.$

Table 1 gives the values of measure of modified slope rotatability ($Q_v(D)$) for second order response surface designs, at different values of 'a'. It can be verified that $Q_v(D)$ is zero, if and only if a design 'D' is modified slope-rotatable. $Q_v(D)$ becomes larger as 'D' deviates from a modified slope rotatable design. Variance of the estimated responses for measure of modified slope rotatability for second order response surface design for different values of 'a' are also included in the table.

7. Conclusions

In this paper, measure of modified slope rotatability for second order response surface designs has been proposed which enables us to assess the degree of slope rotatability for a given response surface design. This measure of modified slope rotatability for second order response surface designs using CCD, $Q_v(D)$ has the value zero, if and only if, the design 'D' is modified slope rotatable design, and becomes larger as 'D' deviates from a modified slope rotatable design. Variances of the estimated response are also obtained.

Table 1 Values of measure of modified slope rotatability for second order response surface designs

$v = 2, N = 36, n_a = 2$		
a	$Q_v(D)$	$V\left(\partial \hat{y} / \partial x_i\right)$
1.0	2.3815×10^{-4}	$0.1250\,\sigma^2 + 0.5625\,d^2\sigma^2$
1.4142*	0.0000	$0.0833\,\sigma^2 + 0.2500\,d^2\sigma^2$
1.5	2.3257×10^{-5}	$0.0769\,\sigma^2 + 0.2130\,d^2\sigma^2$
2.0	2.4387×10^{-3}	$0.0500\,\sigma^2 + 0.0900\,d^2\sigma^2$
2.5	0.0181	$0.0345\,\sigma^2 + 0.0428\,d^2\sigma^2$
3.0	0.0869	$0.0250\,\sigma^2 + 0.0112\,d^2\sigma^2$
3.5	0.2643	$0.0189\,\sigma^2 + 0.0128\,d^2\sigma^2$
4.0	0.7468	$0.0147\,\sigma^2 + 0.0048\,d^2\sigma^2$
4.5	1.8658	$0.0118\,\sigma^2 + 0.0050\,d^2\sigma^2$
5.0	4.2380	$0.0096\,\sigma^2 + 0.0033\,d^2\sigma^2$

$v = 3, N = 32, n_a = 1$		
a	$Q_v(D)$	$V\left(\partial \hat{y} / \partial x_i\right)$
1.0	3.6263×10^{-4}	$0.1000\,\sigma^2 + 0.3200\,d^2\sigma^2$
1.5	1.4827×10^{-4}	$0.0800\,\sigma^2 + 0.2048\,d^2\sigma^2$
2.0*	0.0000	$0.0625\,\sigma^2 + 0.1250\,d^2\sigma^2$
2.5	4.0201×10^{-4}	$0.0488\,\sigma^2 + 0.0762\,d^2\sigma^2$
3.0	2.6286×10^{-3}	$0.0385\,\sigma^2 + 0.0473\,d^2\sigma^2$
3.5	7.6350×10^{-3}	$0.0308\,\sigma^2 + 0.0403\,d^2\sigma^2$
4.0	0.0269	$0.0250\,\sigma^2 + 0.0200\,d^2\sigma^2$
4.5	0.0655	$0.0206\,\sigma^2 + 0.0136\,d^2\sigma^2$
5.0	0.1439	$0.0172\,\sigma^2 + 0.0095\,d^2\sigma^2$
1.0	3.6263×10^{-4}	$0.1000\,\sigma^2 + 0.3200\,d^2\sigma^2$

Table 1 (Continued)

$v=7,\ N=144,\ n_a=2$		
a	$Q_v(D)$	$V\left(\hat{\partial y}/\partial x_i\right)$
1.0	1.1973×10^{-5}	$0.0147\,\sigma^2+0.0311\,d^2\sigma^2$
1.5	8.5787×10^{-6}	$0.0137\,\sigma^2+0.0270\,d^2\sigma^2$
2.0	4.5025×10^{-6}	$0.0125\,\sigma^2+0.0225\,d^2\sigma^2$
2.5	9.5220×10^{-7}	$0.0112\,\sigma^2+0.0182\,d^2\sigma^2$
2.8284*	0.0000	$0.0104\,\sigma^2+0.0156\,d^2\sigma^2$
3.0	3.4900×10^{-7}	$0.0100\,\sigma^2+0.0144\,d^2\sigma^2$
3.5	7.1677×10^{-6}	$0.0089\,\sigma^2+0.0113\,d^2\sigma^2$
4.0	2.9173×10^{-5}	$0.0078\,\sigma^2+0.0088\,d^2\sigma^2$
4.5	7.9180×10^{-5}	$0.0069\,\sigma^2+0.0069\,d^2\sigma^2$
5.0	1.7748×10^{-4}	$0.0061\,\sigma^2+0.0054\,d^2\sigma^2$

$v=8,\ N=144,\ n_a=2$		
a	$Q_v(D)$	$V\left(\hat{\partial y}/\partial x_i\right)$
1.0	1.1973×10^{-5}	$0.0147\,\sigma^2+0.0311\,d^2\sigma^2$
1.5	8.5787×10^{-6}	$0.0137\,\sigma^2+0.0270\,d^2\sigma^2$
2.0	4.5025×10^{-6}	$0.0125\,\sigma^2+0.0225\,d^2\sigma^2$
2.5	9.5220×10^{-7}	$0.0112\,\sigma^2+0.0182\,d^2\sigma^2$
2.8284*	0.0000	$0.0104\,\sigma^2+0.0156\,d^2\sigma^2$
3.0	3.4900×10^{-7}	$0.0100\,\sigma^2+0.0144\,d^2\sigma^2$
3.5	7.1677×10^{-6}	$0.0089\,\sigma^2+0.0113\,d^2\sigma^2$
4.0	2.9173×10^{-5}	$0.0078\,\sigma^2+0.0088\,d^2\sigma^2$
4.5	7.9180×10^{-5}	$0.0069\,\sigma^2+0.0069\,d^2\sigma^2$
5.0	1.7748×10^{-4}	$0.0061\,\sigma^2+0.0054\,d^2\sigma^2$

Table 1 (Continued)

$v = 9$, $N = 200$, $n_a = 1$		
a	$Q_v(D)$	$V\left(\hat{\partial y}/\partial x_i\right)$
1.0	2.8873×10^{-6}	$0.0077\,\sigma^2 + 0.0118\,d^2\sigma^2$
1.5	2.4682×10^{-6}	$0.0075\,\sigma^2 + 0.0114\,d^2\sigma^2$
2.0	1.9252×10^{-6}	$0.0074\,\sigma^2 + 0.0108\,d^2\sigma^2$
2.5	1.3098×10^{-6}	$0.0071\,\sigma^2 + 0.0101\,d^2\sigma^2$
3.0	7.0010×10^{-7}	$0.0068\,\sigma^2 + 0.0094\,d^2\sigma^2$
3.5	2.0955×10^{-7}	$0.0066\,\sigma^2 + 0.0086\,d^2\sigma^2$
4.0^*	0.0000	$0.0063\,\sigma^2 + 0.0078\,d^2\sigma^2$
4.5	2.9742×10^{-7}	$0.0059\,\sigma^2 + 0.0070\,d^2\sigma^2$
5.0	1.4120×10^{-6}	$0.0056\,\sigma^2 + 0.0063\,d^2\sigma^2$

$v = 10$, $N = 200$, $n_a = 1$		
a	$Q_v(D)$	$V\left(\hat{\partial y}/\partial x_i\right)$
1.0	2.8873×10^{-6}	$0.0077\,\sigma^2 + 0.0118\,d^2\sigma^2$
1.5	2.4682×10^{-6}	$0.0075\,\sigma^2 + 0.0114\,d^2\sigma^2$
2.0	1.9252×10^{-6}	$0.0074\,\sigma^2 + 0.0108\,d^2\sigma^2$
2.5	1.3098×10^{-6}	$0.0071\,\sigma^2 + 0.0101\,d^2\sigma^2$
3.0	7.0010×10^{-7}	$0.0068\,\sigma^2 + 0.0094\,d^2\sigma^2$
3.5	2.0955×10^{-7}	$0.0066\,\sigma^2 + 0.0086\,d^2\sigma^2$
4.0^*	0.0000	$0.0063\,\sigma^2 + 0.0078\,d^2\sigma^2$
4.5	2.9742×10^{-7}	$0.0059\,\sigma^2 + 0.0070\,d^2\sigma^2$
5.0	1.4120×10^{-6}	$0.0056\,\sigma^2 + 0.0063\,d^2\sigma^2$

Table 1 (Continued)

$v = 11,\ N = 200,\ n_a = 1$		
a	$Q_v(D)$	$V\left(\hat{\partial y}/\partial x_i\right)$
1.0	2.8873×10^{-6}	$0.0077\,\sigma^2 + 0.0118\,d^2\sigma^2$
1.5	2.4682×10^{-6}	$0.0075\,\sigma^2 + 0.0114\,d^2\sigma^2$
2.0	1.9252×10^{-6}	$0.0074\,\sigma^2 + 0.0108\,d^2\sigma^2$
2.5	1.3098×10^{-6}	$0.0071\,\sigma^2 + 0.0101\,d^2\sigma^2$
3.0	7.0010×10^{-7}	$0.0068\,\sigma^2 + 0.0094\,d^2\sigma^2$
3.5	2.0955×10^{-7}	$0.0066\,\sigma^2 + 0.0086\,d^2\sigma^2$
4.0*	0.0000	$0.0063\,\sigma^2 + 0.0078\,d^2\sigma^2$
4.5	2.9742×10^{-7}	$0.0059\,\sigma^2 + 0.0070\,d^2\sigma^2$
5.0	1.4120×10^{-6}	$0.0056\,\sigma^2 + 0.0063\,d^2\sigma^2$

$v = 12,\ N = 400,\ n_a = 2$		
a	$Q_v(D)$	$V\left(\hat{\partial y}/\partial x_i\right)$
1.0	7.2184×10^{-7}	$0.0039\,\sigma^2 + 0.0059\,d^2\sigma^2$
1.5	6.1705×10^{-7}	$0.0038\,\sigma^2 + 0.0057\,d^2\sigma^2$
2.0	4.8129×10^{-7}	$0.0037\,\sigma^2 + 0.0054\,d^2\sigma^2$
2.5	3.2746×10^{-7}	$0.0036\,\sigma^2 + 0.0051\,d^2\sigma^2$
3.0	1.7503×10^{-7}	$0.0034\,\sigma^2 + 0.0047\,d^2\sigma^2$
3.5	5.2387×10^{-8}	$0.0033\,\sigma^2 + 0.0043\,d^2\sigma^2$
4.0*	0.0000	$0.0031\,\sigma^2 + 0.0039\,d^2\sigma^2$
4.5	7.4355×10^{-8}	$0.0030\,\sigma^2 + 0.0035\,d^2\sigma^2$
5.0	3.5300×10^{-7}	$0.0028\,\sigma^2 + 0.0032\,d^2\sigma^2$

Table 1 (Continued)

$v = 13,\ N = 400,\ n_a = 2$		
a	$Q_v(D)$	$V\left(\partial\hat{y}/\partial x_i\right)$
1.0	7.2184×10^{-7}	$0.0039\,\sigma^2 + 0.0059\,d^2\sigma^2$
1.5	6.1705×10^{-7}	$0.0038\,\sigma^2 + 0.0057\,d^2\sigma^2$
2.0	4.8129×10^{-7}	$0.0037\,\sigma^2 + 0.0054\,d^2\sigma^2$
2.5	3.2746×10^{-7}	$0.0036\,\sigma^2 + 0.0051\,d^2\sigma^2$
3.0	1.7503×10^{-7}	$0.0034\,\sigma^2 + 0.0047\,d^2\sigma^2$
3.5	5.2387×10^{-8}	$0.0033\,\sigma^2 + 0.0043\,d^2\sigma^2$
4.0^*	0.0000	$0.0031\,\sigma^2 + 0.0039\,d^2\sigma^2$
4.5	7.4355×10^{-8}	$0.0030\,\sigma^2 + 0.0035\,d^2\sigma^2$
5.0	3.5300×10^{-7}	$0.0028\,\sigma^2 + 0.0032\,d^2\sigma^2$

$v = 14,\ N = 400,\ n_a = 2$		
a	$Q_v(D)$	$V\left(\partial\hat{y}/\partial x_i\right)$
1.0	7.2184×10^{-7}	$0.0039\,\sigma^2 + 0.0059\,d^2\sigma^2$
1.5	6.1705×10^{-7}	$0.0038\,\sigma^2 + 0.0057\,d^2\sigma^2$
2.0	4.8129×10^{-7}	$0.0037\,\sigma^2 + 0.0054\,d^2\sigma^2$
2.5	3.2746×10^{-7}	$0.0036\,\sigma^2 + 0.0051\,d^2\sigma^2$
3.0	1.7503×10^{-7}	$0.0034\,\sigma^2 + 0.0047\,d^2\sigma^2$
3.5	5.2387×10^{-8}	$0.0033\,\sigma^2 + 0.0043\,d^2\sigma^2$
4.0^*	0.0000	$0.0031\,\sigma^2 + 0.0039\,d^2\sigma^2$
4.5	7.4355×10^{-8}	$0.0030\,\sigma^2 + 0.0035\,d^2\sigma^2$
5.0	3.5300×10^{-7}	$0.0028\,\sigma^2 + 0.0032\,d^2\sigma^2$

Table 1 (Continued)

$v=15,\ N=400,\ n_a=2$		
a	$Q_v(D)$	$V\left(\hat{\partial y}/\partial x_i\right)$
1.0	7.2184×10^{-7}	$0.0039\,\sigma^2+0.0059\,d^2\sigma^2$
1.5	6.1705×10^{-7}	$0.0038\,\sigma^2+0.0057\,d^2\sigma^2$
2.0	4.8129×10^{-7}	$0.0037\,\sigma^2+0.0054\,d^2\sigma^2$
2.5	3.2746×10^{-7}	$0.0036\,\sigma^2+0.0051\,d^2\sigma^2$
3.0	1.7503×10^{-7}	$0.0034\,\sigma^2+0.0047\,d^2\sigma^2$
3.5	5.2387×10^{-8}	$0.0033\,\sigma^2+0.0043\,d^2\sigma^2$
4.0*	0.0000	$0.0031\,\sigma^2+0.0039\,d^2\sigma^2$
4.5	7.4355×10^{-8}	$0.0030\,\sigma^2+0.0035\,d^2\sigma^2$
5.0	3.5300×10^{-7}	$0.0028\,\sigma^2+0.0032\,d^2\sigma^2$

$v=16,\ N=400,\ n_a=2$		
a	$Q_v(D)$	$V\left(\hat{\partial y}/\partial x_i\right)$
1.0	7.2184×10^{-7}	$0.0039\,\sigma^2+0.0059\,d^2\sigma^2$
1.5	6.1705×10^{-7}	$0.0038\,\sigma^2+0.0057\,d^2\sigma^2$
2.0	4.8129×10^{-7}	$0.0037\,\sigma^2+0.0054\,d^2\sigma^2$
2.5	3.2746×10^{-7}	$0.0036\,\sigma^2+0.0051\,d^2\sigma^2$
3.0	1.7503×10^{-7}	$0.0034\,\sigma^2+0.0047\,d^2\sigma^2$
3.5	5.2387×10^{-8}	$0.0033\,\sigma^2+0.0043\,d^2\sigma^2$
4.0*	0.0000	$0.0031\,\sigma^2+0.0039\,d^2\sigma^2$
4.5	7.4355×10^{-8}	$0.0030\,\sigma^2+0.0035\,d^2\sigma^2$
5.0	3.5300×10^{-7}	$0.0028\,\sigma^2+0.0032\,d^2\sigma^2$

* = exact modified slope rotatability values using CCD.

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