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A Comparative Analysis of Robust Moving Average Control Charts for Process Dispersion

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Abstract

In this paper, a comparative analysis of alternative methods for the moving average (MA) control chart for dispersion is developed using robust estimators. To compare the ability and performance of the existing moving average (MA) control charts for dispersion based on the sample standard deviation (S) and the proposed alternative methods based on robust estimators to detect shifts in a process, a Monte Carlo simulation study is used. It is observed from the results of the simulation study that the proposed robust alternative methods are effective in determining small shifts in the process and gives better performance as compared to the existing moving average (MA) control charts for dispersion, i.e. it provides swift indication about shifts in a process. An application numerical example with a real data set is used to illustrate the application and implementation of the control charts considered in this study which also supported the findings of the simulation study to some extent.

Keywords: Moving average control chart, robust dispersion estimator, standard deviation, non-normal distribution, simulation study, average run length.

1. Introduction

The control charts for variables, which first introduced by Walter Shewhart in 1924, are widely used and powerful tools for monitor and detect the variation in the process (Noiplab and Mayureesawan, 2019). They are also provide a quick indication of when the process is shifting to an out-of-control state which can help engineers to bring it back into an under-control state. The Shewhart control charts are simple to apply in industry. However, as Stoumbos et al. (2000) stated, such simple control charts may be inappropriate for detecting a small to moderate shift in the process. To overcome this problem, researchers have been attempting to introduce various control charts that can detect small to moderate changes in a manufacturing process.

The moving average (MA) control chart is one of these introduced methods. It is quite simple to interpret and to apply because it is based on familiar simple averages of the different sizes (Wong et al. 2004). The moving average (MA) control charts have been widely used in industry for monitoring of the process because they use information obtained from entire sequence of points while the Shewhart control chart only use current information (Chen and Yang 2002). Further the moving average (MA) control charts are more sensitive to detect small to moderate shift in the process as compared to Shewhart control chart (see Chen and Yu 2003, Yu and Chen 2005, Montgomery 2009, Chananet et al. 2014, etc). Other recent work on the construction and analysis of MA control charts includes Khoo and Wong (2008), Ghute and Shirke (2013), Ghute and Rajmanya (2014), Pawar and Shirke (2014), Akhundjanov and Pascua (2015), Alghamdi et al. (2017) and reference therein. All these studies identified the importance and use of MA control charts in application for detecting small to moderate shifts as close competitor to EWMA and CUSUM control charts.

Adeoti and Olaomi (2016) proposed a moving average (MA) S-control chart, using the sample standard deviation (S), for quick detection of small shifts in dispersion level of the manufacturing process. The results of their work shows that "the performance of the moving average (MA) S-control chart for varying values of the span w outweigh those of the Shewhart S-control chart for small and moderate shifts in the process variability". Actually, the moving average (MA) S-control chart proposed by Adeoti and Olaomi (2016) depends on the sample standard deviation (S). For normally distributed quality characteristics, the sample standard deviation (S), is the most efficient estimator of dispersion. However, studies have shown that it can be sensitive to departures from normality and outliers, that is not robust, see for examples, Abu-Shawiesh (2009), Aslam (2016), Alghamdi et al. (2017) and Khan et al. (2018). Robustness is a desire property of an efficient control chart. By exploring the literature and to the best of authors' knowledge, there is no work on the design of a moving average (MA) control chart for dispersion using robust scale estimators. Therefore, for efficient monitoring of small changes in the process dispersion, the current study extends the work of Adeoti and Olaomi (2016) to develop a moving average (MA) control chart for monitoring process dispersion using robust scale estimators.

The rest of the paper is organized as follows: the moving average S-control chart for dispersion is presented in Section 2. The robust estimators of process dispersion used in this study are discussed in Section 3. Section 4 gives the design structure of alternative robust moving average (MA) control charts for dispersion proposed in this study. The performance evaluation of the proposed moving average (MA) control charts for dispersion with respect to the average run length (ARL) values of different shift levels have been discussed in Section 5. The Monte-Carlo simulation study is given in Section 5. To illustrate the application and implementation of the control charts discussed in the study, a numerical examples uses a real data set is provided in Section 6. Finally, Section 7 includes summary of the whole study with conclusive remarks.

2. The Moving Average (MA) S-Control Chart

Adeoti and Olaomi (2016) proposed a moving average (MA) control chart based on the sample standard deviation (S) statistic, namely MAS-control chart, for monitoring the small to moderate changes in process dispersion. When the process standard deviation (σ) is unknown, which is the case for many real life applications, then σ is estimated by $\overline{S}/c_4$ where c_4 is a constant that make S be an unbiased estimator of σ. The structure of MAS-control chart as given by Adeoti and Olaomi (2016) is given as follows:

2.1. Design structure

Suppose that, we have a random sample of size n at time i from a normal distribution $N(\mu,\sigma^2)$ and $S_1, S_2, ..., S_i, ...$ be the sample standard deviation of each subgroup *i*. The moving average (MA) statistic of span w at time i denoted by MA_i is defined as follows:

$$MA_i = \begin{cases} \dfrac{\sum_{j=i-w+1}^{i} S_j}{w}, & i \geq w \\ \dfrac{\sum_{j=1}^{i} S_j}{i}, & i < w. \end{cases} \quad (1)$$

The mean for the moving average, $E(MA_i)$, and the variance for the moving average, $Var(MA_i)$, are given as follows (Adeoti and Olaomi 2016):

$$E(MA_i) = c_4\sigma \quad \text{for } i < w \text{ and } i \geq w. \quad (2)$$

$$Var(MA_i) = \begin{cases} \dfrac{\sigma^2(1-c_4^2)}{i}, & i < w \\ \dfrac{\sigma^2(1-c_4^2)}{w}, & i \geq w. \end{cases} \quad (3)$$

The 3σ control limits of MAS-control chart, when σ is estimated by $\overline{S}/c_4$, as proposed by Adeoti and Olaomi (2016) are:

Case (I): For periods $i < w$

$$\begin{aligned} LCL &= c_4\hat{\sigma} - 3\hat{\sigma}\sqrt{\frac{1-c_4^2}{i}} = \overline{S} - 3\left(\frac{\overline{S}}{c_4}\right)\sqrt{\frac{1-c_4^2}{i}} = \left[1 - \frac{3}{c_4}\sqrt{\frac{1-c_4^2}{i}}\right]\overline{S} = D_5^*\overline{S} \\ CL &= c_4\hat{\sigma} = \overline{S} \\ UCL &= c_4\hat{\sigma} + 3\hat{\sigma}\sqrt{\frac{1-c_4^2}{i}} = \overline{S} + 3\left(\frac{\overline{S}}{c_4}\right)\sqrt{\frac{1-c_4^2}{i}} = \left[1 + \frac{3}{c_4}\sqrt{\frac{1-c_4^2}{i}}\right]\overline{S} = D_6^*\overline{S} \end{aligned} \quad (4)$$

Case (II): For periods $i \geq w$

$$\begin{aligned} LCL &= c_4\hat{\sigma} - 3\hat{\sigma}\sqrt{\frac{1-c_4^2}{w}} = \overline{S} - 3\left(\frac{\overline{S}}{c_4}\right)\sqrt{\frac{1-c_4^2}{w}} = \left[1 - \frac{3}{C_4}\sqrt{\frac{1-c_4^2}{w}}\right]\overline{S} = D_7^*\overline{S} \\ CL &= c_4\hat{\sigma} = \overline{S} \\ UCL &= c_4\hat{\sigma} + 3\hat{\sigma}\sqrt{\frac{1-c_4^2}{w}} = \overline{S} + 3\left(\frac{\overline{S}}{c_4}\right)\sqrt{\frac{1-c_4^2}{w}} = \left[1 + \frac{3}{c_4}\sqrt{\frac{1-c_4^2}{w}}\right]\overline{S} = D_8^*\overline{S} \end{aligned} \quad (5)$$

where $D_5^* = 1 - \frac{3}{c_4}\sqrt{\frac{1-c_4^2}{i}}$, $D_6^* = 1 + \frac{3}{c_4}\sqrt{\frac{1-c_4^2}{i}}$, $D_7^* = 1 - \frac{3}{c_4}\sqrt{\frac{1-c_4^2}{w}}$ and $D_8^* = 1 + \frac{3}{c_4}\sqrt{\frac{1-c_4^2}{w}}$.

The constants D_5^*, D_6^*, D_7^* and D_8^* depends on the values of time $i=1,2$, span $w=2,3,4$ and sample size $n=2,3,4,...,15$. These values can be found in Adeoti and Olaomi (2016).

2.2. Out-of-control signals

The MAS-control chart is constructed by plotting the MA_i statistic on the chart against the subgroup i. The probability that a MAS-control chart signals an out-of-control when a point plots outside the control limits is given as $P(MA_i > UCL \ or \ MA_i < LCL)$. If the LCL is calculated to be less than zero, then it is set to be zero.

The MAS-control chart proposed by Adeoti and Olaomi (2016) is based on the sample standard deviation (S), which is not a robust estimator of dispersion. Even that, the sample standard deviation (S) is the most common dispersion estimator that provides a logical point estimate of the population standard deviation (σ), but unfortunately, it is a non-linear function of data and is very sensitive to the presence of outliers in the data (Tukey 1960, Bonett 2006). In this paper, three common alternatives to the sample standard deviation (S) are considered as robust estimators of dispersion to be used in the estimating of the process standard deviation (σ) for the proposed robust moving average (MA) control charts for dispersion.

3. Robust Estimators of Dispersion Alternative to the Sample Standard Deviation

In this section, we describe the three robust dispersion estimators that have been used as alternatives to the sample standard deviation (S) in the construction of the proposed moving average (MA) control charts (see Tiku and Akkaya 2004, Abu-Shawiesh 2008, Abbasi and Miller 2012, Akyüz et al. 2017, etc).

3.1. The median absolute deviation from the sample median estimator

The median absolute deviation from the sample median denoted by MAD is a simple, easy to calculate and robust scale estimator proposed by Hampel (1974). Let X be the quality variable of interest, and let $X_1, X_2, ..., X_n$ be a random sample of size n with a sample median (MD), then the MAD can be calculated as follows:

$$MAD = 1.4826\,MD\{\,|X_i - MD|\,\}\ ; \quad i = 1, 2, 3, \ldots, n \tag{6}$$

where

$$MD = \begin{cases} X_{\left(\frac{n+1}{2}\right)}, & \text{if } n \text{ is odd} \\ \dfrac{X_{\left(\frac{n}{2}\right)} + X_{\left(\frac{n+1}{2}\right)}}{2}, & \text{if } n \text{ is even} \end{cases} \tag{7}$$

Rousseeuw and Croux (1993) showed that the estimate $\hat{\sigma}_{MAD} = b_n MAD$ is an unbiased estimator for the process standard deviation (σ), where b_n is a constant depends on the sample size n given in the literature. Wu et al. (2002) showed that for contaminated normal data, the MAD outperformed some other robust estimators.

3.2. The Rousseeuw and Croux S_n estimator

The S_n estimator was proposed by Rousseeuw and Croux (1993) as a powerful alternative to the MAD. This estimator is very simple and easy to compute. It is based on the use of repeated medians: the inner median and the outer median. Therefore, the S_n estimator can be defined as the median of the n medians of the absolute differences between the values. Let X be the quality variable of interest, and let $X_1, X_2, ..., X_n$ be a random sample of size $n,$ then the S_n estimator can be calculated as follows:

$$S_n = 1.1926\, MD_i \left\{ MD_j \left| X_i - X_j \right| \right\} \; ; \; i, j = 1, 2, 3, \ldots, n. \tag{8}$$

where the factor 1.1926 is for consistency. The statistic $\hat{\sigma}_{S_n} = d_n S_n$ is an unbiased estimator of the process standard deviation (σ) where d_n is a constant factor depends on the sample size n given in the literature.

3.3. The Rousseeuw and Croux Q_n estimator

The Q_n estimator was proposed by Rousseeuw and Croux (1993) as another powerful alternative to the MAD. This estimator is very simple and easy to compute. Let X be the quality variable of interest, and let $X_1, X_2, ..., X_n$ be a random sample of size $n,$ then the Q_n estimator can be calculated as follows:

$$Q_n = 2.2219 \left\{ \left| X_i - X_j \right| ; i < j \right\}_{(g)} ; \; i, j = 1, 2, 3, \ldots, n. \tag{9}$$

where $g = \binom{h}{2} = \frac{h(h-1)}{2}$ and $h = \left[\frac{n}{2}\right] + 1$ where $\left[\frac{n}{2}\right]$ is the integer part of fraction $n/2$. In simple terms, Q_n is the g^{th} order statistic of n-choose-2 interpoint distances. The $\hat{\sigma}_{Q_n} = e_n Q_n$ will be an unbiased estimator of process standard deviation (σ) where e_n is a constant factor depends on the sample size n given in literature.

4. The Proposed Robust Moving Average (MA) Control Charts for Dispersion

In this section, three moving average (MA) control charts, based on robust statistics namely MAD, S_n and Q_n defined in the previous section, are proposed for monitoring small to moderate changes in process dispersion more efficiently. In this study, we will refer to the moving average (MA) control charts for dispersion based on S, MAD, S_n and Q_n as MAS-control chart (proposed by Adeoti and Olaomi (2016)), MAMAD-control chart, MASn-control chart and MAQn-control chart for the rest of this study.

4.1. The MAMAD-control chart

In this section, the structure design of the MAMAD-control chart, proposed as a robust alternative to the MAS-control chart, will be given. Let $MAD_1, MAD_2, ..., MAD_i, ...$ be the median of the absolute deviations from a series of subgroups obtained from normal distribution. The moving average of span w at time i denoted by $MAMAD_i$ is defined as follows:

$$MAMAD_i = \begin{cases} \dfrac{MAD_i + MAD_{i-1} + \ldots + MAD_{i-w+1}}{w}, & i \geq w, \\[2ex] \dfrac{\sum_{j=1}^{i} MAD_j}{i}, & i < w, \end{cases} \tag{10}$$

where $\overline{MAD} = \frac{\sum_{i=1}^{m} MAD_i}{m}$ and $\hat{\sigma} = b_n \overline{MAD}.$ The control limits (LCL and UCL) and the center line (CL) for the MAMAD-control chart will be calculated as follows:

Case (I): For periods $i < w$

$$\begin{aligned} LCL &= c_4\hat{\sigma} - 3\,\hat{\sigma}\sqrt{\frac{1-c_4^2}{i}} = c_4 b_n \overline{MAD} - 3\,(b_n \overline{MAD})\sqrt{\frac{1-c_4^2}{i}} = D_9^* \overline{MAD}, \\ CL &= c_4\hat{\sigma} = c_4 b_n \overline{MAD} = D_{10}^* \overline{MAD}, \\ UCL &= c_4\hat{\sigma} + 3\,\hat{\sigma}\sqrt{\frac{1-c_4^2}{i}} = c_4 b_n \overline{MAD} + 3\,(b_n \overline{MAD})\sqrt{\frac{1-c_4^2}{i}} = D_{11}^* \overline{MAD}, \end{aligned} \tag{11}$$

Case (II): For periods $i \geq w$

$$\begin{aligned} LCL &= c_4\hat{\sigma} - 3\,\hat{\sigma}\sqrt{\frac{1-c_4^2}{w}} = c_4 b_n \overline{MAD} - 3\,(b_n \overline{MAD})\sqrt{\frac{1-c_4^2}{w}} = D_{12}^* \overline{MAD}, \\ CL &= c_4\hat{\sigma} = c_4 b_n \overline{MAD} = D_{10}^* \overline{MAD}, \\ UCL &= c_4\hat{\sigma} + 3\,\hat{\sigma}\sqrt{\frac{1-c_4^2}{w}} = c_4 b_n \overline{MAD} + 3\,(b_n \overline{MAD})\sqrt{\frac{1-c_4^2}{w}} = D_{13}^* \overline{MAD}, \end{aligned} \tag{12}$$

where $D_9^* = b_n\left(c_4 - 3\sqrt{\frac{1-c_4^2}{i}}\right)$, $D_{10}^* = b_n c_4$, $D_{11}^* = b_n\left(c_4 + 3\sqrt{\frac{1-c_4^2}{i}}\right)$, $D_{12}^* = b_n\left(c_4 - 3\sqrt{\frac{1-c_4^2}{w}}\right)$ and $D_{13}^* = b_n\left(c_4 + 3\sqrt{\frac{1-c_4^2}{w}}\right).$

The constants $D_9^*, D_{10}^*, D_{11}^*, D_{12}^*$ and D_{13}^* which are required in the construction of MAMAD-control chart, are also depends on the sample size n and span $w.$ These constants are calculated and provided here in Table 1. These constants are also not monotonic for $n = 2$ as compared to $n > 2.$ The results are expected as discussed by Adekeye and Azubuike (2012).

Table 1 The control limit factors for the MAMAD-control chart

n	$i < w$				$i \geq w$						$i < w$ and $i \geq w$
	i				w						
	$i = 1$		$i = 2$		$w = 2$		$w = 3$		$w = 4$		
	D_9^*	D_{11}^*	D_9^*	D_{11}^*	D_{12}^*	D_{13}^*	D_{12}^*	D_{13}^*	D_{12}^*	D_{13}^*	D_{10}^*
2	0.000	3.117	0.000	2.484	0.000	2.484	0.000	2.203	0.000	2.036	0.954
3	0.000	3.403	0.000	2.794	0.000	2.794	0.125	2.525	0.286	2.364	1.325
4	0.000	2.846	0.131	2.380	0.131	2.380	0.338	2.174	0.461	2.051	1.256
5	0.000	2.368	0.261	2.006	0.261	2.006	0.421	1.846	0.516	1.751	1.134
6	0.034	2.249	0.359	1.925	0.359	1.925	0.502	1.781	0.588	1.696	1.142
7	0.129	2.058	0.412	1.776	0.412	1.776	0.537	1.651	0.611	1.576	1.094
8	0.201	1.978	0.461	1.718	0.461	1.718	0.577	1.602	0.645	1.534	1.089
9	0.256	1.890	0.496	1.650	0.496	1.650	0.602	1.544	0.665	1.481	1.073
10	0.301	1.814	0.522	1.592	0.522	1.592	0.620	1.494	0.679	1.436	1.057
11	0.339	1.764	0.547	1.556	0.547	1.556	0.640	1.463	0.695	1.408	1.051
12	0.371	1.723	0.569	1.525	0.569	1.525	0.657	1.437	0.709	1.385	1.047
13	0.398	1.690	0.587	1.501	0.587	1.501	0.671	1.417	0.721	1.367	1.044
14	0.423	1.658	0.604	1.478	0.604	1.478	0.684	1.397	0.732	1.350	1.041
15	0.444	1.631	0.618	1.457	0.618	1.457	0.695	1.380	0.741	1.334	1.037

Now, the MAMAD-control chart is constructed by plotting the MAMAD_i statistic on the chart against the subgroup i. The probability that a MAMAD-control chart signals an out-of-control when a point plots outside the control limits is given as $P(MAMAD_i > UCL$ or $MAMAD_i < LCL)$. If the LCL is calculated to be less than zero, then it is set to be zero.

4.2. The MASn-control chart

In this section, the structure design of the MASn-control chart, proposed as a robust alternative to the MAS-control chart, will be given. Let $S_{n_1}, S_{n_2}, \ldots, S_{n_i}, \ldots$ be the Rousseeuw and Croux (1993) estimators from a series of subgroups obtained from normal distribution. The moving average (MA) of span w at time i denoted by MAS_{n_i} is defined as follows:

$$MAS_{n_i} = \begin{cases} \dfrac{S_{n_i} + S_{n_{i-1}} + \ldots + S_{n_{i-w+1}}}{w}, & i \geq w, \\ \dfrac{\sum_{j=1}^{i} S_{n_j}}{i}, & i < w, \end{cases} \tag{13}$$

where $\overline{S_n} = \sum_{i=1}^{m} S_{n_i} / m$ and $\hat{\sigma} = d_n \overline{S_n}$. The control limits (LCL and UCL) and the center line (CL) for the MASn-control chart will be calculated as follows:

Case (I): For periods $i < w$

$$\begin{aligned} LCL &= c_4\hat{\sigma} - 3\hat{\sigma}\sqrt{\frac{1-c_4^2}{i}} = c_4 d_n \overline{S_n} - 3(d_n \overline{S_n})\sqrt{\frac{1-c_4^2}{i}} = D_{14}^* \overline{S_n}, \\ CL &= c_4\hat{\sigma} = c_4 d_n \overline{S_n} = D_{15}^* \overline{S_n}, \\ UCL &= c_4\hat{\sigma} + 3\hat{\sigma}\sqrt{\frac{1-c_4^2}{i}} = c_4 d_n \overline{S_n} + 3(d_n \overline{S_n})\sqrt{\frac{1-c_4^2}{i}} = D_{16}^* \overline{S_n}, \end{aligned} \tag{14}$$

Case (II): For periods $i \geq w$

$$\begin{aligned} LCL &= c_4\hat{\sigma} - 3\hat{\sigma}\sqrt{\frac{1-c_4^2}{w}} = c_4 d_n \overline{S_n} - 3(d_n \overline{S_n})\sqrt{\frac{1-c_4^2}{w}} = D_{17}^* \overline{S_n}, \\ CL &= c_4\hat{\sigma} = c_4 d_n \overline{S_n} = D_{15}^* \overline{S_n}, \\ UCL &= c_4\hat{\sigma} + 3\hat{\sigma}\sqrt{\frac{1-c_4^2}{w}} = c_4 d_n \overline{S_n} + 3(d_n \overline{S_n})\sqrt{\frac{1-c_4^2}{w}} = D_{18}^* \overline{S_n}, \end{aligned} \tag{15}$$

where $D_{14}^* = d_n\left(c_4 - 3\sqrt{\frac{1-c_4^2}{i}}\right)$, $D_{15}^* = d_n c_4$, $D_{16}^* = d_n\left(c_4 + 3\sqrt{\frac{1-c_4^2}{i}}\right)$, $D_{17}^* = d_n\left(c_4 - 3\sqrt{\frac{1-c_4^2}{w}}\right)$ and $D_{18}^* = d_n\left(c_4 + 3\sqrt{\frac{1-c_4^2}{w}}\right)$.

The constants $D_{14}^*, D_{15}^*, D_{16}^*, D_{17}^*$ and D_{18}^* which are required in the construction of MASn-control chart, are also depends on the sample size n and span w. These constants are calculated and provided in Table 2.

Now, the MASn-control chart is constructed by plotting the MASn_i statistic on the chart against subgroup i. The probability that a MASn-control chart signals an out-of-control when a point plots outside the control limits is given as $P(MASn_i > UCL \text{ or } MASn_i < LCL)$. If the LCL is calculated to be less than zero, then it is set to be zero.

Table 2 The control limit factors for the MASn-control chart

n	$i<w$				$i \geq w$						$i<w$ and $i \geq w$
	i				w						
	$i=1$		$i=2$		$w=2$		$w=3$		$w=4$		
	D_{14}^*	D_{16}^*	D_{14}^*	D_{16}^*	D_{17}^*	D_{18}^*	D_{17}^*	D_{18}^*	D_{17}^*	D_{18}^*	D_{15}^*
2	0.000	1.936	0.000	1.543	0.000	1.543	0.000	1.369	0.000	1.265	0.593
3	0.000	4.213	0.000	3.460	0.000	3.460	0.155	3.126	0.354	2.927	1.640
4	0.000	1.986	0.092	1.661	0.092	1.661	0.236	1.517	0.321	1.431	0.876
5	0.000	2.653	0.292	2.248	0.292	2.248	0.472	2.068	0.579	1.961	1.270
6	0.028	1.861	0.297	1.593	0.297	1.593	0.416	1.474	0.487	1.403	0.945
7	0.136	2.163	0.433	1.866	0.433	1.866	0.564	1.735	0.643	1.656	1.149
8	0.179	1.761	0.411	1.529	0.411	1.529	0.513	1.426	0.574	1.365	0.970
9	0.262	1.931	0.506	1.686	0.506	1.686	0.615	1.578	0.679	1.513	1.096
10	0.277	1.669	0.480	1.465	0.480	1.465	0.571	1.375	0.625	1.321	0.973
11	0.342	1.782	0.553	1.571	0.553	1.571	0.646	1.478	0.702	1.422	1.062
12	0.346	1.609	0.531	1.424	0.531	1.424	0.613	1.342	0.662	1.293	0.978
13	0.401	1.702	0.592	1.512	0.592	1.512	0.676	1.428	0.727	1.377	1.052
14	0.399	1.563	0.569	1.393	0.569	1.393	0.645	1.317	0.690	1.272	0.981
15	0.447	1.643	0.622	1.468	0.622	1.468	0.700	1.390	0.746	1.344	1.045

4.3. The MAQn-control chart

In this section, the structure design of the MAQn-control chart, proposed as a robust alternative to the MAS-control chart, will be given. Let $Q_{n_1}, Q_{n_2}, \ldots, Q_{n_i}, \ldots$ be the Rousseeuw and Croux (1993) estimators from a series of subgroups obtained from normal distribution. The moving average (MA) of span w at time i denoted by $MAQn_i$ is defined as follows:

$$MAQn_i = \begin{cases} \dfrac{Q_{n_i} + Q_{n_{i-1}} + \ldots + Q_{n_{i-w+1}}}{w}, & i \geq w, \\ \dfrac{\sum_{j=}^{i} Q_{n_j}}{i}, & i < w, \end{cases} \tag{16}$$

where $\overline{Q_n} = \sum_{i=1}^{m} Q_{n_i} / m$ and $\hat{\sigma} = e_n \overline{Q_n}$. The control limits (LCL and UCL) and the center line (CL) for the MAQn-control chart will be calculated as follows:

Case (I): For periods $i < w$

$$\begin{aligned} LCL &= c_4\hat{\sigma} - 3\hat{\sigma}\sqrt{\frac{1-c_4^2}{i}} = c_4 e_n \overline{Q_n} - 3(e_n \overline{Q_n})\sqrt{\frac{1-c_4^2}{i}} = D_{19}^* \overline{Q_n}, \\ CL &= c_4\hat{\sigma} = c_4 e_n \overline{Q_n} = D_{20}^* \overline{Q_n}, \\ UCL &= c_4\hat{\sigma} + 3\hat{\sigma}\sqrt{\frac{1-c_4^2}{i}} = c_4 e_n \overline{Q_n} + 3(e_n \overline{Q_n})\sqrt{\frac{1-c_4^2}{i}} = D_{21}^* \overline{Q_n}, \end{aligned} \tag{17}$$

Case (II): For periods $i \geq w$

$$LCL = c_4\hat{\sigma} - 3\hat{\sigma}\sqrt{\frac{1-c_4^2}{w}} = c_4 e_n \overline{Q_n} - 3(e_n \overline{Q_n})\sqrt{\frac{1-c_4^2}{w}} = D_{22}^* \overline{Q_n},$$
$$CL = c_4\hat{\sigma} = c_4 e_n \overline{Q_n} = D_{20}^* \overline{Q_n}, \tag{18}$$
$$UCL = c_4\hat{\sigma} + 3\hat{\sigma}\sqrt{\frac{1-c_4^2}{w}} = c_4 e_n \overline{Q_n} + 3(e_n \overline{Q_n})\sqrt{\frac{1-c_4^2}{w}} = D_{23}^* \overline{Q_n},$$

where $D_{19}^* = e_n\left(c_4 - 3\sqrt{\frac{1-c_4^2}{i}}\right)$, $D_{20}^* = e_n c_4$, $D_{21}^* = e_n\left(c_4 + 3\sqrt{\frac{1-c_4^2}{i}}\right)$, $D_{22}^* = e_n\left(c_4 - 3\sqrt{\frac{1-c_4^2}{w}}\right)$ and $D_{23}^* = e_n\left(c_4 + 3\sqrt{\frac{1-c_4^2}{w}}\right)$.

The constants $D_{19}^*, D_{20}^*, D_{21}^*, D_{22}^*$ and D_{23}^* which are required in the construction of MAQn-control chart, are also depends on the sample size n and span w. These constants are calculated and provided in Table 3.

Now, the moving average MAQn-control chart is constructed by plotting the MAQn$_i$ statistic on the chart against the sample i. The probability that a moving average MAQn-control chart signals an out-of-control when a point plots outside the control limits is given as $P(MAQn_i > UCL$ or $MAQn_i < LCL)$. If the LCL is calculated to be less than zero, then it is set to be zero.

Table 3 The control limit factors for the MAQn-control chart

n	$i < w$				$i \geq w$						$i < w$ and $i \geq w$
	i				w						
	$i=1$		$i=2$		$w=2$		$w=3$		$w=4$		
	D_{19}^*	D_{21}^*	D_{19}^*	D_{21}^*	D_{22}^*	D_{23}^*	D_{22}^*	D_{23}^*	D_{22}^*	D_{23}^*	D_{20}^*
2	0.000	1.040	0.000	0.829	0.000	0.829	0.000	0.735	0.000	0.679	0.318
3	0.000	2.256	0.000	1.852	0.000	1.852	0.083	1.673	0.190	1.567	0.878
4	0.000	1.069	0.049	0.894	0.049	0.894	0.127	0.817	0.173	0.770	0.472
5	0.000	1.657	0.183	1.404	0.183	1.404	0.295	1.292	0.361	1.225	0.793
6	0.017	1.145	0.183	0.980	0.183	0.980	0.256	0.907	0.299	0.863	0.581
7	0.097	1.547	0.309	1.335	0.309	1.335	0.404	1.241	0.460	1.185	0.822
8	0.119	1.172	0.273	1.018	0.273	1.018	0.342	0.949	0.382	0.909	0.646
9	0.202	1.488	0.390	1.300	0.390	1.300	0.474	1.217	0.524	1.167	0.845
10	0.200	1.210	0.348	1.062	0.348	1.062	0.414	0.997	0.453	0.958	0.705
11	0.279	1.452	0.450	1.280	0.450	1.280	0.527	1.204	0.572	1.158	0.865
12	0.263	1.221	0.403	1.081	0.403	1.081	0.465	1.019	0.502	0.982	0.742
13	0.337	1.431	0.498	1.271	0.498	1.271	0.569	1.200	0.611	1.158	0.884
14	0.314	1.230	0.448	1.096	0.448	1.096	0.508	1.037	0.543	1.001	0.772
15	0.385	1.413	0.535	1.262	0.535	1.262	0.602	1.196	0.642	1.156	0.899

5. Performance Evaluation of Moving Average Control Charts for Dispersion

The performance comparison of the control charts is evaluated by using different measures, among these, the Average Run Length (ARL). The ARL defines as "the average number of samples (*subgroups*) collected before an out-of-control signal is shown". Therefore, the ARL value is of high interest in the development of any control chart scheme (Knoth 2007). The average run length (ARL) comparison for in-control and out-of-control processes have been studied by many authors, see for example the following studies: Crowder (1987), Molnau et al. (2001), Li et al. (2014), Chananet et al. (2014)

and Abu-Shawiesh et al. (2019). In case of in-control process, a large ARL value is desired while a small ARL value is desired when σ_0 shift to $\sigma_1 = \delta\sigma_0\ (\delta > 1)$. Let the in-control process of quality characteristic follows a normal distribution, i.e. $N(\mu, \sigma_0^2)$ and in case of out-of-control, the process follows $N(\mu, \delta\sigma_0^2)$. The ARL for in-control and out-of-control situations is used as a performance of moving average (MA) control charts for dispersion (based on S, MAD, S_n and Q_n) and are calculated using a mathematical approximation proposed by Khoo (2004) and Adeoti and Olaomi (2016). The expression for this mathematical approximation is given as follows:

$$ARL \cong \left\{1-\sum_{i=1}^{w}\left[p\left(Z_1 > \frac{t\sigma+3\sigma\sqrt{\frac{1-t^2}{i}}-T}{\sqrt{\frac{1-t^2}{i}}}\right)+p\left(Z_1 < \frac{t\sigma-3\sigma\sqrt{\frac{1-t^2}{i}}-T}{\sqrt{\frac{1-t^2}{i}}}\right)\right]\right\}$$
$$\times\left\{p\left(Z_2 > \frac{t\sigma+3\sigma\sqrt{\frac{1-t^2}{w}}-T}{\sqrt{\frac{1-t^2}{w}}}\right)+p\left(Z_2 < \frac{t\sigma-3\sigma\sqrt{\frac{1-t^2}{w}}-T}{\sqrt{\frac{1-t^2}{w}}}\right)\right\}^{-1}+(w-1), \tag{19}$$

where T is either S, MAD, S_n or Q_n. The amount of shift values is given as $\delta = \sigma_1/\sigma_0$ where $\delta = \{1.00, 1.25, 1.50, ..., 3.00\}$. For the sack of generalization, the standardized normal distribution is considered here and fixed $L_0 = 370,$ when the process is in-control level. The moving average (MA) control chart for dispersion produces the minimum out-of-control average run length (ARL_1) is declared the more efficient control chart for the fixed in-control average run length (ARL_0).

5.1. Simulation study

To evaluate the performance of various proposed robust moving average (MA) control charts for dispersion with the existing control charts considered in Sections 2 and 4, we performed a comprehensive Monte Carlo simulation study. A total of five dispersion control charts were studied. The run length characteristic is used as an evaluation measure. The Monte Carlo simulation is the most popular scheme for the evaluation of a control chart in the quality control literature which is used when the theoretical approach is difficult to implement. The application of the Monte Carlo simulation for evaluation of the control charts have been studied by many authors including Sullivan and Woodall (1996), Fu and Hu (1999), Testik et al. (2003) and Jones-Farmer et al. (2009). The ARLs are estimated by running the proposed schemes using the R-language program. An algorithm of evaluating ARLs based on the following steps is used:

Step 1. An m subgroups each of sample size n is generated from either normal with the specified parameters.

Step 2. The dispersion estimates and their average values are calculated.

Step 3. The control limits of the moving average (MA) control chart for dispersion using these average estimates are determined.

Step 4. Finally, using the mathematical approximation defined in (19), ARL is calculated for the control limits.

This process from Steps 1-4 is repeated 10,000 times. The mean value of the 10,000 ARLs, with standard error lies within the range 0.005-0.023, is reported as performance measure. The ARLs of moving average (MA) control chart using different estimates of dispersion are calculated for different amount of shifts in the process dispersion with sample sizes $n = 5$, 10 and number of subgroups $m = 25$. The results of the simulation study are reported in Tables 4-5, respectively. Tables 4-5 shows that ARL decreases as n increases from 5 to 10 and ARL decreases more rapidly as w increases from 2 to 4. Also, ARL decreases as the amount of shift values increases.

Now, the comparison between Shewhart and moving average (MA) control charts from Tables 4-5 reveals that

(i) The moving average (MA) control chart for dispersion detects all shifts more quickly as compared to the traditional Shewhart S-control chart because it considers the previous observation along current data. However, its efficiency losses when large shift occur, say $\delta \geq 2$ (see Table 4). The ARL values are inversely proportional to the shift size (δ).

(ii) The performance of moving average (MA) control chart for dispersion directly affected with span size (w) for small to moderate shift in the process dispersion. Therefore, the moving average (MA) control chart for dispersion performs better in case of small to moderate shifts (δ) for all sample sizes.

Table 4 The ARL values for the Shewhart S-control chart

Sample Size (n)	Shift (δ)								
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
5	370.158	31.552	9.052	4.256	2.981	1.965	1.705	1.589	1.352
10	370.455	17.102	4.005	2.565	1.505	1.221	1.124	1.074	1.032

Further, from Table 5 it can be seen that

(a) The ARL results for the MAMAD-control chart shows that for any range of shifts (δ), the MAMAD-control chart consistently gives smaller out-of-control ARL as compared to the Shewhart S-control chart and MAS-control chart for all sample sizes. Therefore, the MAMAD-control chart performs better than the Shewhart S-control chart and MAS-control chart for any range of shifts (δ) in the process dispersion.

(b) The ARL results shows that MAQn-control chart performs better than its competitive robust MAMAD-control chart and MASn-control chart in the detecting small to moderate shifts in process dispersion. Among these, MAMAD-control chart shows the worst performance because of low Gaussian efficiency of MAD.

Hence, it is concluded that the moving average (MA) control charts for dispersion are more efficient in terms of ARLs values and have shown better performance than the Shewhart S-control chart for detecting small to moderate shifts. The MAMAD-control chart, the MASn-control chart and the MAQn-control chart can be treated as strong robust competitors to the MAS-control chart. They have shown at least equal performance to the MAS-control chart for the detection of small to moderate shifts in the process standard deviation (σ). The MAQn-control chart has the best performance followed by MASn-control chart, MAMAD-control chart and MAS-control chart.

Table 5 The ARL values for the moving average control charts for dispersion

Shift (δ)	$n=5$			$n=10$		
MAS-Control Chart	$w=2$	$w=3$	$w=4$	$w=2$	$w=3$	$w=4$
1.00	367.009	365.418	360.372	367.411	364.372	361.371
1.25	20.536	15.332	12.514	10.560	7.562	6.965
1.50	5.245	3.856	3.682	2.855	2.714	2.621
1.75	2.526	2.401	2.461	1.623	1.655	1.766
2.00	1.925	1.945	1.966	1.311	1.322	1.354
2.25	1.445	1.405	1.492	1.172	1.198	1.194
2.50	1.385	1.426	1.478	1.098	1.110	1.117
2.75	1.284	1.320	1.350	1.061	1.069	1.058
3.00	1.215	1.251	1.268	1.041	1.043	1.042
MAMAD-Control Chart						
Shift (δ)	$n=5$			$n=10$		
	$w=2$	$w=3$	$w=4$	$w=2$	$w=3$	$w=4$
1.00	370.715	370.370	370.372	370.371	370.371	370.372
1.25	17.287	12.001	10.645	8.800	6.046	5.748
1.50	3.948	2.995	2.795	2.652	2.654	2.521
1.75	2.251	1.996	2.011	1.512	1.604	1.612
2.00	1.788	1.755	1.767	1.201	1.284	1.287
2.25	1.318	1.308	1.312	1.100	1.145	1.148
2.50	1.286	1.287	1.275	1.051	1.052	1.053
2.75	1.234	1.224	1.228	1.032	1.033	1.031
3.00	1.164	1.160	1.161	1.021	1.023	1.022
MASn-Control Chart						
Shift (δ)	$n=5$			$n=10$		
	$w=2$	$w=3$	$w=4$	$w=2$	$w=3$	$w=4$
1.00	370.721	370.372	370.371	370.370	370.371	370.372
1.25	16.407	12.001	10.645	8.800	6.046	5.748
1.50	3.851	2.995	2.795	2.652	2.634	2.521
1.75	2.217	1.996	1.895	1.512	1.604	1.612
2.00	1.745	1.731	1.728	1.201	1.284	1.287
2.25	1.309	1.295	1.298	1.100	1.145	1.148
2.50	1.271	1.273	1.270	1.051	1.052	1.053
2.75	1.220	1.215	1.224	1.032	1.033	1.031
3.00	1.151	1.152	1.154	1.021	1.023	1.022
MAQn-Control Chart						
Shift (δ)	$n=5$			$n=10$		
	$w=2$	$w=3$	$w=4$	$w=2$	$w=3$	$w=4$
1.00	370.371	370.370	370.370	370.370	370.372	370.372
1.25	15.744	11.958	10.012	8.654	5.895	5.748
1.50	3.834	2.974	2.795	2.601	2.597	2.521
1.75	2.202	1.975	1.895	1.454	1.424	1.612
2.00	1.731	1.730	1.728	1.187	1.210	1.194
2.25	1.294	1.291	1.298	1.084	1.091	1.092
2.50	1.261	1.265	1.270	1.042	1.044	1.043
2.75	1.214	1.210	1.224	1.029	1.030	1.029
3.00	1.138	1.152	1.154	1.020	1.019	1.020

5.2. Effect of contamination/outliers

The above simulation procedure is again adopted in this subsection to see the impact of contamination on the performance of moving average (MA) control charts for dispersion. The 70% observations are drawn from $N(0,1)$ and 30% observations are drawn from $N(0,5)$, the set of observations now is contaminated with outliers. The results are reported here in Tables 6-7 for discussion purposes. Results of Tables 6-7 shows that:

Table 6 The ARL values for the Shewhart S-Control Chart when outliers exist

Sample Size (n)	Shift (δ)								
	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
5	378.501	42.192	15.256	9.120	6.468	4.125	3.450	2.152	1.790
10	380.256	28.153	12.565	7.741	4.526	2.821	1.984	1.574	1.272

Table 7 The ARL values for the MA control charts of dispersion for contamination

MAS-Control Chart						
Shift (δ)	$n=5$			$n=10$		
	$w=2$	$w=3$	$w=4$	$w=2$	$w=3$	$w=4$
1.00	376.405	377.569	376.172	378.813	376.072	375.852
1.25	28.582	21.8522	17.123	22.185	17.152	14.132
1.50	12.205	10.425	8.125	10.552	8.858	6.899
1.75	8.745	7.025	6.001	7.523	5.958	4.452
2.00	5.346	4.321	3.702	4.355	3.152	2.785
2.25	3.156	2.856	2.123	2.985	1.980	1.750
2.50	2.128	1.956	1.715	1.845	1.721	1.612
2.75	1.785	1.589	1.301	1.504	1.350	1.254
3.00	1.542	1.410	1.204	1.421	1.210	1.152
MAMAD-Control Chart						
Shift (δ)	$n=5$			$n=10$		
	$w=2$	$w=3$	$w=4$	$w=2$	$w=3$	$w=4$
1.00	377.132	376.689	376.541	377.459	376.895	375.203
1.25	25.142	20.258	16.440	20.774	16.015	12.912
1.50	11.005	9.450	7.758	9.742	7.985	5.112
1.75	7.801	6.441	5.112	6.412	5.008	3.958
2.00	4.986	3.152	2.852	3.856	2.945	2.245
2.25	3.005	2.124	1.965	2.142	1.850	1.645
2.50	1.988	1.845	1.605	1.720	1.660	1.550
2.75	1.684	1.502	1.264	1.398	1.267	1.205
3.00	1.465	1.375	1.184	1.290	1.208	1.131
MASn-Control Chart						
Shift (δ)	$n=5$			$n=10$		
	$w=2$	$w=3$	$w=4$	$w=2$	$w=3$	$w=4$
1.00	376.789	375.143	375.885	376.485	377.112	376.478
1.25	24.554	18.988	15.850	19.145	15.004	11.0145
1.50	10.558	8.441	6.884	8.855	6.887	4.956
1.75	7.005	5.658	4.562	5.442	4.785	3.152
2.00	4.102	2.879	2.008	2.859	2.152	1.958
2.25	2.905	1.990	1.850	1.960	1.704	1.589
2.50	1.881	1.745	1.550	1.652	1.542	1.475
2.75	1.570	1.398	1.198	1.302	1.205	1.184
3.00	1.387	1.275	1.141	1.201	1.185	1.105
MAQn-Control Chart						
Shift (δ)	$n=5$			$n=10$		
	$w=2$	$w=3$	$w=4$	$w=2$	$w=3$	$w=4$
1.00	378.009	378.150	376.850	374.895	378.002	377.102
1.25	23.112	17.556	14.850	18.441	14.552	10.552
1.50	9.258	7.665	5.995	7.158	5.258	3.885
1.75	6.580	4.485	3.958	4.458	3.885	2.805
2.00	3.458	2.005	1.920	2.005	1.852	1.802
2.25	2.450	1.801	1.782	1.820	1.620	1.456
2.50	1.785	1.675	1.490	1.510	1.452	1.345
2.75	1.490	1.299	1.170	1.258	1.165	1.154
3.00	1.298	1.190	1.120	1.185	1.102	1.009

(a) The performance of the Shewhart S-control chart is highly effected when data contains outliers. It is efficiency almost 73.81% decreases in presence of outliers to detect shifts in dispersion.

(b) The MAS-control chart performance is better than the Shewhart S-control chart in presence of outliers. The efficiency of MAS-control chart also decreases almost 71.43% in presence of outliers for detecting small to moderate shifts.

(c) The MAMAD-control chart performance is better than the Shewhart S-control chart and MAS-control chart. It is efficiency of detecting small to moderate shifts decreases almost 68% in presence of outliers.

(d) The efficiency of MASn-control chart almost 66.54% decreases in detecting small to moderate shifts while the efficiency of MAQn-control chart almost decreases 65.01% in presence of outliers.

(e) The ARL results shows that MAQn-control chart performs better than other moving average (MA) control charts for dispersion under study in the presence of outliers.

This study suggests the use of MAS-control chart in case of data follows normal distribution and no outlier exist in the data. In the case of violation of any assumptions, robust moving average (MA) control charts for dispersion (MAMAD, MASn and MAQn) for monitoring small to moderate shifts in process dispersion is recommended to be used instead of the MAS-control chart.

6. Application Example Using Real Data

In this section, a numerical example is used to illustrate the application of the Shewhart S-control chart and the alternative robust moving average (MA) control charts for dispersion considered in this study with a real data set taken from Yang and Arnold (2015) for $m = 10$ subgroups each of subgroup size $n = 10$. Also, the data will be used to show the out-of-control detection ability for each subgroup. Table 8 shows the data, which represents the service time (in minutes) of a bank branch in Taiwan from new automatic service system of the bank. According to Yang and Arnold (2015), the data is non-normal from an unknown distribution with a variance of 27.805.

Table 8 The service times from 10 counters of a bank branch in Taiwan

Subgroup Number (i)	Service Time Data									
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
1	3.54	0.01	1.33	7.27	5.52	0.09	1.84	1.04	2.91	0.63
2	0.86	1.61	1.15	0.96	0.54	3.05	4.11	0.63	2.37	0.05
3	1.45	0.19	4.18	0.18	0.02	0.70	0.80	0.97	3.60	2.94
4	1.37	0.14	1.54	1.58	0.45	6.01	4.59	1.74	3.92	4.82
5	3.00	2.46	0.06	1.80	3.25	2.13	2.22	1.37	2.13	0.25
6	1.59	3.88	0.39	0.54	1.58	1.70	0.68	1.25	6.83	0.31
7	5.01	1.85	3.10	1.00	0.09	1.16	2.69	2.79	1.84	2.62
8	4.96	0.55	1.43	4.12	4.06	1.42	1.43	0.86	0.67	0.13
9	1.08	0.65	0.91	0.88	2.02	2.88	1.76	2.87	1.97	0.62
10	4.56	0.44	5.61	2.79	1.73	2.46	0.53	1.73	7.02	2.13

The control limits, central line and number of points falling outside the control limits of the process for the Shewhart S-control chart are: LCL = 0.467, CL = 1.645, UCL = 2.823. The Shewhart S-control chart declared that there is no shift occur in the process, as all points lie within the control limits, as shown in Figure 1. Table 9 gives summary information for the values of the dispersion statistics used in this paper.

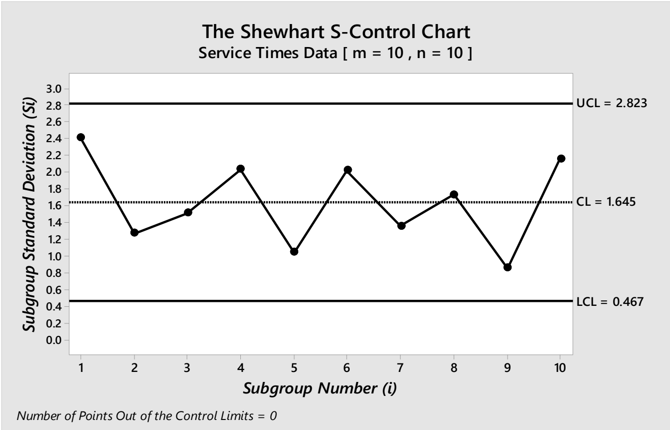

Figure 1 The Shewhart S-control chart for the service times data

Table 9 The scale estimators values for service times data

Subgroup Number (i)	Scale Estimators Values			
	S	MAD	Sn	Qn
1	2.41516	904.27	1.830640	2.93291
2	1.27268	0.793190	0.855690	1.510890
3	1.51761	3782.10	0.9839	1.510890
4	3707.20	237.25	1.76803	2.73294
5	5105.10	0.80802	0.876560	1.688640
6	283.21	1.193490	1.195580	1.910830
7	1.361950	526.15	1.311860	1.88862
8	1.73654	1.20832	107.13	1.688640
9	0.86314	0.85250	0.99284	1.510890
10	2.167410	1.72723	1.896230	2.866250
Average	1.64509	1.27874	1.27221	2.02415

The results regarding the control limits (LCL and UCL), the central line (CL) and the number of points falling outside the control limits for the process are estimated for the moving average (MA) control charts of dispersion MAS-control chart, MAMAD-control chart, MASn-control chart and MAQn-control chart and given in Table 10.

Table 10 Comparison of control charts for the service times data

Span (w) (w)	MA Control Chart	LCL	CL	UCL	Number of Points Out
2	MAS	0.813	1.645	2.477	0
	MAMAD	0.668	1.352	2.036	1
	MASn	0.611	1.238	1.864	0
	MAQn	0.704	1.427	2.150	4
3	MAS	0.966	1.645	2.325	1
	MAMAD	0.793	1.352	1.910	1
	MASn	0.726	1.238	1.749	1
	MAQn	0.838	1.427	2.018	4
4	MAS	1.056	1.645	2.234	1
	MAMAD	0.868	1.352	1.836	1
	MASn	0.795	1.238	1.681	1
	MAQn	0.917	1.427	1.939	7

The MAS-control chart and the MASn-control chart for $w = 2$ declared that there is no shift occur in the process as all points lie within the control limits, while the MAMAD-control chart and the MAQn-control chart are able to detect a shift in dispersion when the shift occur where one point and more plotted outside the control limits for the two control charts. However, the MAS-control chart, the MAMAD-control chart, the MASn-control chart and the MAQn-control chart with span $w = 3$ and 4 are able to detect a shift in dispersion when the shift actually occur. We also notice that the MAQn-control chart produces the maximum number of points falling outside the control limits while the MAS-control chart, the MAMAD-control chart and the MASn-control chart produces the same number of points falling outside the control limits. This means that "the sensitivity of the MAQn-control chart to detect a shift in the process dispersion when the shift occurs is more than that for the other control charts". These results means that the moving average (MA) control charts for dispersion are more efficient and effective for the detection of dispersion shift than the Shewhart S-control chart and the MAS-control chart. Accordingly, it can confirm that the proposed control charts are more effective than the other control charts. Hence the results are consists with the simulation study data. Figures 2-4 shows the moving average (MA) control charts for dispersion together with their respective control limits and central line.

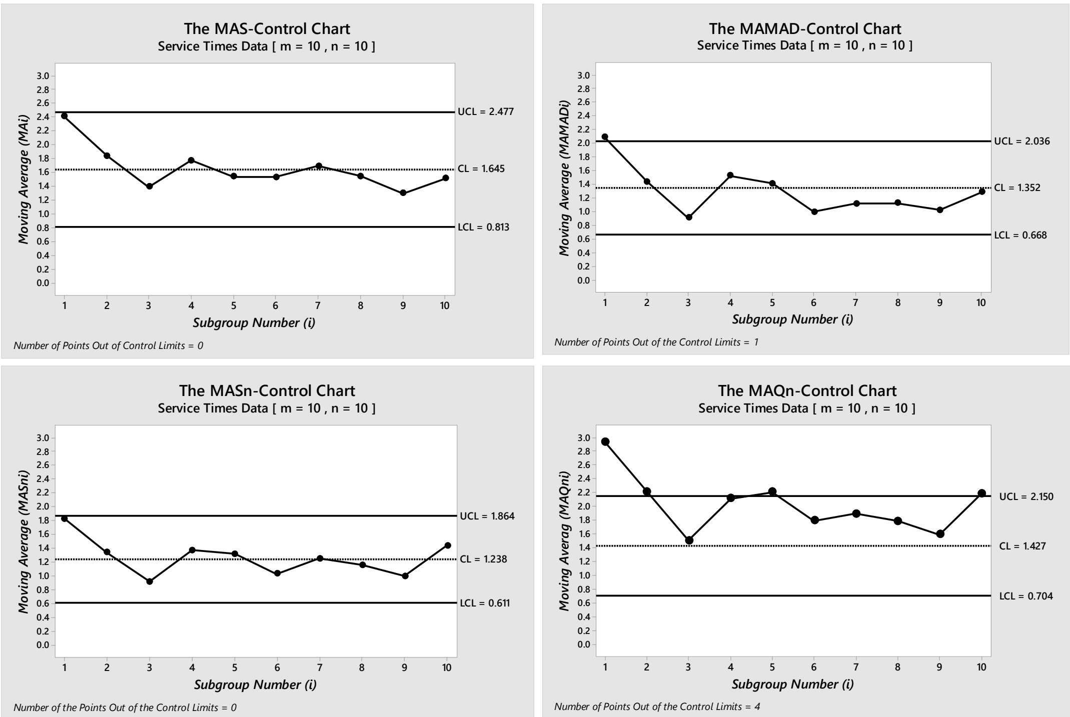

Figure 2 The moving average (MA) control charts for dispersion for $w = 2$

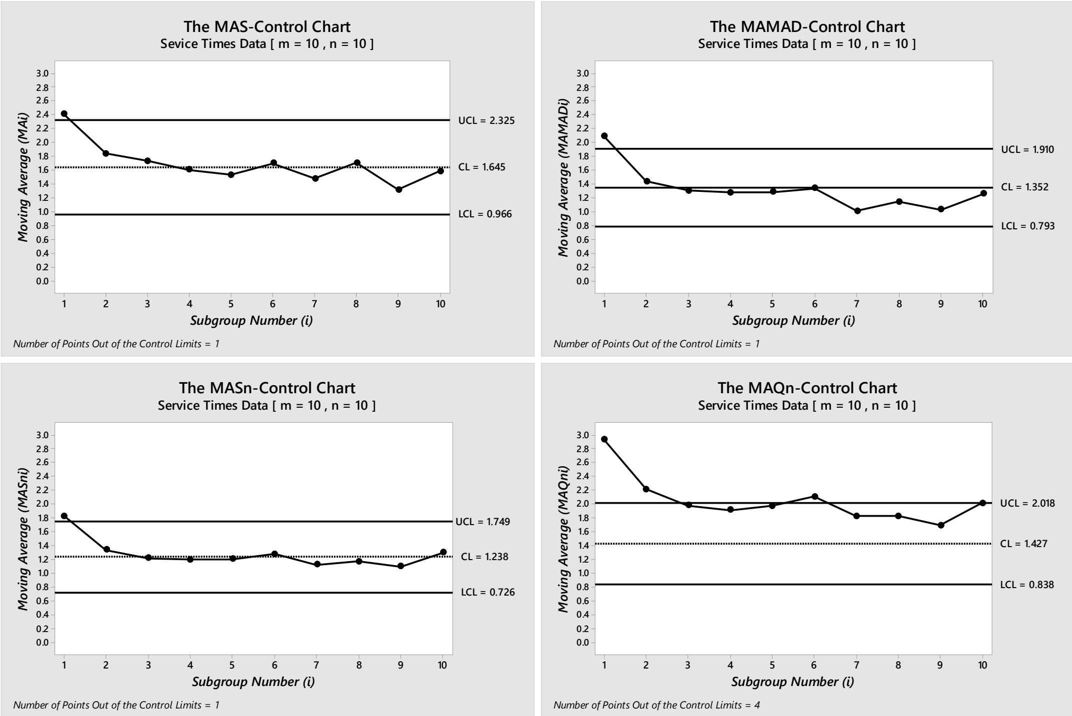

Figure 3 The moving average (MA) control charts for dispersion for $w = 3$

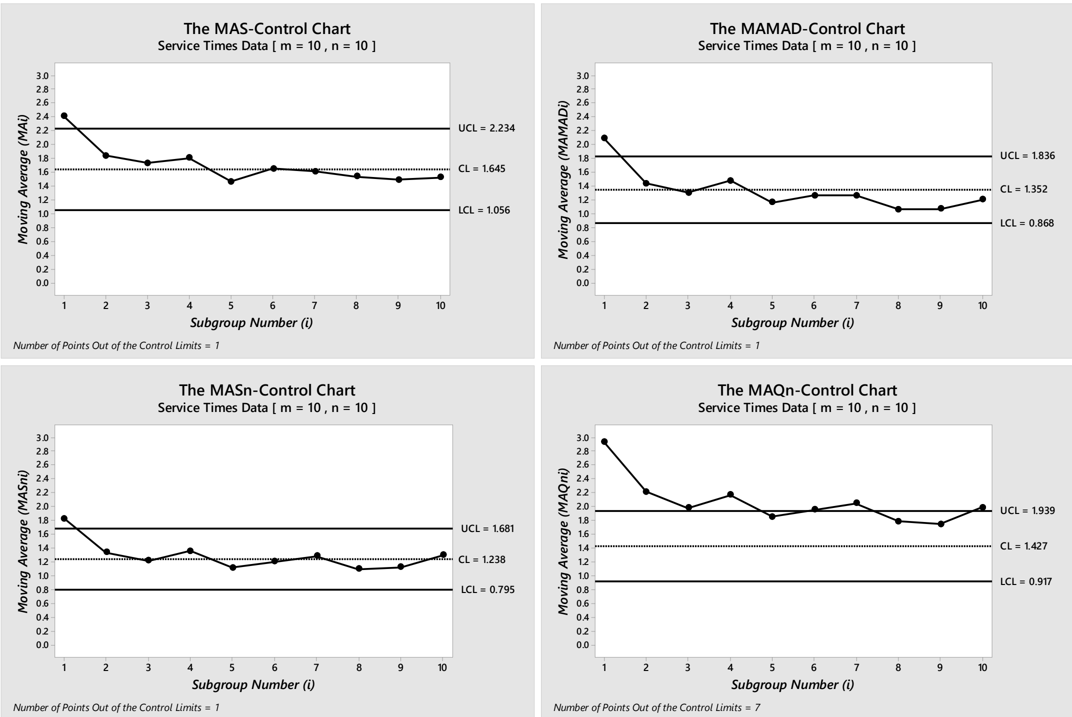

Figure 4 The moving average (MA) control charts for dispersion for $w = 4$

7. Summary and Conclusions

In this paper, we compare the performance of moving average (MA) control charts based on robust dispersion estimators. The proposed robust moving average (MA) control charts for dispersion considered showed better overall performance as compared with the Shewhart S and MAS control charts. Among the proposed control charts, the MAQn-control chart has shown its superiority. The other two proposed methods, namely the MAMAD-control chart and the MASn-control chart, have shown reasonable and similar performance. The MAS-control chart is also performing reasonably well. The Monte-Carlo simulation study suggests the use of MAS-control chart in case of data follows a normal distribution without outliers. In the case of violation of any assumptions, robust moving average (MA) control charts for dispersion (MAMAD, MASn and MAQn) for monitoring small to moderate shifts in process dispersion is recommended to be used instead of MAS-control chart. These results proved that the power of a variability control chart is strongly related to the efficiency of the dispersion estimator used in its construction. Finally, the main conclusion one should draw from the paper's results is that the proposed robust moving average (MA) control charts for dispersion are an additional and viable way of tracking a process and that, under conditions similar to that in the simulation study, will outperform the others.

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