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Improvement of the Holt-Winters Multiplicative Method with a New Initial Value Settings Method

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Abstract

The Holt-Winters method is a well-known and effective approach for forecasting time series, particularly when trends and seasonality exist. The proper settings of the initial values for level, trend, and seasonality play an important role in this method and lead to better forecasting results. In this paper, a new method is proposed to obtain the initial values in the Holt-Winters multiplicative method via setting the initial values of level and trend based on the weighted moving average and the initial value of seasonality based on a decomposition method. 10 real-world datasets were used to evaluate the performance of the proposed method compared to the existing Holt-Winters and Hansun's methods while varying the smoothing parameter from 0.1 to 1 in increments of 0.1. The results of the study show that the proposed method outperformed the existing ones in terms of the mean-absolute-percentage error (MAPE), symmetric mean-absolute-percentage error (sMAPE), root-mean-squared error (RMSE), and the Theil-U statistic.

Keywords: Time series forecasting, initial seasonality, seasonal index, weighted moving average.

1. Introduction

Forecasting a time series is a vital activity in many study areas, so there have been many attempts at obtaining the most accurate forecast for a given time series model. Examples can be found in biology (Bar-Joseph et al. 2003, Fokianos and Promponas 2012), finance (Taylor 2007), economics (Da Veiga et al. 2014), social science (Mitchell 2017), energy industry (Deb et al. 2017), tourism (Thoplan 2014) and climate (Murat et al. 2018). When both trend and seasonality are present in the data, the Holt-Winters (HW) exponential smoothing method is suitable for obtaining predictions (Chatfield and Yar 1988, Chatfield et al. 2001, Kalekar 2004, Holt 2004, Montgomery et al. 2008). Because of its low computational cost and optimal forecasting efficiency, the HW method has been widely used in various time series models (Assis et al. 2014, Momin and Chavan 2017) and in various study areas (Bermúdez et al. 2007, Sudheer and Suseelatha 2015, Wu et al. 2017). Two major problems with the HW method that affect all exponential smoothing methods are the selection of the smoothing parameters and their initial values (Montgomery et al. 2008). Along with other optimization methods (Segura and Vercher 2006), the maximum likelihood method has been used for the estimation of the smoothing parameter and initial values (Ord et al. 1997, Broze and Melard 1990, Hyndman et al. 2002, Hanzák 2008, Osman and King 2015). Meanwhile, De Livera et al. (2011) showed that via the HW

method, setting the initial values could be concentrated out of the likelihood and estimated directly using a regression procedure. The choice of initial values for the forecasting procedure is important for improving the forecasting accuracy (Vercher et al. 2012).

Depending on the form of seasonality being modeled, there are two types of HW approaches: the HW multiplicative method used for non-linear seasonality and the HW additive method for linear seasonality. As for most economic time series, the seasonal variation appears to be proportional to the level of the time series; the multiplicative version is more widely used and usually works better than the additive one (Bermúdez et al. 2006, Kuznets 1932). Thus, the HW multiplicative method is focused on in this study.

In recent research, Hansun (2017) provided new estimation rules for the initial values that gave good results. The idea is based on the principle of the weighted moving average to improve the initial values of the overall smoothing and trend smoothing components. In this study, the focus is on improving the initial values for the HW multiplicative method based on the principle of the weighted moving average and the decomposition method.

The remaining parts of this paper are as follows. The theoretical framework is covered in Section 2. The proposed method of this study is presented in Section 3. Experimentation to prove the efficacy of the proposed method is reported in Section 4. The results and a discussion are provided in Section 5. Finally, conclusions and remarks are presented in Section 6.

2. Theoretical Framework

2.1. The HW multiplicative method

The HW multiplicative method can be used to handle lots of complex seasonal patterns by simply finding the central value, then adding in the effects of slope and seasonality (Brown 1956; Holt 2004; Winters 1960). It is based on three updating equations for level, trend, and seasonality as follows

$$\text{Level: } L_t = \alpha \frac{Y_t}{S_{t-L}} + (1-\alpha)(L_{t-1} + T_{t-1}), \quad (1)$$

$$\text{Trend: } T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}, \quad (2)$$

$$\text{Seasonality: } S_t = \gamma \frac{Y_t}{L_t} + (1-\gamma)S_{t-L}, \quad (3)$$

where L is the seasonality length, L_t is the overall smoothing, T_t is the trend smoothing, S_t is the seasonal smoothing, and Y_t refers to the real data for time period t . The values of the smoothing parameters α, β and γ are set from 0 to 1. These parameters are estimated in such a way that the mean-squared error (MSE) is minimized (Dufour 2008). Meanwhile, the forecast can be obtained from

$$Y_{t+m} = (L_t + T_t m)S_{t-L+m}, \quad (4)$$

where Y_{t+m} is the forecast for the period ahead (m). The HW multiplicative method requires three initial values: level (L_0), trend (T_0) and seasonality ($S_{01}, S_{02}, \dots, S_{0L}$) obtained from

$$L_0 = \frac{Y_1 + Y_2 + Y_3 + \dots + Y_L}{L}, \quad (5)$$

$$T_0 = \frac{1}{L} \left(\frac{Y_{L+1} - Y_1}{L} + \frac{Y_{L+2} - Y_2}{L} + \dots + \frac{Y_{L+L} - Y_L}{L} \right), \quad (6)$$

$$S_{01} = \frac{Y_1}{L_0}, S_{02} = \frac{Y_2}{L_0}, \dots, S_{0L} = \frac{Y_L}{L_0}. \quad (7)$$

The initial level L_0 is the average of the first year of data. The initial trend T_0 is set as the average of the slopes for each period in the first two years. The initial seasonality $S_{01}, S_{02}, \dots, S_{0L}$ is computed by dividing each data item in the first year by the initial value.

2.2. Hansun's initial value settings for the HW multiplicative method

The initial value settings for the HW multiplicative method were suggested by Hansun (2017) using the basic principle of the weighted moving average to give more weight to more recent data for L_0 and T_0 . The initial seasonality is estimated via (7), i.e. it is still the same as for the HW method.

Hansun's initial level L_0^H and the initial trend T_0^H are respectively calculated as

$$L_0^H = \frac{LY_L + (L-1)Y_{L-1} + \dots + (L-m+2)Y_{L-m+2} + (L-m+1)Y_{L-m+1}}{L + (L-1) + \dots + (L-m+2) + (L-m+1)}, \quad (8)$$

$$T_0^H = \frac{1}{L^2} \left(\frac{2LY_{2L} + (2L-1)Y_{2L-1} + \dots + (L+2)Y_{L+2} + (L+1)Y_{L+1}}{2L + (2L-1) + \dots + (L+2) + (L+1)} - \frac{LY_L + (L-1)Y_{L-1} + \dots + 2Y_2 + Y_1}{L + (L-1) + \dots + 2 + 1} \right). \quad (9)$$

It is worth noting that Hansun's method performs better than the original HW method (Hansun 2017).

2.3. Performance metrics

Evaluating the performance of the forecast methods is achieved by comparing the actual and predicted values. A typical approach is to use specific criteria to measure the error of the predicted value, the performance of which is assessed based on the closeness of the predicted and actual values. As such, a variety of different evaluation statistics are employed to evaluate the forecasting accuracy in this study four criteria were used: the mean absolute percentage error (MAPE), the root mean squared error (RMSE), the symmetric mean absolute percentage error (sMAPE), and the Theil-U statistic defined as follows:

$$\text{MAPE} = \frac{100\%}{n} \sum_{t=1}^n \left(\left| \frac{Y_t - \hat{Y}_t}{Y_t} \right| \right), \quad \text{RMSE} = \sqrt{\frac{\sum_{t=1}^n (Y_t - \hat{Y}_t)^2}{n}},$$

$$\text{sMAPE} = \frac{100\%}{n} \sum_{t=1}^n \frac{|Y_t - \hat{Y}_t|}{(|Y_t| + |\hat{Y}_t|)/2}, \quad \text{Theil-U} = \sqrt{\frac{\sum_{t=1}^{n-1} \left(\frac{\hat{Y}_{t+1} - Y_{t+1}}{Y_t} \right)^2}{\sum_{t=1}^{n-1} \left(\frac{Y_{t+1} - Y_t}{Y_t} \right)^2}},$$

where Y_t is the true value at time t , \hat{Y}_t is the predicted value at time t , and n is the number of data points.

3. The Proposed Method

The HW multiplicative method is improved using the decomposition method with the ratio-to-moving-average method to obtain the seasonal indices (S_1, S_2, \dots, S_L) and using them to represent the initial seasonality ($S_{01}^{New}, S_{02}^{New}, \dots, S_{0L}^{New}$), as in (10), while the initial level (L_0^H) and the initial trend (T_0^H) are obtained using the idea proposed by Hansun (2017). The mathematical expression for the multiplicative decomposition approach is $Y_t = T_t \times S_t \times C_t \times I_t$, where Y_t is the time series value (actual

data) at period t . T_t, S_t, C_t and I_t are the trend component, the seasonal component (index), the cycle component, and the irregular (remainder) component for period t , respectively.

The seasonal indices are obtained as follows:

- 1) Calculate the moving average using the seasonality length (L).
- 2) Center the moving average when L is an even number.
- 3) Calculate the seasonal index for each period t using the actual data as a proportion of the centered moving average.
- 4) Adjust the total of the seasonal indices to equal L , i.e. $\sum_{t=1}^L S_t = L$.

The obtained seasonal indices from the above procedure were set as the initial seasonality as follows

$$S_{01}^{New} = S_1, S_{02}^{New} = S_2, \dots, S_{0L}^{New} = S_L. \quad (10)$$

The three initial values proposed are expressed as

$$\text{The initial level: } L_0^H = \frac{LY_L + (L-1)Y_{L-1} + \dots + (L-m+2)Y_{L-m+2} + (L-m+1)Y_{L-m+1}}{L + (L-1) + \dots + (L-m+2) + (L-m+1)},$$

$$\text{The initial trend: } T_0^H = \frac{1}{L^2} \left(\frac{2LY_{2L} + (2L-1)Y_{2L-1} + \dots + (L+2)Y_{L+2} + (L+1)Y_{L+1}}{2L + (2L-1) + \dots + (L+2) + (L+1)} \right. \\ \left. - \frac{LY_L + (L-1)Y_{L-1} + \dots + 2Y_2 + Y_1}{L + (L-1) + \dots + 2 + 1} \right),$$

$$\text{The initial seasonality: } S_{01}^{New} = S_1, S_{02}^{New} = S_2, \dots, S_{0L}^{New} = S_L.$$

4. Experimental Study

The various time series data in 10 real datasets were used in this study to determine the performance of the proposed method. The data in quarterly time periods with both trend and seasonality were taken from the Statistics of New Zealand Information Centre (<http://www.stats.govt.nz>). Details of the 10 datasets are provided in Table 1.

Table 1 Description of the datasets used in the study

| Dataset | Description | Time Period | Size |
|---------|--|---------------|------|
| TS1 | The number of Australian visitors to New Zealand | 1998Q4–2012Q1 | 54 |
| TS2 | The number of New Zealand travelers to India | 2000Q1–2012Q3 | 51 |
| TS3 | The number of visitors to New Zealand for visiting | 1998Q4–2012Q1 | 54 |
| TS4 | The number of visitor arrivals to New Zealand | 2000Q1–2012Q1 | 49 |
| TS5 | The motel occupancy rate in New Zealand | 1996Q3–2012Q4 | 65 |
| TS6 | The number of United Kingdom visitors to New Zealand | 1998Q4–2012Q1 | 54 |
| TS7 | The number of visitors to New Zealand for all purposes | 1998Q4–2012Q1 | 54 |
| TS8 | The number of New Zealand travelers to foreign | 2000Q1–2012Q3 | 51 |
| TS9 | Beer sales data in USA | 2000Q1–2017Q4 | 72 |
| TS10 | Wine consumed in New Zealand | 2000Q1–2012Q3 | 51 |

The smoothing parameters α, β and γ were set from 0 to 1 in increments of 0.1, so there were 1,000 sets of conditions in total for each dataset (Table 2). The experimental study was conducted via the steps shown in Figure 2 by R program version 3.5.2 (R Core Team 2019), to measure the performance of the proposed method comparing to the HW method and Hansun's method. The initial

values obtained from the HW, Hansun's, and the proposed methods were applied when evaluating the 10 datasets. The performance of the three methods was evaluated by using RMSE, MAPE, sMAPE, and the Theil-U statistic.

Table 2 Smoothing parameter settings with varying smoothing parameter α, β and γ values

| Setting | α | β | γ |
|----------|----------|----------|----------|
| 1 | 0.1 | 0.1 | 0.1 |
| 2 | 0.1 | 0.1 | 0.2 |
| 3 | 0.1 | 0.1 | 0.3 |
| \vdots | \vdots | \vdots | \vdots |
| 1,000 | 1.0 | 1.0 | 1.0 |

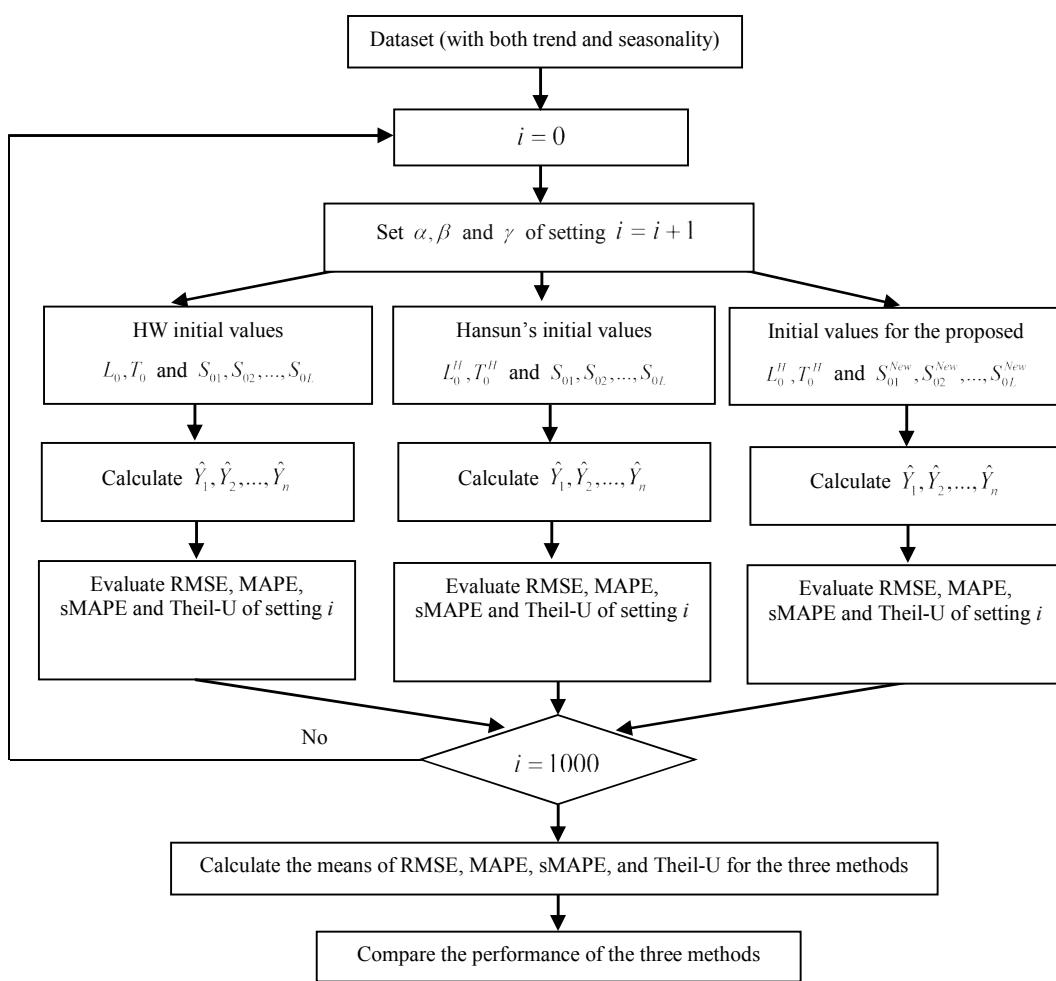


Figure 2 Flow chart of the experimental study

5. Results and Discussion

It can be seen that setting the initial values for each method produced varying average accuracy levels in terms of MAPE, RMSE, sMAPE, and the Theil-U statistic (Tables 5 and 6). The results show that for all 10 real-world datasets (TS1–TS10), the HW, Hansun's, and proposed methods produced MAPE values of 5.697, 5.641, and 4.827, RMSE values of 9234.278, 9316.868, and 7184.859, sMAPE

values of 5.7465, 5.8063, and 4.9498, and Theil-U values of 0.2575, 0.2575, and 0.2126, respectively. Thus, the proposed method achieved the lowest values for the four accuracy measures, and it is evident that it quite considerably outperformed the other two methods.

Table 5 Average MAPE and RMSE values of the three methods

| Dataset | Average MAPE (%) | | | Average RMSE | | |
|---------|------------------|-----------------|-----------------|--------------|-----------------|-----------------|
| | HW Method | Hansun's Method | Proposed Method | HW Method | Hansun's Method | Proposed Method |
| TS1 | 4.580 | 4.596 | 3.975* | 1671.007 | 1689.454 | 1394.675* |
| TS2 | 10.142 | 10.135 | 9.463* | 1062.626 | 1063.912 | 967.355* |
| TS3 | 4.128 | 4.166 | 3.614* | 6192.297 | 6277.042 | 5493.579* |
| TS4 | 5.378 | 5.397 | 4.266* | 44609.420 | 44833.023 | 32093.613* |
| TS5 | 2.876 | 2.806 | 2.234* | 5.846 | 5.798 | 4.498* |
| TS6 | 7.412 | 7.490 | 7.035* | 1981.942 | 1995.491 | 1894.898* |
| TS7 | 3.465 | 3.461 | 3.258* | 16844.486 | 16854.437 | 15674.613* |
| TS8 | 5.801 | 6.058 | 4.105* | 20069.708 | 20434.139 | 14313.619* |
| TS9 | 3.193 | 3.207 | 2.556* | 15.147 | 15.086 | 11.489* |
| TS10 | 8.875 | 9.100 | 7.765* | 0.301 | 0.301 | 0.250* |
| Average | 5.697 | 5.641 | 4.827 | 9245.278 | 9316.868 | 7184.859 |

*The best performance in terms of average MAPE and RMSE values.

Table 6 Average sMAPE and Theil-U values of the three methods

| Dataset | Average sMAPE (%) | | | Average Theil-U | | |
|---------|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| | HW Method | Hansun's Method | Proposed Method | HW Method | Hansun's Method | Proposed Method |
| TS1 | 4.5761 | 4.5982 | 3.9653* | 0.2339 | 0.2346 | 0.1969* |
| TS2 | 10.2711 | 10.2668 | 9.5292* | 0.2301 | 0.2306 | 0.2052* |
| TS3 | 4.1054 | 4.1558 | 3.6116* | 0.1375 | 0.1357 | 0.1178* |
| TS4 | 5.3778 | 5.4076 | 4.2677* | 0.2625 | 0.2626 | 0.2040* |
| TS5 | 2.8740 | 2.8010 | 2.2347* | 0.1950 | 0.1941 | 0.1473* |
| TS6 | 7.3795 | 7.4663 | 7.0059* | 0.1839 | 0.1812 | 0.1704* |
| TS7 | 3.4541 | 3.4559 | 3.2572* | 0.1472 | 0.1427 | 0.1341* |
| TS8 | 5.7674 | 6.0300 | 4.1094* | 0.4421 | 0.4504 | 0.3162* |
| TS9 | 4.5476 | 4.5623 | 3.5876* | 0.3164 | 0.3155 | 0.2520* |
| TS10 | 9.1121 | 9.3189 | 7.9291* | 0.4268 | 0.4271 | 0.3822* |
| Average | 5.7465 | 5.8063 | 4.9498 | 0.2575 | 0.2575 | 0.2126 |

*The best performance in terms of average sMAPE and Theil-U values.

Bar charts of the average MAPE, sMAPE, and Theil-U values obtained by the three methods for each dataset are shown in Figure 3 (a)-(c), respectively. The results clearly show that the proposed method performed much better than the HW and Hansun's methods.

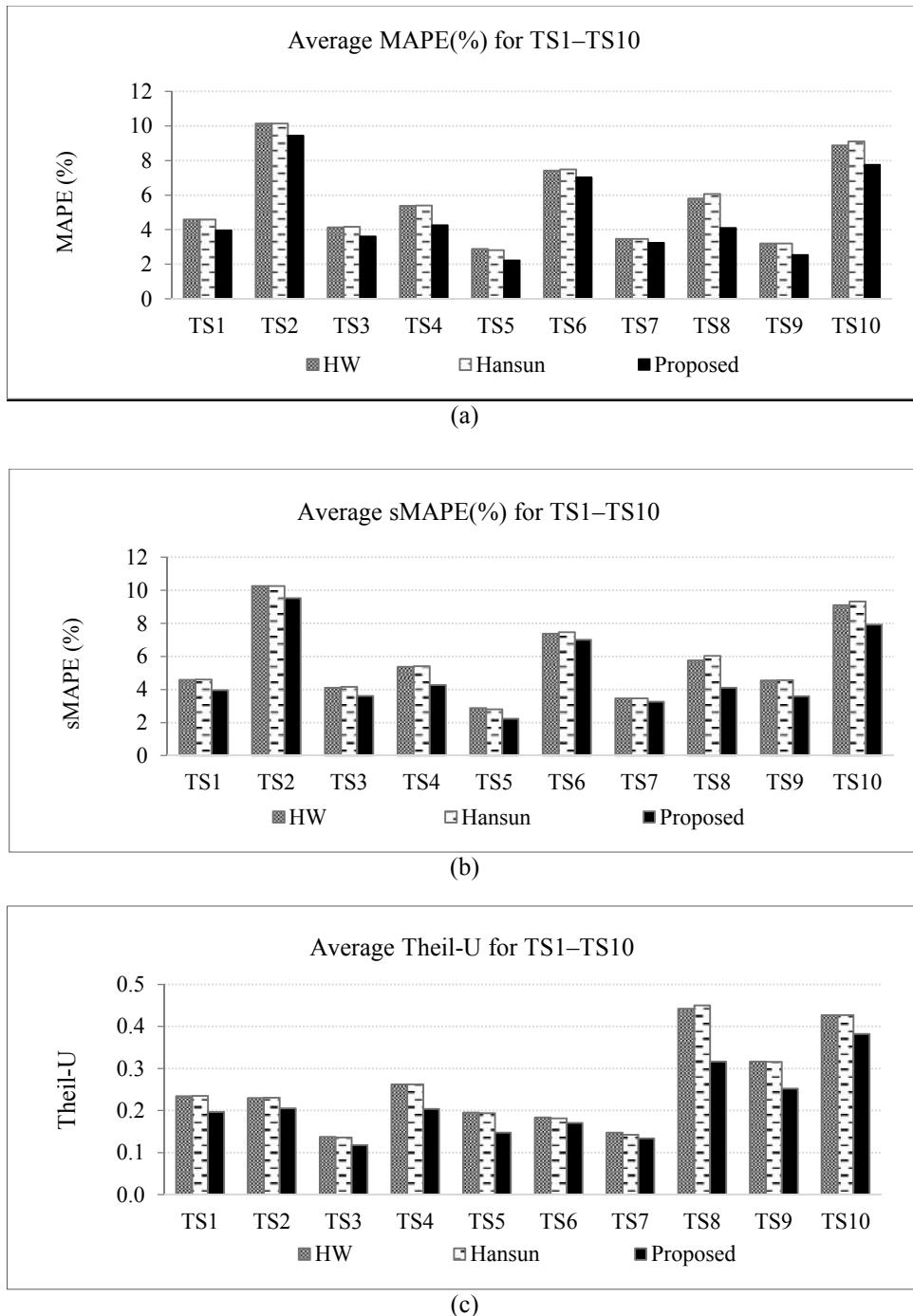


Figure 3 Average accuracy values for the three methods with the 10 datasets:
 (a) MAPE, (b) sMAPE, and (c) Theil-U

The improvement in terms of MAPE, RMSE, sMAPE, and Theil-U by the proposed method were calculated relative to the HW and Hansun's methods (Tables 7 and 8). The results show that the improvements were 14.81% and 15.38% in terms of MAPE, 16.90% and 17.26% for RMSE, 14.91% and 15.54% for sMAPE, and 16.32% and 15.92% for the Theil-U statistic, respectively. These results

exhibit the same trend as is evident in Tables 5 and 6, and it is once again evident that the proposed method outperformed the other two.

Table 7 MAPE and RMSE improvements of the proposed method compared to the HW and Hansun's methods

| Dataset | MAPE Improvement (%) | | RMSE Improvement (%) | |
|---------|----------------------|-----------------------|----------------------|-----------------------|
| | Proposed/ HW | Proposed/ Hansun's | Proposed/ HW | Proposed/ Hansun's |
| TS1 | 13.22 | 13.53 | 16.54 | 17.45 |
| TS2 | 6.69 | 6.62 | 8.97 | 9.08 |
| TS3 | 12.45 | 13.24 | 11.28 | 12.48 |
| TS4 | 20.66 | 20.94 | 28.06 | 28.42 |
| TS5 | 22.34 | 20.40 | 23.06 | 22.42 |
| TS6 | 5.08 | 6.07 | 4.39 | 5.04 |
| TS7 | 5.98 | 5.86 | 6.95 | 7.00 |
| TS8 | 29.24 | 32.24 | 28.68 | 29.95 |
| TS9 | 19.94 | 20.28 | 24.15 | 23.84 |
| TS10 | 12.50 | 14.67 | 16.94 | 16.94 |
| Average | 14.81 | 15.38 | 16.90 | 17.26 |

Table 8 sMAPE and Theil-U improvements of the proposed method compared to the HW and Hansun's methods

| Dataset | sMAPE Improvement (%) | | Theil-U Improvement (%) | |
|---------|-----------------------|-----------------------|-------------------------|-----------------------|
| | Proposed/ HW | Proposed/ Hansun's | Proposed/ HW | Proposed/ Hansun's |
| TS1 | 13.35 | 13.76 | 15.81 | 16.07 |
| TS2 | 7.22 | 7.18 | 10.82 | 11.04 |
| TS3 | 12.03 | 13.10 | 14.32 | 13.17 |
| TS4 | 20.64 | 21.08 | 22.27 | 22.32 |
| TS5 | 22.24 | 20.22 | 24.47 | 24.11 |
| TS6 | 5.06 | 6.17 | 7.32 | 5.96 |
| TS7 | 5.70 | 5.75 | 8.92 | 6.07 |
| TS8 | 28.75 | 31.85 | 28.48 | 29.80 |
| TS9 | 21.11 | 21.36 | 20.36 | 20.13 |
| TS10 | 12.98 | 14.91 | 10.45 | 10.51 |
| Average | 14.91 | 15.54 | 16.32 | 15.92 |

For the 1,000 sets of conditions for each dataset, the number of times that each method achieved the lowest of MAPE, RMSE, sMAPE, and Theil-U for the same set of conditions of (α, β, γ) are shown in Tables 9 and 10. For example, for TS1, the proposed, HW, and Hansun's methods achieved the lowest MAPE value 783, 17 and 200 times, the lowest RMSE value 841, 49 and 110 times, the lowest sMAPE value 785, 21 and 194 times, and the lowest Theil-U value 985, 7 and 8 times, respectively. This trend was the same for the other datasets except for TS10, where the occurrence of the lowest MAPE value for the HW method (507) was higher than that of the proposed method (493). Although the difference is quite small, this could have been because this time series did not conform

closely enough to the multiplicative form or that the MAPE test was not specific enough in this case. These results support the finding that the proposed method outperformed the other two.

Table 9 The lowest MAPE and RMSE value frequencies of the three methods

| Dataset | MAPE | | | RMSE | | |
|---------|-----------|-----------------|-----------------|-----------|-----------------|-----------------|
| | HW Method | Hansun's Method | Proposed Method | HW Method | Hansun's Method | Proposed Method |
| TS1 | 17 | 200 | 783* | 49 | 110 | 841* |
| TS2 | 87 | 324 | 589* | 185 | 56 | 759* |
| TS3 | 109 | 108 | 783* | 89 | 43 | 868* |
| TS4 | 220 | 205 | 575* | 145 | 2 | 853* |
| TS5 | 29 | 262 | 709* | 68 | 91 | 841* |
| TS6 | 39 | 217 | 744* | 83 | 112 | 805* |
| TS7 | 158 | 300 | 542* | 76 | 246 | 678* |
| TS8 | 347 | 14 | 639* | 391 | 0 | 609* |
| TS9 | 304 | 89 | 607* | 80 | 224 | 696* |
| TS10 | 507* | 0 | 493 | 354 | 66 | 580* |
| Average | 181.7 | 171.9 | 646.4 | 152 | 95 | 753 |

*The best performance in terms of the number of times that each method achieved the lowest MAPE and RMSE values in 1,000 sets of conditions

Table 10 The lowest sMAPE and Theil-U value frequencies of the three methods

| Dataset | sMAPE | | | Theil-U | | |
|---------|-----------|-----------------|-----------------|-----------|-----------------|-----------------|
| | HW Method | Hansun's Method | Proposed Method | HW Method | Hansun's Method | Proposed Method |
| TS1 | 21 | 194 | 785* | 7 | 8 | 985* |
| TS2 | 95 | 318 | 587* | 169 | 70 | 761* |
| TS3 | 122 | 108 | 770* | 50 | 3 | 947* |
| TS4 | 273 | 161 | 566* | 72 | 13 | 915* |
| TS5 | 29 | 265 | 706* | 61 | 10 | 929* |
| TS6 | 54 | 214 | 732* | 38 | 37 | 925* |
| TS7 | 158 | 304 | 538* | 58 | 160 | 782* |
| TS8 | 386 | 0 | 614* | 217 | 1 | 782* |
| TS9 | 319 | 69 | 612* | 115 | 190 | 695* |
| TS10 | 493 | 0 | 507* | 342 | 139 | 519* |
| Average | 195.0 | 163.3 | 641.7 | 112.9 | 63.1 | 824.0 |

*The best performance in terms of the number of times that each method achieved the lowest sMAPE and Theil-U values in 1,000 sets of conditions

From the results in Tables 9 and 10, the frequency of each method was then analyzed using the chi-squared goodness-of-fit test. The null hypothesis states that the number of times each method achieves the lowest MAPE, RMSE, sMAPE, or Theil-U is the same. The results of the chi-squared goodness-of-fit test are given in Tables 11 and 12. For example, for the first dataset TS1, the chi-squared statistic for MAPE is 960.134, with a p-value of 0.000, which leads to the conclusion that the number of times each method achieves the lowest MAPE is significantly different; the RMSE, sMAPE, and the Theil-U results can be interpreted in the same way. After testing the hypothesis with

the chi-squared goodness-of-fit test, there was a significant difference between all three methods in terms of the number of times they achieved the lowest error.

Table 11 Chi-squared goodness-of-fit test results under the null hypothesis that the number of times each method achieves the lowest MAPE or RMSE value is the same

| Dataset | MAPE | | RMSE | |
|---------|-------------|---------|-------------|---------|
| | chi-squared | p-value | chi-squared | p-value |
| TS1 | 960.134 | 0.000 | 1165.350 | 0.000 |
| TS2 | 378.398 | 0.000 | 840.326 | 0.000 |
| TS3 | 909.902 | 0.000 | 1289.582 | 0.000 |
| TS4 | 263.150 | 0.000 | 1245.910 | 0.000 |
| TS5 | 716.498 | 0.000 | 1160.560 | 0.000 |
| TS6 | 806.438 | 0.000 | 1002.370 | 0.000 |
| TS7 | 226.184 | 0.000 | 577.928 | 0.000 |
| TS8 | 586.778 | 0.000 | 571.286 | 0.000 |
| TS9 | 406.358 | 0.000 | 622.976 | 0.000 |
| TS10 | 500.294 | 0.000 | 398.216 | 0.000 |

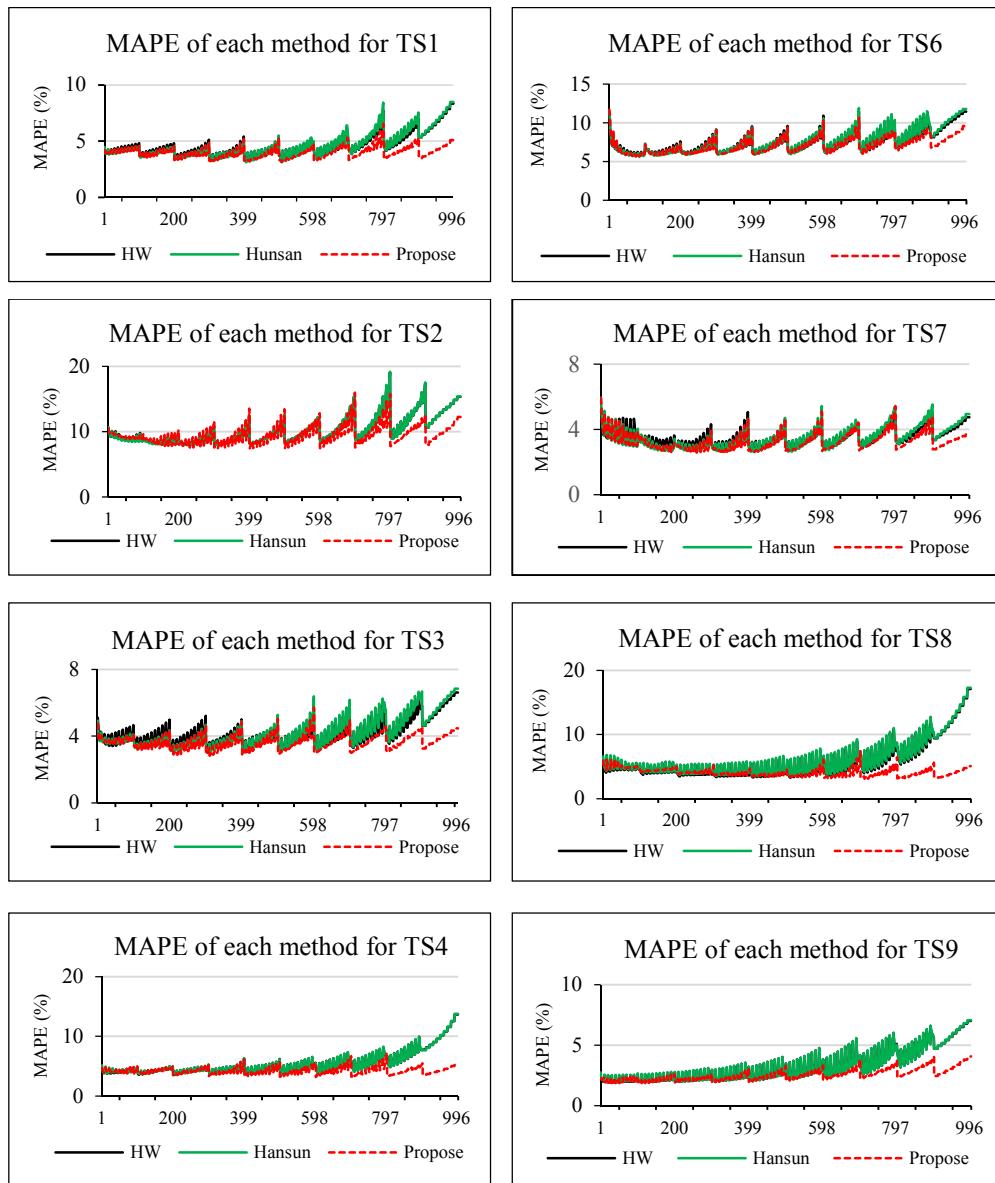
Table 12 Chi-squared goodness-of-fit test results under the null hypothesis that the number of times each method achieves the lowest sMAPE or Theil-U value is the same

| Dataset | sMAPE | | Theil-U | |
|---------|-------------|---------|-------------|---------|
| | chi-squared | p-value | chi-squared | p-value |
| TS1 | 962.906 | 0.000 | 1911.014 | 0.000 |
| TS2 | 364.154 | 0.000 | 837.746 | 0.000 |
| TS3 | 858.344 | 0.000 | 1697.954 | 0.000 |
| TS4 | 262.418 | 0.000 | 1527.734 | 0.000 |
| TS5 | 708.506 | 0.000 | 1600.586 | 0.000 |
| TS6 | 753.608 | 0.000 | 1575.314 | 0.000 |
| TS7 | 220.472 | 0.000 | 921.464 | 0.000 |
| TS8 | 577.976 | 0.000 | 975.842 | 0.000 |
| TS9 | 443.198 | 0.000 | 597.050 | 0.000 |
| TS10 | 500.294 | 0.000 | 216.938 | 0.000 |

The chi-squared goodness-of-fit test results lead to the same conclusion that the proposed method outperformed the others. It can be seen from the results in Table 9 that on average in 1,000 sets of conditions, there were 646.4 sets where the proposed method attained the minimum MAPE and 753 where it attained the minimum RMSE. Similarly, the results in Table 10 clearly show that on average in 1,000 sets, the proposed method achieved the most lowest sMAPE values 641.7 times and Theil-U values 824 times.

Each setting had smoothing parameters α, β , and γ ranging from 0 to 1 ($\alpha = \{0.1, 0.2, \dots, 1\}$, $\beta = \{0.1, 0.2, \dots, 1\}$, and $\gamma = \{0.1, 0.2, \dots, 1\}$), and $(\alpha, \beta, \gamma) = (0.1, 0.1, 0.1), (0.1, 0.1, 0.2), \dots, (1, 1, 1)$ were used for the 1 to 1,000 sets. Figure 4 shows plots of the MAPE values with the HW, Hansun's, and proposed methods versus the sets of conditions for TS1–TS10. For the same set of (α, β, γ) conditions, the MAPE values for the proposed method were lower than those for the other two

methods. It is evident that for each dataset, the MAPE plot for the proposed method usually stayed under those of the HW and Hunsan's methods. The latter two methods usually had similar values for MAPE for the same set of (α, β, γ) conditions and their plotted lines tended to overlap under all sets of conditions (1 to 1000) for every data set and showed similar performances. Meanwhile, the lower MAPE values for the proposed method became more obvious as the settings of conditions increased. These results are consistent with the other findings and lead to the same conclusion that the proposed method performed better than the others.



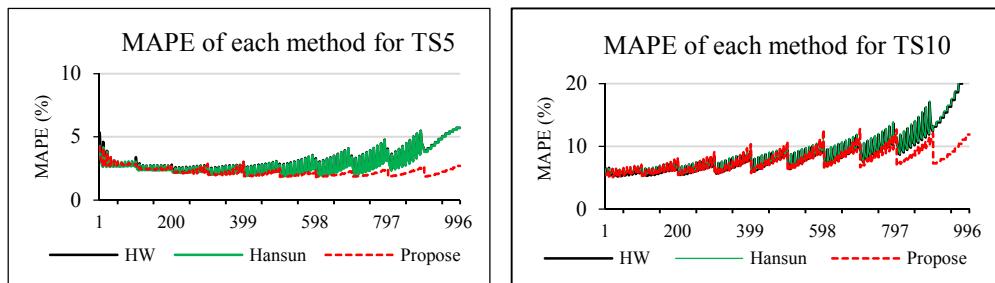


Figure 4 MAPE of the three methods for 1,000 sets of conditions for datasets TS1–TS10

6. Conclusions and Remarks

A method for setting the initial parameter values to improve the HW multiplicative method is proposed herein. The initial values consist of three components: the initial level, the initial trend, and the initial seasonality. The initial level and trend are obtained by using the weighted moving average method, while the initial seasonality is obtained by using decomposition with the ratio-to-moving-average method.

The proposed method was experimentally compared with the HW and Hansun's methods. The performances of the methods were compared in terms of MAPE, RMSE, sMAPE, and the Theil-U statistic using 10 real-world datasets. Three smoothing parameters (α, β, γ) were set from 0 to 1 with increments of 0.1 in the experimental study. From the results, it can be seen that the proposed method performed better than the HW and Hansun's methods in terms of MAPE, RMSE, sMAPE, and Theil-U values in almost all cases. The proposed method reduced MAPE, RMSE, sMAPE, and Theil-U compared to HW and Hansun's methods by 14%–16% and 15%–17% on average, respectively. Therefore, the proposed method is a plausible alternative to estimate the initial values for the HW multiplicative method.

As the approach proposed in this study is only valid for the HW multiplicative model, it will not work when the time series conforms to the additive model or when the magnitude of the seasonal pattern does not depend on the magnitude of the data. Hence, proposing a new approach for the HW additive method could be of interest and should be studied in the future.

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