



Thailand Statistician
April 2021; 19(2): 361-373
<http://statassoc.or.th>
Contributed paper

Sensitivity of Priors in the Presence of Collinearity in Vector Autoregressive Model: A Monte Carlo Study

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Received: 6 May 2020

Revised: 9 July 2020

Accepted: 7 August 2020

Abstract

The use of prior distribution plays a great role in Bayesian approach. This work examines the sensitivity of priors in Bayesian Vector Autoregressive (BVAR) model when the time series data are correlated. The three scenarios of collinearity namely; Low Collinearity (LC), Moderate Collinearity (MC) and High Collinearity (HC) were considered while the forecast evaluation criteria were used to judge the performance of these BVAR priors. Results from Monte Carlo experiment showed that all the forecast criteria evaluation for all the scenarios of collinearity have the same pattern of performance. However, Normal-Wishart prior performed best in the three scenarios of collinearity especially in small sample sizes.

Keywords: Cholesky, conjugate, hyper-parameters, posterior distribution.

1. Introduction

Vector Autoregressive (VAR) model is a forecasting method in econometric analysis. It is a flexible time series model and takes into account of complex dynamic interrelationships among macroeconomic variables. Some many works had been carried out on VAR model in frequentist way (see Fong et al. 2007, Lazariu et al. 2011). However, the dense parameterization in VAR model can leads to unstable inference and inaccurate out-of sample forecasts, especially when many variables are considered using a frequentist way (Giannone et al. 2012).

The Bayesian method to statistics provides a general approach for combining a modeller's beliefs with the evidence contained in the data. Notables works on the use of Bayesian inference in VAR model is recorded in the works of (Spencer 1993, Kenny et al. 1998, Ni and Sun 2005, Lu et al. 2018, Giannone et al. 2010, Bloor and Matheson 2009).

Ni and Sun (2005) examined the frequentist risks of Bayesian estimates of VAR regression coefficient and error covariance matrices under loss functions with different non-informative priors. It was demonstrated that choice of prior has significant effect on the estimates than the form of estimator.

A Bayesian hierarchical vector autoregressive (VAR) model to predict medical and psychological conditions using multivariate time series data was proposed by Lu et al. (2018). Their

model demonstrated higher accuracy in terms of prediction of future observations in for both point and interval estimates due to the pooling effect of the hierarchical model specification.

In recent times, choice of prior has been a hot debate among Bayesian econometricians (see Jeffreys 1961, Bernardo 1979, Kass and Wasserman 1996). A proper Bayesian analysis must be able to incorporate genuine prior information, which will help to strengthen inferences about the true value of the parameter, and ensure that any relevant information about it is not wasted (Garthwaite et al. 2005). The imposition of Bayesian priors should be considerably improves the forecasting performance in VAR models. Many priors have been developed in BVAR framework (see Villani 2009, Karlsson 2013, Korobilis and Pettenuzzo 2016). These developed priors were able to minimize error in the estimation of VAR model.

There are limited works that had discussed the problem of collinearity in time series analysis. Examples are Salinas and Hillmer (1987), Adenomon et al. (2015) explored the short term forecast when there was problem of limited data in time series analysis. They evaluated the performances of both the classical VAR and Bayesian VAR (BVAR) for short term series at different levels of collinearity. Their results showed that the BVAR models outperformed the classical VAR for time series length of 8 for all levels of collinearity, while the classical VAR was effective for time series length of 16 for all collinearity levels.

However, since it has been demonstrated that BVAR outperformed the classical VAR in literature, it is therefore important that this research efforts be directed toward the construction of models by examining the kind of prior which are optimal in forecasting of VAR model especially when there is collinearity in time series data.

Therefore, this work hereby extends the work of Adenomon et al. (2015) by examining the sensitivity of priors by focussing on four different kind of priors distributions namely; Litterman/Minnesota prior, Normal-Wishart prior, Normal-Wishart prior by Sims-Zha and Normal-flat prior by Sims-Zha in Bayesian Vector Autoregressive (BVAR) model when the time series data are correlated under three scenarios of collinearity; High Collinearity (HP), Moderate Collinearity (MC), Low Collinearity (LC) in VAR model.

The rest of the paper is structured as: Section 2 gives the specification of VAR model; Section 3 presents the methods and different kinds of priors in VAR model. Monte Carlo experiment and forecast evaluation criteria were described in Section 4 while Section 5 discusses the results and Section 6 renders the conclusions remark.

2. VAR Model Specification

Let us assume a p -dimensional multiple time series $y_1, \dots, y_T$ with $y_t = (y_{1t}, \dots, y_{pt})$ which can be generated by VAR(k) process given as

$$y_t = c + B_1 y_{t-1} + \dots + B_k y_{t-k} + u_t, \quad (1)$$

where y_t is simply called a VAR(k) process. Equation (1) can also be written as

$$y_t = c + \sum_{j=1}^k B_j y_{t-j} + u_t, \quad (2)$$

where y_t for $t=1, \dots, T$ is a $M \times 1$ vector containing observations on M time series variable, u_t is $M \times 1$ vector of shocks, c is $M \times 1$ vector of intercepts and B is $M \times M$ matrix of coefficients. It is assumed that $u_t \sim N(0, \Sigma)$ and $B = (c, \dots, B_1, \dots, B_k)'$.

3. Materials and Methods

There are so many priors that have been applied in literature to derive the posterior distribution of VAR model. In this section, these existing priors will be reviewed. However, the posterior distribution will not be derived here since it had been done in literature (see Kadiyala and Karlsson 1997, Sims and Zha 1998, Sims 2000 for details). The aim of this study is to examine the sensitivity of these priors. These priors are

3.1. Litterman/Minnesota (LM) prior

This prior was originally developed by Litterman (1986), it was developed primarily for macroeconomic and financial applications. The prior assumes to have the following coefficients $B = B_1, \dots, B_k$ to be a priori independent and follow a normal distribution with the following moments:

$$E[(B_\ell)_{ij} | \Sigma] = \begin{cases} \delta_i ; i = j, \ell = 1, \\ 0 ; \text{otherwise,} \end{cases} \quad (3)$$

$$\text{var}[(B_\ell)_{ij} | \Sigma] = \begin{cases} \frac{\lambda_1^2}{f(\ell)} & \text{for } i = j, \forall \ell, \\ \frac{\lambda_1^2}{f(\ell)} \frac{\Sigma_{ij}}{\omega_j^2} & \text{for } i \neq j, \forall \ell, \end{cases} \quad (4)$$

where $f(\ell) = (k^{\lambda_3})^2$, $\Sigma_{ij} = \lambda_2^2 \omega_j^2$ and λ_1, λ_2 and λ_3 are hyper-parameters to be chosen by the researcher.

3.2. Normal-Wishart (NW) prior

This is kind of prior is a conjugate class of prior. It can be simply stated as

$$B \sim N(B, \Omega), \quad (5)$$

$$\Sigma \sim W(S, \nu), \quad (6)$$

where $\Omega = \Sigma \otimes \Psi$, S is a fixed (non-random) positive definite matrix, ν is a scalar degrees of freedom parameter and B is a vector of means.

3.3. Normal-Wishart (Sims-Zha) prior (NWSZ)

It was proposed by Sims and Zha (1998) and is a kind of prior that has gained popularity in recent times. It is a unique form of prior that can be used on coefficients of multiple-equation linear model that is computationally tractable and substantively appealing. The major advantage of this prior is that, it has capability to extend Bayesian methods to larger models and to models with over identifying restrictions.

Hence, the Normal-Wishart prior by Sims and Zha (1998) is given by

$$B | B_0 \sim N(B_{SZ}, H_{SZ} \otimes \Sigma). \quad (7)$$

H_{SZ} has a diagonal matrix and the diagonal elements corresponding to endogenous variables, i, j at lag k are specified by

$$H'_{SZ,ij} = \left(\frac{\lambda_0 \lambda_1}{\sigma_j k^{\lambda_3}} \right)^2, \quad \text{for } i = j, \quad (8)$$

where λ_0, λ_1 and λ_3 are hyper-parameters chosen by the researcher, and σ_j is the square root of the corresponding $(i, i)^{\text{th}}$ element of an initial estimate of Σ .

3.4. Normal-flat (Sims-Zha) (NFSZ)

This kind of prior was observed by (Sims 1996, 2000). The prior can be stated as

$$B | \Sigma \sim N(B_N, H_N \otimes \Sigma), \quad (9)$$

where $H_N = d_3 I_M$ and d_3 is chosen hyper-parameter. As such, the Normal-flat prior do not depend on initial estimate of the disturbance covariance.

3.5. Specification of hyperparameters for the priors

The main problem in the estimation of BVAR is the specification of hyperparameters. These specifications of hyperparameters determine the tightness (both overall and relative) and decay parameters. Hence, the specification of hyperparameters for this study is as follows:

Litterman/Minnesota (LM) prior:

$$\mu_1 = 0, \quad \lambda_1 = 0.1, \quad \lambda_2 = 0.99, \quad \lambda_3 = 1,$$

Normal-Wishart (NW) prior:

$$\mu_1 = 0, \quad \lambda_1 = 0.1,$$

Normal-Wishart (Sims-Zha) prior (NWSZ):

$$\mu_5 = 0.1, \quad \mu_6 = 0.1, \quad \lambda_1 = 0.1, \quad \lambda_3 = 1, \quad \lambda_0 = 1,$$

Normal-flat (Sims-Zha) (NFSZ):

$$\mu_5 = 0.1, \quad \mu_6 = 0.1, \quad \lambda_1 = 0.1, \quad \lambda_3 = 1, \quad \lambda_0 = 1.$$

4. Monte Carlo Experiment

The main obstacle of VAR modelling is how to select the appropriate lag length. However, it had been observed that model with few number of lags may be preferable in VAR modelling. (Gujarati and Porter 2009). In this work, EViews version 10.0 was used for analysis, hence we will fit a VAR(2) which is given as:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix} + \begin{bmatrix} 0.5 & 0.1 \\ 0.4 & 0.5 \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \end{bmatrix} + \begin{bmatrix} 2.5 & 6.5 \\ 1.4 & 3.0 \end{bmatrix} \begin{bmatrix} y_{1t-2} \\ y_{2t-2} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}. \quad (10)$$

The steps for Monte Carlo experiment are follows as:

(a) Generation of time series data, these time series data were generated from multivariate Gaussian distribution with means and specified variance-covariance that have different scenarios of collinearity using a Cholesky decomposition method (Diebold and Mariano 2002).

The scenarios of collinearity that will be considered are:

High Collinearity (HC); $\rho = 0.99, 0.95$ and 0.80

Moderate Collinearity (MC); $\rho = 0.49, 0.45$ and 0.35

Low Collinearity (LC); $\rho = 0.20, 0.16$ and 0.13 .

(b) The innovations, u_{1t} and u_{2t} were simulated from Gaussian distribution.

(c) In order to obtain stability of results, steps (a) and (b) were replicated 100,000 times for different time series lengths.

(d) All the priors that will be considered then generate forecasts for different scenarios of collinearity at each sample sizes.

4.1. Forecast evaluation

The forecast evaluation criteria that will be considered are; Root Mean Squares Forecast Error (RMSFE), Mean Absolute Forecast Error (MAFE), Mean Absolute Percentage Forecast Error

(MAPFE) and Theil Inequality (1961). Sample sizes are $T = 10, 25, 100, 500$ and $2,000$ each replicated at $100,000$ with 25% of samples as in-sample forecasts.

1) Root Mean Squares Forecast Error (RMSFE)

The RMSFE is simply defined as

$$\text{RMSFE} = \sqrt{\frac{1}{n} \sum_{t=1}^n (y_t - y_t^f)^2},$$

where y_t^f is predicted in-sample of y , y_t is the time series data, and n is the number of observation.

2) Mean Absolute Forecast Error (MAFE)

This MAFE can be computed as

$$\text{MAFE} = \frac{1}{n} \sum_{t=1}^n |y_t^f - y_t|.$$

3) Mean Absolute Percentage Forecast Error (MAPFE)

MAPFE can be obtained as

$$\text{MAPFE} = 100 \sum_{t=1}^n \left| \frac{y_t^f - y_t}{y_t} \right|.$$

4) Theil Inequality (TI)

The Theil inequality is given as

$$\text{Theil Inequality (TI)} = \frac{\sqrt{\frac{1}{n} \sum_{t=1}^n (y_t y_t - y_t^f)^2}}{\sqrt{\frac{1}{n} \sum_{t=1}^n y_t^f} \sqrt{\frac{1}{n} \sum_{t=1}^n y_t}} = \frac{\text{RMSFE}}{\sqrt{\frac{1}{n} \sum_{t=1}^n y_t^f} \sqrt{\frac{1}{n} \sum_{t=1}^n y_t}}.$$

The inequality coefficient always lies between 0 and 1. The smaller these forecast evaluation criterion the better the candidate model represents well the data.

5. Results and Discussion

The results of the Monte Carlo experiment are presented in Tables 1-12 in the Appendix. These results are given in terms of forecast evaluation estimates for different scenarios of collinearity for the BVAR priors. The forecast estimates are Root Mean Squares Forecast Error (RMSFE), Mean Absolute Forecast Error (MAFE), Mean Absolute Percentage Forecast Error (MAPFE) and Theil Inequality (1961). Table 13 gives the summary of performance of the BVAR priors for different sample sizes in all scenarios of collinearity as shown in the Appendix.

The scenarios of collinearity are classified as: Scenario 1 is Low collinearity (LC) (0.13, 0.16, and 0.20), Scenario 2 is Moderate Collinearity (MC) (0.35, 0.45, and 0.49) and Scenario 3 is High Collinearity (HC) (0.80, 0.95, 0.99).

In Scenario 1, The RMSFE, MAFE, MAPFE and Theil inequality coefficient for the Normal-Wishart prior are the lowest especially for sample size of 10 while Litterman/Minnesota prior has lowest estimates for sample size of 25. For large samples sizes in Scenario 1, all the BVAR priors have the same performances.

It was observed that Normal-Wishart prior performed well for sample sizes 10 and 25 in scenario 2 while all the BVAR priors have the same performance in sample sizes of 100, 500 and 2,000 in Scenario 2. In Scenario 3, Normal-Wishart and Normal-Wishart (Sims-Zha) performed well in sample sizes 10 and 25, respectively while all the BVAR priors have the same performance in large sample sizes.

It is apparent that all the coefficient of Theil inequality for all the priors across all scenarios of collinearity are close to zero which means all the priors is good forecast. Theil inequality estimates of Scenario 3 are larger than Scenarios 1 and 2 which mean collinearity are more prominent in the data sets.

For sample size of 10, Normal-Wishart prior is a preferred prior for all the scenarios of collinearity considered. There is a mixed result for sample size of 25, Litterman/Minnesota prior performed best for low collinearity while Normal-Wishart and Normal-Wishart by Sims and Zha performed best for moderate collinearity and high collinearity, respectively.

It can be deduced that all the priors, Normal-Wishart, Litterman/Minnesota, Normal-Wishart (Sims-Zha) and Normal-flat (Sims-Zha) have the same performance in all the scenarios of collinearity considered in large sample sizes (100, 500 and 2,000).

6. Conclusions

Previous research has focused on evaluation of the performances of both the classical VAR and Bayesian VAR (BVAR) for short term series at different levels of collinearity. However, since it had been justified that BVAR outperformed the classical VAR, it is therefore important that research efforts be directed toward the construction of models by examining the kind of prior which are optimal in forecasting in VAR model especially when there is collinearity in time series data.

The sensitivity priors of Bayesian Vector Autoregressive (BVAR) model when the time series data are correlated was considered using forecast evaluation as criteria. Four priors considered were Litterman/Minnesota, Normal-Wishart, Normal-Wishart by Sims-Zha and Normal-flat by Sims-Zha. The forecast criteria used were Root Mean Squares Forecast Error (RMSFE), Mean Absolute Forecast Error (MAFE), Mean Absolute Percentage Forecast Error (MAPFE) and Theil Inequality (TI). Five different time series data sets were simulated through a Monte Carlo process for all the scenarios.

It was observed that forecast estimates of Root Mean Squares Forecast Error (RMSFE), Mean Absolute Forecast Error (MAFE), Mean Absolute Percentage Forecast Error (MAPFE) and Theil Inequality (TI) gave a good and same pattern forecast for the priors considered. It was also observed that the performance of all the four priors are different for small sample sizes (10 and 25) in all scenarios of collinearity while forecasts estimates for all the priors are the same for large sample sizes (100, 500 and 2,000) across all scenarios of collinearity for the four different kind of prior distributions considered. Also, low collinearity has the smallest values of forecast for all the forecast criteria used followed by moderate collinearity.

However, Normal-Wishart prior was observed to perform best in the three scenarios of collinearity especially in small samples while all the BVAR priors have the same performance in large samples which means sufficient large sample should be taken in Bayesian Vector Autoregressive modelling, this will make the researcher to be at liberty to make any choice of prior.

Therefore, it is advisable to employ a Normal-Wishart prior for forecasting in Bayesian Vector Autoregressive analysis with small sample sizes because it will guarantee efficient results while any form of prior can be used in large sample sizes especially when the time series data sets are correlated. A natural extension of this work is the case where autocorrelation of innovations is introduced, an interesting connection to the issue of autocorrelation may be considered in future work.

Acknowledgements

The author would like to thank both the referees and editor for their fruitful suggestions and comments, which led to the overall improvement of this manuscript.

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Appendix

Scenario 1: Low Collinearity (LC).

Table 1 RMSFE for LC

Sample	Collinearity levels	LM	NW	NWSZ	NFSZ
10	0.13	0.2798	0.2845	0.5059	0.2796
	0.16	0.5496	0.4675	0.7670	0.5497
	0.20	0.6030	0.5344	0.5989	0.6029
25	0.13	0.5635	0.5637	0.5945	0.5636
	0.16	0.6052	0.6018	0.6981	0.6050
	0.20	0.5953	0.5938	0.5823	0.5954
100	0.13	0.5891	0.5891	0.5891	0.5891
	0.16	0.5922	0.5922	0.5922	0.5922
	0.20	0.5915	0.5915	0.5915	0.5915
500	0.13	0.5530	0.5529	0.5535	0.5530
	0.16	0.5794	0.5794	0.5793	0.5794
	0.20	0.5530	0.5529	0.5535	0.5530
2,000	0.13	0.5808	0.5808	0.5808	0.5808
	0.16	0.5889	0.5889	0.5889	0.5889
	0.20	0.5833	0.5833	0.5833	0.5833

Table 2 MAFE for LC

Sample	Collinearity levels	LM	NW	NWSZ	NFSZ
10	0.13	0.2482	0.2439	0.4189	0.2479
	0.16	0.4938	0.3861	0.6941	0.4944
	0.20	0.4764	0.4758	0.4748	0.4761
25	0.13	0.4867	0.4829	0.4905	0.4868
	0.16	0.5298	0.5271	0.5968	0.5296
	0.20	0.4873	0.4923	0.4900	0.4874
100	0.13	0.4917	0.4917	0.4917	0.4917
	0.16	0.4940	0.4940	0.4940	0.4940
	0.20	0.4957	0.4957	0.4957	0.4957
500	0.13	0.4801	0.4801	0.4804	0.4801
	0.16	0.5020	0.5020	0.5019	0.5020
	0.20	0.4801	0.4801	0.4804	0.4801
2,000	0.13	0.4993	0.4993	0.4993	0.4993
	0.16	0.5085	0.5086	0.5086	0.5085
	0.20	0.5056	0.5056	0.5055	0.5056

Table 3 MAPFE for LC

Sample	Collinearity levels	LM	NW	NWSZ	NFSZ
10	0.13	13.1645	12.9815	29.8092	13.1474
	0.16	12.0968	12.1470	15.2509	12.0998
	0.20	21.4683	21.0195	23.7771	21.4789
25	0.13	29.0206	29.8373	32.6760	29.0288
	0.16	28.1018	28.0904	32.0915	28.0960
	0.20	26.8125	27.4051	28.3252	28.1420
100	0.13	24.1130	24.1143	24.1133	24.1130
	0.16	24.3211	24.3213	24.3213	24.3211
	0.20	24.3444	24.3457	24.3445	24.3446
500	0.13	28.1418	28.1473	28.1883	28.1420
	0.16	30.0600	30.0694	30.0553	30.0600
	0.20	28.1418	28.1473	28.1617	28.1418
2,000	0.13	29.4349	29.4362	24.4352	29.4348
	0.16	29.5918	29.5936	29.5926	29.5918
	0.20	28.5701	28.5715	28.5690	28.5700

Table 4 TI for LC

Sample	Collinearity levels	LM	NW	NWSZ	NFSZ
10	0.13	0.0737	0.0755	0.1440	0.0737
	0.16	0.0716	0.0718	0.0873	0.1217
	0.20	0.1337	0.0967	0.1373	0.1337
25	0.13	0.1665	0.1680	0.1848	0.1666
	0.16	0.1565	0.1561	0.1751	0.1565
	0.20	0.1596	0.1602	0.1627	0.1597
100	0.13	0.1415	0.1415	0.1415	0.1415
	0.16	0.1428	0.1428	0.1428	0.1428
	0.20	0.1423	0.1423	0.1423	0.1423
500	0.13	0.1580	0.1580	0.1582	0.1580
	0.16	0.1686	0.1686	0.1685	0.1686
	0.20	0.1580	0.1580	0.1582	0.1580
2,000	0.13	0.1665	0.1665	0.1665	0.1665
	0.16	0.1666	0.1666	0.1666	0.1666
	0.20	0.1606	0.1606	0.1606	0.1606

Scenario 2: Moderate Collinearity (MC)**Table 5** RMSFE for MC

Sample	Collinearity levels	LM	NW	NWSZ	NFSZ
10	0.35	0.3914	0.4083	0.4183	0.3960
	0.45	0.5572	0.4825	0.5343	0.5565
	0.49	0.6030	0.5344	0.5988	0.6029
25	0.35	0.5238	0.5070	0.5394	0.5245
	0.45	0.6718	0.6627	0.7165	0.6715
	0.49	0.5477	0.5423	0.6566	0.5470
100	0.35	0.5915	0.5915	0.5915	0.5915
	0.45	0.5849	0.5849	0.5849	0.5849
	0.49	0.5863	0.5863	0.5863	0.5863
500	0.35	0.5530	0.5529	0.5535	0.5530
	0.45	0.5530	0.5535	0.5182	0.5530
	0.49	0.5530	0.5529	0.5535	0.5530
2,000	0.35	0.5732	0.5732	0.5732	0.5732
	0.45	0.5754	0.5754	0.5754	0.5754
	0.49	0.5823	0.5823	0.5823	0.5823

Table 6 MAFE for MC

Sample	Collinearity levels	LM	NW	NWSZ	NFSZ
10	0.35	0.3538	0.3658	0.3789	0.3599
	0.45	0.5001	0.3834	0.4700	0.4997
	0.49	0.4764	0.4758	0.4748	0.4761
25	0.35	0.4649	0.4400	0.4687	0.4654
	0.45	0.5854	0.5770	0.6114	0.5850
	0.49	0.4623	0.4565	0.5329	0.4610
100	0.35	0.4957	0.4957	0.4957	0.4957
	0.45	0.4887	0.4887	0.4887	0.4887
	0.49	0.4893	0.4893	0.4893	0.4893
500	0.35	0.4801	0.4801	0.4804	0.4801
	0.45	0.4801	0.4804	0.4804	0.4801
	0.49	0.4801	0.4801	0.4804	0.4801
2,000	0.35	0.4859	0.4859	0.4859	0.4859
	0.45	0.4823	0.4823	0.4823	0.4823
	0.49	0.4861	0.4861	0.4861	0.4861

Table 7 MAPFE for MC

Sample	Collinearity levels	LM	NW	NWSZ	NFSZ
10	0.35	18.6632	19.8614	22.1190	18.9867
	0.45	31.9336	26.1771	27.7056	31.8876
	0.49	21.4683	16.5234	23.7771	21.4789
25	0.35	20.9193	19.8245	20.3132	20.9379
	0.45	31.4994	31.0979	35.9272	31.4764
	0.49	23.6345	23.4793	27.8245	23.5633
100	0.35	24.3444	24.3457	24.3445	24.3446
	0.45	23.8427	23.8441	23.8431	23.8428
	0.49	23.8196	23.8209	23.8197	23.8196
500	0.35	28.1418	28.1473	28.1617	28.1420
	0.45	28.1418	28.1617	28.1617	28.1420
	0.49	28.1418	28.1473	28.1617	28.1420
2,000	0.35	25.0842	25.0857	25.0852	25.0843
	0.45	23.8837	23.8847	23.8836	23.8837
	0.49	23.6905	23.6918	23.6905	23.6906

Table 8 TI for MC

Sample	Collinearity levels	LM	NW	NWSZ	NFSZ
10	0.35	0.1022	0.1064	0.1146	0.1034
	0.45	0.1683	0.1523	0.1607	0.1683
	0.49	0.1337	0.1168	0.1373	0.1337
25	0.35	0.1161	0.1128	0.1175	0.1163
	0.45	0.1735	0.1728	0.1928	0.1733
	0.49	0.1373	0.1365	0.1646	0.1371
100	0.35	0.1423	0.1423	0.1423	0.1423
	0.45	0.1399	0.1399	0.1399	0.1399
	0.49	0.1399	0.1399	0.1399	0.1399
500	0.35	0.1580	0.1580	0.1582	0.1580
	0.45	0.1580	0.1582	0.1582	0.1580
	0.49	0.1580	0.1580	0.1582	0.1580
2,000	0.35	0.1448	0.1448	0.1448	0.1448
	0.45	0.1397	0.1397	0.1397	0.1397
	0.49	0.1392	0.1392	0.1392	0.1392

Scenario 3: High Collinearity (HC)**Table 9** RMSFE for HC

Sample	Collinearity levels	LM	NW	NWSZ	NFSZ
10	0.80	0.5420	0.4917	0.7382	0.5513
	0.95	0.5889	0.5222	0.6788	0.5752
	0.99	0.6091	0.5504	0.6751	0.6069
25	0.80	0.6115	0.5850	0.5917	0.6100
	0.95	0.4835	0.4586	0.4520	0.4844
	0.99	0.5684	0.5653	0.6172	0.5676
100	0.80	0.5867	0.5867	0.5867	0.5867
	0.95	0.5882	0.5882	0.5882	0.5882
	0.99	0.5932	0.5932	0.5932	0.5932
500	0.80	0.5839	0.5839	0.5836	0.5839
	0.95	0.5772	0.5772	0.5772	0.5772
	0.99	0.5763	0.5763	0.5763	0.5763
2,000	0.80	0.5675	0.5675	0.5675	0.5675
	0.95	0.5827	0.5827	0.5827	0.5827
	0.99	0.5835	0.5835	0.5834	0.5835

Table 10 MAFE for HC

Sample	Collinearity levels	LM	NW	NWSZ	NFSZ
10	0.80	0.4884	0.4586	0.5909	0.4941
	0.95	0.4811	0.4217	0.5265	0.4774
	0.99	0.5329	0.4830	0.5548	0.5315
25	0.80	0.5240	0.5052	0.4965	0.5231
	0.95	0.3976	0.3817	0.3746	0.3983
	0.99	0.5006	0.5653	0.5332	0.5004
100	0.80	0.4896	0.4896	0.4896	0.4896
	0.95	0.4916	0.4916	0.4916	0.4916
	0.99	0.4954	0.4954	0.4954	0.4954
500	0.80	0.4829	0.4829	0.4824	0.4829
	0.95	0.4944	0.4944	0.4943	0.4943
	0.99	0.4945	0.4945	0.4945	0.4945
2,000	0.80	0.4694	0.4694	0.4694	0.4694
	0.95	0.4996	0.4996	0.4997	0.4996
	0.99	0.5048	0.5048	0.5047	0.5048

Table 11 MAPFE for HC

Sample	Collinearity levels	LM	NW	NWSZ	NFSZ
10	0.80	19.6397	18.5015	28.0709	19.9453
	0.95	24.7999	22.7897	23.0149	25.0027
	0.99	36.6076	32.6524	45.59260	36.4154
25	0.80	24.6820	23.7158	23.9382	24.6151
	0.95	22.3225	21.5042	21.2081	22.3672
	0.99	30.2602	30.3854	34.6662	30.2683
100	0.80	23.7513	23.7527	23.7512	23.7514
	0.95	24.0553	24.0566	24.0566	24.0553
	0.99	24.3681	24.3694	24.3681	24.3681
500	0.80	22.6283	22.6352	22.6103	22.6280
	0.95	26.0682	26.0766	26.0793	26.0683
	0.99	29.5543	29.5604	29.5604	29.5541
2,000	0.80	22.3345	22.3358	22.3338	22.3345
	0.95	26.5716	26.5727	26.5721	26.5716
	0.99	29.7375	29.7383	29.7374	29.7376

Table 12 TI for HC

Sample	Collinearity levels	LM	NW	NWSZ	NFSZ
10	0.80	0.1083	0.0977	0.1551	0.1103
	0.95	0.1493	0.1338	0.1580	0.1467
	0.99	0.2006	0.1807	0.2386	0.1997
25	0.80	0.1411	0.1351	0.1384	0.1407
	0.95	0.1333	0.1268	0.1246	0.1336
	0.99	0.1670	0.1664	0.1842	0.1667
100	0.80	0.1395	0.1395	0.1395	0.1395
	0.95	0.1411	0.1411	0.1411	0.1411
	0.99	0.1429	0.1429	0.1429	0.1429
500	0.80	0.1343	0.1344	0.1343	0.1343
	0.95	0.1488	0.1488	0.1489	0.1488
	0.99	0.1674	0.1674	0.1674	0.1674
2,000	0.80	0.1326	0.1326	0.1326	0.1326
	0.95	0.1514	0.1514	0.1514	0.1514
	0.99	0.1671	0.1671	0.1671	0.1671

Table 13 Summary of performance of the priors for all the scenarios

Sample sizes	Low collinearity	Moderate collinearity	High collinearity
10	Normal-Wishart	Normal-Wishart	Normal-Wishart
25	Litterman/Minnesota	Normal-Wishart	Normal-Wishart (Sims-Zha)
100	Same performance	Same performance	Same performance
500	Same performance	Same performance	Same performance
2,000	Same performance	Same performance	Same performance