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The Design of GM-F Sampling Plan for Continuous Processes

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Abstract

This paper presents a plan for inspection of continuous production line namely, GM-F plan. The formulas for performance measures of the GM-F plan have been derived by using Markov Chain, namely Average Fraction Inspected (AFI), Average Outgoing Quality (AOQ), Average Outgoing Quality Limit (AOQL) and Average Fraction of the Total Produced Accepted on Sampling Basis (Pa). The accuracy of the performance measure formulas has been tested by comparing the values computed from the formulas with the values for the performance measures obtained from extensive simulations for each $p = 0.005, 0.008, 0.01, 0.02, 0.04$ and 0.05 when p is the value of incoming fraction of nonconforming unit on production line. The result found that the formulas for all performance measures are valid for all simulations. The GM-F plan also has been compared with a CSP-F-L and a MCSP-F-L plan for the recommendation of the operator to use the sampling plan that appropriate to their production line.

Keywords: Continuous sampling plans, average fraction inspected, average outgoing quality.

1. Introduction

When production is continuous, a continuous sampling plans (CSPs) are often used for inspecting units are produced item by item on a continuous flow of product and the result of the inspection is either conforming or nonconforming. The conforming unit is accepted and passed on to the customer. CSPs consist of alternating sequences of sampling inspection and 100% inspection. There are two main types of CSPs: single-level continuous sampling plans and multi-level continuous sampling plans. Both types of CSPs are different at sampling inspection in that single level continuous sampling plans are only one level of sampling inspection but multilevel continuous sampling plans are more than one level of sampling inspection. A continuous sampling plan is not the same as for a lot-by-lot sampling plan. In continuous sampling, there are no specific lots. Consequently, a different measure of evaluation of the performance of CSP is needed. The most common performance measure is the Average Fraction of the Total Produced Accepted on Sampling Basis (Pa). Other performance measures of CSP plans are the Average Fraction Inspected (AFI), the Average Outgoing Quality (AOQ) and the Average Outgoing Quality Limit (AOQL).

Continuous sampling plans were first proposed by Dodge (1943). Dodge's initial plan is called CSP-1 plan. The CSP-1 is a single-level continuous sampling plan has simplest and most commonly procedure that starts with 100% inspection until i successive conforming units are found (the number of units i is usually called the clearance number), as soon as the clearance number has been reached, sampling inspection is instituted, and only a fraction (f) of the units are inspected. These sampling units are selected one at a time at random from the flow of production. Sampling inspection continues until a nonconforming sample unit is found then the procedure switches back to 100% inspection. All nonconforming units found are replaced with conforming units. The procedure of Dodge CSP-1 has been developed as other plans such as CSP-2 and CSP-3 by Dodge and Torrey (1977) that subsequent developments represent extensions and variations in his basic procedure. Another common objection to continuous sampling plans is the abrupt transition between sampling inspection and 100% inspection.

Lieberman and Solomon (1955) have designed multilevel continuous sampling plans (CSP-M) to overcome the objection of Dodge's plan. The CSP-M begins with 100% inspection as the CSP-1 plan. Under sampling inspection at rate f, a run of i consecutive sampling units is conforming, then sampling continues at the rate f^2. If a further run of i consecutive units is conforming, then sampling may continue at the rate f^3. This reduction in sampling frequency may be continued as far as the sampling agency wishes. If at any time sampling inspection reveals a nonconforming, return is immediately made to the next lower level of sampling. Much of the work on continuous sampling plans has done. A review of various CSPs can be seen in many statistical quality control textbooks; see Stephens (2001), Montgomery (2009), Schilling and Neubauer (2017).

Derman et al. (1957) developed three tightened multilevel plans, namely MLP-T or CSP-T. The principal difference is a return to 100% inspection upon finding a nonconforming unit at any of the sampling levels. It is further simplified over CSP-M by having the number of sampling levels fixed at three. Additionally, the sampling rates are reduced geometrically by one half between the levels rather than exponentially as in CSP-M. The modified of the MLP-T plan is designated as MLP-T-2 by Kandaswamy and Govindaraju (1993). The procedure of the MLP-T-2 plan alternates between screening and sampling inspection with two sampling levels. Kandasamy and Govindaraju derived the performance measures of the MLP-T-2 plan using the Markov Chain approach. Balamurali and Govindaraju (2000) have developed the Modified MLP-T-2 plan that the operating procedure start with 100% inspection. When the first i consecutive conforming units are found, then switch to the sampling inspection at level 2 (f_2). Otherwise the 100% inspection is continued until any run of i successive conforming units are found and then switch to the sampling inspection at level 1 $(f_1,\ f_1 > f_2)$. When a nonconforming unit is found on either sampling level, immediately revert to the 100% inspection. A prominent point of the Modified MLP-T-2 plan over MLP-T-2 plan is that one cannot go from one level of sampling inspection to another without going back to 100% inspection.

Guayjarernpanishk (2014) developed a fractional sampling plan, namely CSP-F-L, based on Modified MLP-T-2. The purpose of developing CSP-F-L is to reduce the number of units inspected of Modified MLP-T-2. The difference between the two plans is in the beginning of inspection. The Modified MLP-T-2 starts with 100% inspection but the procedure of CSP-F-L starts with sampling inspection at level 1 with a rate f_1 of the units. For the CSP-F-L plan, the inspection is continued k consecutive units. If the first k consecutive units are found clear of nonconforming, then switch to sampling inspection at level 2 $(f_2, f_2 < f_1)$. Otherwise, switch to 100% inspection of units in the order of production. During at the 100% inspection, if the first i consecutive units are found clear of nonconforming discontinue 100% inspection and switch to sampling inspection at level 2. Otherwise, continue 100% inspection until i successive units are found clear of nonconforming then proceed to sampling inspection at level 1 begins. When a nonconforming unit is found at level 2, immediately revert to the sampling inspection at level 1. Guayjarernpanishk (2014) derived the performance measures of the CSP-F-L plan using the Markov Chain approach, such as the Average Fraction Inspected (AFI), the Average Fraction of the Total Produced Accepted on Sampling Basis (Pa), the Average Outgoing Quality (AOQ) and the Average Outgoing Quality Limit (AOQL).

Guayjarernpanishk and Mayureesawan (2015) presented the MCSP-F-L for the concept of a fractional sampling plan that has been developed from the CSP-F-L plan. The attractive feature of the MCSP-F-L plan is that addition a maximum allowable number of inspected units (l) for prevention long length of inspection at level 2 in the procedure of CSP-F-L plan. The operating procedure of the MCSP-F-L plan is different from CSP-F-L plan for deciding when switch from the phase of sampling inspection at level 2 to the phase of sampling inspection at level 1, during the inspection at level 2, if a nonconforming unit is found then revert immediately to sampling inspection at level 1 as in the CSP-F-L plan, but in the MCSP-F-L plan, if continue sampling inspection till l conforming units are found then revert to sampling inspection at level 1. The conventional measures of performance have been derived using a Markov Chain model as in the CSP-F-L plan. The performance measures of the MCSP-F-L plan were compared with the CSP-F-L plan and the Modified MLP-T-2 plan at various levels of incoming quality levels and plan parameters.

In this paper, we developed a sampling plan for continuous production line, namely, GM-F plan. The GM-F plan has two sampling inspection levels as the CSP-F-L and MCSP-F-L plans also begins with sampling inspection at level 1 as in CSP-F-L and MCSP-F-L plans. The transition between sampling inspection and 100% inspection of the three plans are different. A detailed operating procedure of the GM-F plan is given in section 2. In Section 3, the GM-F procedure as a Markov Chain is described. Section 4 presents performance measure formulas of the GM-F plan derived from Markov Chain, such as the Average Fraction Inspected, the Average Fraction of the Total Produced Accepted on Sampling Basis, the Average Outgoing Quality and the Average Outgoing Quality Limit. Section 5 describes accuracy of these performance measure formulas, and Section 5 describes criterion for testing the accuracy of the performance measure formulas of the GM-F plan. In Section 6, the comparison of performance of GM-F plan with CSP-F-L and MCSP-F-L plans is described. In Section 7, we compare analytical results obtained through extensive Monte Carlo simulations. Finally, the discussions and conclusions of the study are provided in Section 8.

2. The Operating Procedure of the GM-F Plan

The operating procedure of the GM-F plan is given below and shown in Figure 1.

1) The procedure starts with sampling inspection at a moderate level 1 with a rate f_1 of the units $(f_1 = 1/r)$, selecting individual units one at a time in the order of production randomly.

1.1) If a nonconforming unit found before g consecutive units inspected, then switch to 100% inspection of units in the order of production $(g = ri)$.

1.2) Otherwise, switch to sampling inspection at level 2 with a rate f_2 of the units $(f_2 = 1/(r+1))$.

2) During the 100% inspection, continue 100% inspection until i consecutive units are inspected, then proceed to sampling inspection at level 1 and continues as in step 1.

3) During the sampling inspection at level 2.

3.1) If a nonconforming unit found before m consecutive units inspected, switch to 100% inspection of units in the order of production and then continues as in step 2 $(m = (r+1)i)$.

3.2) Otherwise, the inspection switch to sampling inspection at level 1 and then the inspection continues as in step 1.

4) Replace or correct all the nonconforming units found with conforming units.

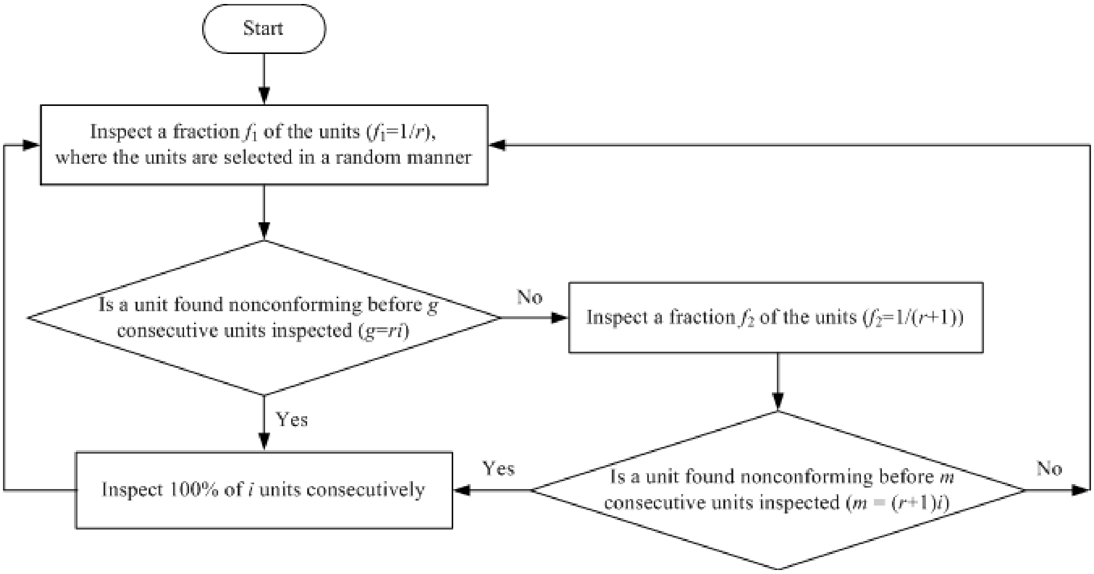

Figure 1 The operating procedure of GM-F plan

3. The GM-F Procedure as a Markov Chain

The formulation of the GM-F procedure as a Markov Chain is given, assuming that the production process is in statistical control. The performance measures such as the Average Fraction Inspected (AFI), the Average Fraction of the Total Produced Accepted on Sampling Basis (Pa) and the Average Outgoing Quality (AOQ) are derived and are given in the following.

Let $[X_t]$; $t = 1,2,...$ denote a discrete-parameter Markov Chain with finite state space (S_j) ; $j = 1, 2,...,3g+i+3m+1$. The states of the process are defined, in a same way Roberts (1965) and Lasater (1970), as follows:

$S_{3k+1} = f_1 \mathrm{N}_{k+1}$; $k = 0,1,2,...,g-1$

= Sampling inspection at level 1 is in effect and the k units submitted for inspection were all found to be conforming but the last unit was not selected for inspection.

$S_{3k+2} = f_1 \mathrm{In}_{k+1}$; $k = 0,1,2,...,g-1$

= Sampling inspection at level 1 is in effect and the $k+1$ units submitted for inspection were all found to be conforming.

$S_{3k+3} = f_1 \mathrm{Id}_{k+1}$; $k = 0,1,2,...,g-1$

= Sampling inspection at level 1 is in effect, the $k+1$ units submitted for inspection and only unit $k+1$ was found to be nonconforming.

$S_{3g+1} = \mathrm{A}_0$

= Nonconforming unit is found on 100% inspection.

$S_{3g+j+1} = \mathrm{A}_j \; ; j = 1,2,...,i$

= j consecutive conforming units found during 100% inspection after having a nonconforming unit is found on 100% inspection.

$S_{3g+i+3l+2} = f_2 \mathrm{N}_{l+1} \; ; l = 0,1,2,...,m-1$

= Sampling inspection at level 2 is in effect and the l units submitted for inspection were all found to be conforming but the last unit was not selected for inspection.

$S_{3g+i+3l+3} = f_2 \, \mathrm{In}_{l+1} \; ; l = 0,1,2,...,m-1$

= Sampling inspection at level 2 is in effect and the $l+1$ units submitted for inspection were all found to be conforming.

$S_{3g+i+3l+4} = f_2 \, \mathrm{Id}_{l+1} \; ; l = 0,1,2,...,m-1$

= Sampling inspection at level 2 is in effect, the $l+1$ units submitted for inspection and only unit $l+1$ was found to be nonconforming.

The set of $(3g+i+3m+1)$ states defined above completely describe the mutually exclusive phases of inspection for the GM-F plan procedure. A flow chart showing the description of the process by means of states and transition is given in Figure 2 and the one-step transition probability matrix for the process is given in Table 1. The transition probability matrix reveals that the process is a discrete-parameter, finite, recurrent, irreducible, aperiodic (DFRIA) Markov Chain; see Karlin and Taylor (2012), Lasater (1970).

4. The Performance Measures of the GM-F Plan

The conventional measures of performance for GM-F plan have been derived using a Markov Chain model, namely the Average Fraction Inspected (AFI), the Average Fraction of Total Produced Accepted on Sampling Basis (Pa), the Average Outgoing Quality (AOQ) and the Average Outgoing Quality Limit (AOQL). Letting p be the probability of a unit produced by the process being nonconforming and q be the probability of a unit produced by the process being conforming, the following performance measures may be obtained.

The average number of units inspected in a 100% screening sequence following the finding of a nonconforming unit, u:

$$u = \frac{(1-q^i)(1-q^{g+m})}{pq^i}. \tag{1}$$

The average number of units passed under the sampling inspection, v:

$$v = \frac{f_1 q^g (1-q^m) + f_2 (1-q^g)}{f_1 f_2 p}. \tag{2}$$

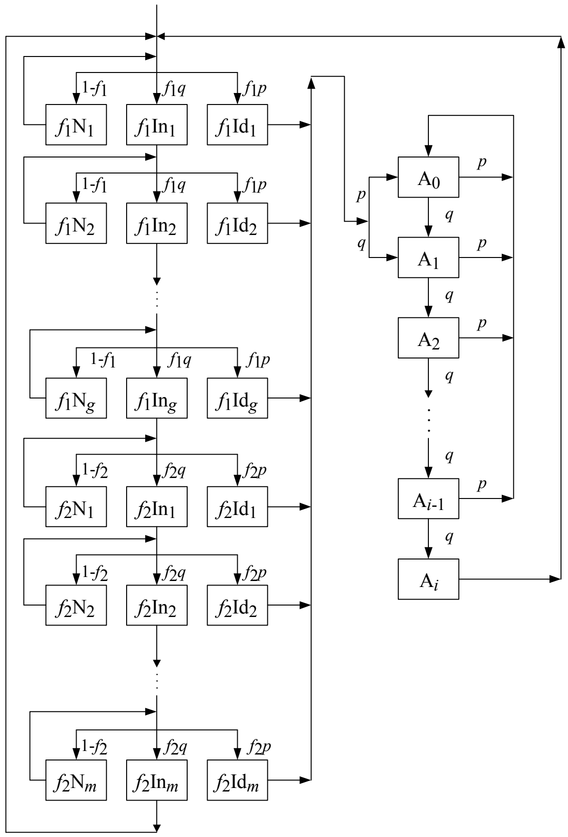

Figure 2 States and transitions of the GM-F procedure

The Average Fraction Inspected, AFI:

$$\text{AFI} \;=\; \frac{f_1 f_2(1-q^{g+m})}{f_1 q^{i+g}(1-q^m) + f_1 f_2(1-q^i)(1-q^{g+m}) + f_2 q^i(1-q^g)}. \tag{3}$$

The Average Fraction of the Total Produced Accepted on Sampling Basis, Pa:

$$\text{Pa} \;=\; \frac{q^i[f_1 q^g(1-q^m) + f_2(1-q^g)]}{f_1 q^{i+g}(1-q^m) + f_1 f_2(1-q^i)(1-q^{g+m}) + f_2 q^i(1-q^g)}. \tag{4}$$

The Average Outgoing Quality, AOQ:

$$\text{AOQ} \;=\; \frac{pq^i[f_1 q^g(1-q^m) - f_1 f_2(1-q^{g+m}) + f_2(1-q^g)]}{f_1 q^{i+g}(1-q^m) + f_1 f_2(1-q^i)(1-q^{g+m}) + f_2 q^i(1-q^g)}. \tag{5}$$

The Average Outgoing Quality Limit, AOQL that is the maximum of AOQ for all values of p. A detailed derivation of these performance measures based on Markov Chain formulation is given in the Appendix.

Table 1 One-step transition probability matrix of the GM-F plan

	$f_1\mathrm{N}_1$	$f_1\mathrm{In}_1$	$f_1\mathrm{Id}_1$	$f_1\mathrm{N}_2$	…	$f_1\mathrm{Id}_g$	A_0	A_1	…	A_i	$f_2\mathrm{N}_1$	$f_2\mathrm{In}_1$	$f_2\mathrm{Id}_1$	$f_2\mathrm{N}_2$	…	$f_2\mathrm{Id}_m$
$f_1\mathrm{N}_1$	$1-f_1$	f_1q	f_1p	0	…	0	0	0	…	0	0	0	0	0	…	0
$f_1\mathrm{In}_1$	0	0	0	$1-f_1$	…	0	0	0	…	0	0	0	0	0	…	0
$f_1\mathrm{Id}_1$	0	0	0	0	…	0	p	q	…	0	0	0	0	0	…	0
$f_1\mathrm{N}_2$	0	0	0	$1-f_1$	…	0	0	0	…	0	0	0	0	0	…	0
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
$f_1\mathrm{In}_g$	0	0	0	0	…	0	0	0	…	0	$1-f_2$	f_2q	f_2p	0	…	0
$f_1\mathrm{Id}_g$	0	0	0	0	…	0	p	q	…	0	0	0	0	0	…	0
$f_2\mathrm{N}_1$	0	0	0	0	…	0	0	0	…	0	$1-f_2$	f_2q	f_2p	0	…	0
$f_2\mathrm{In}_1$	0	0	0	0	…	0	0	0	…	0	0	0	0	$1-f_2$	…	0
$f_2\mathrm{Id}_1$	0	0	0	0	…	0	p	q	…	0	0	0	0	0	…	0
$f_2\mathrm{N}_2$	0	0	0	0	…	0	0	0	…	0	0	0	0	$1-f_2$	…	0
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
$f_2\mathrm{In}_m$	$1-f_1$	f_1q	f_1p	0	…	0	0	0	…	0	0	0	0	0	…	0
$f_2\mathrm{Id}_m$	0	0	0	0	…	0	p	q	…	0	0	0	0	0	…	0
A_0	0	0	0	0	…	0	p	q	…	0	0	0	0	0	…	0
A_1	0	0	0	0	…	0	p	0	…	0	0	0	0	0	…	0
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
A_i	$1-f_1$	f_1q	f_1p	0	…	0	0	0	…	0	0	0	0	0	…	0

5. Test of the Accuracy of Performance Measures for the GM-F Plan

For testing the accuracy of the performance measure formulas that defined for the GM-F, the results from the formulas were compared with the values obtained from extensive simulations. Six different levels were examined for the probability p of nonconforming units produced on the line 0.005, 0.008, 0.01, 0.02, 0.04 and 0.05. For each p, values of $i = 10, 20, 30, 40$ and 50, values of $f_1 = 1/r$, values of $f_2 = 1/(r+1)$ when $r = 4$ and 6, and values of $g = ri$ and $m = (r+1)i$. For each value of p, the simulation with R program was repeated 500 different product lines (R Core Team 2017), and for each set of values of p, i and r, a simulation with MATLAB program was carried out to inspect the product line and compute the fraction of units inspected (MATLAB 2017), the fraction of the total produced accepted on sampling basis and the fraction of outgoing nonconforming units. The simulation was repeated 500 different product lines and the values of the Average Fraction Inspected (AFI), the Average Fraction of the Total Produced Accepted on Sampling Basis (Pa) and the Average Outgoing Quality (AOQ) were calculated and then compared with the values of AFI, Pa and AOQ computed from the formulas given in (3), (4) and (5), respectively.

When DAFI, DPa and DAOQ were defined by

$$\mathrm{DAFI} = \left|\mathrm{AFI\_F} - \mathrm{AFI\_S}\right| \tag{6}$$

$$\mathrm{DPa} = \left|\mathrm{Pa\_F} - \mathrm{Pa\_S}\right| \tag{7}$$

and

$$\mathrm{DAOQ} = \left|\mathrm{AOQ\_F} - \mathrm{AOQ\_S}\right|, \tag{8}$$

where AFI_F = the AFI values of GM-F plan from the formula, AFI_S = the AFI values of GM-F plan from the simulation,

Pa_F = the Pa values of GM-F plan from the formula,

Pa_S = the Pa values of GM-F plan from the simulation,

AOQ_F = the AOQ values of GM-F plan from the formula,

AOQ_S = the AOQ values of GM-F plan from the simulation.

The AFI and Pa formulas are accepted as the accurate formulas if DAFI and DPa were less than or equal to 0.02. The AOQ formula is accepted as an accurate formula if DAOQ was less than or equal to 0.002. The accuracy of the formulas was then compared for each set of values of p, i and $r,$ the results are presented in Section 7.1.

6. Comparison of GM-F Plan with CSP-F-L and MCSP-F-L Plans

In order to compare the performance of GM-F with other CSP plans, the GM-F is a plan with two sampling inspection levels as the CSP-F-L and MCSP-F-L plans also start with sampling inspection at level 1 with a rate f_1 of the units $(f_1 = 1/r),$ as does CSP-F-L and MCSP-F-L plans. But the transition between sampling inspection and 100% inspection is different. Therefore, in this paper, the performance measure such as, the AFI, the AOQ and the Pa values of the GM-F plan were compared with CSP-F-L and MCSP-F-L plans. These performance measures were obtained from extensive simulations when the values of $p =$ 0.005, 0.008, 0.01, 0.02, 0.04 and 0.05, and $i =$ 10, 20, 30, 40 and 50, and $r =$ 4 and 6. For the MCSP-F-L plan, the values of $i = k = l.$ The results are presented in Section 7.2.

7. Comparison of GM-F Plan with CSP-F-L and MCSP-F-L Plans

7.1. The accuracy of performance measures for GM-F plan

For all sets of p, i and r values, the difference of the AFI values from the formula and from the simulations (DAFI) are shown in Table 2, the difference of the AOQ values from the formula and from the simulations (DAOQ) are shown in Table 3 and the difference of the Pa values from the formula and from the simulations (DPa) are shown in Table 4. From the highlights in the final column of Table 2 and 4, it was found that the DAFI and DPa values, respectively were less than 0.02 for all sets of $p,$ i and r values, then from the condition of the accuracy of the formulas in Section 5, the AFI and Pa formulas are accepted as the accurate formulas. From the highlights in the last column of Table 3, it was also found that the DAOQ values was less than 0.002 for all sets of p, i and r values, then the AOQ formula is accepted as an accurate formula. So the simulations signified that the AFI, the Pa and the AOQ formulas are accurate for all sets of p, i and r values.

7.2. The comparison of the performance measures

Figures 3 and 4 show a comparison of the AFI curves for the GM-F, CSP-F-L and MCSP-F-L plans for all sets of p and i when $r = 4$ and 6, respectively. It is observed that the shape of AFI curves at $r = 4$ and $r = 6$ are similar. For all set of p, i and r values, the AFI values of the GM-F plan are clearly higher than the others. For all sets of r values, at good and moderate quality levels $(p = 0.005, 0.008,$ 0.01 and 0.02), it can be found that the difference of the AFI values between the GM-F plan and the other two plans become go up as the value of i is increased. However, at poor quality levels $(p = 0.05),$ it is observed that the AFI values of GM-F plan are slightly higher than the others when $i = 50.$

Figures 5 and 6 show a comparison of the AOQ curves for the GM-F, CSP-F-L and MCSP-F-L plans for all sets of p and i when $r=4$ and 6, respectively. The principle of AOQ and AFI values will be adverse, then for all set of p,i and r values, it is observed that the feature of AOQ curves are opposite of the AFI curves. The AOQ values of the GM-F plan are clearly lower than the MCSP-F-L and the CSP-F-L plans, and the shape of AOQ curves at $r=4$ and $r=6$ are similar.

Figures 7 and 8 show a comparison of the Pa curves for the GM-F, CSP-F-L and MCSP-F-L plans for all sets of p and i when $r=4$ and 6, respectively. It is observed that the Pa and AOQ curves are similar appearance and the shape of Pa curves at $r=4$ and $r=6$ look alike. The Pa values of the GM-F plan are lower than the other two plans for all sets of p,i and r values. For all levels of r when $p=0.005, 0.008$ and 0.01, the Pa values of the GM-F plan are clearly lower than the MCSP-F-L and the CSP-F-L plans while the value of i is increased, but the Pa values of the GM-F plan are slightly lower than the other plans for the case of moderate and poor quality levels ($p=0.02, 0.04$ and 0.05).

Table 2 The AFI_F, AFI_S and DAFI values of GM-F plan

i	r	$p=0.005$			$p=0.008$			$p=0.01$		
		AFI_F	AFI_S	DAFI	AFI_F	AFI_S	DAFI	AFI_F	AFI_S	DAFI
10	4	0.23103	0.23003	0.00099	0.23818	0.23980	0.00163	0.24304	0.24044	0.00259
	6	0.16153	0.16158	0.00004	0.16688	0.16583	0.00105	0.17051	0.17175	0.00124
20	4	0.24298	0.24124	0.00174	0.25785	0.26063	0.00279	0.26801	0.26630	0.00171
	6	0.17047	0.17028	0.00019	0.18154	0.18126	0.00029	0.18907	0.18955	0.00048
30	4	0.25529	0.25539	0.00010	0.27820	0.28175	0.00356	0.29379	0.29277	0.00102
	6	0.17964	0.17922	0.00043	0.19658	0.19416	0.00241	0.20802	0.20539	0.00264
40	4	0.26788	0.27013	0.00225	0.29892	0.30550	0.00658	0.31988	0.31233	0.00755
	6	0.18897	0.18893	0.00004	0.21178	0.21049	0.00129	0.22719	0.22603	0.00117
50	4	0.28067	0.28204	0.00137	0.31978	0.32568	0.00590	0.34598	0.33871	0.00727
	6	0.19840	0.19766	0.00074	0.22712	0.22743	0.00031	0.24668	0.24543	0.00125
i	r	$p=0.02$			$p=0.04$			$p=0.05$		
		AFI_F	AFI_S	DAFI	AFI_F	AFI_S	DAFI	AFI_F	AFI_S	DAFI
10	4	0.26827	0.26498	0.00329	0.32150	0.32508	0.00358	0.34868	0.34070	0.00798
	6	0.18926	0.18752	0.00174	0.22839	0.22960	0.00121	0.24872	0.24966	0.00094
20	4	0.32041	0.33213	0.01171	0.42688	0.43220	0.00533	0.48046	0.48521	0.00475
	6	0.22759	0.22527	0.00232	0.31121	0.30670	0.00450	0.35801	0.35524	0.00277
30	4	0.37277	0.37839	0.00562	0.53085	0.54393	0.01307	0.60814	0.61211	0.00397
	6	0.26727	0.25787	0.00940	0.40494	0.42001	0.01506	0.48234	0.47537	0.00697
40	4	0.42474	0.42551	0.00077	0.63036	0.62792	0.00245	0.72172	0.72967	0.00795
	6	0.30941	0.30139	0.00801	0.50586	0.51826	0.01241	0.60881	0.62156	0.01276
50	4	0.47645	0.47807	0.00162	0.71958	0.72301	0.00342	0.81245	0.81690	0.00444
	6	0.35440	0.34538	0.00901	0.60627	0.61134	0.00507	0.72217	0.73787	0.01570

Table 3 The AOQ_F, AOQ_S and DAOQ values of GM-F plan

i	r	$p = 0.005$			$p = 0.008$			$p = 0.01$		
		AOQ_F	AOQ_S	DAOQ	AOQ_F	AOQ_S	DAOQ	AOQ_F	AOQ_S	DAOQ
1	4	0.00384	0.00370	0.00015	0.00609	0.00621	0.00011	0.00757	0.00693	0.00064
	6	0.00419	0.00395	0.00024	0.00666	0.00686	0.00020	0.00829	0.00788	0.00041
2	4	0.00379	0.00370	0.00008	0.00594	0.00599	0.00006	0.00732	0.00700	0.00032
	6	0.00415	0.00389	0.00025	0.00655	0.00674	0.00019	0.00811	0.00761	0.00050
3	4	0.00372	0.00356	0.00016	0.00577	0.00593	0.00015	0.00706	0.00660	0.00046
	6	0.00410	0.00394	0.00016	0.00643	0.00663	0.00020	0.00792	0.00767	0.00025
4	4	0.00366	0.00339	0.00027	0.00561	0.00585	0.00024	0.00680	0.00633	0.00047
	6	0.00406	0.00390	0.00016	0.00631	0.00647	0.00016	0.00773	0.00735	0.00038
5	4	0.00360	0.00333	0.00027	0.00544	0.00567	0.00023	0.00654	0.00618	0.00036
	6	0.00401	0.00387	0.00014	0.00618	0.00645	0.00026	0.00753	0.00735	0.00018

i	r	$p = 0.02$			$p = 0.04$			$p = 0.05$		
		AOQ_F	AOQ_S	DAOQ	AOQ_F	AOQ_S	DAOQ	AOQ_F	AOQ_S	DAOQ
1	4	0.01463	0.0148	0.0001	0.02714	0.0265	0.0005	0.03257	0.0322	0.0003
	6	0.01621	0.0159	0.0003	0.03086	0.0311	0.0002	0.03756	0.0355	0.0020
2	4	0.01359	0.0124	0.0011	0.02292	0.0227	0.0001	0.02598	0.0252	0.0006
	6	0.01545	0.0150	0.0004	0.02755	0.0278	0.0002	0.03210	0.0306	0.0014
3	4	0.01254	0.0120	0.0005	0.01877	0.0183	0.0004	0.01959	0.0191	0.0004
	6	0.01465	0.0143	0.0003	0.02380	0.0231	0.0006	0.02588	0.0249	0.0009
4	4	0.01151	0.0109	0.0005	0.01479	0.0149	0.0001	0.01391	0.0134	0.0004
	6	0.01381	0.0134	0.0003	0.01977	0.0192	0.0004	0.01956	0.0180	0.0015
5	4	0.01047	0.0099	0.0004	0.01122	0.0111	0.0001	0.00938	0.0090	0.0002
	6	0.01291	0.0126	0.0002	0.01575	0.0155	0.0002	0.01389	0.0124	0.0014

Table 4 The Pa_F, Pa_S and DPa values of GM-F plan

i	r	$p = 0.005$			$p = 0.008$			$p = 0.01$		
		Pa_F	Pa_S	DPa	Pa_F	Pa_S	DPa	Pa_F	Pa_S	DPa
10	4	0.98871	0.98989	0.00118	0.98162	0.98020	0.00141	0.97676	0.97942	0.00266
	6	0.99210	0.99212	0.00002	0.98712	0.98811	0.00099	0.98370	0.98279	0.00091
20	4	0.97682	0.97872	0.00190	0.96173	0.95923	0.00250	0.95120	0.95361	0.00241
	6	0.98374	0.98380	0.00006	0.97306	0.97313	0.00008	0.96557	0.96480	0.00077
30	4	0.96436	0.96461	0.00025	0.94043	0.93673	0.00370	0.92353	0.92591	0.00238
	6	0.97492	0.97520	0.00028	0.95791	0.95995	0.00204	0.94585	0.94845	0.00260
40	4	0.95133	0.94916	0.00217	0.91786	0.91056	0.00730	0.89411	0.90250	0.00839
	6	0.96567	0.96551	0.00015	0.94181	0.94285	0.00104	0.92479	0.92566	0.00087
50	4	0.93778	0.93652	0.00126	0.89423	0.88816	0.00607	0.86334	0.87148	0.00814
	6	0.95602	0.95633	0.00032	0.92488	0.92480	0.00008	0.90256	0.90409	0.00152

i	r	$p = 0.02$			$p = 0.04$			$p = 0.05$		
		Pa_F	Pa_S	DPa	Pa_F	Pa_S	DPa	Pa_F	Pa_S	DPa
10	4	0.95093	0.95347	0.00255	0.89224	0.88995	0.00230	0.86009	0.86944	0.00936
	6	0.96538	0.96700	0.00162	0.92345	0.92216	0.00129	0.90020	0.89899	0.00120
20	4	0.89350	0.88151	0.01199	0.76180	0.75620	0.00561	0.69178	0.68626	0.00552
	6	0.92435	0.92691	0.00256	0.82635	0.83155	0.00520	0.77033	0.77371	0.00338
30	4	0.83057	0.82532	0.00525	0.62514	0.60810	0.01705	0.52239	0.51709	0.00530
	6	0.87852	0.88933	0.01080	0.71405	0.69597	0.01808	0.62119	0.62956	0.00837
40	4	0.76457	0.76514	0.00057	0.49279	0.49611	0.00332	0.37103	0.36044	0.01059
	6	0.82850	0.83831	0.00981	0.59297	0.57808	0.01489	0.46943	0.45413	0.01531
50	4	0.69706	0.69552	0.00155	0.37388	0.36933	0.00455	0.25006	0.24414	0.00592
	6	0.77466	0.78554	0.01087	0.47248	0.46639	0.00608	0.33340	0.31456	0.01884

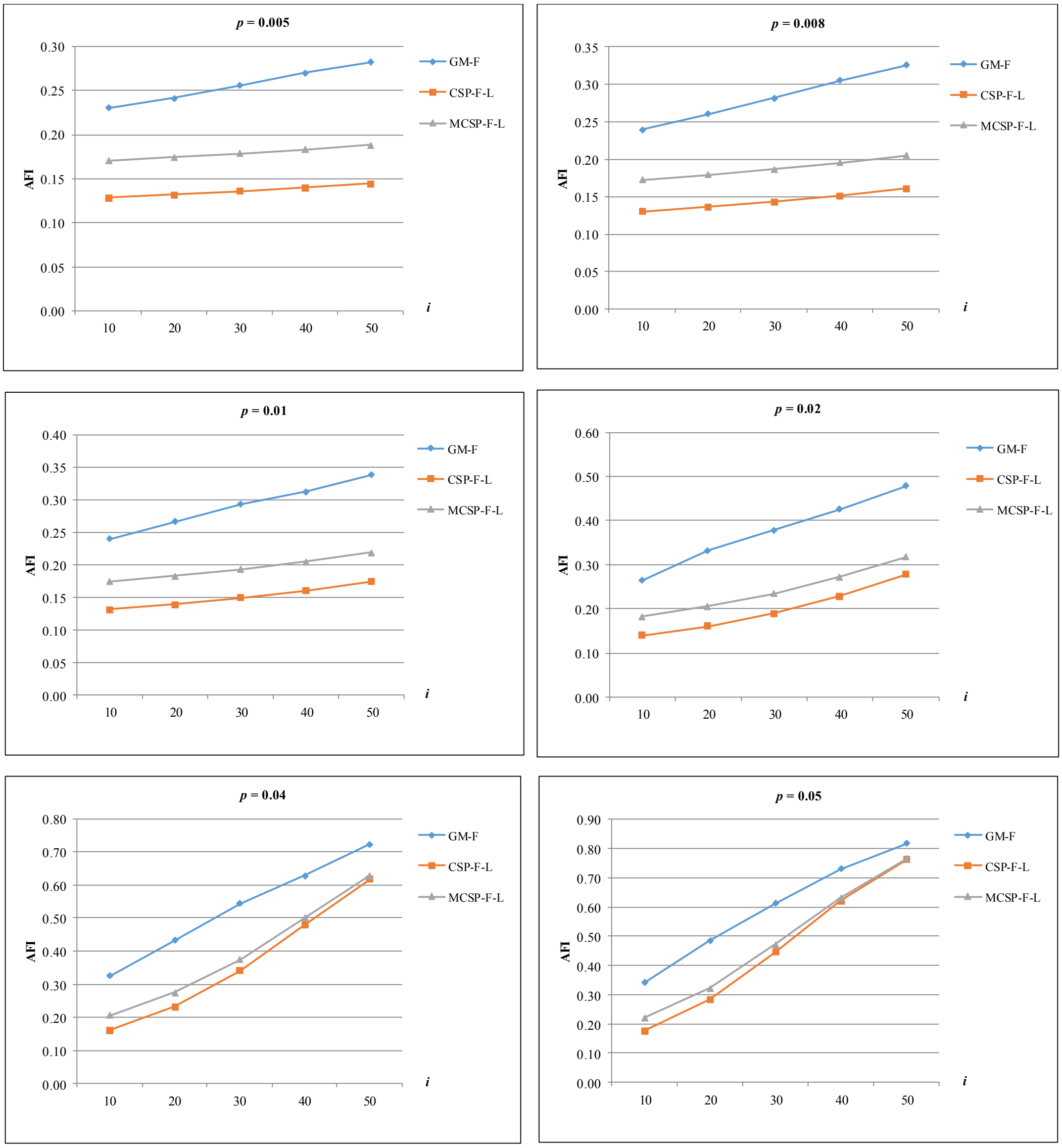

Figure 3 AFI curves of GM-F, CSP-F-L and MCSP-F-L plans for $r = 4$

8. Discussions and Conclusions

In this paper, the GM-F plan for inspection of continuous production line has been developed. A operating procedure of GM-F plan is shown in Figure 1. The formulas have been derived for performance measures using a Markov Chain model such as the average fraction inspected (AFI), the average fraction of total produced accepted on sampling basis (Pa), the average outgoing quality (AOQ) and the average outgoing quality limit (AOQL). The accuracy of the performance measures has been tested by extensive simulations. The difference of the AFI, AOQ and Pa values from the formula and from the simulations were found to agree within target values in all simulations. Extensive simulations have been carried out to compare the AFI, AOQ and Pa values obtained from the GM-F plan with AFI, AOQ and Pa values from the CSP-F-L and the MCSP-F-L plan. The attractive feature of the GM-F plan is that a smaller AOQ and Pa values for all of incoming quality levels (p) and parameters when compared to the CSP-F-L and the MCSP-F-L plan. The differences of the performance measures between the GM-F plan and the two plans are increased as the value of i is increased, especially at low level of p. However, the differences of the performance measures of the three plans are small as the value of p is increased. Figures provided in Section 7.2 are shown a comparison of the performance measures from the three plans will be useful for the recommendation of the operator to use the sampling plan that appropriate to their production line.

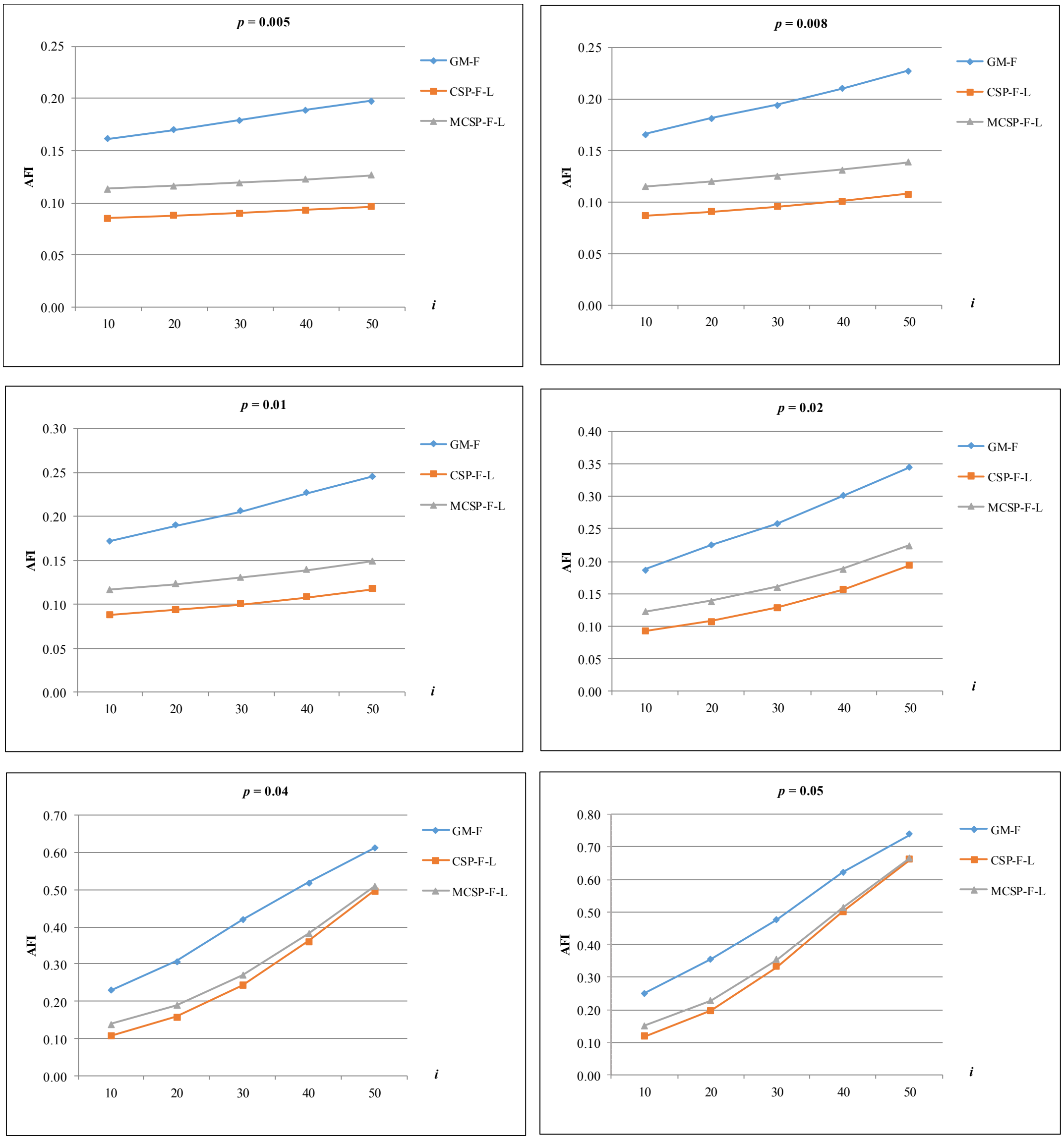

Figure 4 AFI curves of GM-F, CSP-F-L and MCSP-F-L plans for $r = 6$

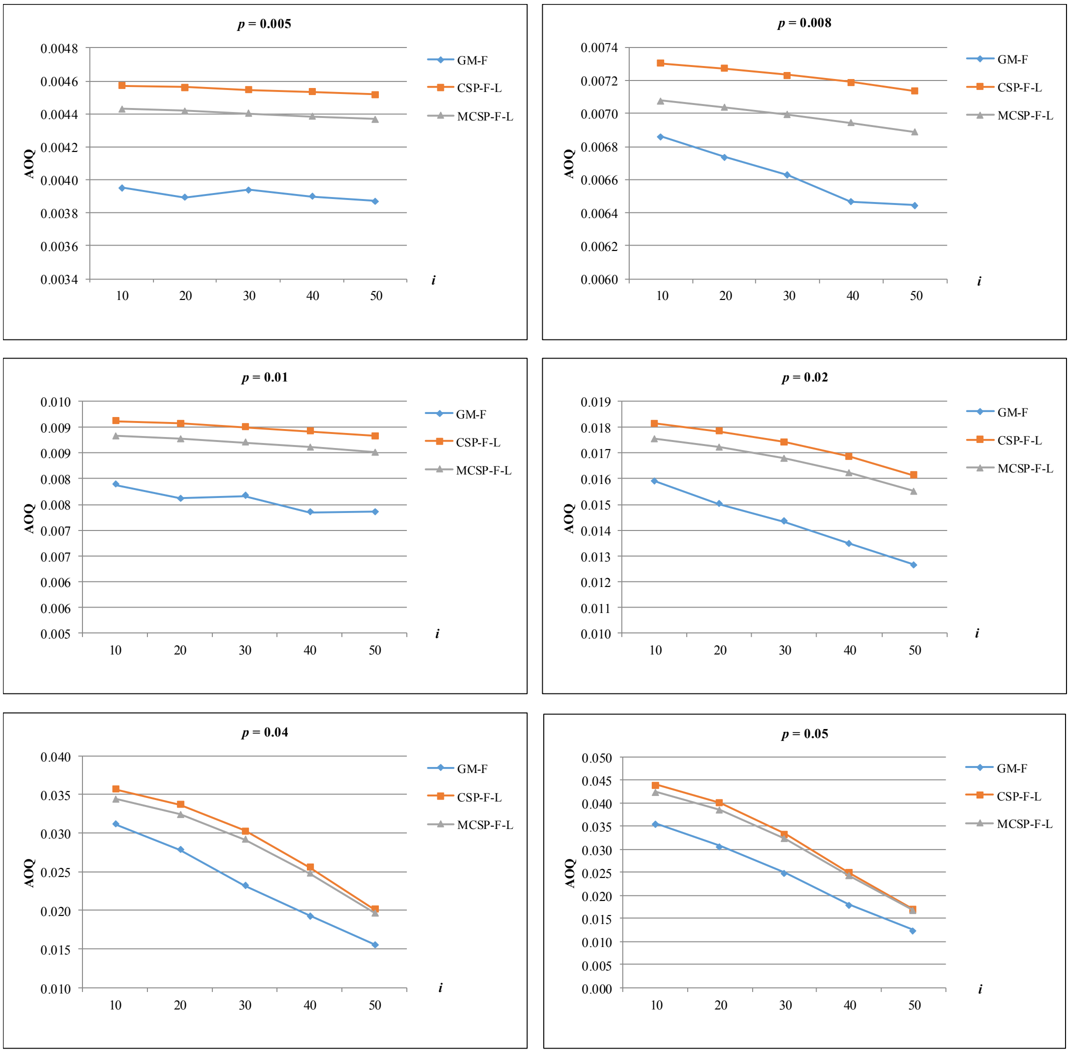

Figure 5 AOQ curves of GM-F, CSP-F-L and MCSP-F-L plans for $r = 4$

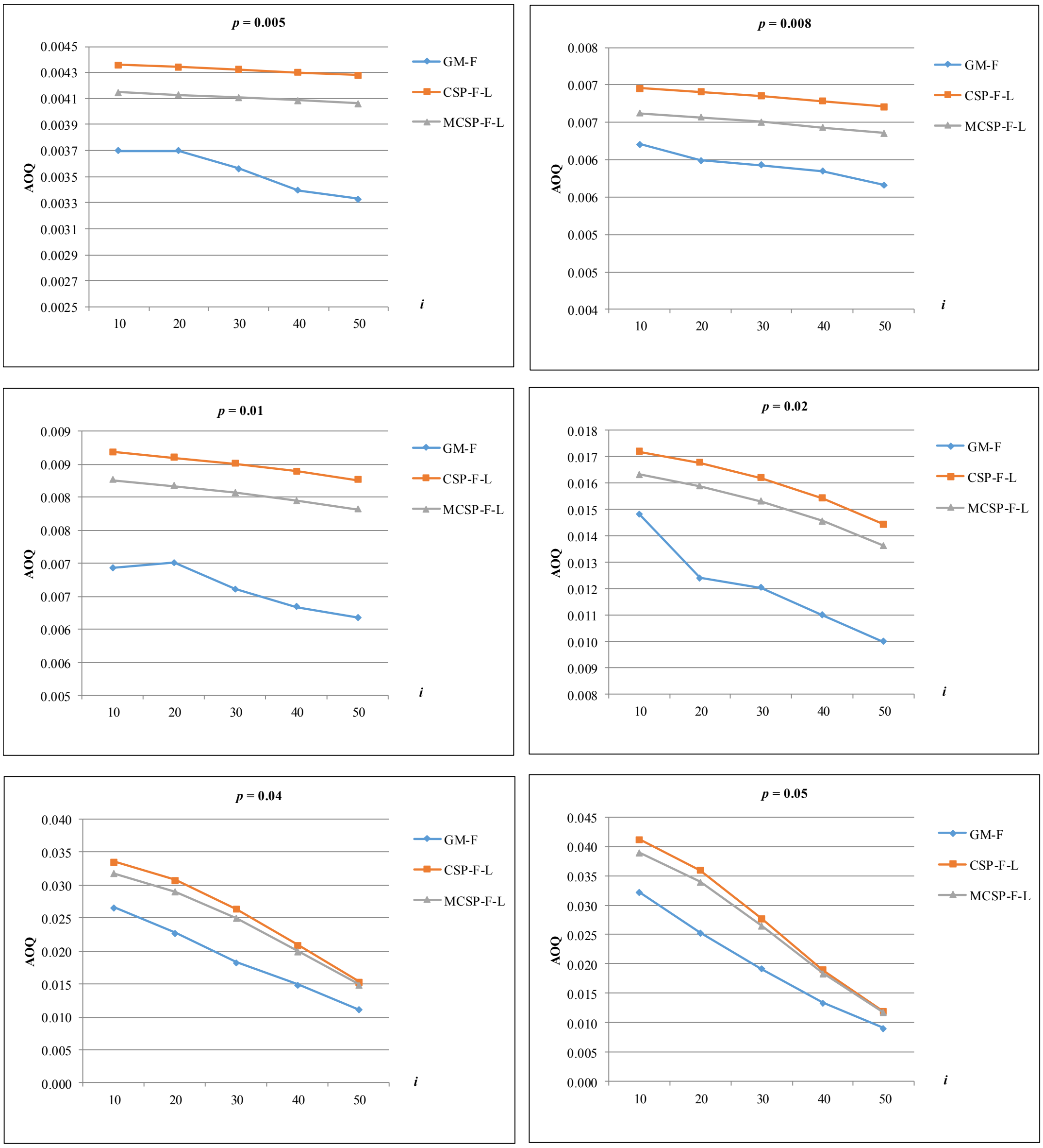

Figure 6 AOQ curves of GM-F, CSP-F-L and MCSP-F-L plans for $r = 6$

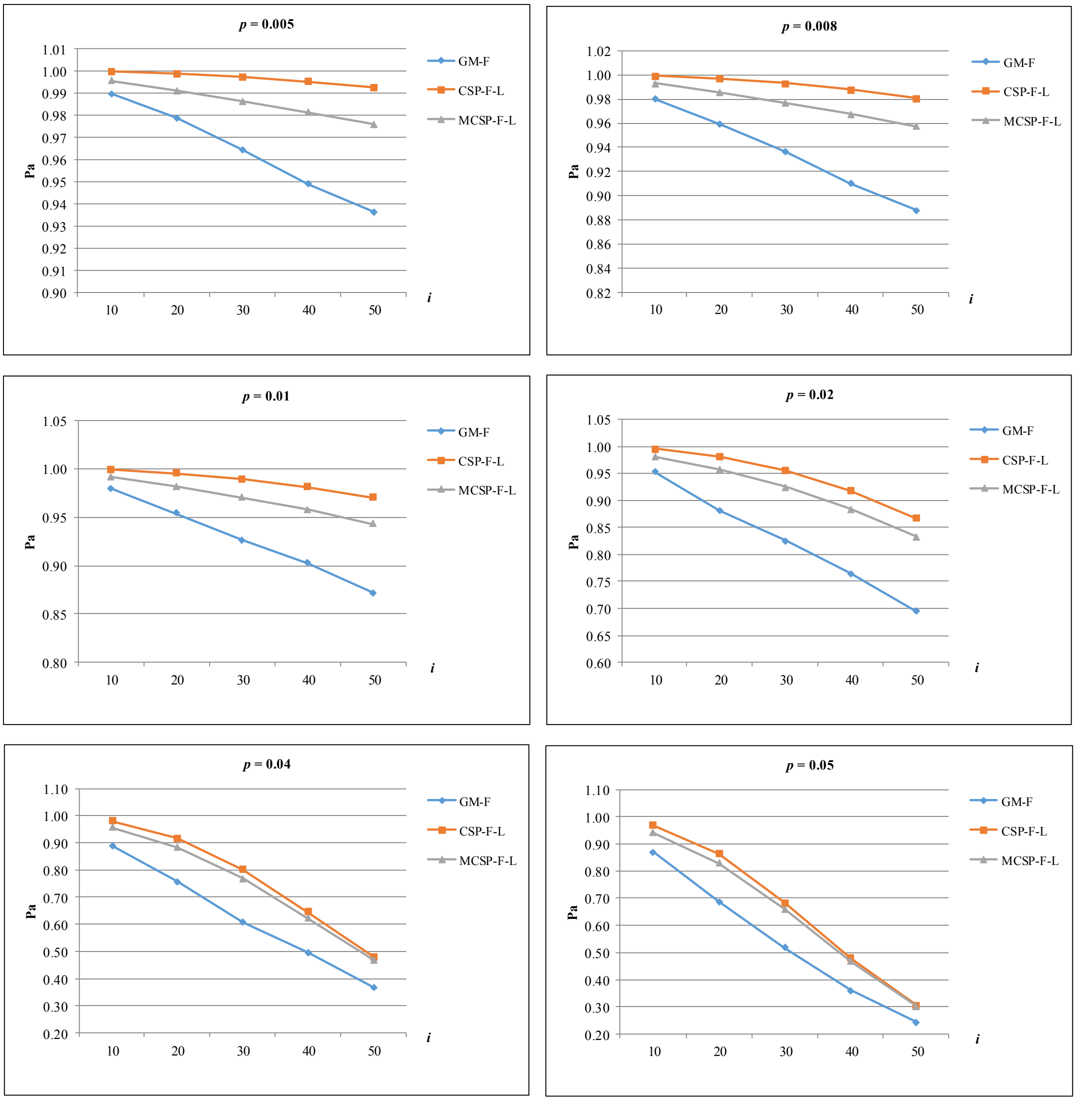

Figure 7 Pa curves of GM-F, CSP-F-L and MCSP-F-L plans for $r = 4$

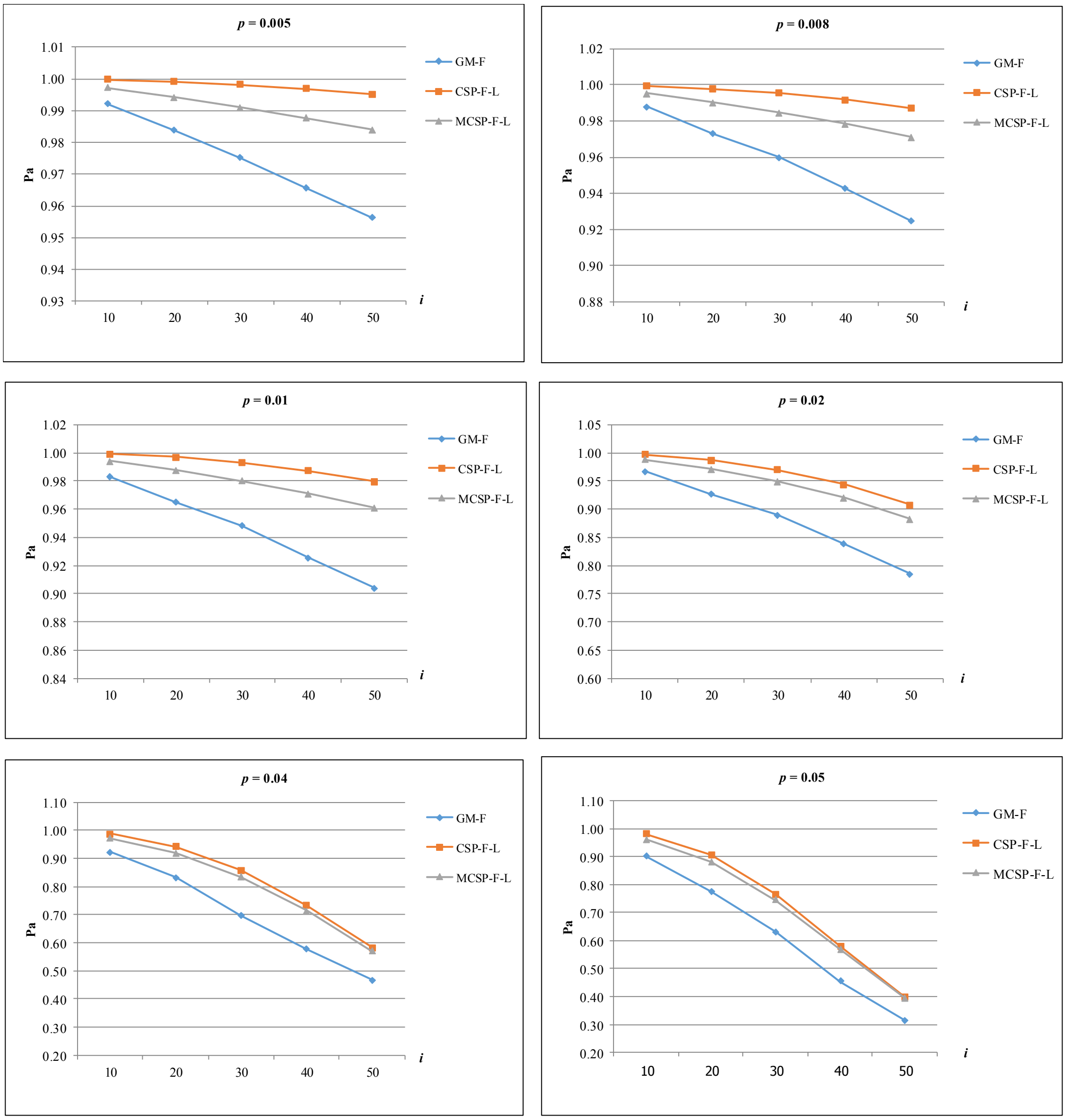

Figure 8 Pa curves of GM-F, CSP-F-L and MCSP-F-L plans for $r = 6$

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Appendix

Glossary of symbols:

S_n = the n^{th} state of the process,

$P(S_n)$ = the steady-state probability for the state S_n,

p_{in} = the probability that the process transits from state S_i to S_n in one step.

Derivation of Performance Measures of the GM-F CSP plan:

The formulation of the GM-F CSP using the Markov Chain development is similar to Stephens (2001). Let $[X_t]$; $t = 1, 2, \ldots$ denote a discrete-parameter Markov Chain with finite state space (S_n) ; $n = 1, 2, \ldots, 3g + i + 3m + 1$. The states of the process are defined, in a way similar to that of Roberts (1965).

These steady-state probabilities $P(S_n)$ satisfy the following conditions:

$$P(S_n) \geq 0 \text{ for } n = 1, 2, \ldots, 3g + i + 3m + 1 \tag{9}$$

$$P(S_n) = \sum_{x=1}^{3g+i+3m+1} P(S) p_{xn} \text{ for } n = 1, 2, \ldots, 3g + i + 3m + 1 \tag{10}$$

$$\sum_{\text{all } n} P(S) = 1. \tag{11}$$

From conditions (10) and (11), we acquire the following:

$$P(f_1N_n) = \frac{q^{n-1}(1-f_1)}{f_1}[P(f_2Id_m)+P(A_i)] \quad ;n=1,2,...,g \tag{12}$$

$$P(f_1In_n) = q^n[P(f_2Id_m)+P(A_i)] \quad ;n=1,2,...,g \tag{13}$$

$$P(f_1Id_n) = pq^{n-1}[P(f_2Id_m)+P(A_i)] \quad ;n=1,2,...,g \tag{14}$$

$$P(A_0) = p[\sum_{n=1}^{g}P(f_1Id_n)+\sum_{n=1}^{m}P(f_2Id_n)+P(A_0)+\sum_{n=1}^{i}P(A_n)] \tag{15}$$

$$P(A_n) = q^n[P(A_0)+P(A_i)] \quad ;n=1,2,...,i \tag{16}$$

$$P(f_2N_n) = \frac{q^{g+n-1}(1-f_2)}{f_2}[P(f_2Id_m)+P(A_i)] \quad ;n=1,2,...,m \tag{17}$$

$$P(f_2In_n) = q^{g+n}[P(f_2Id_m)+P(A_i)] \quad ;n=1,2,...,m \tag{18}$$

$$P(f_2Id_n) = pq^{g+n-1}[P(f_2Id_m)+P(A_i)] \quad ;n=1,2,...,m \tag{19}$$

$$\sum_{n=1}^{g}[P(f_1N_n)+P(f_1In_n)+P(f_1Id_n)]+\sum_{n=0}^{i}P(A_n)+\sum_{n=1}^{m}[P(f_2N_n)+P(f_2In_n)+P(f_2Id_n)] = 1. \tag{20}$$

By Equations (12) to (20), (15) can be written as

$$P(A_0) = \frac{f_1f_2p(1-q^i)(1-q^{g+m})}{D},$$

where $D = f_1q^{i+g}(1-q^m)+f_1f_2(1-q^i)(1-q^{g+m})+f_2q^i(1-q^g)$.

The steady-state probabilities can be written as follows:

$$\mathrm{P(A}_n) = \frac{f_1f_2pq^n(1-q^{g+m})}{D} \quad ;n=1,2,...,i$$

$$\mathrm{P}(f_1\mathrm{N}_n) = \frac{(1-f_1)f_2pq^{i+n-1}}{D} \quad ;n=1,2,...,g$$

$$\mathrm{P}(f_1\mathrm{In}_n) = \frac{f_1f_2pq^{i+n}}{D} \quad ;n=1,2,...,g$$

$$\mathrm{P}(f_1\mathrm{Id}_n) = \frac{f_1f_2p^2q^{i+n-1}}{D} \quad ;n=1,2,...,g$$

$$\mathrm{P}(f_2\mathrm{N}_n) = \frac{(1-f_2)f_1pq^{i+g+n-1}}{D} \quad ;n=1,2,...,m$$

$$\mathrm{P}(f_2\mathrm{In}_n) = \frac{f_1f_2pq^{i+g+n}}{D} \quad ;n=1,2,...,m$$

$$\mathrm{P}(f_2\mathrm{Id}_n) = \frac{f_1f_2p^2q^{i+g+n-1}}{D} \quad ;n=1,2,...,m$$

then

$$u = \frac{\sum_{n=0}^{i}P(A_n)}{P(f_1In_g)+P(A_i)-\sum_{n=1}^{m}P(f_2Id_n)}$$

$$v = \frac{\sum_{n=1}^{g}[P(f_1N_n)+P(f_1In_n)+P(f_1Id_n)]+\sum_{n=1}^{m}[P(f_2N_n)+P(f_2In_n)+P(f_2Id_n)]}{P(f_1In_g)+P(A_i)-\sum_{n=1}^{m}P(f_2Id_n)}$$

$$AFI = 1-\sum_{n=1}^{g}P(f_1N_n)-\sum_{n=1}^{m}P(f_2N_n)$$

$$\text{Pa} = 1-\sum_{n=0}^{i}P(A_n)$$

$$\text{AOQ} = p[\sum_{n=1}^{g}P(f_1N_n)+\sum_{n=1}^{m}P(f_2N_n)].$$

By simplifying the above equations, we can get the performance measures of a GM-F CSP plan which are given in (3) to (5).