Thailand Statistician
July 2021; 19(3): 536-546
http://statassoc.or.th
Contributed paper

Power Exponentiated Family of Distributions with Application on Two Real-Life Datasets

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Received: 1 August 2019
Revised: 9 November 2019
Accepted: 24 February 2020

Abstract

In this paper we introduce new power exponentiated family of continuous univariate probability distributions with application on two real-life datasets. The proposed distribution possesses density function with three parameters and constant hazard rate function. We studied the nature of distribution with the help of its mathematical and statistical properties. Probability density function of order statistics for this distribution is also obtained. We perform classical estimation of parameters by using the technique of maximum likelihood estimate. Application of the model on two real data sets is finally presented and compared to the fit attained by some other well-known distributions.

Keywords: Exponential distribution, moments, Renyi entropy, order statistics, maximum likelihood estimation.

1. Introduction

In recent years, an impressive set of new statistical distributions has been explored by statisticians. The need to generate new distributions arise either due to theoretical considerations or practical applications or both. The necessity to develop an extended class of classical distributions is even more in areas such as survival data analysis, insurance, finance and risk modelling, modelling weather data etc. so as to increase its flexibility to acquire high degree of skewness and kurtosis. A considerable progress has been made towards the generalization of some well-known distributions and their successful application to problems in these areas.

The modifications in the classical distribution has been proved useful in exploring tail properties and also for improving the goodness-of-fit of the family under study. Recently many researchers are working upon this area and have proposed new methods to develop improved probability distributions with utility. For instance, we can refer, method of skew distributions by Azzalini (1985), method of adding parameters to an existing distribution by Mudholkar and Srivastava (1993) and Marshall and Olkin (1997), beta-generated method by Eugene et al. (2002), transformed-transformer method by Alzaatreh et al. (2013), composite method by Cooray and Ananda (2005). Length-biased weighted Maxwell distribution by Modi and Gill (2015). Inverse probability integral transformation method by Ferreira and Steel (2006), compounding approach by Barreto-Souza et al. (2011), alpha logarithmic transformed (αLT) method by Pappas et al. (2012). Distribution of product and ratio of random variables approach by Modi and Joshi (2012). Logarithmic transformed method was given by Maurya et al. (2016).

The aim of this paper is to discuss some properties of the power exponentiated exponential distribution. These include the shapes of the density and hazard rate functions, the moments and some associated measures and the limiting distributions of order statistics. Maximum likelihood estimators of the model parameter are derived. Further the efficient fitting of data with power exponentiated exponential distribution is also shown over the other well-known classical distributions. The following lemmas will also be needed to complete the derivations.

Lemma 1 *From Equation (1.110) of Gradshteyn and Ryzhik (2007, p.25), if* α *is a positive real non integer and* $|x| \le 1,$ *then by binomial series expansion we have*

$$(1-x)^{\alpha-1} = \sum_{j=0}^{\infty} (-1)^j \binom{\alpha-1}{j} x^j .$$

Lemma 2 *From Equation (1.211.2) of Gradshteyn and Ryzhik (2007, p.26),*

$$a^x = \sum_{k=0}^{\infty} \frac{(x \ln a)^k}{k!},$$

where a *and* x *are any real numbers.*

Lemma 3 *From Equation (3.312.1) of Gradshteyn and Ryzhik (2007, p.335), if* $\operatorname{Re}\beta > 0,$ $\operatorname{Re}\nu > 0$ *and* $\operatorname{Re}\mu > 0,$

$$\int_0^{\infty} \left(1-e^{-\frac{x}{\beta}}\right)^{\nu-1} e^{-\mu x} dx = \beta B\left(\beta\mu,\nu\right),$$

where $B\left(\beta\mu,\nu\right) = \dfrac{\Gamma\left(\beta\mu\right)\Gamma\nu}{\Gamma\left(\beta\mu+\nu\right)}.$

Lemma 4 *From Equation (3.381.4) of Gradshteyn and Ryzhik (2007, p.346), for* $\operatorname{Re} p > 0$ *and* $\operatorname{Re} c > 0,$

$$\int_0^{\infty} x^{c-1} e^{-px} dx = \frac{\Gamma(c)}{p^c}.$$

2. Power Exponentiated Family

We introduce a new power exponentiated family of probability distributions to model lifetime data or survival data. The cdf $F(x)$ and pdf $f(x)$ of power exponentiated family are, respectively, given by

$$F(x) = \frac{\nu^{\{G(x)\}^{\beta}} - 1}{\nu - 1}, \tag{1}$$

and

$$f\left(x\right) = \frac{\beta \nu^{(G(x))^{\beta}} \ln \nu \left(G(x)\right)^{\beta-1} g(x)}{\nu - 1}, \quad x > 0, \nu > 0, \beta > 0. \tag{2}$$

Using Lemma 2, we get

$$f(x)=\frac{\beta\ln\nu}{\nu-1}\sum_{i=0}^{\infty}\frac{(\ln\nu)^{i}}{i!}\left(G(x)\right)^{\beta i+\beta-1}g(x).$$

3. Exponential Distribution

The exponential distribution is a well-known distribution and has its importance in study of growth, lifetime data, etc. It's modification and generalization in the form of exponentiated exponential (EE) distribution and generalized exponential distribution (GED) had been given by different mathematicians and statisticians. A continuous random variable X has Exponential distribution, if its cdf $G(x)$ and pdf $g(x)$ are, respectively, given by

$$G(x)=1-e^{-px} \tag{3}$$

and

$$g(x)=pe^{-px},\ \ x>0,p>0. \tag{4}$$

4. CDF and PDF of Power Exponentiated Exponential Distribution

For the power exponentiated family using the cdf and pdf defined in (3) and (4) respectively, we propose a new power exponentiated exponential distribution. Thus, the cdf of the power exponentiated exponential distribution with β and ν as shape parameters and p as scale parameter, can be defined as

$$F(x)=\frac{\nu^{\{G(x)\}^{\beta}}-1}{\nu-1}, \tag{5}$$

and its corresponding pdf is given by

$$f(x)=\frac{\beta p\nu^{\{1-e^{-px}\}^{\beta}}\ln\nu\left(1-e^{-px}\right)^{\beta-1}e^{-px}}{\nu-1},\ \ x>0,\nu>0,\beta>0,p>0. \tag{6}$$

Using Lemma 2 and then using Lemma 1, we obtain

$$f(x)=\frac{\beta p\ln\nu}{\nu-1}\sum_{i=0}^{\infty}\frac{(\ln\nu)^{i}}{i!}(1-e^{-px})^{\beta i+\beta-1}e^{-px}=\frac{\beta p\ln\nu}{\nu-1}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^{j}(\ln\nu)^{i}}{i!}\binom{\beta i+\beta-1}{j}e^{-px(1+j)}. \tag{7}$$

5. Hazard Rate Function and Survival Function

The hazard rate function given by $h(x)=\dfrac{f(x)}{1-F(x)}$ is an important measure for characterizing life phenomenon. It measures the conditional probability of a failure given the system is currently working. For the cdf and pdf defined in (5) and (6) respectively, $h(x)$ takes the form

$$h(x)=\frac{\beta p\ln\nu.\nu^{\{1-e^{-px}\}^{\beta}}(1-e^{-px})^{\beta-1}e^{-px}}{\nu-\nu^{(1-e^{-px})^{\beta}}}, \tag{8}$$

and its survival function is given by

$$S(x)=1-F(x)=1-\frac{\nu^{(1-e^{-px})^{\beta}}-1}{\nu-1}=\frac{\nu-\nu^{\{1-e^{-px}\}^{\beta}}}{\nu-1}. \tag{9}$$

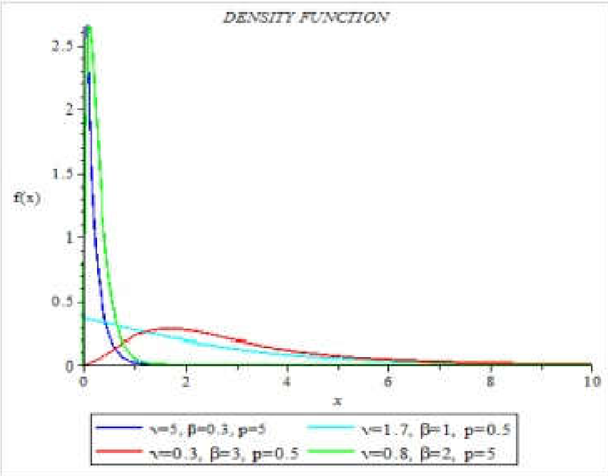

Figure 1 Graph of density function of power exponentiated exponential distribution for different combination of values of its parameters ν, p and β

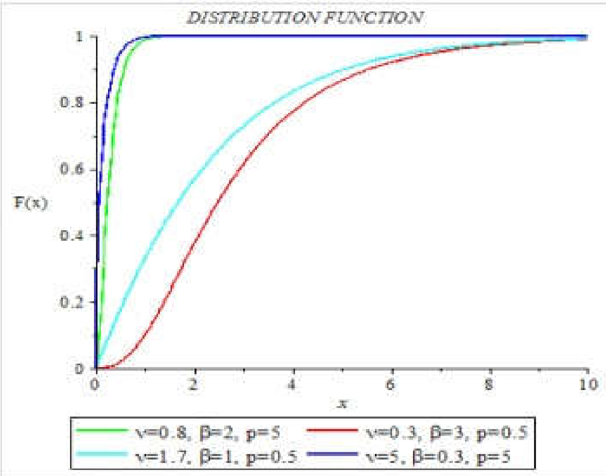

Figure 2 Graph of distribution function of power exponentiated exponential distribution for different combination of values of its parameters ν, p and β

6. Mode and Median

If a random variable X has the pdf given by (6), then the corresponding mode is given by $f'(x) = 0$.

$$f(x) = \frac{\beta p \ln \nu \; \nu^{(1-e^{-px})^{\beta}} (1-e^{-px})^{\beta-1} e^{-px}}{\nu - 1},$$

$$f'(x) = \frac{\beta p \ln \nu \; \nu^{(1-e^{-px})^{\beta}} (1-e^{-px})^{\beta-2} e^{-px} \left(-p + p\beta e^{-px} + \left(1-e^{-px}\right)\ln \nu\right)}{\nu - 1} = 0,$$

$$\Rightarrow -p + p\beta e^{-px} + \left(1-e^{-px}\right)\ln \nu = 0$$

$$\Rightarrow Mode = x = \frac{1}{p}\ln\left(\frac{p\beta - \ln \nu}{p - \ln \nu}\right). \tag{10}$$

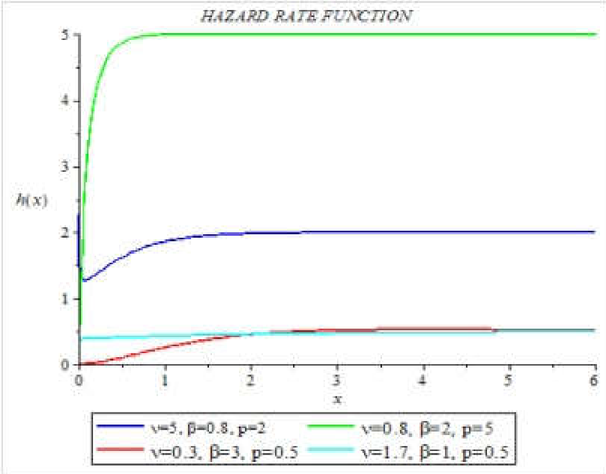

Figure 3 Graph of hazard rate function of power exponentiated exponential distribution for different combination of values of its parameters ν, p and β

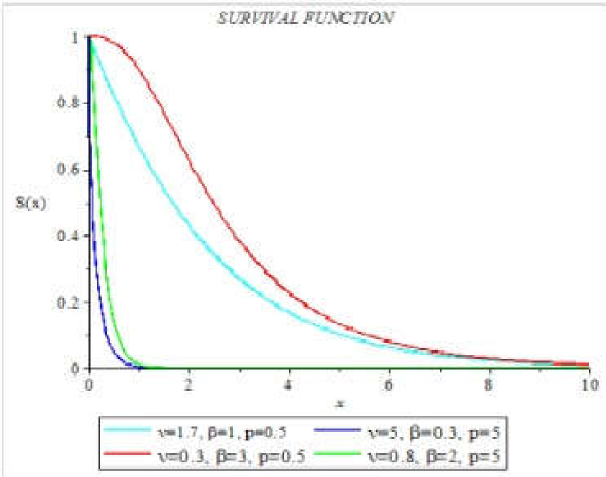

Figure 4 Graph of survival function of power exponentiated exponential distribution for different combination of values of its parameters ν, p and β

Median is obtained by

$$\int_0^m f(x)dx = \frac{1}{2}$$

$$\frac{\beta p \ln \nu}{\nu - 1} \int_0^m \nu^{(1-e^{-px})^\beta} (1-e^{-px})^{\beta-1} e^{-px} dx = \frac{1}{2}$$

$$\int_0^m \nu^{(1-e^{-px})^\beta} (1-e^{-px})^{\beta-1} e^{-px} dx = \frac{\nu - 1}{2\beta p \ln \nu}.$$

Put $(1-e^{-px})^\beta = t \quad \Rightarrow \beta p (1-e^{-px})^{\beta-1} e^{-px} dx = dt$

$$\int_0^{\left(1-e^{-pm}\right)^{\beta}} \nu^t dt = \frac{\nu-1}{2\beta^2 p^2 \ln \nu}$$

$$\left[\nu^{\left(1-e^{-pm}\right)^{\beta}} - 1\right] = \frac{\nu-1}{2\beta^2 p^2}$$

$$(1-e^{-pm})^{\beta} = \frac{\ln\left(\nu - 1 + 2\beta^2 p^2\right) - \ln(2\beta^2 p^2)}{\ln \nu}$$

$$e^{-pm} = 1 - \left[\frac{\ln\left(\nu - 1 + 2\beta^2 p^2\right) - \ln(2\beta^2 p^2)}{\ln \nu}\right]^{1/\beta}. \tag{11}$$

Thus, median for proposed distribution can be obtained by solving above equation form.

7. Moments

If a random variable X has the pdf given by (7), then the corresponding r^{th} moment is given by

$$\mu_r' = E(X^r) = \int_0^{\infty} x^r f(x) dx$$

$$= \frac{\beta p \ln \nu}{\nu - 1} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{(\ln \nu)^i (-1)^j}{i!} \binom{\beta i + \beta - 1}{j} \int_0^{\infty} x^r e^{-px(1+j)} dx.$$

Using Lemma 4, we get

$$\mu_r' = \frac{\beta p \ln \nu}{\nu - 1} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{(\ln \nu)^i (-1)^j}{i!} \binom{\beta i + \beta - 1}{j} \frac{\Gamma(r+1)}{\left[\lambda(1+j)\right]^{(r+1)}}. \tag{12}$$

8. Moment Generating Function

If a random variable X has the pdf given by (7), then the corresponding r^{th} moment is given by

$$E(e^{tX}) = \int_0^{\infty} e^{tx} f(x) dx$$

$$= \frac{\beta p \ln \nu}{\nu - 1} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{(\ln \nu)^i (-1)^j}{i!} \binom{\beta i + \beta - 1}{j} \int_0^{\infty} e^{-x(p+pj-t)} dx.$$

Solving integral, we obtain

$$E(e^{tX}) = \frac{\beta p \ln \nu}{\nu - 1} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{(\ln \nu)^i (-1)^j}{i!} \binom{\beta i + \beta - 1}{j} \frac{1}{(p + pj - t)}. \tag{13}$$

9. Renyi Entropy

The variation of the uncertainty is measured by the entropy of a random variable. The Renyi entropy defined by Renyi (1961) as

$$E_X(\nu) = \frac{1}{(1-\nu)} \ln\left(\int_{-\infty}^{\infty} \left(f_X(x)\right)^{\nu} dx\right), \nu > 0, \nu \neq 1.$$

Using pdf defined in equation (6), we get

$$E_X(\nu)=\frac{1}{(1-\nu)}\ln\left(\int_0^\infty\left(\frac{\beta p\ln\nu\;\nu^{(1-e^{-px})^\beta}(1-e^{-px})^{\beta-1}e^{-px}}{\nu-1}\right)^\nu dx\right)$$
$$=\frac{1}{(1-\nu)}\ln\left(\left(\frac{\beta p\ln\nu}{\nu-1}\right)^\nu\int_0^\infty\nu^{\nu(1-e^{-px})^\beta}(1-e^{-px})^{\nu(\beta-1)}e^{-\nu px}dx\right)$$
$$=\frac{1}{(1-\nu)}\ln\left(\left(\frac{\beta p\ln\nu}{\nu-1}\right)^\nu\sum_{i=0}^\infty\frac{(\ln\nu)^i}{i!}\int_0^\infty\nu^i\left(1-e^{-px}\right)^{\nu(\beta-1)+i\beta}e^{-\nu px}dx\right).$$

Using Lemma 3,

$$E_X(\nu)=\frac{1}{(1-\nu)}\ln\left(\left(\frac{\beta p\ln\nu}{\nu-1}\right)^\nu\sum_{i=0}^\infty\frac{\nu^i(\ln\nu)^i}{i!\,p}B(\nu,\nu-\nu\beta+i\beta+1)\right). \tag{14}$$

10. Order Statistics

In this section, we derive closed form expression for the pdf of the i^{th} order statistic of the power exponentiated exponential distribution. Let $X_1,\ldots,X_n$ be a simple random sample from power exponentiated exponential distribution with cdf and pdf given by (5) and (6), respectively. Let $X_{(1:n)}\le X_{(2:n)}\le\ldots\le X_{(n:n)}$ denote the order statistics obtained from this sample. We now give the probability density function of $X_{r:n}$, say $f_{r:n}(x)$ of $X_{r:n}$, $i=1,2,\ldots,n.$ The probability density function of the r^{th} order statistics $X_{r:n}$, $r=1,2,\ldots,n$ given by (David 1981)

$$f_{r:n}(x)=C_{r:n}\left[F(x;\alpha,\beta,p)\right]^{r-1}\left[1-F(x;\alpha,\beta,p)\right]^{n-r}f(x;\alpha,\beta,p),\quad x>0,$$

where $F(\cdot)$ and $f(\cdot)$ are given by (5) and (6) respectively, and $C_{r:n}=\dfrac{n!}{(r-1)!(n-r)!}.$

Thus, using binomial expansion given in Lemma 1, we get

$$f_{r:n}(x)=C_{r:n}\sum_{s=0}^\infty(-1)^s\binom{n-r}{s}\left[F(x;\nu,\beta,p)\right]^{r+s-1}f(x;\nu,\beta,p)$$
$$=\frac{C_{r:n}\beta p\ln\nu}{\nu-1}\sum_{s=0}^\infty(-1)^s\binom{n-r}{s}\left[\frac{\nu^{(1-e^{-px})^\beta}-1}{\nu-1}\right]^{(r+s-1)}\nu^{(1-e^{-px})^\beta}(1-e^{-px})^{\beta-1}e^{-px}$$
$$=\frac{\beta p\ln\nu}{\nu-1}C_{r:n}\sum_{s=0}^\infty(-1)^{r+2s-1}\binom{n-r}{s}\frac{1}{(\nu-1)^{(r+s-1)}}\left[1-\nu^{(1-e^{-px})^\beta}\right]^{(r+s-1)}\nu^{(1-e^{-px})^\beta}(1-e^{-px})^{\beta-1}e^{-px}$$
$$=\beta p\ln\nu C_{r:n}\sum_{s=0}^\infty\sum_{q=0}^\infty(-1)^{r+q+2s-1}\binom{n-r}{s}\binom{r+s-1}{q}\frac{1}{(\nu-1)^{(r+s)}}\nu^{(1-e^{-px})^{\beta(q+1)}}(1-e^{-px})^{\beta-1}e^{-px}. \tag{15}$$

11. Maximum Likelihood Estimators

Let X is a random variable having the pdf of power exponentiated exponential distribution defined as

$$f(x)=\frac{\beta p\nu^{(1-e^{-px})^\beta}\ln\nu(1-e^{-px})^{\beta-1}e^{-px}}{\nu-1}.$$

Then its log-likelihood function is given by

$$L(x;\nu,\beta,p)=n\ln p+n\ln\beta+n\ln(\ln\nu)-n\ln(\nu-1)-p\sum_{i=0}^{\infty}x_i+(\beta-1)\sum_{i=0}^{\infty}\ln(1-e^{-px_i})$$
$$+\ln\nu\sum_{i=0}^{\infty}(1-e^{-px_i})^{\beta}. \tag{16}$$

Thus, the non-linear normal equations are given as follows:

$$\frac{\partial L(x;\nu,\beta,p)}{\partial p}=\frac{n}{p}-\sum_{i=0}^{\infty}x_i+(\beta-1)\sum_{i=0}^{\infty}\frac{x_ie^{-px_i}}{1-e^{-px_i}}+\beta\ln\nu\sum_{i=0}^{\infty}(1-e^{-px_i})^{\beta-1}x_ie^{-px_i}, \tag{17}$$

$$\frac{\partial L(x;\nu,\beta,p)}{\partial \nu}=\frac{n}{\nu\ln\nu}-\frac{n}{\nu-1}+\sum_{i=0}^{\infty}\frac{(1-e^{-px_i})^{\beta}}{\nu}, \tag{18}$$

$$\frac{\partial L(x;\nu,\beta,p)}{\partial \beta}=\frac{n}{\beta}+\ln\nu\sum_{i=0}^{\infty}(1-e^{-px_i})^{\beta}\ln(1-e^{-px_i})+\sum_{i=0}^{\infty}\ln(1-e^{-px_i}). \tag{19}$$

We can estimate of the unknown parameter by the method of maximum likelihood by setting these above non-linear equation (17)-(19) to zero and solving them simultaneously.

12. Application to Real Life Data

In this section, the proposed power exponentiated exponential distribution is applied on two real data sets. We observe its flexibility over some well-known existing distributions. The results for the analysis in this present study are obtained using R software. We have also calculated the Akaike Information Criteria (AIC) and p-value for the considered distributions to observe their fit. Meanwhile, the distribution with the highest log-likelihood value or the lowest AIC is considered the best. The pdf of the distributions taken are as follows.

Power exponentiated exponential distribution:

$$f(x)=\frac{\beta p\nu^{(1-e^{-px})^{\beta}}\ln\nu(1-e^{-px})^{\beta-1}e^{-px}}{\nu-1},$$

Log-logistic distribution:

$$f(x)=\frac{\beta\left(\frac{x}{\alpha}\right)^{\beta-1}}{\alpha\left(1+\left(\frac{x}{\alpha}\right)^{\beta}\right)^{2}},$$

Exponentiated exponential distribution:

$$f(x)=\alpha\theta(1-e^{-\theta x})^{\alpha-1}e^{-\theta x},$$

Exponentiated gamma distribution:

$$f(x)=\theta\lambda^{2}xe^{-\lambda x}\left[1-e^{-\lambda x}(\lambda x+1)\right]^{\theta-1}$$

Let us assume the hypothesis at α =1% LOS,

H_0 : The data follow the power exponentiated exponential distribution

H_1 : The data do not follow the power exponentiated exponential distribution

Data set I: The data set is obtained from Smith and Naylor (1987). The data are the strengths of 1.5 cm glass fibres, measured at the National Physical Laboratory England. The data set is: 0.55, 0.93, 1.25, 1.36, 1.49, 1.52, 1.58, 1.61, 1.64, 1.68, 1.73, 1.81, 2.00, 0.74, 1.04, 1.27, 1.39, 1.49, 1.53, 1.59, 1.61, 1.66, 1.68, 1.76, 1.82, 2.01, 0.77, 1.11, 1.28, 1.42, 1.50, 1.54, 1.6, 1.62, 1.66, 1.69, 1.76, 1.84, 2.24, 0.81, 1.13, 1.29, 1.48, 1.50, 1.55, 1.61, 1.62, 1.66, 1.70, 1.77, 1.84, 0.84, 1.24, 1.30, 1.48, 1.51, 1.55, 1.61, 1.63, 1.67, 1.70, 1.78, 1.89.

Data set II: In this example we use uncensored data set from Nichols and Padgett (2006). The data gives 100 observations on breaking stress of carbon fibres (in Gba):
3.70, 2.74, 2.73, 2.50, 3.6, 3.11, 3.27, 2.87, 1.47, 3.11, 4.42, 2.41, 3.19, 3.22, 1.69, 3.28, 3.09, 1.87, 3.15, 4.9, 3.75, 2.43, 2.95, 2.97, 3.39, 2.96, 2.53, 2.67, 2.93, 3.22, 3.39, 2.81, 4.20, 3.33, 2.55, 3.31, 3.31, 2.85, 2.56, 3.56, 3.15, 2.35, 2.55, 2.59, 2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36, 0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19, 1.57, 0.81, 5.56, 1.73, 1.59, 2.00, 1.22, 1.12, 1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69, 1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.70, 2.03, 1.80, 1.57, 1.08, 2.03, 1.61, 2.12, 1.89, 2.88 ,2.82, 2.05, 3.65.

Table 1 The MLEs of the power exponentiated exponential distribution parameters and AIC value for data set 1

Distributions	Estimates	Log-likelihood	D	p-value	AIC
Power exponentiated exponential distribution	$\nu = 14.732890$ $\beta = 23.421452$ $p = 3.004398$	−26.26765	0.20207	0.01166	58.53530
Exponentiated gamma distribution	$\theta = 12.905928$ $\lambda = 03.202199$	−30.08129	0.22878	0.00274	64.16258
Exponentiated exponential distribution	$\alpha = 26.781075$ $\theta = 2.485061$	−31.52905	0.21806	0.00500	67.05810
Log exponential distribution	$\alpha = 0.455181$	−97.02823	0.41165	1.067e-09	194.05646

Table 2 The MLEs of the power exponentiated exponential distribution parameters and AIC value for data set 2

Distributions	Estimates	Log-likelihood	D	p-value	AIC
Power exponentiated exponential distribution	$\nu = 16.893490$ $\beta = 5.583429$ $p = 1.246344$	−142.9224	0.09026	0.38910	291.84480
Log-logistic distribution	$\alpha = 2.498696$ $\theta = 4.118730$	−146.2795	0.09016	0.39050	296.55900
Exponentiated gamma distribution	$\theta = 3.468442$ $\lambda = 2.485061$	−145.0898	0.83509	2.2e-16	294.17960
Exponentiated exponential distribution	$\alpha = 7.826723$ $\theta = 1.015015$	−146.1826	0.10767	0.19660	296.36520

From Table1 and Table 2, the power exponentiated exponential distribution has the highest log-likelihood value from other two parameter distributions and lowest AIC value, thus making it to be fitted better than the log-logistic distribution, exponentiated gamma distribution and exponentiated exponential distribution. The AIC can be calculated using AIC = $-2\log_e L + 2k$, where $\log_e L$ denotes the log-likelihood function calculated at the maximum likelihood estimates, k is the number of parameters. Since p-value $> \alpha$, we cannot reject the null hypothesis and hence assume that data follows power exponentiated exponential distribution.

13. Conclusions

In this paper, we propose the cdf and pdf of power exponentiated family in Section 2. Further the expressions for the cdf and pdf of power exponentiated exponential distribution with two shape and one scale are derived in Section 4. We have considered the mathematical and statistical properties for the derived distribution. From the graphs drawn for pdf of derived distribution, we observe that derived distribution is unimodal and positively skewed. The hazard rate function, survival function and their graphs for new distribution are given in Section 5. Derived distribution has constant hazard rate function, as depicted in graphs. The expressions for finding mode and median are given in Section 6. The expression for its r^{th} moment of derived distribution is given in (12). We have also derived the expressions for the Renyi entropy and the pdf of its r^{th} order statistics in (14) and (15), respectively. The method of MLE to estimate its parameters is discussed in Section 11. Moreover, the derived distribution is applied on two real data sets and compared with the other well-known distributions in Section 12. Results show that the power exponentiated exponential distribution provides a better fit than other well-known distributions.

Acknowledgments

The author is grateful to the reviewers and editor for carefully reading the manuscript and for offering substantial suggestions for article improvement.

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