Thailand Statistician
July 2021; 19(3): 565-582
http://statassoc.or.th
Contributed paper

The Effect on Forecasting Accuracy of the Holt-Winters Method When Using the Incorrect Model on a Non-Stationary Time Series

Chantha Wongoutong
Department of Statistics, Faculty of Science, Kasetsart University, Bangkok, Thailand.
Corresponding author; email: fscictw@ku.ac.th

Received: 15 April 2020
Revised: 20 June 2020
Accepted: 22 July 2020

Abstract

The Holt-Winters method is one of the most popular forecasting techniques for time series, particularly with trend and seasonal components. There are two variations of the Holt-Winters method depending on the nature or type of the seasonal component: additive and multiplicative, and the type of seasonality is required to select the appropriate one. Unfortunately, time-series data are sometimes ambiguous, which can lead to incorrect identification of the model resulting in erroneous predicted values. In this study, the effect on forecasting accuracy when using the incorrect seasonal model in the Holt-Winters method was considered. Ten simulated datasets, five of which contained additive seasonality and the other five multiplicative seasonality, were used to study the effect of using the incorrect model on the forecasting accuracy. Five real datasets, in which it was difficult to distinguish the type of seasonal component, were used in the experimental study. Each dataset was examined using both additive and multiplicative models while varying the three smoothing parameters of the Holt-Winters method from 0.1 to 1 in increments of 0.1. The forecasting accuracy was evaluated in terms of the mean-absolute-percentage error and the root-mean-squared error. The results confirm the significance of the correct identification of the type of seasonality. For the ambiguous time-series data in which identifying whether to apply the additive or multiplicative model is not simple, the results show that utilizing the multiplicative model achieved significantly higher accuracy than utilizing the additive model.

Keywords: Forecasting method, seasonality, additive model, multiplicative model.

1. Introduction

Time series forecasting can be applied to a wide range of disciplines, such as in the business, economic, social science, biomedical, and engineering fields. When analyzing a time series, one searches for structures and patterns to describe and explain the underlying process and based on fitting adequate models, to forecast future values or to predict results from alternative scenarios (Chatfield and Yar 1988). Thus, there have been many attempts at obtaining the most accurate forecast for a given time-series model. The Holt-Winters method (Holt 1957, and Winters 1960) is one of the most popular and effective approaches for forecasting a time series, particularly when trends and seasonality exist (Brockwell et al. 1991, and Chatfield 1978). This method belongs to a class of exponential smoothing methods that aims to capture the behavior of a time series by identifying trends, seasonality, and error terms. There are two variations of the Holt-Winters method depending on the nature or type of the seasonal component: additive and multiplicative (Montgomery et al. 2008), and many empirical studies have previously been undertaken to show forecasting performances of these two methods. Goodwin (2010) confirmed the Holt-Winters approach to exponential smoothing, with it then being 50 years old and still going strong. Many older studies have reported that the Holt-Winters method often performs well in actual applications (Groff 1973, Huss 1985, Makridakis and Hibon 1979, and Makridakis et al. 1984). Likewise, it has been applied more recently to create forecasting models in various study areas, such as clinical, finance, economics, energy industry, tourism, and climate (Cuicui and Jun 2012, Dantas et al. 2017, Jere et al. 2019, Linden 2018, Rahman et al. 2016, Tirkes et al. 2017, Valakevicius and Brazenas 2015, and Wu et al. 2017).

For time-series data with a trend and exhibiting an explicit type of seasonality, accurate forecasts can be obtained by using the additive or multiplicative Holt-Winters methods accordingly. However, in a situation where the seasonality of the time-series data is ambiguous, both the additive and multiplicative forms can be used to build the forecasting model. For example, Heydari et al. (2020) built models using both the additive and multiplicative procedures and discovered that the multiplicative form of the Holt-Winters time-series method resulted in 4% less mean-absolute-percentage error (MAPE) overall compared to the additive one. Similarly, Wei and Song (2013) showed that volatility is best predicted by a simplified version of the multiplicative Holt-Winters model. Furthermore, Da Fonseca et al. (2016) used the methodology of Holt-Winters without a trend, or with linear or exponential trends, and without seasonality, or with additive or multiplicative seasonality. After applying all of the Holt-Winters model variations, they found that the best model was the one with a linear trend series and multiplicative seasonality. Likewise, Bermúdez et al. (2006), and Kuznets (1932), concluded that for most economic time series, the seasonal variation appears to be proportional to the level of the time series and that the multiplicative version usually works better than the additive one.

Despite the use of the Holt-Winters method is widespread, the effect of using the incorrect model has not yet been clarified. Hence, there is a strong need for further study in this area. In this study, the effect of forecasting accuracy when using the incorrect model by the Holt-Winters method is concentrated on. Both a simulation study and real datasets were used to investigate the effect of using the incorrect model on forecasting accuracy.

The remaining parts of this paper are as follows. The theoretical framework is covered in Section 2. The datasets of this study are presented in Section 3. The experimental study is reported in Section 4. The results and discussion are provided in Section 5. Finally, conclusions is presented in Section 6.

2. Theoretical Framework

In the Holt-Winters method, three exponential smoothing formulas, collectively called triple exponential smoothing (Holt 1957; Winters 1960), are applied to the series. First, the mean is smoothed to give a local average value for the series; second, the trend is smoothed; and last, each seasonal sub-series is smoothed separately to give a seasonal estimate for each season. The exponential smoothing formula can be applied to a series with a trend and constant seasonal component using the additive and multiplicative methods (Box et al. 1994). The additive method is used when the seasonal variations are roughly constant throughout the series, while the multiplicative method is used when the seasonal variation changes proportionally to the level of the series and the pattern of the magnitude of the seasonal variation in the data depends on the magnitude of the data used. The pattern of the magnitude of the seasonal in data does not depend on the magnitude of data used, the additive model.

2.1. The multiplicative Holt-Winters method

This method can be used to handle many complex seasonal patterns by simply finding the central value, then adding in the effects of slope and seasonality. It is based on three updating equations for level, trend, and seasonality, respectively:

$$L_t = \alpha \frac{Y_t}{S_{t-L}} + (1-\alpha)(L_{t-1} + T_{t-1}),$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1},$$

$$S_t = \gamma \frac{Y_t}{L_t} + (1-\gamma)S_{t-L},$$

where L is the seasonality length, L_t is the overall smoothing, T_t is trend smoothing, S_t is seasonal smoothing, and Y_t refers to the real data at the time period t. The smoothing parameters α, β and γ are set from 0 to 1. These parameters are estimated in such a way that the mean-squared error (MSE) is minimized. Meanwhile, the forecast can be obtained from

$$Y_{t+m} = (L_t + T_t m)S_{t-L+m},$$

where Y_{t+m} is the forecast for the period ahead (m). The multiplicative Holt-Winters method requires three initial values: level (L_0), trend (T_0), and seasonality $(S_{01}, S_{02}, ..., S_{0L})$, which is obtained from

$$L_0 = \frac{Y_1 + Y_2 + Y_3 + ... + Y_L}{L},$$

$$T_0 = \frac{1}{L}\left(\frac{Y_{L+1} - Y_1}{L} + \frac{Y_{L+2} - Y_2}{L} + ... + \frac{Y_{L+L} - Y_L}{L}\right),$$

$$S_{01} = \frac{Y_1}{L_0}, S_{02} = \frac{Y_2}{L_0}, ..., S_{0L} = \frac{Y_L}{L_0}.$$

The initial level is the average of the first year of data. The initial trend is set as the average of the slopes for each period in the first two years. The initial seasonality is computed by dividing each data item in the first year by the initial level.

2.2. The additive Holt-Winters method

This method is identical to the multiplicative model except that the seasonality is considered to be additive. This means that the forecasted value for each data element is the sum of the baseline, trend, and seasonality components. The sum of the seasonality components for L consecutive time periods is approximately L (not 1 as in the multiplicative model). The recursive approach to the additive model is based on three updating equations for level, trend, and seasonality, respectively:

$$L_t = \alpha(Y_t - S_{t-L}) + (1-\alpha)(L_{t-1} + T_{t-1}),$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1},$$

$$S_t = \gamma(Y_t - L_t) + (1-\gamma)S_{t-L},$$

where L is the seasonality length, L_t is the overall smoothing, T_t is trend smoothing, S_t is seasonal smoothing, and Y_t refers to the real data at time period t. The smoothing parameters α, β and γ are set from 0 to 1. These parameters are estimated in such a way that the MSE is minimized. Meanwhile, the forecast can be obtained from

$$Y_{t+m} = L_t + T_t m + S_{t-L+m},$$

where Y_{t+m} is the forecast for the period ahead (m). The additive Holt-Winters method requires three initial values: level (L_0), trend (T_0; similar to multiplicative), and seasonality $(S_{01}, S_{02}, ..., S_{0L})$, respectively obtained from

$$S_{01} = Y_1 - L_0, S_{02} = Y_2 - L_0, ..., S_{0L} = Y_L - L_0.$$

The initial seasonality is computed by subtracting each data item in the first year from the initial level.

2.3. Performance metrics

Evaluating the performance of forecast methods is achieved by comparing the actual and predicted values. A typical approach is to use specific criteria to measure the error of the predicted value, the performance of which is assessed based on the closeness of the predicted and actual values. Two criteria error measurements were used in this study: MAPE and the root-mean-squared error (RMSE), which are respectively defined as follows:

$$MAPE = \frac{\sum_{t=1}^{n}\left(\left|Y_t - \hat{Y}_t\right| / Y_t\right)}{n} \times 100, \ RMSE = \sqrt{\frac{\sum_{t=1}^{n}\left(Y_t - \hat{Y}_t\right)^2}{n}},$$

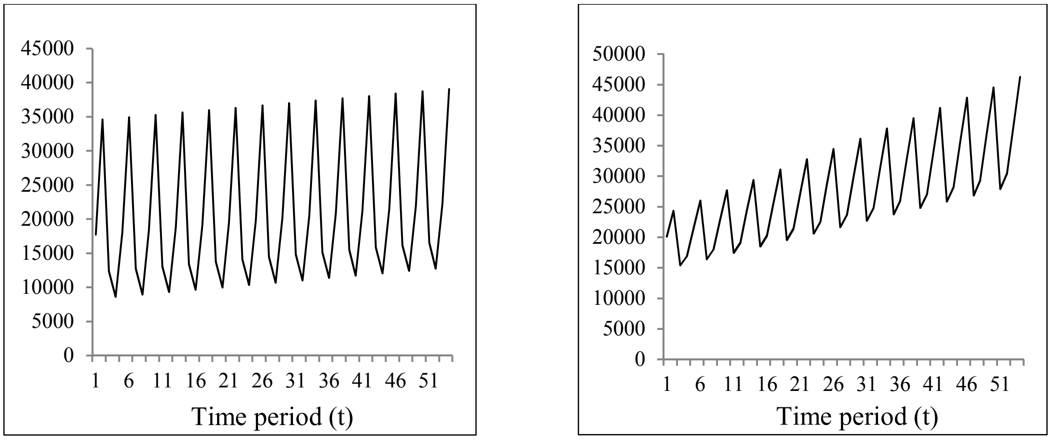
where Y_t is the true value, $\hat{Y}_t$ is the predicted value, $Y_t - \hat{Y}$ gives the forecasting error, and n is the number of forecasting errors.

3. Datasets

Both simulated and real datasets were used in this study. There were five simulated datasets with additive seasonality (A1–A5) and five with multiplicative seasonality (M1–M5). Details of all 10 simulated datasets are reported in Table 1, and time-series plots for some of them are presented in Figure 1.

Table 1. Details of the 10 simulated datasets used in the study

Seasonality	Dataset	Simulation Model with $\varepsilon_t \sim N(0,1)$	S1	S2	S3	S4	Size
Additive	A1	$18{,}246 + 335.3 \times t + S_i + \varepsilon_t$	1,688.8	7,878.73	−5,430.45	−4,137.08	54
	A2	$225.29 + 2.8656 \times t + S_i + \varepsilon_t$	6.4469	−39.7906	−24.5781	57.9219	72
	A3	$161.05 + 5.56 \times t + S_i + \varepsilon_t$	−17.9547	0.9203	36.4766	-19.4422	64
	A4	$469{,}978 + 4{,}179 \times t + S_i + \varepsilon_t$	129,140	−137,739	−105,091	113,690	49
	A5	$7{,}596 - 70.87 \times t + S_i + \varepsilon_t$	−80.36	1196.2	-1426.92	311.08	54
Multiplicative	M1	$(18{,}408 + 329.4 \times t) \times S_i + \varepsilon_t$	1.0738	1.2781	0.7922	0.8560	54
	M2	$(1{,}519.93 + 50.011 \times t) \times S_i + \varepsilon_t$	0.9811	1.7417	0.7416	0.5356	54
	M3	$(573.13 + 52.677 \times t) \times S_i + \varepsilon_t$	1.6154	0.9730	0.4709	0.9407	49
	M4	$(7{,}584 - 70.48 \times t) \times S_i + \varepsilon_t$	0.9855	1.1990	0.7597	1.0558	54
	M5	$(46{,}9794 + 4{,}167 \times t) \times S_i + \varepsilon_t$	1.2183	0.7721	0.8160	1.1936	49



(a) A1 with additive seasonality (b) M1 with multiplicative seasonality

Figure 1 Time-series plots for some of the simulated datasets

Five real datasets (TS1–TS5) with ambiguous seasonality (i.e., it was difficult to determine whether their models should be additive or multiplicative) were used in this study. Dataset TS1 was obtained from Data Market (https://datamarket.com), datasets TS2, TS3, and TS5 from Statistics of New Zealand Information Centre (https://www.stats.govt.nz), and dataset TS4 from RPubs (https://rpubs.com). A brief description of the datasets is provided in Table 2.

Table 2 Description of the real datasets used in the study

Dataset	Description	Time Period	Size
TS1	Quarterly electricity production in Australia: million kilowatts	1956Q1–1965Q4	40
TS2	Quarterly total visitor arrivals in New Zealand	2000Q1–2012Q1	49
TS3	Quarterly motel occupancy rates in New Zealand	1996Q3–2012Q3	65
TS4	Quarterly beer sales data in the US	2000Q1–2017Q4	72
TS5	Quarterly number of visitors to the UK	1998Q4–2012Q1	54

Both additive and multiplicative models were applied to all of the datasets used in this study to determine the forecasting accuracy.

4. Experimental Study

The smoothing parameters applied for each dataset ranged from 0 to 1 in increments of 0.1 (i.e. $\alpha = \{0.1, 0.2, ..., 1\}$, $\beta = \{0.1, 0.2, ..., 1\}$ and $\gamma = \{0.1, 0.2, ..., 1\}$), thus there were 1,000 settings in total (Table 3).

The performance with each dataset was determined by computing the MAPE and RMSE values for each set of parameters. The flow chart in Figure 2 illustrates the steps conducted in the experimental study using the R program version 3.5.2 (The R Foundation 2020).

Table 3 Settings for the Holt-Winters model smoothing parameters

Setting	α	β	γ
1	0.1	0.1	0.1
2	0.1	0.1	0.2
3	0.1	0.1	0.3
⋮	⋮	⋮	⋮
998	1.0	1.0	0.8
999	1.0	1.0	0.9
1,000	1.0	1.0	1.0

5. Results and Discussion

5.1. The simulated datasets

1,000 settings by varying the smoothing parameter (α,β,γ) were performed on each dataset using the additive and multiplicative Holt-Winters methods. Their performances were evaluated by comparing the average MAPE and RMSE values for each dataset.

Table 4 reports the average MAPE and RMSE values for the additive seasonality datasets (A1–A5) using both Holt-Winters models. As an example, dataset A1 has additive seasonality and applying the correct (additive) model produced average MAPE and RMSE values of 0.396 and 106.750 compared to 3.992 and 1,382.266, respectively, using the incorrect (multiplicative) model. The results for the other additive datasets are similar. Bar charts of the average MAPE values obtained by the additive and multiplicative Holt-Winters methods for datasets A1–A5 are shown in Figure 3. The results very clearly show that the additive Holt-Winters method performed much better than the multiplicative one for all five datasets. Furthermore, the number of times that each method achieved the lowest of MAPE and RMSE for the same set of conditions (α,β,γ) are summarized in Table 5. Using A1 as an example, the additive and multiplicative Holt-Winters methods achieved the lowest MAPE and RMSE values 1,000 and 0 times, respectively. This trend was the same for the other datasets. Thus, using the correct (additive) Holt-Winters method on the datasets with additive seasonality significantly outperformed the multiplicative one.

Table 4 Average MAPE and RMSE values for the simulated datasets with additive seasonality (A1–A5)

Simulated Dataset	Correct Model		Incorrect Model	
	Additive Holt-Winters		Multiplicative Holt-Winters	
	Average MAPE	Average RMSE	Average MAPE	Average RMSE
A1	0.396	106.750	3.992	1,382.266
A2	0.829	3.461	3.119	17.758
A3	1.650	6.936	4.544	25.223
A4	0.619	5,298.182	3.432	29,623.685
A5	0.956	88.098	4.486	319.347

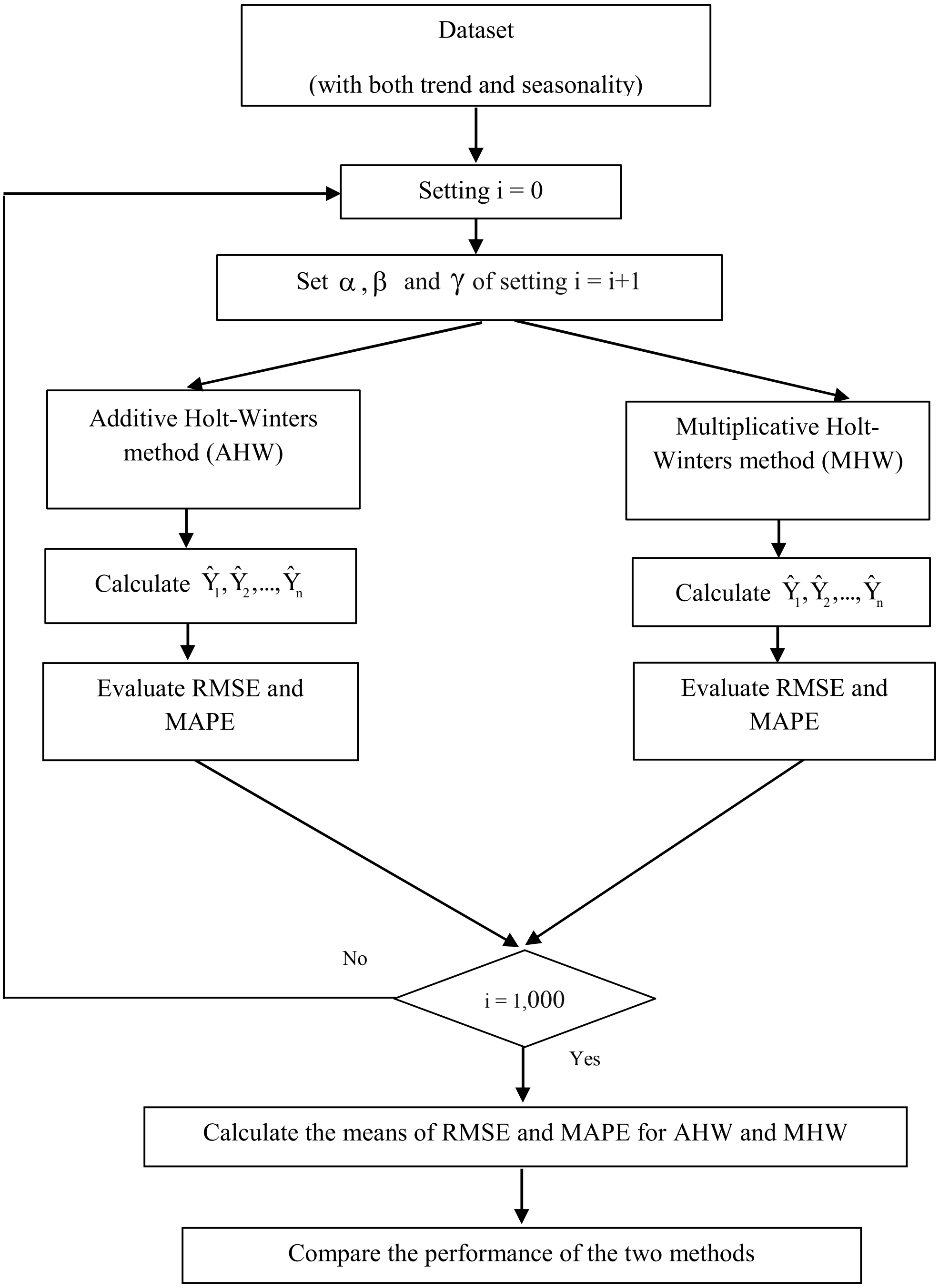


Figure 2 Flow chart of the experimental study

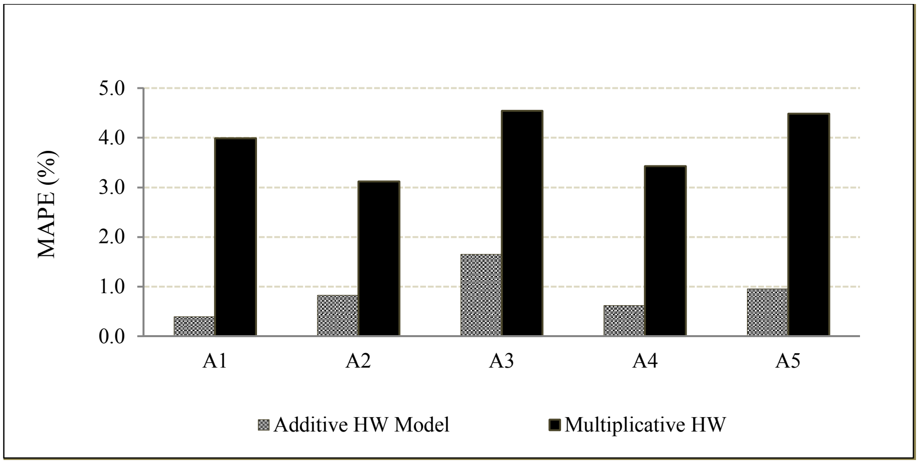


Figure 3 Average MAPE values for datasets A1–A5 with additive seasonality

Table 5 Number of lowest MAPE and RMSE occurrences for datasets A1–A5 with additive seasonality

Simulated Dataset	Correct Model		Incorrect Model	
	Additive Holt-Winters		Multiplicative Holt-Winters	
	Number of lowest MAPE	Number of lowest RMSE	Number of lowest MAPE	Number of lowest RMSE
A1	1,000	1,000	0	0
A2	1,000	1,000	0	0
A3	1,000	1,000	0	0
A4	1,000	1,000	0	0
A5	1,000	1,000	0	0

Table 6 reports the average MAPE and RMSE values for the simulated datasets with multiplicative seasonality (M1–M5) using both Holt-Winters models. As an example, the correct (multiplicative) Holt-Winters model for dataset M1 produced average MAPE and RMSE values of 1.308 and 571.685 compared to 4.987 and 1,974.994 using the additive Holt-Winters model, respectively. Bar charts of the average MAPE values obtained with the additive and multiplicative Holt-Winters methods for datasets M1–M5 are shown in Figure 4. The results indicate very clearly that the multiplicative Holt-Winters method performed much better than the additive one for all datasets. Moreover, 1,000 sets of conditions for M1 to M5, the number of times that each method achieved the lowest MAPE and RMSE values for the same (α, β, γ) values are shown in Table 7. As an example using M1, the additive and multiplicative Holt-Winters methods achieved the lowest MAPE and RMSE values 0 and 1,000 times, respectively, as was found for the other datasets. These results indicate that using the correct (multiplicative) model when the type of seasonality is multiplicative significantly outperformed the additive Holt-Winters method.

Table 6 Average MAPE and RMSE values for datasets M1–M5 with multiplicative seasonality

Simulated Dataset	Incorrect Model		Correct Model	
	Additive Holt-Winters		Multiplicative Holt-Winters	
	Average MAPE	Average RMSE	Average MAPE	Average RMSE
M1	4.987	1,974.994	1.308	571.685
M2	16.264	671.553	2.313	111.241
M3	22.712	407.226	7.157	173.141
M4	4.647	361.940	0.755	75.437
M5	2.811	22,373.727	0.697	6,664.612

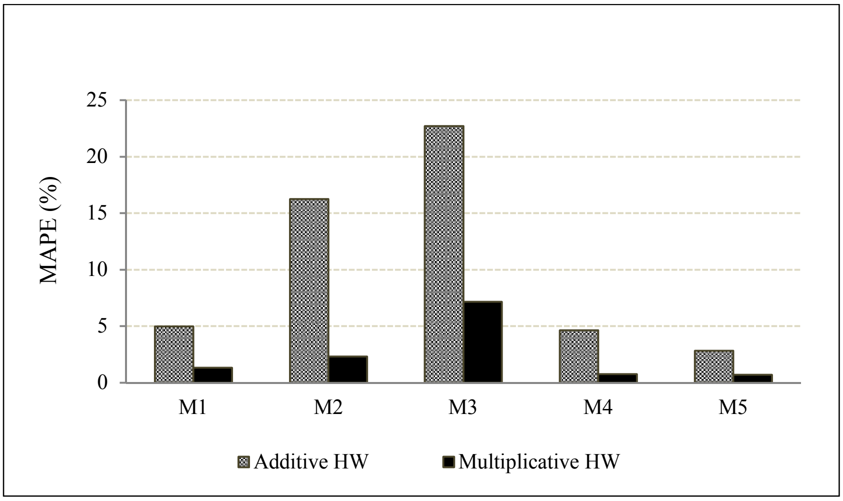


Figure 4 Average MAPE values for datasets M1–M5 with multiplicative seasonality

Table 7 Number of lowest MAPE and RMSE occurrences for datasets M1–M5 with multiplicative seasonality

Simulated Dataset	Incorrect Model		Correct Model	
	Additive Holt-Winters		Multiplicative Holt-Winters	
	Number of lowest	Number of lowest	Number of lowest	Number of lowest
M1	0	0	1,000	1,000
M2	0	0	1,000	1,000
M3	0	0	1,000	1,000
M4	0	0	1,000	1,000
M5	0	0	1,000	1,000

The smoothing parameters α, β and γ were varied from 0 to 1 ($\alpha = \{0.1, 0.2, ..., 1\}$, $\beta = \{0.1, 0.2, ..., 1\}$ and $\gamma = \{0.1, 0.2, ..., 1\}$), thus $(\alpha, \beta, \gamma) = (0.1, 0.1, 0.1)$, $(0.1, 0.1, 0.2)$, …, $(1,1,1)$ for both Holt-Winters methods. Figure 5 shows plots of the MAPE values with the additive and multiplicative Holt-Winters methods versus the sets of conditions for A1–A5 and M1–M5. When A1-A5 had the same set of (α, β, γ) conditions, the MAPE values for the additive Holt-Winters method were lower than those of the multiplicative Holt-Winters method, as evidenced by the MAPE plots for the additive Holt-Winters method staying under those for the multiplicative Holt-Winters method for all of the datasets. These results are consistent with the other findings and lead to the same conclusion that the additive Holt-Winters method performed better than the multiplicative Holt-Winters method under these conditions. On the other hand, when M1–M5 had the same set of (α, β, γ) conditions, the MAPE values for the multiplicative Holt-Winters method were lower than those of the additive Holt-Winters method, as illustrated by the MAPE plots for the multiplicative Holt-Winters method remaining under those of the additive Holt-Winters method for all of the datasets. These results lead to the same conclusion that the multiplicative Holt-Winters method performed better than the additive Holt-Winters method under these conditions.

Two null hypotheses, the difference between the medians of the MAPE (RMSE) values of the additive and multiplicative Holt-Winters methods on the simulated datasets at the 5% significance level, were tested by using the Mann-Whitney U test. The results in Table 8 indicated that all of the datasets (A1–A5 and M1–M5) were the difference between the median MAPE values of the additive and multiplicative Holt-Winters methods at the 5% significance level. However, the 95% confidence intervals (CIs) were all positive for A1–A5 and negative for M1–M5. For A1–A5 can be interpreted as when the lower and upper limits are positive values, the MAPE median in the first group (multiplicative Holt-Winters) is significantly higher than that of the second group (additive Holt-Winters). This result leads to the conclusion that the additive Holt-Winters method significantly outperformed the multiplicative one when the type of seasonality was additive. Likewise, for M1-M5 can be interpreted as when the lower and upper limits are negative values, the MAPE median in the first group (multiplicative Holt-Winters) is significantly lower than that of the second group (additive Holt-Winters). This result leads to the conclusion that the multiplicative Holt-Winters method significantly outperformed the additive one when the type of seasonality was multiplicative. The results in Table 9 using the average RMSE in the analysis can be interpreted similarly to those in Table 8. These findings support the conclusion that the additive Holt-Winters method is significantly more effective than the multiplicative one when the type of seasonality is additive and vice versa when the type of seasonality is multiplicative.

5.2. The real datasets

Each experiment was performed with 1,000 settings of (α, β, γ) on each dataset using the additive and multiplicative Holt-Winters methods. Their performances were compared as the average MAPE and RMSE values for each dataset (Table 10). As an illustration, dataset TS1 produced average MAPE and RMSE values of 2.8107 and 226.4925, and 1.8481 and 142.0190, with the additive and multiplicative Holt-Winters methods, respectively. The results of TS2 to TS5 were similar. Bar charts of the average MAPE values obtained by the additive and multiplicative Holt-Winters methods for datasets TS1 to TS5 are shown in Figure 6. The results show that the multiplicative Holt-Winters method performed much better than the additive one for all of the datasets. Furthermore, the number of times that each method achieved the lowest MAPE and RMSE values for the same set of conditions (α, β, γ) are reported in Table 11. For example, the additive and multiplicative Holt-Winters methods achieved the lowest MAPE value 14 and 986 times, and the lowest RMSE value 0, and 1,000 times for TS1, respectively, the other datasets can be explained similarly. Overall, the additive and multiplicative Holt-Winters methods achieved the lowest MAPE value 103.2 and 896.8 times, and the lowest RMSE value 55 and 945 times, respectively. These results support that when the seasonality pattern is ambiguous, using the multiplicative Holt-Winters method is the best choice.

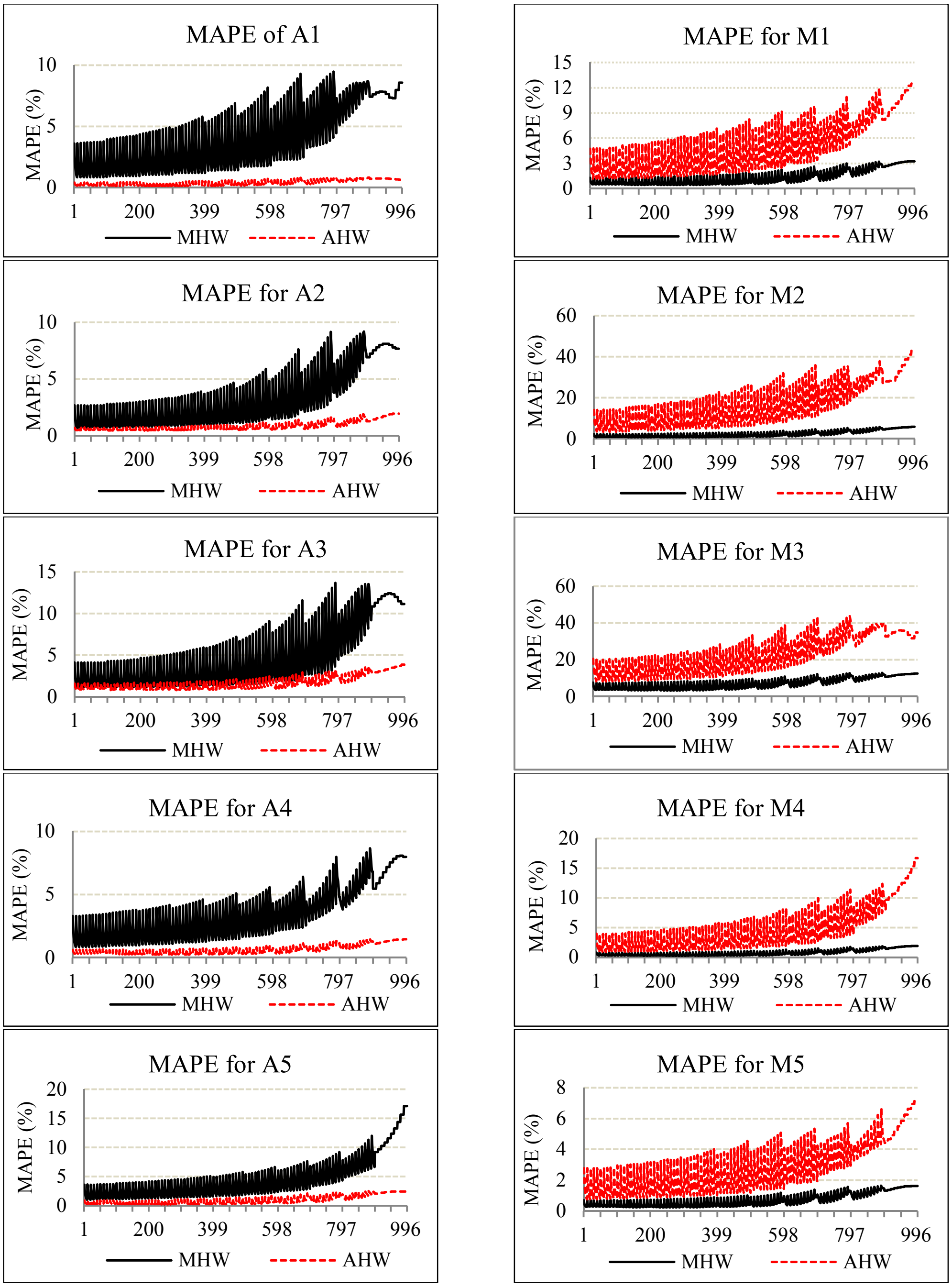


Figure 5 MAPE plots for datasets A1–A5 (left) and M1–M5 (right); MHW, multiplicative Holt-Winters; AHW, additive Holt-Winters

Table 8 The difference between the median MAPE values of the multiplicative and additive Holt-Winters methods at the 5% significance level for the simulated datasets

Seasonality	Simulation Dataset	p-value	95% CIs for ($Median_{MHW} - Median_{AHW}$)
Additive	A1	< 0.001	[2.6327, 3.0832]
	A2	< 0.001	[1.2163, 1.4982]
	A3	< 0.001	[1.1959, 1.5669]
	A4	< 0.001	[2.2191, 2.5424]
	A5	< 0.001	[2.3301,2.6543]
Multiplicative	M1	< 0.001	[−3.1887, −2.7263]
	M2	< 0.001	[−11.906, −10.231]
	M3	< 0.001	[−15.013, −13.285)
	M4	< 0.001	[−2.7564, −2.3624]
	M5	< 0.001	[−1.9079, −1.6557]

CIs: confidence intervals

Table 9 The difference between the median RMSE values of the multiplicative and additive Holt-Winters methods at the 5% significance level for the simulated datasets

Seasonality	Simulation Dataset	p-value	95% CIs for ($Median_{MHW} - Median_{AHW}$)
Additive	A1	< 0.001	[880.1, 1015.6]
	A2	< 0.001	[5.145, 6.424]
	A3	< 0.001	[4.758, 6.460]
	A4	< 0.001	[13937, 16604]
	A5	< 0.001	[110.72, 129.06]
Multiplicative	M1	< 0.001	[−878.9, −722.7]
	M2	< 0.001	[−373.00, −318.90]
	M3	< 0.001	[−206.93, −176.24]
	M4	< 0.001	[−148.94, −125.22)
	M5	< 0.001	[−10170, −8415]

CIs: confidence intervals

Table 10 Average MAPE and RMSE values with the real datasets

Real Dataset	Additive Holt-Winters		Multiplicative Holt-Winters	
	Average MAPE	Average RMSE	Average MAPE	Average RMSE
TS1	2.8107	226.4925	1.8481	142.0190
TS2	5.1462	36,658.1575	4.3980	33,036.7666
TS3	2.4809	4.8766	2.2260	4.4889
TS4	3.9616	16.6119	3.8421	15.9475
TS5	15.3466	3,440.7040	8.6536	2,219.0715

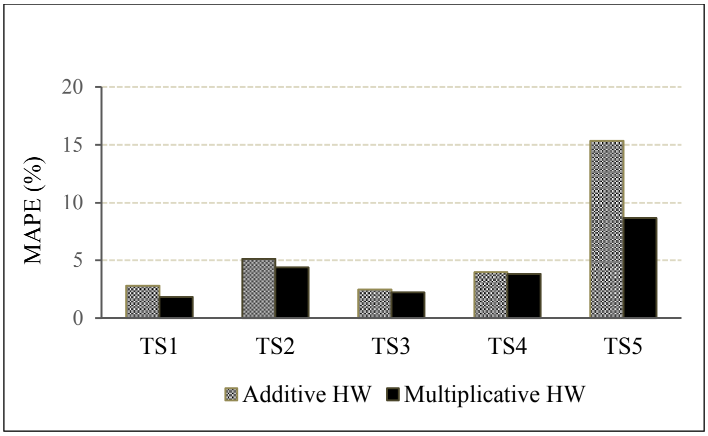


Figure 6 Average MAPE values for the real datasets using the additive and multiplicative Holt-Winters methods

Table 11 Number of lowest MAPE and RMSE occurrences with the real datasets

Real Dataset	MAPE		RMSE	
	Additive Holt-Winters	Multiplicative Holt-Winters	Additive Holt-Winters	Multiplicative Holt-Winters
TS1	14	986	0	1000
TS2	36	964	98	902
TS3	12	988	197	803
TS4	204	796	221	779
TS5	9	991	0	1000
Average	55	945	103.2	896.8

Figures 7-11 show plots of the MAPE values with the additive and multiplicative Holt-Winters methods versus the sets of conditions for TS1–TS5 with the same set of (α, β, γ) conditions. Each dataset setting had smoothing parameters α, β and γ ranging from 0 to 1 ($\alpha = \{0.1, 0.2, ..., 1\}$, $\beta = \{0.1, 0.2, ..., 1\}$, and $\gamma = \{0.1, 0.2, ..., 1\}$), thus (α, β, γ) = (0.1,0.1,0.1), (0.1,0.1,0.2), …, (1,1,1) were used to generate the 1,000 unique sets. The graphs visually compare the performance of the additive and multiplicative Holt-Winters methods according to the MAPE values. It can be seen that for the datasets except for TS4, the MAPE plots for the multiplicative Holt-Winters method usually stayed under those of the additive Holt-Winters method. When examining the time-series plot for TS4 in Figure 10, it is evident that the type of seasonality tends toward additive rather than multiplicative. From the results in Table 10 using the additive Holt-Winters model on TS4, the average MAPE and RMSE values are 3.9616 and 16.6119, respectively, while those using the multiplicative Holt-Winters are 3.8421 and 15.9475, respectively, which signify almost the same performance. Moreover, the performance in terms of the lowest MAPE and RMSE show that the additive and multiplicative Holt-Winters methods achieved the lowest MAPE value 204 and 796 times, and the lowest RMSE value 221 and 779 times, respectively. These results support the earlier finding that when the type of seasonality is not clear, using the multiplicative Holt-Winters method is the preferable.

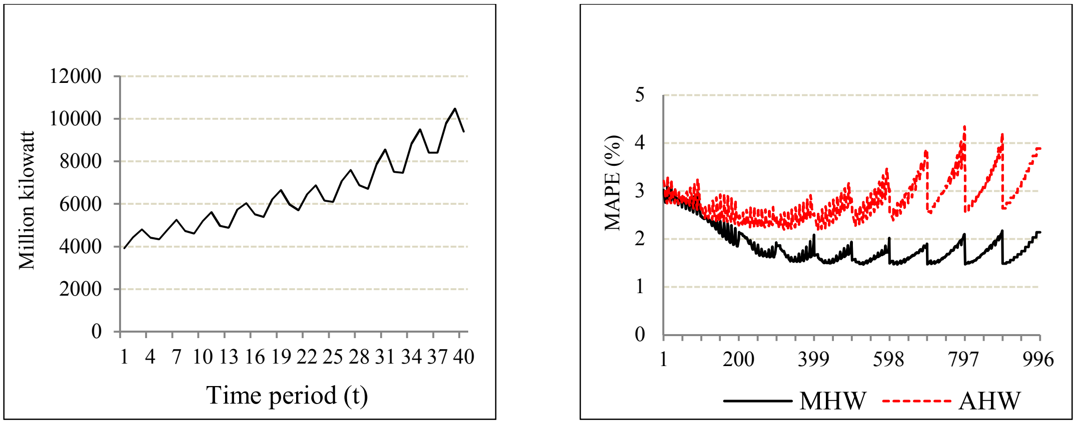


Figure 7 Time-series (left) and the MAPE (right) plots for the additive (AHW) and multiplicative Holt-Winters (MHW) method with TS1

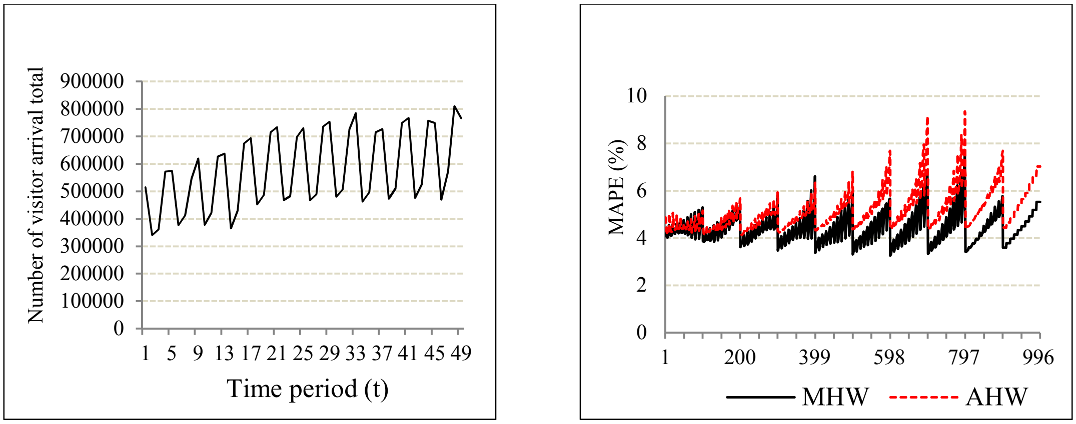


Figure 8 Time-series (left) and MAPE (right) plots for the additive (AHW) and multiplicative Holt-Winters (MHW) methods with TS2

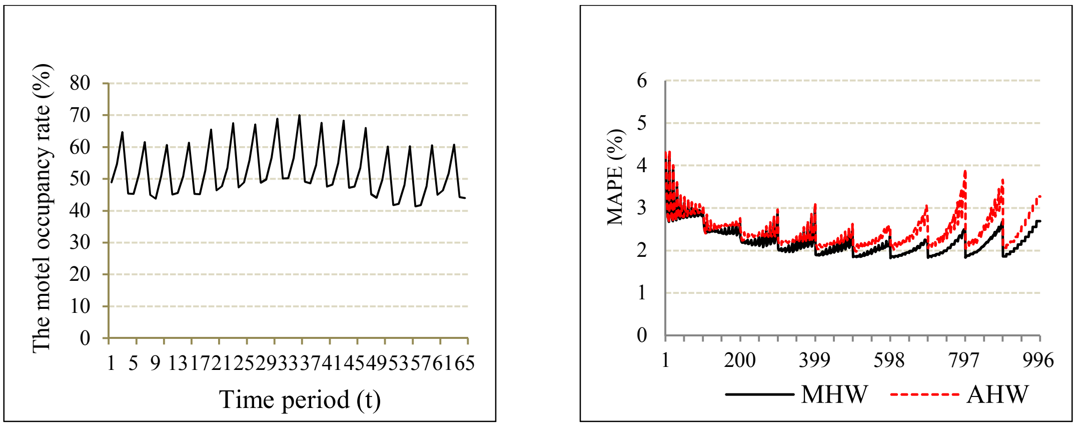


Figure 9 Time-series (left) and MAPE (right) plots for the additive (AHW) and multiplicative Holt-Winters (MHW) methods with TS3

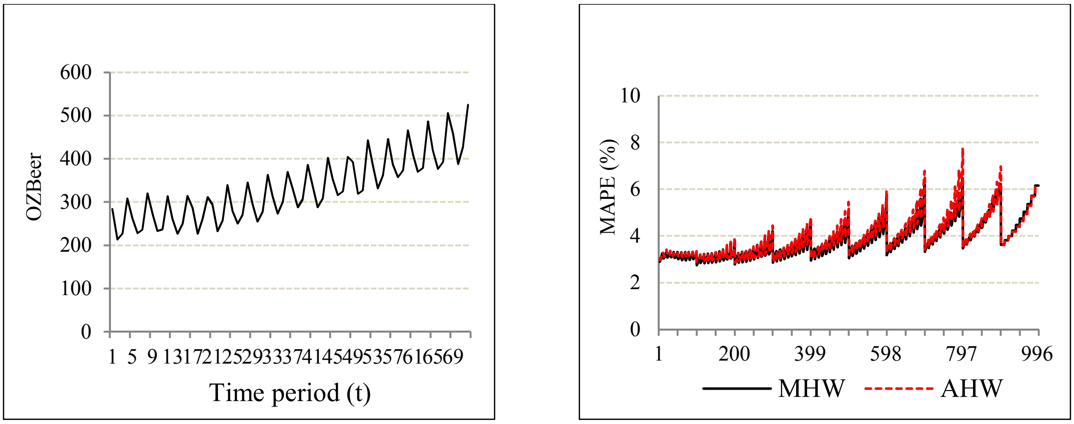


Figure 10 Time-series (left) and MAPE (right) plots for the additive (AHW) and multiplicative Holt-Winters (MHW) methods with TS4

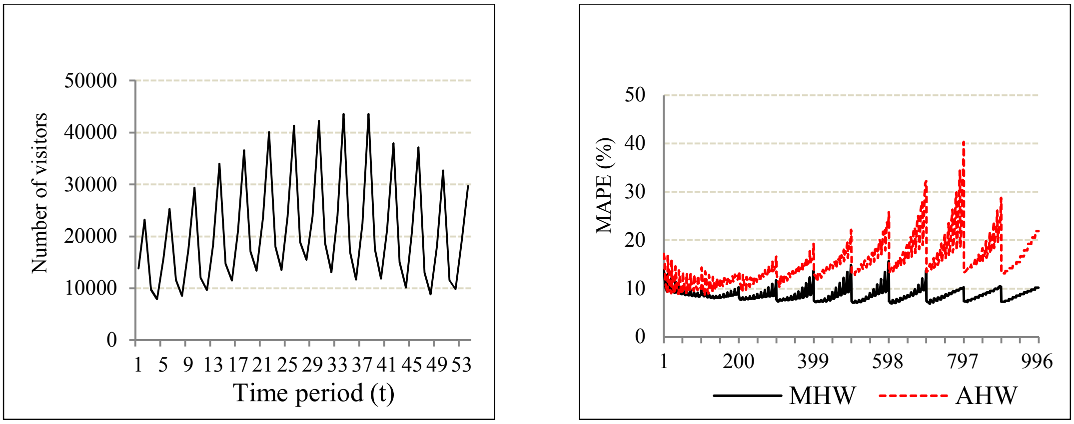


Figure 11 Time-series (left) and MAPE (right) plots for the additive (AHW) and multiplicative Holt-Winters (MHW) methods with TS5

Hypothesis testing of two null hypotheses, there is no difference between the medians of MAPE (RMSE) of the two methods at the 5% significance level, was conducted using the Mann-Whitney U test. The results in Tables 12 and 13 indicate that there are significant differences between them in terms of MAPE and RMSE for all of the real datasets. Besides, the 95% CIs for the difference in the median values show that both the lower and the upper limits are negative for all of the datasets. However, it is again clear that the multiplicative Holt-Winters method significantly outperformed the additive Holt-Winters method for all of the datasets. Furthermore, the improvement in terms of MAPE and RMSE by the multiplicative Holt-Winters method was calculated in relation to the additive Holt-Winters method (Table 14); these were 21.14% and 18.93%, respectively. These results exhibit the same trend as is evident in Tables 12 and 13, i.e. the multiplicative Holt-Winters method outperformed the additive one on all occasions. These results support that when the type of seasonality is ambiguous, the multiplicative Holt-Winters method is the preferred choice.

Table 12 The difference between the median MAPE values of the multiplicative and additive Holt-Winters methods at the 5% significance level for the real datasets

Real Dataset	p-value	95% CIs for ($Median_{MHW} - Median_{AHW}$)
TS1	< 0.001	[−0.9799, −0.9191]
TS2	< 0.001	[−0.7116, −0.6057]
TS3	< 0.001	[−0.2652, −0.2181]
TS4	0.005	[−0.1355, −0.0236]
TS5	< 0.001	[−6.1220, −5.6890]

CIs: confidence intervals

Table 13 The difference between the median RMSE values of the multiplicative and additive Holt-Winters methods at the 5% significance level for the real datasets

Real Dataset	p-value	95% CIs for ($Median_{MHW} - Median_{AHW}$)
TS1	<0.001	[−80.57, −74.83]
TS2	<0.001	[−3144, −2345]
TS3	<0.001	[−0.4337, −0.3353]
TS4	0.004	[−0.556, −0.104]
TS5	<0.001	[−1050.7, −950.3]

CIs: confidence intervals

Table 14 Forecasting accuracy improvement when using the multiplicative over the additive Holt-Winters methods for the real datasets

Real Dataset	Improvement (multiplicative/additive Holt-Winters)	
	MAPE (%)	RMSE (%)
TS1	34.25	37.30
TS2	14.54	9.88
TS3	10.27	7.95
TS4	3.02	4.00
TS5	43.61	35.51
Average	21.14	18.93

6. Conclusions

In this study, the effect on forecasting accuracy when using the incorrect model for seasonality (additive or multiplicative) in the Holt-Winters method was investigated. Ten simulated and five real datasets were used to compare the performances of the multiplicative and additive Holt-Winters methods. The seasonality of the simulated datasets (five datasets of each type) was set to test the theory, after which the investigation was shifted to apply the methods to the real datasets. The additive and multiplicative Holt-Winters methods were tested by using three smoothing parameters ranging from 0 to 1 with increments of 0.1, thus there were 1,000 different settings applied to each dataset.

The results obtained with the simulated datasets are expressed as the MAPE and RMSE of the predictions calculated as accuracy indicators to compare the methods. With the additive simulation datasets when using the incorrect (multiplicative) and correct (additive) Holt-Winters models with 1,000 different settings, the latter achieved the lowest MAPE and RMSE values for all of the cases. Likewise, the reverse was true for the simulated multiplicative datasets. On the other hand, the real datasets had ambiguous seasonality, and the results show that using the multiplicative Holt-Winters method significantly outperformed using the additive one. Furthermore, the improvements in MAPE and RMSE by applying the multiplicative Holt-Winters method were 21% and 19%, on average respectively. Hence, applying the multiplicative Holt-Winters method when the seasonality of the dataset is ambiguous is a plausible forecasting approach.

Acknowledgments

The author is grateful to the reviewers for carefully reading the manuscript and for offering substantial suggestions to improve the manuscript. In addition, the author would like to thank Kasetsart University for providing the facilities to conduct the research.

References

Bermúdez JD, Segura JV, Vercher E. A decision support system methodology for forecasting of time series based on soft computing. Comput Stat Data Anal. 2006; 51(1): 177-191.

Box GEP, Jenkins GM, Reinsel GC. Time series analysis: forecasting and control, 3rd edition. Englewood Cliffs, New Jersey: Prentice-Hall; 1994.

Brockwell PJ, Davis RA, Fienberg SE. Time series: theory and methods. Berlin/Heidelberg, Germany: Springer Science & Business Media; 1991.

Chatfield C. The Holt-Winters forecasting procedure. J R Stat Soc Ser C Appl Stat. 1978; 27(3): 264-279.

Chatfield C, Yar M. Holt-Winters forecasting: some practical issues. Statistician. 1988; 37(2): 129-140.

Cuicui L, Jun Y. The comparative analysis of economic forecasting model on China's logistics engineering industry. Syst Eng. 2012; 5:366-371.

Da Fonseca LS, Russo SL, Fabris JP, Camargo ME. Holt-Winters forecasting investigation in Brazil patent deposits. Bus Manag Dynam. 2016; 5(9): 22-32.

Dantas TM, Oliveira FL, Repolho HM. Air transportation demand forecast through bagging Holt-Winters methods. J Air Transp Manag. 2017; 59: 116-123.

Goodwin P. The Holt-Winters approach to exponential smoothing: 50 years old and going strong. Foresight. 2010; 19: 30-33.

Groff GK. Empirical comparison of models for short range forecasting. Manage Sci. 1973; 20(1): 22-31.

Heydari M, Ghadim HB, Rashidi M, Noori M. Application of holt-winters time series models for predicting climatic parameters (case study: Robat Garah-Bil Station, Iran). Pol J Environ Stud. 2020; 29(1): 617-627.

Holt CC. Forecasting seasonals and trends by exponentially weighted moving averages, ONR memorandum, Vol 52. Pittsburgh, Pennsylvania: Carnegie Institute of Technology; 1957.

Huss WR. The teachers/practitioners corner comparative analysis of load forecasting techniques at a southern utility. J Forecast. 1985; 4(1): 99-107.

Jere S, Banda A, Kasense B, Siluyele I, Moyo E. Forecasting annual international tourist arrivals in Zambia using Holt-Winters exponential smoothing. Open J Stat. 2019; 9(2):258-267.

Kuznets S. Seasonal pattern and seasonal amplitude: measurement of their short-time variations. J Am Stat Assoc. 1932; 27(177): 9-20.

Linden A. Using forecast modelling to evaluate treatment effects in single-group interrupted time series analysis. J Eval Clin Pract. 2018; 24(4): 695-700.

Makridakis S, Hibon M. Accuracy of forecasting: an empirical investigation. J R Stat Soc Ser A. 1979; 142(2): 97-125.

Makridakis SG, Anderson A, Carbone R, Fildes R, Hibon M, Lewandowski R, et al. The forecasting accuracy of major time series methods. Hoboken, New Jersey: John Wiley & Sons, Inc; 1984.

Montgomery DC, Jennings CL, Kulahci M. Introduction to time series analysis and forecasting, Wiley series in probability and statistics. Hoboken, New Jersey: John Wiley & Sons, Inc; 2008.

Rahman MH, Salma U, Hossain MM, Khan MT. Revenue forecasting using Holt–Winters exponential smoothing. Res Rev J Stat. 2016; 5(3): 19-25.

The R Foundation. The R project for statistical computing. 2020 [cited 2020 Apr 14]. Available from: http://www.r-project.org/

Tirkes G, Guray C, Celebi N. Demand forecasting: A comparison between the Holt-Winters, trend analysis and decomposition models. Teh Vjesn. 2017; 24(S2): 503-510.

Valakevicius E, Brazenas M. Application of the seasonal Holt-Winters model to study exchange rate volatility. Inz Ekon. 2015; 26(4): 384-390.

Winters PR. Forecasting sales by exponentially weighted moving averages. Manage Sci. 1960; 6(3): 324-342.

Wei ZM, Song HC. Prediction scheme of railway passenger flow based on multiplicative holt-winters model. Appl Mech Mater. 2013; 416: 1949-1953.

Wu L, Gao X, Xiao Y, Liu S, Yang Y. Using grey Holt-Winters model to predict the air quality index for cities in China. Nat Hazards. 2017; 88(2): 1003-1012.