Thailand Statistician
July 2021; 19(3): 606-616
http://statassoc.or.th
Contributed paper

Variance Estimators in the Presence of Measurement Errors Using Auxiliary Information

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Received: 17 March 2020
Revised: 11 July 2020
Accepted: 22 August 2020

Abstract

Estimation of variance is very important in the area of sample survey. It provides the information on the accuracy of the estimators and allows drawing valid conclusion about the true value of the population parameter. Measurement error is a serious problem in survey sampling which arises when the answer provide by the respondents departs from the true value. This paper discussed the estimation problem of finite population variance using the information of the auxiliary variable in presence of measurement error under simple random sampling (SRS) design. Here some estimators for population variance have been suggested for the study variable *Y*. The proposed estimators based on arithmetic mean, harmonic mean and geometric mean of the sample variance estimator, ratio estimator and exponential ratio estimator. The approximate expressions of biases and mean square errors (MSEs) derived and calculated for the suggested estimators up to the first order using Taylor expansion. The efficiency comparisons of the suggested estimators have been made with the existing estimators. A numerical study also conducted to support the performance of suggested estimators.

Keywords: Mean square error, measurement error, ratio-type estimators, percentage relative efficiency.

1. Introduction

The estimation of variance for finite population has large significance in many fields of life like agriculture, biological sciences, medical and industry. For example blood pressure, pulse rate, body temperature and sugar level of blood are very basic diagnosis where treatment prescribed is to control their variation. Many authors including Das and Tripathi (1978), Isaki (1983), Bahl and Tuteja (1991), Kadilar and Cingi (2006), Singh et al. (2011), Yadav and Kadilar (2013), Singh et al. (2014), Muili et al. (2019), Qureshi et al. (2019), Yadav et al. (2019) estimated the variance of population using auxiliary information.

It is a common assumption of most of statistical analysis that the collected observations are free from any error. However, Cochran (1963, 1968) and Biemer et al. (1991) suggested that mostly this assumption is violated and collected information is contaminated with measurement error due to many reasons. Measurement errors are the discrepancy between the observed value and true value. Such errors increased variability and bias of the estimators of population parameters. Therefore, it is essential to study the measurement errors. Measurement errors distributed normally with zero mean

Biemer et al. (1991). Many authors, together with Das and Tripathi (1978), Srivastava and Jhajj (1980), Fuller (1995), Diana and Giordan (2012), Sharma and Singh (2013), Misra et al. (2016), Khalil et al. (2018 and 2019), Zahid and Shabbir (2019), Nguyen and Tran (2020) and Singh et al. (2020) considered the estimation of population parameters with measurement errors using auxiliary information under different sampling designs.

In this present article, the estimation of population variance considered when the observations subjected to the measurement errors. After the brief introduction of variance estimation with some relevant literature, the rest of article is as follows. Section 2 is based on the sampling methodology, basic notations and some associated classical estimators. In Section 3, we have suggested exponential ratio estimator in the presence of measurement error. In the same section, we have also proposed some estimators using the combinations of arithmetic mean, geometric mean and harmonic mean of the sample mean, classical ratio and exponential ratio estimators in the presence of measurement error. Mathematical comparisons are obtained in Section 4. An extensive numerical study is conducted in Section 5 to assess the performance of the estimators mentioned in this paper. Conclusion is given in Section 6.

2. Methodology and Notations

Let us assume that the study variable Y and the auxiliary variable X be defined on N certain and distinct unit of finite population $U = \{U_1, U_2, ..., U_N\}$.. Let x_i and y_i be the recorded instead of the true values of X_i and Y_i for the i^{th} sampling unit. The measurement or observational errors defined as

$$u_i = (y_i - Y_i) \text{ and } v_i = (x_i - X_i),$$

where the associated measurement errors are u_i and v_i with zero mean S_u^2 and S_v^2 variances respectively. Following Singh and Karpe (2009), it is assumed that both u_i and v_i are independent from each other and also uncorrelated with Y_i and X_i. This implies $Cov(X,Y) \neq 0$ and $Cov(X,U) = Cov(X,V) = Cov(U,Y) = Cov(V,Y) = Cov(U,V) = 0$. It is also assume that the finite population correction may be neglected.

Let $(\bar{Y}, \bar{X})$ and (S_y^2, S_x^2) be the means and variances of population variables Y and X respectively and ρ be the correlation coefficient between (Y, X). Let $\bar{y} = \frac{1}{n}\sum_{i=0}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=0}^{n} x_i$ be the sample means which are unbiased estimators of $(\bar{Y}, \bar{X})$, respectively. The sample variance of variables Y and X are respectively defined as

$$s_y^2 = 1/(n-1)\sum_{i=0}^{n}(y_i - \bar{y})^2.$$

$$s_x^2 = 1/(n-1)\sum_{i=0}^{n}(x_i - \bar{x})^2.$$

The expected values of s_y^2 and s_x^2 under the measurement error are given by

$$E(s_y^2) = S_y^2 + S_u^2 .$$

$$E(s_x^2) = S_x^2 + S_v^2 .$$

Let S_u^2 and S_v^2 be the error variances associated with Y and X respectively, which are assume to be known. For this situation, the unbiased estimators of S_y^2 and S_x^2 are respectively given by

$$\hat{s}_y^2 = s_y^2 - S_u^2 > 0.$$
$$\hat{s}_x^2 = s_x^2 - S_v^2 > 0.$$

Let the sampling errors for the study and auxiliary variables are defined as $\hat{s}_y^2 = S_y^2(1+e_o)$ and $\hat{s}_x^2 = S_x^2(1+e_1)$. Such that $E(e_o) = E(e_1) = 0,\ E(e_o^2) = A_y / n,\ \ E(e_1^2) = A_x / n$ and $E(e_o e_1) = \dfrac{\delta - 1}{n}$, where $A_y = \gamma_{2y} + \gamma_{2u}(S_u^4)/(S_y^4) + 2(1+(S_u^2 / S_y^2))^2$, $A_x = \gamma_{2x} + \gamma_{2v}(S_v^4)/(S_x^4) + 2(1+(S_v^2 / S_x^2))^2$, $\gamma_{2z} = \beta_{2z} - 3,\ \beta_{2z} = \dfrac{\mu_{4z}}{\mu_{2z}^2},\ \mu_{rz} = E(z_i - \mu_z)^r,\ \theta_x = (S_x^2)/(S_x^2 - S_v^2)$ and $\delta = (\mu_{22}(X,Y))/(S_x^2 S_y^2)$, where $z = Y, X, U, V$.

The usual sample variance estimator for the estimation of finite population variance in the presence of measurement errors is

$$t_1 = \hat{s}_y^2.$$

The expression of MSE of t_1 is

$$MSE(t_1) = A_y \frac{S_y^4}{n}.$$

Isaki's (1983) ratio estimator in presence of measurement errors is

$$t_2 = \hat{s}_y^2 \frac{S_x^2}{\hat{s}_x^2}.$$

The approximate bias and MSE of t_2 is

$$Bias(t_2) = \frac{S_y^2}{n}(1 + A_x - \delta).$$

and

$$MSE(t_2) = \frac{S_y^4}{n}(2 + A_y + A_x - 2\delta).$$

3. Suggested Estimators

In this paper, we have proposed some variance estimators using arithmetic mean (AM), geometric mean (GM) and harmonic mean (HM) based on sample variance, ratio and exponential ratio estimators. The following combinations of the estimators (t_1, t_2), (t_1, t_3), (t_2, t_3) and (t_1, t_2, t_3) are used. We first suggested t_3 by following Singh et al. (2011) in the presence of ME as

$$t_3 = \hat{s}_y^2 \exp\left(\frac{S_x^2 - \hat{s}_x^2}{S_x^2 + \hat{s}_x^2}\right).$$

To obtain the approximate expression of bias and MSE, we may re-write t_3 in terms of e's as

$$t_3 = S_y^2(1+e_o)\exp\left(\frac{S_x^2 - S_x^2(1+e_1)}{S_x^2 + S_x^2(1+e_1)}\right).$$

We get the bias of t_3 up to the first-order approximation

$$Bias(t_3) = \frac{S_y^2}{2n}\left(\frac{3A_x}{4} - (\delta - 1)\right).$$

The approximate MSE of t_3 is

$$MSE(t_3) = \frac{S_y^4}{n}\left(A_y + \frac{A_x}{4} - (\delta - 1)\right).$$

3.1. Estimators on the basis of t_1 and t_2

The respective estimator t_4 based on AM, GM and HM of the estimators t_1 and t_2 in the presence of ME are given as

$$t_4^{AM} = \frac{t_1 + t_2}{2} = \frac{\hat{s}_y^2}{2(1 + (S_x^2 / \hat{s}_x^2))},$$

$$t_4^{GM} = (t_1 t_2)^{(1/2)} = \hat{s}_y^2 \left(\frac{S_x^2}{\hat{s}_x^2}\right)^{(1/2)},$$

and

$$t_4^{HM} = \frac{2}{\left(\frac{1}{t_1} + \frac{1}{t_2}\right)} = \frac{2\hat{s}_y^2}{\left(1 + \frac{\hat{s}_x^2}{S_x^2}\right)}.$$

The expressions of biases and MSEs of t_4^{AM}, t_4^{GM} and t_4^{HM} up to first-order are

$$Bias(t_4^{AM}) = \frac{S_y^2}{2n}\left(A_x - (\delta - 1)\right),$$

$$Bias(t_4^{GM}) = \frac{S_y^2}{8n}\left(3A_x - 4(\delta - 1)\right),$$

$$Bias(t_4^{HM}) = \frac{S_y^2}{4n}\left(A_x - 2(\delta - 1)\right),$$

and

$$MSE(t_4^{AM}) = MSE(t_4^{GM}) = MSE(t_4^{HM}) = \frac{S_y^4}{4n}\left(A_x + 4(A_y - (\delta - 1))\right).$$

3.2. Estimators on the basis of t_1 and t_3

The estimator t_5 based on AM, GM and HM of the estimators t_1 and t_3 in the presence of ME are given as

$$t_5^{AM} = \frac{t_1 + t_3}{2} = \frac{\hat{s}_y^2}{2}(1 + \exp\left(\frac{S_x^2 - \hat{s}_x^2}{S_x^2 + \hat{s}_x^2}\right),$$

$$t_5^{GM} = (t_1 t_3)^{(1/2)} = \hat{s}_y^2 \left(\exp\left(\frac{S_x^2 - \hat{s}_x^2}{S_x^2 + \hat{s}_x^2}\right)\right)^{(1/2)},$$

and

$$t_5^{HM} = \frac{2}{\frac{1}{t_1} + \frac{1}{t_3}} = \frac{2\hat{s}_y^2}{1 + \frac{1}{\exp\left(\frac{S_x^2 - \hat{s}_x^2}{S_x^2 + \hat{s}_x^2}\right)}}.$$

The approximate expressions of biases and MSEs of t_5^{AM}, t_5^{GM} and t_5^{HM} up to first-order are

$$Bias(t_5^{AM}) = \frac{S_y^2}{8n}\left(3A_x - 4(\delta-1)\right),$$

$$Bias(t_5^{GM}) = \frac{S_y^2}{4n}\left(\frac{5A_x}{8} - (\delta-1)\right),$$

$$Bias(t_5^{HM}) = \frac{S_y^2}{4n}\left(\frac{3A_x}{4} - (\delta-1)\right),$$

and

$$MSE(t_5^{AM}) = MSE(t_5^{GM}) = MSE(t_5^{HM}) = \frac{S_y^4}{4n}\left(4A_y + \frac{A_x}{4} - 2(\delta-1)\right).$$

3.3. Estimators on the basis of t_2 and t_3

The respective estimator t_6 based on AM, GM and HM of the estimators t_2 and t_3 in the presence of ME given as

$$t_6^{AM} = \frac{(t_2+t_3)}{2} = \frac{\hat{s}_y^2}{2}\left(\frac{S_x^2}{\hat{s}_x^2} + \exp\left(\frac{S_x^2 - \hat{s}_x^2}{S_x^2 + \hat{s}_x^2}\right)\right),$$

$$t_6^{GM} = (t_2 t_3)^{(1/2)} = \hat{s}_y^2\left(\frac{S_x^2}{\hat{s}_x^2}\exp\left(\frac{S_x^2 - \hat{s}_x^2}{S_x^2 + \hat{s}_x^2}\right)\right)^{(1/2)},$$

$$t_6^{HM} = \frac{2}{\left(\frac{1}{t_2} + \frac{1}{t_3}\right)} = \frac{2\hat{s}_y^2}{\frac{1}{(S_x^2/\hat{s}_x^2)} + \frac{1}{\exp\left(\frac{S_x^2 - \hat{s}_x^2}{S_x^2 + \hat{s}_x^2}\right)}}.$$

The approximate expressions of biases and MSEs of t_6^{AM}, t_6^{GM} and t_6^{HM} up to first-order are

$$Bias(t_6^{AM}) = \frac{S_y^2}{8n}\left(\frac{11A_x}{2} - 3(\delta-1)\right),$$

$$Bias(t_6^{GM}) = \frac{S_y^2}{4n}\left(\frac{21A_x}{8} - 3(\delta-1)\right),$$

$$Bias(t_6^{HM}) = \frac{S_y^2}{4n}\left(\frac{5A_x}{2} - 3(\delta-1)\right),$$

and

$$MSE(t_6^{AM}) = MSE(t_6^{GM}) = MSE(t_6^{HM}) = \frac{S_y^4}{4n}\left(4A_y + \frac{9A_x}{4} - 6(\delta-1)\right).$$

3.4. Estimators based on t_1, t_2 and t_3

The estimator t_7 based on AM, GM and HM of the estimators t_1, t_2 and t_3 in the presence of *ME*

$$t_7^{AM} = \frac{t_1+t_2+t_3}{3} = \frac{\hat{s}_y^2}{3}\left(1 + \frac{S_x^2}{\hat{s}_x^2} + \exp\left(\frac{S_x^2 - \hat{s}_x^2}{S_x^2 + \hat{s}_x^2}\right)\right),$$

$$t_7^{GM} = (t_1 t_2 t_3)^{(1/3)} = \hat{s}_y^2 \left(\frac{S_x^2}{\hat{s}_x^2} \exp\left(\frac{S_x^2 - \hat{s}_x^2}{S_x^2 + \hat{s}_x^2} \right) \right)^{(1/3)},$$

and
$$t_7^{HM} = \frac{3}{\left(\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} \right)} = \frac{3\hat{s}_y^2}{1 + \frac{1}{(S_x^2 / \hat{s}_x^2)} + \frac{1}{\exp\left(\frac{S_x^2 - \hat{s}_x^2}{S_x^2 + \hat{s}_x^2} \right)}}.$$

The approximate expressions of biases and MSEs of t_7^{AM}, t_7^{GM} and t_7^{HM} up to first-order are

$$Bias(t_7^{AM}) = \frac{S_y^2}{2n}\left(\frac{11A_x}{12} - (\delta - 1) \right),$$

$$Bias(t_7^{GM}) = \frac{S_y^2}{2n}\left(\frac{11A_x}{4} - (\delta - 1) \right),$$

$$Bias(t_7^{HM}) = \frac{S_y^2}{2n}\left(\frac{A_x}{3} - (\delta - 1) \right),$$

and
$$MSE(t_7^{AM}) = MSE(t_7^{GM}) = MSE(t_7^{HM}) = \frac{S_y^4}{n}\left(A_y + \frac{A_x}{4} - (\delta - 1) \right).$$

4. Efficiency Comparisons

The efficiency comparisons of $t_2, t_3, t_4^j, t_5^j, t_6^j, t_7^j$ (where j = AM, GM and HM) with t_1 are respectively given by

$$MSE(t_2) - MSE(t_1) < 0,$$

$$\frac{S_y^4}{n}(2 + A_y + A_x - 2\delta) - A_y \frac{S_y^4}{n} < 0, \text{ if } 2\delta - A_x > 2.$$

As proven t_3, t_4^j and t_7^j all are equally efficient as

$$MSE(t_3) = MSE(t_4^j) = MSE(t_7^j),$$

$$MSE(t_3) - MSE(t_1) = MSE(t_4^j) - MSE(t_1) = MSE(t_7^j) - MSE(t_1) < 0,$$

$$\frac{S_y^4}{n}\left(A_y + \frac{A_x}{4} - (\delta - 1) \right) - A_y \frac{S_y^4}{n} < 0, \text{ if } 4\delta - A_x > 4,$$

$$MSE(t_5^j) - MSE(t_1) < 0,$$

$$\frac{S_y^4}{4n}\left(4A_y + \frac{A_x}{4} - 2(\delta - 1) \right) - A_y \frac{S_y^4}{n} < 0, \text{ if } 8\delta - A_x > 8,$$

$$MSE(t_6^j) - MSE(t_1) < 0,$$

$$\frac{S_y^4}{4n}\left(4A_y + \frac{9A_x}{4} - 6(\delta - 1) \right) - A_y \frac{S_y^4}{n} < 0, \text{ if } \delta - 3A_x > 1.$$

The efficiency comparisons of $t_3, t_4^j, t_5^j, t_6^j, t_7^j$ with t_2 are respectively given by

$$MSE(t_3) - MSE(t_2) = MSE(t_4^j) - MSE(t_2) = MSE(t_7^j) - MSE(t_2) < 0,$$

$$\frac{S_y^4}{n}\left(A_y+\frac{A_x}{4}-(\delta-1)\right)-\frac{S_y^4}{n}(2+A_y+A_x-2\delta)<0,\ \text{if}\ 4\delta-3A_x<4.$$

$$MSE(t_5^j)-MSE(t_2)<0.$$

$$\frac{S_y^4}{4n}\left(4A_y+\frac{A_x}{4}-2(\delta-1)\right)-\frac{S_y^4}{n}(2+A_y+A_x-2\delta)<0,\ \text{if}\ 8\delta-5A_x<8.$$

$$MSE(t_6^j)-MSE(t_2)<0.$$

$$\frac{S_y^4}{4n}\left(4A_y+\frac{9A_x}{4}-6(\delta-1)\right)-\frac{S_y^4}{n}(2+A_y+A_x-2\delta)<0,\ \text{if}\ 8\delta-7A_x<8.$$

The efficiency comparisons of $t_4^j, t_5^j, t_6^j, t_7^j$ with t_3 are respectively given by

$$MSE(t_4^j)-MSE(t_3)=MSE(t_7^j)-MSE(t_3)=0,$$

$$MSE(t_5^j)-MSE(t_3)<0,$$

$$\frac{S_y^4}{4n}\left(4A_y+\frac{A_x}{4}-2(\delta-1)\right)-\frac{S_y^4}{n}\left(A_y+\frac{A_x}{4}-(\delta-1)\right)<0,\ \text{if}\ 8\delta-3A_x<8.$$

$$MSE(t_6^j)-MSE(t_3)<0,$$

$$\frac{S_y^4}{4n}\left(4A_y+\frac{9A_x}{4}-6(\delta-1)\right)-\frac{S_y^4}{n}\left(A_y+\frac{A_x}{4}-(\delta-1)\right)<0,\ \text{if}\ 8\delta-5A_x>8.$$

5. Numerical Study

In this section, the performance of the suggested estimators has demonstrated over the mentioned estimators. The simulation study is carried through R software by generating samples using SRSWOR from the three populations each of size $N = 5{,}000$. The details of three population parameters are given by:

POP 1: $\bar{Y}=6, \bar{X}=5,\ S_y^2=160, S_x^2=144, S_u^2=9, S_v^2=9$ and $\rho_{yx}=0.95$,

POP 2: $\bar{Y}=6, \bar{X}=5,\ S_y^2=170.58, S_x^2=144, S_u^2=9, S_v^2=9$ and $\rho_{yx}=0.92$,

POP 3: $\bar{Y}=6, \bar{X}=5,\ S_y^2=109, S_x^2=100.3, S_u^2=9, S_v^2=9$ and $\rho_{yx}=0.96$.

The absolute relative biases (ARBs) and percent relative efficiencies (PREs) have computed by using the following formulas

$$ARBs(t_i)=\left|\frac{Bias(t_i)}{Var(y)}\right|,$$

and

$$PREs(t_i,t_1)=\frac{MSE(t_1)}{MSE(t_i)}\times 100,$$

where $t_i = t_2, t_3, t_4^j, t_5^j, t_6^j, t_7^j$.

The simulated results of ARBs and PREs are summarized in Tables 1-2, respectively. The results of ARBs and PREs presented in Tables 1-2 shows that the suggested estimators are more efficient than the existing ones. The observed amount ARBs of the suggested estimators decrease by increases the sample sizes. The PREs of all the estimators increases especially for the large samples. It is also notice that the ratio estimator performs better than the existing estimators. Further, the suggested estimators belong to t_6^j has the greater PRE as compare to the estimators s_y^2, $t_2, t_3, t_4^j, t_5^j, t_6^j, t_7^j$ for all populations. However, the ARBs show mixed behavior as t_3, t_4 and t_2 has minimum ARBs for population I-III respectively. Finally, we concluded that the suggested estimators belong to t_6^j performs outstandingly as compare to the mentioned estimators for all sample sizes.

Table 1 The ARBs of different estimators in the presence of measurement error

Sample Size	Estimator	The ARB of Different Estimators with ME		
		POP 1	POP 2	POP 3
50	t_2	0.002670	0.012358	0.186209
	t_3	0.195954	0.279562	0.221497
	t_4^{AM}	0.146935	0.340915	0.246754
	t_5^{AM}	0.243577	0.486875	0.264399
	t_6^{AM}	0.099312	0.133602	0.203853
	t_7^{AM}	0.163274	0.320464	0.238335
100	t_2	0.032915	0.009862	0.059026
	t_3	0.037812	0.027322	0.044283
	t_4^{AM}	0.009814	0.054494	0.013946
	t_5^{AM}	0.012263	0.073087	0.065601
	t_6^{AM}	0.035364	0.008730	0.007371
	t_7^{AM}	0.019147	0.045437	0.024058
200	t_2	0.005728	0.053249	0.026498
	t_3	0.096753	0.100524	0.078129
	t_4^{AM}	0.062983	0.061818	0.047231
	t_5^{AM}	0.114224	0.138705	0.099545
	t_6^{AM}	0.045512	0.023637	0.025815
	t_7^{AM}	0.074240	0.074720	0.057530
500	t_2	0.010654	0.022994	0.000907*
	t_3	0.073808	0.039565	0.048533
	t_4^{AM}	0.043415	0.011005	0.022351
	t_5^{AM}	0.074991	0.042285	0.047072
	t_6^{AM}	0.042231	0.008285	0.023813
	t_7^{AM}	0.053546	0.020525	0.031079
1000	t_2	0.007282	0.015177	0.054582
	t_3	0.002287*	0.030769	0.001617
	t_4^{AM}	0.025801	0.004033*	0.024643
	t_5^{AM}	0.021016	0.011829	0.003457
	t_6^{AM}	0.002497	0.022973	0.026483
	t_7^{AM}	0.016438	0.012945	0.015889

Table 2 The PREs of different estimators in the presence of measurement errors

The PREs of Different Estimators				
Estimator	Sample Size	PREs		
		POP 1	POP 2	POP 3
t_2	50	404.74	268.16	198.39
	100	424.58	278.74	204.49
	200	425.64	282.31	214.46
	500	428.10	291.06	219.54
	1000	434.50	295.77	222.19
t_3	50	265.45	232.97	205.72
	100	267.99	234.00	207.27
	200	268.50	236.08	209.33
	500	268.84	237.25	210.55
	1000	272.07	241.07	212.93
t_4^j	50	277.05	239.60	208.65
	100	278.29	240.35	210.93
	200	279.41	242.74	213.41
	500	279.75	244.13	214.99
	1000	282.51	248.02	217.60
t_5^j	50	159.05	152.81	146.66
	100	159.55	153.25	147.21
	200	159.62	153.77	147.58
	500	159.76	153.92	147.73
	1000	160.54	154.93	148.71
t_6^j	50	418.69	308.03	239.02
	100	425.76	310.07	242.89
	200	429.36	315.08	249.66
	500	432.43	320.19	254.49
	1000	436.67	327.61	256.05
t_7^j	50	273.54	237.53	207.96
	100	274.85	238.56	209.85
	200	275.94	240.59	212.12
	500	276.13	241.85	213.53
	1000	279.01	245.73	216.08

6. Conclusions

In this article, we have considered the problem of estimation of finite population variance in the presence of ME. We have suggested an exponential estimator with ME for population variance by following Singh et al. (2011) under SRS design. We have also suggested some estimators based on the AM, GM, and HM using the combinations of sample variance estimator, ratio estimator and exponential ratio estimator. The optimum conditions are also obtained in which the suggested estimators may perform better than the existing estimators. We have considered three populations to examine the performance of all the estimators considered in this paper. The results of ARB and PRE of simulation study show that the suggested estimators performed better than other estimators available in the literature. The PREs of the suggested estimators increase and the ARBs decreases by increases the sample sizes. The estimators t_6^j based on the averages (AM, GM and HM) of ratio and exponential ratio estimators found to be more efficient as compare to the other suggested and existing estimators in terms of high PRE. Based on the numerical findings, we recommend the suggested estimators for the estimation of population variance.

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