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Valuation of Portfolio Risk and Performance of Several Blue Chip Stocks in Indonesia using Value-at-Risk based on n-Dimensional Geometric Brownian Motion

Di Asih I Maruddani*[a] and Trimono Trimono [b]

[a] Department of Statistics, Universitas Diponegoro, Kota Semarang, Jawa Tengah, Indonesia.

[b] Department of Data Science, UPN Veteran Jawa Timur, Indonesia.

*Corresponding author; e-mail: maruddani@live.undip.ac.id

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Abstract

Blue Chips are stocks of leading companies with good management and strong financial performance. History of financial market data shows that Blue Chip stocks have stable stock price volatility, have a strong effect on the Composite Stock Price Index (CSPI), but the performance of these companies is mutually independent. Therefore, modeling the prices of Blue Chip stocks uses n-dimensional geometric Brownian motion which describes volatility and data which follows stochastic processes. In addition, by the presence of negative relationship between stock prices, it is necessary to build portfolio to minimize risks. This article aimed to model the stocks of BCA, PT Telekomunikasi Indonesia, and PT Unilever. This year, these stocks were recorded to increase the CSPI which fell dramatically in mid-October 2019. Stock data were taken from 30 November 2018 to 29 November 2019. The results showed that forecast which used 3-dimensional GBM was very accurate to predict stock prices, evident by MAPE less than 5%. Portfolio risk was measured using VaR, showing a value of 0.003295758% at a confidence level of 95%. In addition, the Sharpe Index resulted in a value of 20.437, indicating profits that exceeded investment in a risk-free interest rate. This indicates that it is very appropriate for investors to invest in BCA, TLKM and UNVR stock portfolios with the proportions of 0.000006%, 10.265098% and 89.734895%, respectively.

Keywords: MVEP, MAPE, sharpe index.

1. Introduction

Blue Chips are stocks of leading companies with good management and strong financial performance. These stocks are known to have the ability to survive in difficult market conditions and provide high returns in good market conditions. Based on the history of financial market data, Blue Chip stocks have stable stock price volatility and co-movement, and have a strong effect on the Composite Stock Price Index (CSPI). On October 16, 2019 the CSPI prices were very fluctuating, with a quite dramatic decline (Figure 1).



Figure 1 The Movements of Composite Stock Price Index (CSPI) on 16 October 2019

Source: <https://www.cnbcindonesia.com/>

Six leading stocks, commonly referred to as blue chips, could make the stock price index back up. These stocks have a very significant influence on CSPI, namely PT Unilever Indonesia Tbk (UNVR), PT Bank Rakyat Indonesia Tbk (BBRI), PT Bank Central Asia Tbk (BBCA), and PT Semen Indonesia Tbk (SMGR). Two other stocks which increases the stock market at the last moment of trading are PT Bank Negara Indonesia Tbk (BBNI) and PT Bank Mandiri Tbk (BMRI). The jump of these six leading stocks could help increase the Composite Stock Price Index (CSPI) because this occurs at the last minute of stock market trading, or injury time (<https://www.cnbcindonesia.com/>).

In addition to these six stocks, two other stocks considered very stable are PT Telekomunikasi Indonesia Tbk. (TLKM) and PT Indofood CBP Sukses Makmur Tbk. (ICBP). In the last ten years, the dividend payout of TLKM increased from 40% to 90% in profits, indicating a very healthy condition. Meanwhile, the financial indicators of ICBP show a consistent performance, in the form of revenue growth followed by consistent profit growth over year in line with market growth.

In modeling data sets that describe co-movements and follow stochastic processes, n -dimensional geometric Brownian motion can be used (Platen 2009). To minimize risk in investment, building portfolios of several stocks with negative correlations is appropriate. The nature of inverse relationship will eliminate the effect of risks.

A number of studies have applied geometric Brownian motion in stock price valuations. Abidin and Jaffar (2014) analyzed stock price modeling using geometric Brownian motion in several companies in Bursa Malaysia. Marathe and Ryan (2005) discuss the process for checking whether a given time series follows the GBM process. They found that of the four industries they studied, the time series for usage of established services met the criteria for a GBM process; while the data form growth of emergent services did not. Reddy and Clinton (2016) simulate Australian Companies' stock prices using GBM. Geometric Brownian motion with jump was performed by Maruddani and Trimono (2017) in the case of stock data containing extreme values of PT Astra Argo Lestari Tbk in 2017. Yunita et al. (2015) used the multidimensional geometric Brownian motion to build an optimal portfolio with the Markowitz method on three stocks with multidimensional motion. Maruddani and Trimono (2018) modeled the stock price data of Matahari Department Store Ltd and Telekomunikasi Indonesia Ltd using 2-dimensional geometric Brownian motion in portfolios and concluded that stock price modeling based on multidimensional model is more accurate than modeling with one-dimensional model. Hoyyi et al. (2019) made a more accurate prediction of the stock price of

Telekomunikasi Indonesia Ltd. (TLKM) by using the CSPI as stocks with co-movement with TLKM stocks.

Based on some of previous studies, this article further analyzed the building of portfolios of several stocks that are suspected to have co-movement in their position as leading stocks. Based on Figure 2, UNVR is the stock with the strongest influence on the CSPI, making it selected in the building of the portfolio. In addition, BBKA and TLKM stocks were selected as an element to build the portfolio because they were indicated to have a negative correlation with UNVR stocks, so it was expected to eliminate risks. Portfolio weight was determined based on the minimum variance efficient portfolio method and the portfolio performance was tested using the Sharpe index.



Figure 2 The weight of Blue Chips Stocks on CSPI (Source: <https://www.cnbcindonesia.com/>)

2. Theoretical Framework

2.1. n-dimensional GBM

Suppose $\{X_1(t), X_2(t), \dots, X_n(t)\}$ is solution of stochastic differential equation and $g(t, x_1, x_2, \dots, x_n)$ is continuously differentiable function on t and continuously differentiable to x_1, \dots, x_n , (Kloeden and Platen, 1992; Brigo, 2008). Then suppose the function $S(X) = \log X$ and $X(0)$ is the starting point of X . Based on n -dimensional Ito's Lemma it is obtained (Lin 2006):

$$\begin{aligned}
 X_1(t) &= X_1(t-1) \exp\left(\mu_1 - \frac{1}{2} \sum_{j=1}^n \sigma_{1j}^2\right) dt + \left(\sum_{j=1}^n \sigma_{1j}\right) \sqrt{dt} Z_{t-1} \\
 X_2(t) &= X_2(t-1) \exp\left(\mu_2 - \frac{1}{2} \sum_{j=1}^n \sigma_{2j}^2\right) dt + \left(\sum_{j=1}^n \sigma_{2j}\right) \sqrt{dt} Z_{t-1} \\
 &\vdots \\
 X_n(t) &= X_n(t-1) \exp\left(\mu_n - \frac{1}{2} \sum_{j=1}^n \sigma_{nj}^2\right) dt + \left(\sum_{j=1}^n \sigma_{nj}\right) \sqrt{dt} Z_{t-1}.
 \end{aligned} \tag{1}$$

2.2. Minimum variance efficient portfolio

MVEP is defined as a portfolio that has a minimum variance compared to all the possible portfolios that can be built. This is the same as optimizing weight $\mathbf{w} = [w_1 \dots w_N]^T$ based on the maximum mean return of the given variance. More formally, weight vector \mathbf{w} is searched, so the portfolio built has a minimum variance based on two constraints, namely:

1. The initial specification of the mean return μ_p has to be achieved, i.e. $\mathbf{w}^T \boldsymbol{\mu}$.
2. The proportion of the portfolio built is equal to 1, i.e. $\mathbf{w}^T \mathbf{1}_N = 1$, where $\mathbf{1}_N$ is vector 1 with a dimension of $N \times 1$.

The optimization problem can be solved using Lagrange function as follows

$$L = \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} + \lambda_1 (\mu_p - \mathbf{w}^T \boldsymbol{\mu}) + \lambda_2 (1 - \mathbf{w}^T \mathbf{1}_N), \quad (2)$$

with L is the Lagrange function and λ is the Lagrange multiplier. In the case of a portfolio with efficient variance, there is no limitation on the mean portfolio ($\lambda_1 = 0$), so the weight in mean variance efficient portfolio with return $\mathbf{X} \sim N_N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ is

$$\mathbf{w} = \frac{\boldsymbol{\Sigma}^{-1} \mathbf{1}_N}{\mathbf{1}_N^T \boldsymbol{\Sigma}^{-1} \mathbf{1}_N}, \quad (3)$$

where $\boldsymbol{\Sigma}^{-1}$ is the inverse variance-covariance matrix.

2.3. Value-at-Risk

VaR is a method used to measure the maximum potential loss that might occur due to owning certain assets for a specified period with a degree of confidence. The return of each asset at time t , namely $R_{1,t}$ and $R_{2,t}$ is used to calculate portfolio return at time t , i.e. (Wilmott 2000)

$$R_{p,t} = w_1 R_{1,t} + w_2 R_{2,t}, \quad (4)$$

where $R_{p,t}$ is the portfolio return at time t , w_1 is the size of composition or proportion of the first asset, w_2 is the size of composition or proportion of the second asset.

Looking for an estimate of the maximum potential loss at a degree of confidence $(1 - \alpha)$ is as α^{th} quantile of empirical distribution of portfolio returns denoted by R^* . Calculating VaR at a degree of confidence $(1 - \alpha)$ at period of time t days is

$$VaR_{(1-\alpha)}(t) = V_0 R^* \sqrt{t}. \quad (5)$$

2.4. Sharpe index

Portfolio performance is measured by comparing the portfolio risk premium (the difference between the mean return of portfolio and risk-free rate) with portfolio risk denoted by standard deviation (total risk). In equation, the Sharpe index is formulated as follows:

$$S_p = \frac{\bar{R}_p - \bar{R}_f}{SD_p}, \quad (6)$$

where S_p is the Sharpe index of i^{th} portfolio, \bar{R}_p is the mean return of i^{th} portfolio, \bar{R}_f = mean risk-free rate and SD_p is the standard deviation of return of i^{th} portfolio. The higher the ratio of portfolio risk premiums to standard deviations, the better the portfolio performance.

3. Method

3.1. Data source

The data used in this study were the daily closing price data of three stocks categorized as Blue Chips, namely PT Bank Central Asia Tbk, PT Telekomunikasi Indonesia Tbk, PT Unilever Indonesia Tbk from 30 November 2018 to 29 November 2019. There were 262 stock data divided into two

groups, namely the in-sample group consisting of 232 data and the out-sample group consisting of the remaining 30 data. These three stocks were selected in the price and portfolio valuation because BBKA and TLKM were indicated to have independent performance with UNVR, thus the building of a portfolio would minimize risks. The data were taken from the website <https://finance.yahoo.com/>.

3.2. Analysis method

The steps taken to analyze the data are: (1) Calculating the descriptive statistics of each of the stock data; (2) Performing a normality test using the Kolmogorov-Smirnov test for each of the stock data; (3) Developing a model for the three stock data with 3-dimensional GBM, (4) making prediction of each stock price data with 3-dimensional GBM, (5) Calculating MAPE, (6) Converting daily closing stock price data into return data, (7) Performing multivariate normality tests for the three stocks using the Shapiro-Wilk Test, (8) Building a portfolio of all the three stocks, by determining the weight of each stock using the MVEP method, (9) Doing test for normality of portfolios return data using Kolmogorov-Smirnov test (10) Calculating the risk of the portfolio with Value-at-Risk using Monte Carlo simulation method, and (10) Measuring portfolio performance using the Sharpe index.

4. Results and Discussion

In mid-October 2019, the CSPI weakened with a very dramatic decline. However, several leading stocks helped strengthen the CSPI. These stocks were the ones with stable prices, moved in the same direction as the CSPI, and brought a significant contribution to the CSPI. In fact, the stocks of UNVR had the highest weight in terms of effect on the CSPI, making it a very good choice for investment. In order to minimize the potential risk of making investment in UNVR stocks, investment is done in the form of a portfolio. For the purpose of building a portfolio, it would be more appropriate to invest in stocks that are inversely related to UNVR stocks. BBKA and TLKM are two blue chip stocks whose fluctuations are considered to have a negative correlation with UNVR.

Therefore, stock portfolios in this study were built based on a co-movement model, i.e., with 3-dimensional GBM. The descriptive statistics of the three stocks data are presented in Table 1.

Table 1 Descriptive statistics

	BBKA	TLKM	UNVR
Mean	28,478.90	3,981.34	46,381.82
Standard of Deviation	1,610.73	225.06	2,200.10
Variance	2,594,452.15	50,651.70	4,840,227.77
Coefficient of Variation	5.66%	5.65%	4.74%
Minimum	25,325	3,510	41,600
Maximum	31,450	4,470	50,025

The stock price of UNVR was the highest price compared to the other two stocks, with a high standard deviation as well. But in the other hand, UNVR has the lowest coefficient of variation. This shows that a low fluctuation in the stock price data of UNVR indicates a low risk as well. Coefficient of variation for BBKA and TLKM had the same level, so we can say that the two stocks have the same risk of investment.

Both BBKA and TLKM stocks were indicated as the stocks whose movement was negatively correlated to UNVR, shown by negative correlation and covariance, namely BBKA stocks and TLKM stocks. This means that if there is an increase in UNVR stocks, then BBKA and TLKM stocks decline. The correlation among these three stocks is shown in Table 2. Negative correlation indicates that the

influential stocks move inversely, which will eliminate each other’s risks in building portfolios. Negative effects can also be demonstrated by covariance. The covariance among the three stocks is presented in Table 3.

Table 2 Correlation coefficients

	BBCA	TLKM	UNVR
BBCA	1.0000	0.7975	-0.4453
TLKM	0.7975	1.0000	-0.1334
UNVR	-0.4453	-0.1334	1.000

Table 3 Covariance coefficients

	BBCA	TLKM	UNVR
BBCA	317164.0	312767.6	-1.874.280.0
TLKM	312767.6	48491.5	-69397.3
UNVR	-1.874.280.0	-69397.3	5584742.0

Stock price model of BBCA based on multidimensional GBM is

$$\begin{aligned}
 X_1(t) &= X_1(t-1) \exp\left(\mu_1 - \frac{1}{2} \sum_{j=1}^3 \sigma_{1j}^2\right) dt + \left(\sum_{j=1}^3 \sigma_{1j}\right) \sqrt{dt} Z_{t-1} \\
 &= X_1(t-1) \exp\left(28,478.89 - \frac{1}{2} (1,610.73^2 + 312,088.23^2 + (-950,358.20)^2)\right) \\
 &\quad + (1,610.73 + 312,088.23 - 950,358.20) Z_{t-1}.
 \end{aligned}$$

Stock price model of TLKM based on multidimensional GBM is

$$\begin{aligned}
 X_2(t) &= X_2(t-1) \exp\left(\mu_2 - \frac{1}{2} \sum_{j=1}^3 \sigma_{2j}^2\right) dt + \left(\sum_{j=1}^3 \sigma_{2j}\right) \sqrt{dt} Z_{t-1} \\
 &= X_2(t-1) \exp\left(3,981.35 - \frac{1}{2} (312,088.20^2 + 225.06^2 + (-40,737.45)^2)\right) \\
 &\quad + (312,088.20 + 255.06 - 40,737.45) Z_{t-1}.
 \end{aligned}$$

Stock price model of UNVR based on multidimensional GBM is

$$\begin{aligned}
 X_3(t) &= X_3(t-1) \exp\left(\mu_3 - \frac{1}{2} \sum_{j=1}^3 \sigma_{3j}^2\right) dt + \left(\sum_{j=1}^3 \sigma_{3j}\right) \sqrt{dt} Z_{t-1} \\
 &= X_3(t-1) \exp\left(46,381.82 - \frac{1}{2} ((-950,358.20)^2 + (-40,737.45)^2 + 2,200.05^2)\right) \\
 &\quad + (-950,358.20 - 40,737.45 + 2,200.05) Z_{t-1}.
 \end{aligned}$$

The prediction of stock prices for the next 30 periods is given in Table 4. The time series plots between actual and prediction data using 3-dimensional GBM for BBCA, TLKM, and UNVR shown by Figure 3.

Table 4 Prediction of stock prices of the next 30 periods

Date	BBCA	TLKM	UNVR	Date	BBCA	TLKM	UNVR
31/10/2019	31144,51	3931,84	41799,30	15/11/2019	32584,70	3959,51	41788,79
01/11/2019	30961,97	3933,68	41798,60	16/11/2019	32500,21	3961,36	41788,09
02/11/2019	31596,69	3935,52	41797,90	17/11/2019	32520,97	3963,22	41787,39
03/11/2019	31918,21	3937,36	41797,20	18/11/2019	32780,77	3965,07	41786,69
04/11/2019	32343,32	3939,20	41796,50	19/11/2019	32706,63	3966,92	41785,99
05/11/2019	32021,72	3941,04	41795,80	20/11/2019	32869,82	3968,78	41785,29
06/11/2019	31771,80	3942,88	41795,10	21/11/2019	33176,54	3970,64	41784,59
07/11/2019	32094,87	3944,73	41794,40	22/11/2019	32732,01	3972,49	41783,89
08/11/2019	32625,23	3946,57	41793,70	23/11/2019	33490,24	3974,35	41783,19
09/11/2019	32861,79	3948,42	41793,00	24/11/2019	33729,91	3976,21	41782,49
10/11/2019	33022,44	3950,27	41792,30	25/11/2019	33820,60	3978,07	41781,79
11/11/2019	33115,91	3952,11	41791,60	26/11/2019	34196,88	3979,93	41781,09
12/11/2019	32725,02	3953,96	41790,89	27/11/2019	34369,12	3981,79	41780,39
13/11/2019	32122,55	3955,81	41790,19	28/11/2019	34211,23	3983,65	41779,69
14/11/2019	32450,88	3957,66	41789,49	29/11/2019	34167,95	3985,52	41778,99

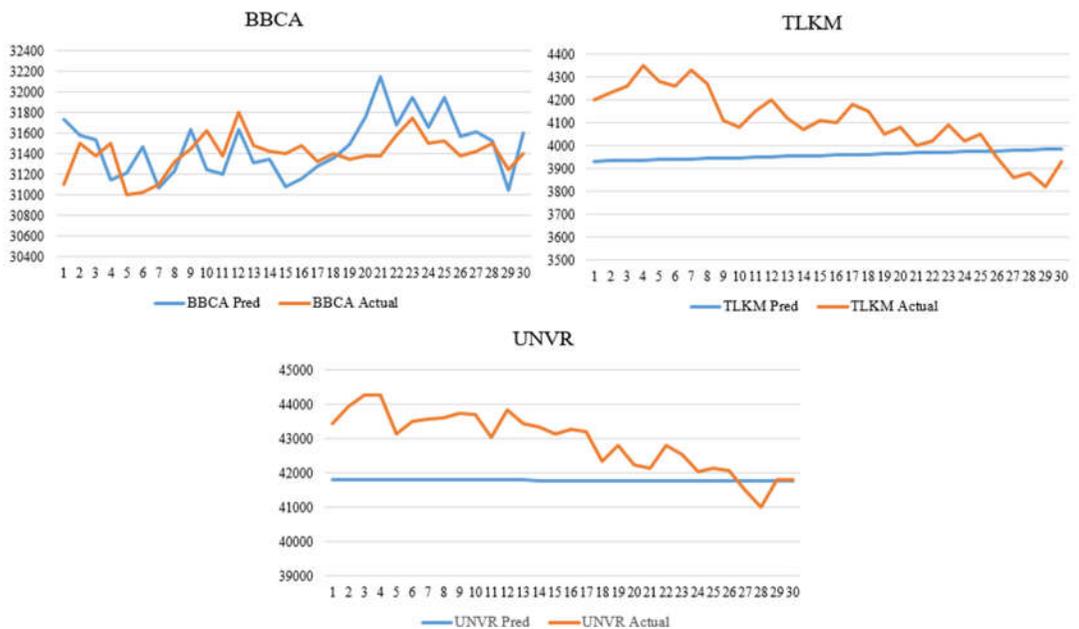


Figure 3. Actual and prediction value of BBCA, TLKM, and UNVR

From Figure 3 above, a visual inspection of the time series plots indicates that there is no significant difference between actual and prediction data. This implies that 3-dimensional GBM is one of the best model for predict stock price of BBCA, TLKM, and UNVR. The MAPE of the three stocks is a very small values, less than 10%, meaning that the stock price prediction for the next 30 days using the out-samples can be said to be very accurate. The accuracy of the prediction based on MAPE is presented in Table 5.

Table 5 MAPE

BBCA	4,417854
TLKM	4,298760
UNVR	2,775402

Based on the accuracy of the 3-dimensional GBM model, portfolio risk measurement can be done using predictive data. Portfolio weights were determined using the Minimum-Variance Efficient Portfolio (MVEP) method. The assumption needed in this method is that stock return data meet the assumption of a multivariate normal distribution. Returns were calculated using continuously compounding returns. Normal distribution was checked using the Shapiro-Wilk test whose result showed that the stock return data had multivariate normal distribution at an error rate of 5%. The results are shown in Table 6.

Table 6 Shapiro-Wilk multivariate normality test output

Statistics	Value
W	0.93894
p-value	0.07715
Decision	H_0 is accepted

Once the stock return data were proven to have a multivariate normal distribution, then the weight of each stock in the portfolio could be determined. The results of weighting using the MVEP method are presented in Table 7.

Table 7 The Weight of each stock in the portfolio

BBCA	0.00000007
TLKM	0.10265098
UNVR	0.89734895

Based on the MAPE obtained, it can be said that the investment proportion of each stock in the portfolio was 0.000000006 or 0.000006% for BBCA stocks, 0.10265098 or 10.265098% for TLKM stocks, and 0.89734895 or 89.734895% for UNVR stocks. It can be seen that the contribution of UNVR stocks in the portfolio is dominant, while BBCA stocks does not significantly affect the portfolio. Applying these weights to each stock, the portfolio return can be obtained and is given in Table 8.

Table 8 Portfolio return for the next 30 periods

Date	Portfolio Return	Date	Portfolio Return	Date	Portfolio Return
31/10/2019		10/11/2019	-0.00001156032	20/11/2019	-0.00001153458
01/11/2019	-0.00001158683	11/11/2019	-0.00001158766	21/11/2019	-0.00001153444
02/11/2019	-0.00001158545	12/11/2019	-0.00001179833	22/11/2019	-0.00001156305
03/11/2019	-0.00001158617	13/11/2019	-0.00001156214	23/11/2019	-0.00001153388
04/11/2019	-0.00001158611	14/11/2019	-0.00001156055	24/11/2019	-0.00001153497
05/11/2019	-0.00001158762	15/11/2019	-0.00001156104	25/11/2019	-0.00001153538
06/11/2019	-0.00001158762	16/11/2019	-0.00001156158	26/11/2019	-0.00001153498
07/11/2019	-0.00001155962	17/11/2019	-0.00001153444	27/11/2019	-0.00001153549
08/11/2019	-0.00001158645	18/11/2019	-0.00001156121	28/11/2019	-0.00001153623
09/11/2019	-0.00001156005	19/11/2019	-0.00001156196	29/11/2019	-0.00001150907

Normal distribution test for portfolio return was checked using the Kolmogorov-Smirnov test whose result showed that the portfolio return data had normal distribution at an error rate of 5%. The results are shown in Table 9.

Table 9 Kolmogorov-Smirnov normality test output

Statistics	Value
KS	0,19594
p-value	0.18490
Decision	H ₀ is accepted

The portfolio investment risk using the Monte Carlo simulation is presented in Table 10.

Table 10 VaR of some significant level

99%	97.5%	95%	90%
0.00003295745	0.0000329575	0.00003295758	0.00003295776

The results showed that the investment risk is very small and for a confidence level of 95%, the maximum possible losses of making investment in the portfolio consisting of BBKA, TLKM, and UNVR stocks were 0.00003295758 or 0.003295758% of the investments made in the following one day.

In addition to a very small risk, portfolio performance can be evaluated according to the Sharpe index. The mean portfolio return was 0.00115665 and the standard deviation of the portfolio was 0.00000000498. The risk-free interest rate based on the BI 7-day (Reverse) Repo Rate in November 2019 was 5% p.a. or 0.01389%, so the Sharpe index is

$$S_p = \frac{0.00115665 - 0.0001389}{0.0000000498} = \frac{0.00101775}{0.0000000498} = 20,436.75.$$

A positive value indicates that the portfolio returns had better returns than investments in risk-free interest rates. The results showed that the performance score was quite high, indicating that making investment in the portfolio of these three stocks promises profit with small risks.

5. Conclusions

The fact that there are many methods to predict stock prices required the researchers select the right method based on the characteristics of stock data. This study shows that modeling using 3-dimensional GBM results in accurate prediction results by considering low MAPE. BBKA, TLKM, and UNVR stocks with the right characteristics to build a portfolio can be proven by a very low VaR risk, which is equal to 0.003295758% of the investments made in the following one day with a confidence level of 95%. The weight of BBKA stocks is 0.000006%, that of TLKM stocks is 10.265098%, and that of UNVR stocks is 89.734895%, showing a very good portfolio performance, exceeding investments in risk-free interest rates. In this portfolio, UNVR stock contribution is dominant, while BBKA stock contribution is very low, allowing BBKA stocks to be excluded from this portfolio and only TLKM or UNVR stocks are included. However, it is necessary to carry out valuation first in order to determine whether the risk will be lower.

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