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Bayesian Estimation for the Scale Parameter of a Family of Lifetime Distributions under Different Priors

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Abstract

In this study, the Bayes estimators for the scale parameter of a family of lifetime distributions are considered under the assumptions of non-informative and conjugate priors. The uniform and inverted gamma priors are observed to obtain the posterior distribution for the scale parameter of this family of lifetime distributions. Considerations are given to three loss functions, namely, Squared Error Loss Function (SELF), Quadratic Loss Function (QLF) and Precautionary Loss Function (PLF). The performance of the estimator is assessed on the basis of its relative posterior risk. Markov Chain Monte Carlo (MCMC) Simulation Techniques are used to compare the performance of these estimators.

Keywords: Bayesian estimation, posterior risk, Monte Carlo simulation, family of lifetime distributions.

1. Introduction

In the present paper, we consider a family of lifetime distributions proposed by Chaturvedi and Rani (1997). Let the random variable X follows the family of lifetime distributions presented by the probabilistic model

$$f(x;\theta,a,b,c)=\frac{cx^{ac-1}e^{(-x^c/\theta^b)}}{\theta^{ab}\Gamma a};x>0,\quad a,b,c,\theta>0, \tag{1}$$

where $\theta^{b/c}$ is the scale parameter and a, b, c are the shape parameters. The model given at (1) covers various lifetime distributions as specific cases, for example, one-parameter exponential distribution, gamma distribution, generalized gamma distribution, erlang distribution, Weibull distribution, half-normal distribution, Rayleigh distribution, chi-square distribution and Maxwells failure distribution may obtain through assigning the different values of a, b and c.

In the Bayesian decision theory, loss function plays a very significant role in obtaining a good estimator. The performance of a Bayes estimator mainly depends upon the assumed prior distribution and loss function. In the decision theory, most widely used symmetric unbounded loss function is called the Squared Error Loss Function (SELF) proposed by Legendre (1805) and Gauss (1810). This loss function is suggested for the situations when overestimation and underestimation are given equal importance. Another more general type of loss function is the Quadratic Loss Function (QLF) (see Taguchi 1986). In those situations where overestimation is more serious than underestimation it is suggested to consider the asymmetric loss function in place of SELF (see, Ferguson 1967, Varian 1975, Berger 1980, Zellner 1986). Norstrom (1996) introduced an asymmetric loss function called Precautionary Loss Function (PLF). This loss function is very simple to use and a good alternative for other asymmetric loss functions.

The SELF is defined as

$$L(\theta, \theta_{SELF}) = (\theta - \theta_{SELF})^2.$$

The QLF is defined as

$$L(\theta, \theta_{QLF}) = \left(\frac{\theta - \theta_{QLF}}{\theta}\right)^2.$$

The PLF is defined as

$$L(\theta, \theta_{PLF}) = \frac{(\theta_{PLF} - \theta)^2}{\theta},$$

where θ_{SELF}, θ_{QLF} and θ_{PLF} are the Bayes estimate of the parameter θ for different loss functions.

The objective of this paper is to find the Bayes estimators and the Posterior risks for the scale parameter of the family of lifetime distributions under different loss functions and priors. Further, on utilizing theses Bayes estimators and posterior risks, for all the specific cases are obtained for the purpose of their comparisons. The paper is organized as: in Section 2, we obtain the posterior distributions in case of uniform and inverted gamma priors. Using these posterior distributions, the Bayes estimates are evaluated under the different loss functions i.e. SELF, QLF and PLF in Section 3. In Section 4, the posterior risk is deduced with the help of the Bayes estimators under different loss functions as mentioned in Section 3 for both the priors. In Section 5, a simulation study is done based on the deduced results and is demonstrated through Tables and Graphs. Section 6, contains the conclusion and a brief summary of the results.

2. Posterior Distributions under the Assumption of Different Priors

Posterior distributions for scale parameter of the family of lifetime distributions given at 1 are obtained, assuming the non-informative and informative priors.

Theorem 1 *Posterior distribution for the scale parameter $\theta^{b/c}$ using the uniform prior, which is defined as $P(\theta) = k$ (where k is constant) given by*

$$P(\theta|x) = \frac{b\theta^{-nab} e^{-\sum {x_i}^c/\theta^b} \left(\sum x_i\right)^{na - \dfrac{1}{b}}}{\Gamma\left(na - \dfrac{1}{b}\right)}, \tag{2}$$

where $x = (x_1, x_2, \ldots, x_n)$.

Proof: The likelihood function of the family of lifetime distribution is

$$\begin{aligned} L(\theta|x) &= \frac{c^n e^{(-\sum {x_i}^c/\theta^b)}}{\theta^{nab}\left(\Gamma a\right)^n} \prod_{i=1}^{n} x_i^{ac-1} \\ &\propto \theta^{-nab} e^{(-\sum {x_i}^c/\theta^b)}. \end{aligned}$$

Posterior distribution is obtained as

$$\begin{aligned} P(\theta|x) &= \frac{p(\theta)L(\theta|x)}{\int_0^\infty p(\theta)L(\theta|x)} \\ &= \frac{\theta^{-nab} e^{-\sum x_i^c/\theta^b}}{\int_0^\infty \theta^{-nab} e^{-\sum x_i^c/\theta^b} d\theta}. \end{aligned}$$

Thus, we get

$$P(\theta|x) = \frac{b\theta^{-nab} e^{-\sum x_i^c/\theta^b} \left(\sum x_i^c\right)^{na-\frac{1}{b}}}{\Gamma\left(na - \frac{1}{b}\right)} .$$

Hence, the theorem follows.

Theorem 2 *Posterior distribution of scale parameter $\theta^{b/c}$ using inverted gamma prior which is defined as $g(\theta) = \frac{b\tau^{v/b}\theta^{-(v+1)}e^{-\tau/\theta^b}}{\Gamma(v/b)}; \nu, \tau > 0$ given by*

$$P(\theta|x) = \frac{b\left(\tau + \sum x_i^c\right)^{na+\frac{v}{b}} \theta^{-(nab+v+1)} e^{-(\tau+\sum x_i^c)/\theta^b}}{\Gamma\left(na + \frac{v}{b}\right)} , \tag{3}$$

where $x = (x_1, x_2, ..., x_n)$.

Proof: Posterior distribution of scale parameter using inverted gamma prior is

$$P(\theta|x) = \frac{g(\theta)L(\theta|x)}{\int_0^\infty g(\theta)L(\theta|x)}$$

$$P(\theta|x) = \frac{\theta^{-(nab+v+1)} e^{-(\tau+\sum x_i^c)/\theta^b}}{\int_0^\infty \theta^{-(nab+v+1)} e^{-(\tau+\sum x_i^c)/\theta^b} d\theta} .$$

Thus, we get

$$P(\theta|x) = \frac{b\left(\tau + \sum x_i^c\right)^{na+\frac{v}{b}} \theta^{-(nab+v+1)} e^{-(\tau+\sum x_i^c)/\theta^b}}{\Gamma\left(na + \frac{v}{b}\right)} .$$

Hence, the theorem follows.

3. Bayesian Estimation under Three Loss Functions

Theorem 3 *Bayes estimator for the scale parameter under SELF using uniform prior is given*

$$\left\{\theta^{b/c}\right\}_{SELF} = \frac{\left(\sum {x_i}^c\right)^{1/c} \Gamma\left(na - \frac{1}{b} - \frac{1}{c}\right)}{\Gamma\left(na - \frac{1}{b}\right)} , \tag{4}$$

where $x = (x_1, x_2, ..., x_n)$.

Proof: The Bayes estimate under SELF can be obtained by using the expression given as follows

$$\left\{\theta^{b/c}\right\}_{SELF} = E\left(\theta^{b/c}|x\right)$$

$$E\left(\theta^{b/c}|x\right) = \int_0^\infty \theta^{b/c} P(\theta|x) d\theta. \tag{5}$$

Substituting the value of $P(\theta|x)$ from (2) in (5), we get

$$E\left(\theta^{b/c}|x\right) = \int_0^\infty \frac{b\theta^{b/c}\theta^{-nab}e^{-(\sum x_i^c)/\theta^b}\left(\sum x_i^c\right)^{na-\dfrac{1}{b}}}{\Gamma\left(na-\dfrac{1}{b}\right)}d\theta$$

$$\{\theta^{b/c}\}_{SELF} = \frac{(\sum {x_i}^c)^{1/c}\,\Gamma\left(na-\dfrac{1}{b}-\dfrac{1}{c}\right)}{\Gamma\left(na-\dfrac{1}{b}\right)}.$$

Hence, the theorem follows.

Theorem 4 *Bayes estimator for the scale parameter under SELF using the inverted gamma prior is given as*

$$\left\{\theta^{b/c}\right\}_{SELF} = \frac{(\tau+\sum {x_i}^c)^{1/c}\Gamma\left(na+\dfrac{\nu}{b}-\dfrac{1}{c}\right)}{\Gamma\left(na+\dfrac{\nu}{b}\right)}, \tag{6}$$

where $x=(x_1,x_2,...,x_n)$.

Proof: Bayes estimator for the scale parameter under SELF can be evaluated through the expression by substituting the value of $P(\theta|x)$ from (3) in (5), we get

$$\left\{\theta^{b/c}\right\}_{SELF} = \int_0^\infty \frac{b\theta^{b/c}\left(\tau+\sum x_i\right)^{na+\dfrac{v}{b}}\theta^{-(nab+v+1)}e^{-(\tau+\sum x_i)/\theta^b}d\theta}{\Gamma\left(na+\dfrac{v}{b}\right)}$$

$$\left\{\theta^{b/c}\right\}_{SELF} = \frac{(\tau+\sum {x_i}^c)^{1/c}\Gamma\left(na+\dfrac{\nu}{b}-\dfrac{1}{c}\right)}{\Gamma\left(na+\dfrac{\nu}{b}\right)}.$$

Hence, the theorem follows.

Theorem 5 *Bayes estimator of the scale parameter under QLF using uniform prior is given as*

$$\left\{\theta^{b/c}\right\}_{QLF} = \frac{(\sum {x_i}^c)^{1/c}\,\Gamma\left(na-\dfrac{1}{b}+\dfrac{1}{c}\right)}{\Gamma\left(na-\dfrac{1}{b}+\dfrac{2}{c}\right)},$$

where $x=(x_1,x_2,...,x_n)$.

Proof: The Bayes estimate of the scale parameter under QLF is given by

$$\left\{\theta^{b/c}\right\}_{QLF} = \frac{E\left(\theta^{-b/c}\right)}{E\left(\theta^{-2b/c}\right)}, \tag{7}$$

$$E\left(\theta^{-b/c}\right) = \int_0^\infty \theta^{-b/c}P(\theta|x)d\theta, \tag{8}$$

$$E\left(\theta^{-2b/c}\right) = \int_0^\infty \theta^{-2b/c} P(\theta|x) d\theta. \tag{9}$$

Substituting the value of $P(\theta|x)$ from (2) in (8) and (9), we get

$$E\left(\theta^{-b/c}\right) = \frac{\left(\sum x_i{}^c\right)^{-1/c} \Gamma\left(na - \frac{1}{b} + \frac{1}{c}\right)}{\Gamma\left(na - \frac{1}{b}\right)}, \tag{10}$$

$$E\left(\theta^{-2b/c}\right) = \frac{\left(\sum x_i{}^c\right)^{-2/c} \Gamma\left(na - \frac{1}{b} + \frac{2}{c}\right)}{\Gamma\left(na - \frac{1}{b}\right)}. \tag{11}$$

Substituting the values from (10) and (11) in (7), we get

$$\left\{\theta^{b/c}\right\}_{QLF} = \frac{\left(\sum x_i{}^c\right)^{1/c} \Gamma\left(na - \frac{1}{b} + \frac{1}{c}\right)}{\Gamma\left(na - \frac{1}{b} + \frac{2}{c}\right)}.$$

Hence, the theorem follows.

Theorem 6 *Bayes estimator of the scale parameter under QLF using inverted gamma prior is given as*

$$\left\{\theta^{b/c}\right\}_{QLF} = \frac{\left(\tau + \sum x_i{}^c\right)^{1/c} \Gamma\left(na + \frac{v}{b} + \frac{1}{c}\right)}{\Gamma\left(na + \frac{v}{b} + \frac{2}{c}\right)},$$

where $x = (x_1, x_2, ..., x_n)$.

Proof: To obtain the Bayes estimate of the scale parameter under QLF the expression in (7) is used and we have

$$E\left(\theta^{-b/c}\right) = \frac{\left(\tau + \sum x_i{}^c\right)^{-1/c} \Gamma\left(na + \frac{v}{b} + \frac{1}{c}\right)}{\Gamma\left(na + \frac{v}{b}\right)}, \tag{12}$$

$$E\left(\theta^{-2b/c}\right) = \frac{\left(\tau + \sum x_i{}^c\right)^{-2/c} \Gamma\left(na + \frac{v}{b} + \frac{2}{c}\right)}{\Gamma\left(na + \frac{v}{b}\right)}. \tag{13}$$

Substituting the values from (12) and (13) in (7), we get

$$\left\{\theta^{b/c}\right\}_{QLF} = \frac{\left(\tau + \sum x_i{}^c\right)^{1/c} \Gamma\left(na + \frac{v}{b} + \frac{1}{c}\right)}{\Gamma\left(na + \frac{v}{b} + \frac{2}{c}\right)}.$$

Hence, the theorem follows.

Theorem 7 *Bayes estimator for the scale parameter under PLF using uniform prior is given by*

$$\left\{\theta^{b/c}\right\}_{PLF} = \left\{\frac{\Gamma\left(na+\frac{1}{b}-\frac{2}{c}\right)}{\Gamma\left(na+\frac{1}{b}\right)}\right\}^{1/2} \left(\sum {x_i}^c\right)^{1/c}, \tag{14}$$

where $x = (x_1, x_2, ..., x_n)$.

Proof: Bayes estimator under PLF using uniform prior is computed as

$$\left\{\theta^{b/c}\right\}_{PLF} = \left\{E\left(\theta^{2b/c}|x\right)\right\}^{1/2} \tag{15}$$

$$E\left(\theta^{2b/c}|x\right) = \int_0^\infty \theta^{2b/c} P(\theta|x) d\theta \, . \tag{16}$$

Substituting the value of $P(\theta|x)$ from (2) in (16), we get

$$E\left(\theta^{2b/c}|x\right) = \left\{\frac{\Gamma\left(na+\frac{1}{b}-\frac{2}{c}\right)}{\Gamma\left(na+\frac{1}{b}\right)}\right\} \left(\sum {x_i}^c\right)^{2/c} . \tag{17}$$

Substituting the value from (17) in (15), we get

$$\left\{\theta^{b/c}\right\}_{PLF} = \left\{\frac{\Gamma\left(na+\frac{1}{b}-\frac{2}{c}\right)}{\Gamma\left(na+\frac{1}{b}\right)}\right\}^{1/2} \left(\sum {x_i}^c\right)^{1/c} .$$

Hence, the theorem follows.

Theorem 8 *Bayes estimator under PLF using inverted gamma prior is given by*

$$\left\{\theta^{b/c}\right\}_{PLF} = \left\{\frac{\Gamma\left(na+\frac{v}{b}-\frac{2}{c}\right)}{\Gamma\left(na+\frac{v}{b}\right)}\right\}^{1/2} \left(\tau+\sum {x_i}^c\right)^{1/c}, \tag{18}$$

where $x = (x_1, x_2, ..., x_n)$.

Proof: Bayes estimator under PLF using inverted gamma prior is computed from (15), we have

$$E\left(\theta^{2b/c}|x\right) = \left\{\frac{\Gamma\left(na+\frac{v}{b}-\frac{2}{c}\right)}{\Gamma\left(na+\frac{v}{b}\right)}\right\} \left(\tau+\sum {x_i}^c\right)^{2/c} . \tag{19}$$

Substituting the value from (19) in (15), we get

$$\left\{\theta^{b/c}\right\}_{PLF} = \left\{\frac{\Gamma\left(na+\frac{v}{b}-\frac{2}{c}\right)}{\Gamma\left(na+\frac{v}{b}\right)}\right\}^{1/2} \left(\tau+\sum {x_i}^c\right)^{1/c} .$$

Hence, the theorem follows.

4. Posterior Risks under Different Loss Functions

Considering both the priors for the scale parameter of the family of lifetime distributions and after obtaining the Bayes estimators under different loss functions, the posterior risks are obtained under the assumptions of SELF, QLF and PLF.

4.1. Posterior risk of Bayes estimator under different loss functions using uniform prior

I. Under SELF, the expression for the posterior risk of the Bayes estimator is

$$P\left[\left\{\theta^{b/c}\right\}_{SELF}\right] = E\left(\theta^{2b/c}|x\right) - \left\{E\left(\theta^{b/c}|x\right)\right\}^2. \tag{20}$$

Substituting the values from (4) and (17) in (20), we get

$$P\left[\left\{\theta^{b/c}\right\}_{SELF}\right] = \left\{\Gamma\left(na - \frac{1}{b} - \frac{2}{c}\right) - \frac{\left(\Gamma\left(na - \frac{1}{b} - \frac{1}{c}\right)\right)^2}{\Gamma\left(na - \frac{1}{b}\right)}\right\} \frac{\left(\sum x_i{}^c\right)^{2/c}}{\Gamma\left(na - \frac{1}{b}\right)}.$$

II. Under QLF, the expression for the posterior risk of the Bayes estimator is

$$P\left[\left\{\theta^{b/c}\right\}_{QLF}\right] = 1 - \frac{\left\{E\left(\theta^{-b/c}|x\right)\right\}^2}{E\left(\theta^{-2b/c}|x\right)}. \tag{21}$$

On utilizing the values from (19) and (11) in (21), we get

$$P\left[\left\{\theta^{b/c}\right\}_{QLF}\right] = 1 - \left\{\frac{\left(\Gamma\left(na - \frac{1}{b} + \frac{1}{c}\right)\right)^2}{\Gamma\left(na - \frac{1}{b}\right)\Gamma\left(na - \frac{1}{b} + \frac{2}{c}\right)}\right\}.$$

III. Under PLF, the expression for the posterior risk of the Bayes estimator is

$$P\left[\left\{\theta^{b/c}\right\}_{PLF}\right] = 2\left\{\left\{\theta^{b/c}\right\}_{PLF} - E\left(\theta^{b/c}|x\right)\right\}. \tag{22}$$

Substituting the values from (4) and (14) in (22), we get

$$P\left[\left\{\theta^{b/c}\right\}_{PLF}\right] = 2\left[\left\{\frac{\Gamma\left(na - \frac{1}{b} - \frac{2}{c}\right)}{\Gamma\left(na - \frac{1}{b}\right)}\right\}^{1/2} \left(\sum x_i{}^c\right)^{1/c} - \frac{\Gamma\left(na - \frac{1}{b} - \frac{1}{c}\right)}{\Gamma\left(na - \frac{1}{b}\right)} \left(\sum x_i{}^c\right)^{1/c}\right],$$

where $x = (x_1, x_2, ..., x_n)$.

4.2. Posterior risk of Bayes estimator under different loss functions using inverted gamma prior

I. Under SELF, utilising the results from (6) and (19) in (20), we get

$$P\left[\left\{\theta^{b/c}\right\}_{SELF}\right] = \left\{\Gamma\left(na + \frac{v}{b} - \frac{2}{c}\right) - \frac{\left(\Gamma\left(na + \frac{v}{b} - \frac{1}{c}\right)\right)^2}{\Gamma\left(na + \frac{v}{b}\right)}\right\} \frac{\left(\tau + \sum x_i{}^c\right)^{2/c}}{\Gamma\left(na + \frac{v}{b}\right)}. \tag{23}$$

II. Under(QLF), utilising the results from (12) and (13) in (21), we get

$$P\left[\left\{\theta^{b/c}\right\}_{QLF}\right]=1-\left\{\frac{\left(\Gamma\left(na+\frac{v}{b}+\frac{1}{c}\right)\right)^{2}}{\Gamma\left(na+\frac{v}{b}\right)\Gamma\left(na+\frac{v}{b}+\frac{2}{c}\right)}\right\}.$$

III. Under PLF, utilising the results from (6) and (18) in (22), we get

$$P\left[\left\{\theta^{b/c}\right\}_{PLF}\right]=2\left[\left\{\frac{\Gamma\left(na+\frac{v}{b}-\frac{2}{c}\right)}{\Gamma\left(na+\frac{v}{b}\right)}\right\}^{1/2}-\frac{\Gamma\left(na+\frac{v}{b}-\frac{1}{c}\right)}{\Gamma\left(na+\frac{v}{b}\right)}\right]\left(\tau+\sum x_i{}^{c}\right)^{1/c},$$

where, $x=(x_1,x_2...x_n)$.

5. Simulation Study

To generate the random numbers for Model 1, we fail to use the method of inverse transformation since the cdf is not in a closed form, therefore another method called the accept-reject method is used. In this method, we only need to know the functional form of the probability density function $f(x)$, which is known as the target density. For this target density $f(x)$, a simpler density $g(x)$ for simulation called the instrumental density or candidate density is developed. Two constraints are imposed on the candidate density $g(x)$, firstly, $f(x)$ and $g(x)$ have compatible support (i.e. $g(x)>0$, when $f(x)>0$); secondly, there must exist a constant M with $f(x)/g(x)\geq M,\forall x$, called the normalising constant. R software is used for the purpose of computational work and simulation is done through MCMC techniques. The behaviour of the instrumental density and the target density is shown in Figure 1.

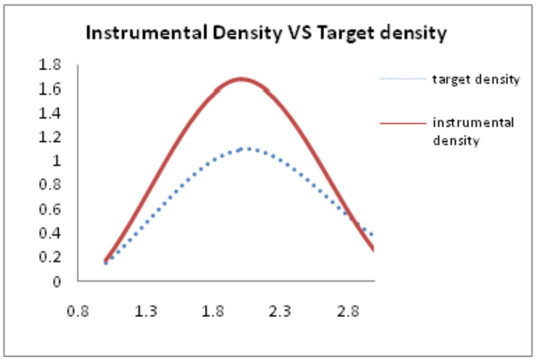

Figure 1 Plots of Instrumental density and Target density

We have generated 5,000 random numbers from Model 1, further, random numbers for all the specific distributions are obtained for different values of parameter θ and a, b and c. The Bayesian Estimates and posterior risks under 10,000 replications and averages of the 10,000 outputs so produced are presented in tables and graphs.

Table 1 Bayes estimator (posterior risk) for one-parameter exponential distribution ($a = b = c = 1$)

		Uniform Prior		
θ	n	SELF	QLF	PLF
0.5	10	0.616334 (0.059599)	0.493067 (0.100000)	0.658889 (0.085109)
	20	0.549284 (0.018589)	0.494355 (0.050000)	0.565208 (0.031849)
	35	0.525709 (0.008877)	0.495668 (0.028571)	0.533859 (0.016302)
	40	0.519791 (0.007485)	0.493802 (0.025000)	0.526769 (0.013955)
	50	0.515860 (0.005773)	0.495226 (0.020000)	0.521319 (0.010918)
1.0	10	1.263462 (0.251220)	1.010770 (0.100000)	1.350698 (0.174471)
	20	1.126900 (0.078401)	1.014210 (0.050000)	1.159570 (0.065341)
	35	1.075154 (0.037189)	1.013716 (0.028571)	1.091824 (0.033340)
	40	1.069618 (0.031685)	1.016137 (0.025000)	1.083976 (0.028716)
	50	1.056978 (0.024239)	1.014699 (0.020000)	1.068163 (0.022370)
1.5	10	1.909606 (0.571681)	1.527685 (0.100000)	2.041455 (0.263697)
	20	1.698386 (0.178077)	1.528547 (0.050000)	1.747624 (0.098478)
	35	1.618531 (0.084189)	1.526043 (0.028571)	1.643626 (0.050189)
	40	1.604996 (0.071344)	1.524746 (0.025000)	1.626541 (0.043089)
	50	1.590025 (0.054837)	1.526424 (0.020000)	1.606851 (0.033652)
2.0	10	2.508897 (0.985467)	2.007117 (0.100000)	2.682123 (0.346453)
	20	2.235502 (0.308219)	2.011951 (0.050000)	2.300312 (0.129621)
	35	2.130332 (0.145832)	2.008599 (0.028571)	2.163362 (0.066061)
	40	2.114622 (0.123849)	2.008891 (0.025000)	2.143007 (0.056771)
	50	2.093400 (0.095750)	2.009664 (0.020000)	2.115553 (0.044306)
		Inverted Gamma Prior		
θ	n	SELF	QLF	PLF
0.5	10	0.559675 (0.039803)	0.462339 (0.086957)	0.591681 (0.064014)
	20	0.522054 (0.015394)	0.473491 (0.046512)	0.535978 (0.027847)
	35	0.506421 (0.007860)	0.478672 (0.027397)	0.513924 (0.015006)
	40	0.502808 (0.006719)	0.478576 (0.024096)	0.509296 (0.012976)
	50	0.499151 (0.005233)	0.479767 (0.019417)	0.504271 (0.010239)
1.0	10	1.092519 (0.153092)	0.902515 (0.086957)	1.154998 (0.124959)
	20	1.038407 (0.060973)	0.941811 (0.046512)	1.066103 (0.055391)
	35	1.014651 (0.031582)	0.959053 (0.027397)	1.029684 (0.030065)
	40	1.014259 (0.027364)	0.965379 (0.024096)	1.027347 (0.026175)
	50	1.004832 (0.021217)	0.965809 (0.019417)	1.015138 (0.020617)
1.5	10	0.756853 (0.073398)	0.625226 (0.086956)	0.800136 (0.086566)
	20	0.708624 (0.028428)	0.642706 (0.046512)	0.727524 (0.037799)
	35	0.692715 (0.014717)	0.654758 (0.027397)	0.702978 (0.020526)
	40	0.690960 (0.012708)	0.657661 (0.024096)	0.699876 (0.017832)
	50	0.686315 (0.009905)	0.659662 (0.019417)	0.693354 (0.014079)
2.0	10	0.577460 (0.042439)	0.477033 (0.086956)	0.610484 (0.066047)
	20	0.537899 (0.016341)	0.487862 (0.046512)	0.552246 (0.028693)
	35	0.518565 (0.008245)	0.490150 (0.027397)	0.526248 (0.015366)
	40	0.517110 (0.007109)	0.492189 (0.024096)	0.523783 (0.013345)
	50	0.513094 (0.005533)	0.493168 (0.019417)	0.513095 (0.010525)

Table 2 Bayes estimator (posterior risk) for gamma distribution ($b = c = 1$)

θ	n	SELF	QLF	PLF
Uniform Prior				
0.5	10	0.619987 (0.060192)	0.495989 (0.100000)	0.662794 (0.085614)
	20	0.550574 (0.018701)	0.495516 (0.050000)	0.566535 (0.031924)
	35	0.523382 (0.008798)	0.493474 (0.028571)	0.531497 (0.016229)
	40	0.520069 (0.007488)	0.494066 (0.025000)	0.527051 (0.013962)
	50	0.516158 (0.005778)	0.495512 (0.020000)	0.521620 (0.010924)
1.0	10	1.258947 (0.248857)	1.007157 (0.100000)	1.345871 (0.173848)
	20	1.123378 (0.077946)	1.011040 (0.050000)	1.155946 (0.065136)
	35	1.074761 (0.037124)	1.013346 (0.028571)	1.091425 (0.033327)
	40	1.063472 (0.031331)	1.010298 (0.025000)	1.077747 (0.028551)
	50	1.055541 (0.024181)	1.013319 (0.020000)	1.066711 (0.022341)
1.5	10	1.899472 (0.566154)	1.519578 (0.100000)	2.030621 (0.262298)
	20	1.699482 (0.178351)	1.529534 (0.050000)	1.748752 (0.098541)
	35	1.612568 (0.083570)	1.520421 (0.028571)	1.637571 (0.050005)
	40	1.599385 (0.070872)	1.519416 (0.025000)	1.620854 (0.042938)
	50	1.589286 (0.054817)	1.525714 (0.020000)	1.606104 (0.033636)
2.0	10	2.528018 (0.989834)	2.022415 (0.100000)	2.702565 (0.349094)
	20	2.233305 (0.307941)	2.009974 (0.050000)	2.298051 (0.129494)
	35	2.128148 (0.145475)	2.006539 (0.028571)	2.161144 (0.065993)
	40	2.113259 (0.123618)	2.007596 (0.025000)	2.141626 (0.056734)
	50	2.092234 (0.095005)	2.008544 (0.020000)	2.114374 (0.044281)
Inverted Gamma Prior				
θ	n	SELF	QLF	PLF
0.5	10	0.565307 (0.040675)	0.466993 (0.086956)	0.597636 (0.064658)
	20	0.523231 (0.015455)	0.474558 (0.046512)	0.537186 (0.027910)
	35	0.507180 (0.007883)	0.479389 (0.027397)	0.514694 (0.015028)
	40	0.503008 (0.006724)	0.478767 (0.024097)	0.509499 (0.012981)
	50	0.499413 (0.005245)	0.480018 (0.019418)	0.019417 (0.010244)
1.0	10	1.097167 (0.154226)	0.906355 (0.086956)	1.159912 (0.125490)
	20	1.039202 (0.061116)	0.942531 (0.046512)	1.066919 (0.055433)
	35	1.019085 (0.031831)	0.963245 (0.027397)	1.034184 (0.030196)
	40	1.013597 (0.027325)	0.964748 (0.024096)	1.026676 (0.026158)
	50	0.971392 (0.021464)	0.971392 (0.019417)	1.021006 (0.020731)
1.5	10	0.760731 (0.074034)	0.628430 (0.086956)	0.804236 (0.087009)
	20	0.713280 (0.028788)	0.646928 (0.046511)	0.732304 (0.038048)
	35	0.692619 (0.014710)	0.654667 (0.027397)	0.702881 (0.020523)
	40	0.690799 (0.012692)	0.657507 (0.024096)	0.699713 (0.017828)
	50	0.685946 (0.009895)	0.659307 (0.019417)	0.692982 (0.014071)
2.0	10	0.577604 (0.042347)	0.477151 (0.086957)	0.610636 (0.066064)
	20	0.536252 (0.016244)	0.486368 (0.046512)	0.550555 (0.028605)
	35	0.519546 (0.008277)	0.491078 (0.027397)	0.527244 (0.015394)
	40	0.516446 (0.007095)	0.491557 (0.024097)	0.523110 (0.013328)
	50	0.511550 (0.005497)	0.491684 (0.019417)	0.516797 (0.010493)

Table 3 Bayes estimator (posterior risk) for generalised gamma distribution ($b = c = 1$)

		Uniform Prior		
θ	n	SELF	QLF	PLF
0.5	10	0.618316 (0.060048)	0.494652 (0.100000)	0.661007 (0.085383)
	20	0.550183 (0.018653)	0.495164 (0.050000)	0.566133 (0.031901)
	35	0.524468 (0.008829)	0.494498 (0.028571)	0.532599 (0.016263)
	40	0.520957 (0.007515)	0.494909 (0.025000)	0.527949 (0.013986)
	50	0.515889 (0.005772)	0.495254 (0.020000)	0.521349 (0.010919)
1.0	10	1.264785 (0.251754)	1.011828 (0.100000)	1.352112 (0.174654)
	20	1.122264 (0.077861)	1.010038 (0.050000)	1.154801 (0.065073)
	35	1.071760 (0.036958)	1.010516 (0.028571)	1.088377 (0.033234)
	40	1.067494 (0.031574)	1.014120 (0.025000)	1.081824 (0.028658)
	50	1.054250 (0.024118)	1.012080 (0.020000)	1.065407 (0.022313)
1.5	10	1.904788 (0.569712)	1.52383 (0.100000)	2.036304 (0.263032)
	20	1.691720 (0.176556)	1.522548 (0.050000)	1.740766 (0.098091)
	35	1.618215 (0.084095)	1.525745 (0.028571)	1.643305 (0.050181)
	40	1.601838(0.071091)	1.521746 (0.025000)	1.623340 (0.043004)
	50	1.587035 (0.054649)	1.523554 (0.020000)	1.603830 (0.033589)
2.0	10	2.518618 (0.995297)	2.014894 (0.100000)	2.692516 (0.347796)
	20	2.237152 (0.309052)	2.013437 (0.050000)	2.302011 (0.129717)
	35	2.129976 (0.145719)	2.008263 (0.028571)	2.163001 (0.066049)
	40	2.121500 (0.124607)	2.015425 (0.025000)	2.149977 (0.056956)
	50	2.091698 (0.094919)	2.008030 (0.020000)	2.113833 (0.044269)
		Inverted Gamma Prior		
θ	n	SELF	QLF	PLF
0.5	10	0.561705 (0.040110)	0.464017 (0.086957)	0.593828 (0.064246)
	20	0.523895 (0.015495)	0.475161 (0.046512)	0.537868 (0.027946)
	35	0.504856 (0.007816)	0.477193 (0.027397)	0.512336 (0.014959)
	40	0.502558 (0.006711)	0.478338 (0.024096)	0.509043 (0.012969)
	50	0.499059 (0.005231)	0.479675 (0.019417)	0.504175 (0.010237)
1.0	10	1.095272 (0.153718)	0.904789 (0.086956)	1.157908 (0.125273)
	20	1.043062 (0.061566)	0.946033 (0.046511)	1.070881 (0.055639)
	35	1.020020 (0.031897)	0.964129 (0.027397)	1.035133 (0.030224)
	40	1.013221 (0.027299)	0.964391 (0.024096)	1.026296 (0.026148)
	50	1.007588 (0.021339)	0.968458 (0.019417)	1.017922 (0.020669)
1.5	10	0.756921 (0.073290)	0.625282 (0.086956)	0.800207 (0.086574)
	20	0.714881 (0.028918)	0.648380 (0.046511)	0.733947 (0.038134)
	35	0.693802 (0.014768)	0.655786 (0.027397)	0.704082 (0.020558)
	40	0.689429 (0.012648)	0.656199 (0.024096)	0.698321 (0.017792)
	50	0.686690 (0.009913)	0.660022 (0.019417)	0.693733 (0.014086)
2.0	10	0.576835 (0.042367)	0.476516 (0.086956)	0.609824 (0.065976)
	20	0.536937 (0.016294)	0.486989 (0.046512)	0.551258 (0.028641)
	35	0.521229 (0.008327)	0.492669 (0.027397)	0.528952 (0.015444)
	40	0.517312 (0.007114)	0.492381 (0.024096)	0.523986 (0.013351)
	50	0.513402 (0.005538)	0.493464 (0.019417)	0.518667 (0.010532)

Table 4 Bayes estimator (posterior risk) for Erlang distribution ($a > 0, b = c = 1$)

θ	n	SELF	QLF	PLF
		Uniform Prior		
0.5	10	0.618993 (0.060134)	0.495195 (0.100000)	0.661732 (0.085477)
	20	0.548955 (0.018594)	0.494059 (0.050000)	0.564868 (0.031830)
	35	0.524356 (0.008834)	0.494393 (0.028571)	0.532486 (0.016260)
	40	0.520586 (0.007494)	0.494557 (0.025000)	0.527574 (0.013976)
	50	0.515081 (0.005756)	0.494483 (0.020000)	0.520537 (0.010901)
1.0	10	1.265817 (0.251529)	1.012654 (0.100000)	1.353215 (0.174797)
	20	1.123568 (0.077909)	1.011211 (0.050000)	1.156142 (0.065148)
	35	1.074347 (0.037087)	1.012955 (0.028571)	1.091004 (0.033315)
	40	1.065961 (0.031465)	1.012663 (0.025000)	1.080270 (0.028618)
	50	1.057226 (0.024266)	1.014937 (0.020000)	1.068414 (0.022376)
1.5	10	1.913619 (0.573794)	1.530895 (0.100000)	2.045745 (0.264252)
	20	1.697824 (0.177912)	1.528042 (0.050000)	1.747046 (0.098445)
	35	1.618863 (0.084234)	1.526357 (0.028571)	1.643963 (0.050200)
	40	1.602879 (0.071170)	1.522735 (0.025000)	1.624395 (0.043032)
	50	1.592441 (0.055025)	1.528743 (0.020000)	1.609292 (0.033703)
2.0	10	2.507497 (0.985658)	2.005998 (0.100000)	2.680627 (0.346260)
	20	2.243270 (0.310479)	2.018943 (0.050000)	2.308306 (0.130072)
	35	2.132715 (0.146059)	2.010846 (0.028571)	2.165783 (0.066135)
	40	2.116199 (0.123993)	2.010389 (0.025000)	2.144605 (0.056813)
	50	2.096589 (0.095379)	2.012725 (0.020000)	2.118775 (0.044373)
		Inverted Gamma Prior		
θ	n	SELF	QLF	PLF
0.5	10	0.561165 (0.040065)	0.463571 (0.086956)	0.593257 (0.064184)
	20	0.523645 (0.015506)	0.474934 (0.046512)	0.537611 (0.027932)
	35	0.505983 (0.007844)	0.478258 (0.027397)	0.513479 (0.014993)
	40	0.503124 (0.006729)	0.478877 (0.024096)	0.509616 (0.012985)
	50	0.498664 (0.005222)	0.479298 (0.019417)	0.503778 (0.010229)
1.0	10	1.089719 (0.152235)	0.900203 (0.086957)	1.152038 (0.124638)
	20	1.039060 (0.061074)	0.942403 (0.046512)	1.066774 (0.055426)
	35	1.014910 (0.031581)	0.959298 (0.027397)	1.029946 (0.030073)
	40	1.013759 (0.027345)	0.964901 (0.024096)	1.026840 (0.026162)
	50	1.008705 (0.021385)	0.969532 (0.019417)	1.019051 (0.020692)
1.5	10	0.754716 (0.072926)	0.623461 (0.086956)	0.797877 (0.086322)
	20	0.713766 (0.028839)	0.647369 (0.046512)	0.732803 (0.038074)
	35	0.692037 (0.014687)	0.654117 (0.027397)	0.702290 (0.020506)
	40	0.689773 (0.012655)	0.656531 (0.024096)	0.698673 (0.017801)
	50	0.686413 (0.009906)	0.659756 (0.019417)	0.693453 (0.014081)
2.0	10	0.577418 (0.042485)	0.476997 (0.086956)	0.610439 (0.066043)
	20	0.536998 (0.016281)	0.487045 (0.046512)	0.551320 (0.028645)
	35	0.522087 (0.008351)	0.493479 (0.027397)	0.529822 (0.015470)
	40	0.518337 (0.007143)	0.493357 (0.024096)	0.525026 (0.013377)
	50	0.513238 (0.005536)	0.493306 (0.019417)	0.518502 (0.010528)

Table 5 Bayes estimator (posterior risk) for Weibull distribution ($a = 1, b = c$)

		Uniform Prior		
θ	n	SELF	QLF	PLF
0.5	10	0.626289 (0.061418)	0.501031 (0.100000)	0.669531 (0.085947)
	20	0.559029 (0.019286)	0.503126 (0.050000)	0.575236 (0.031852)
	35	0.533863 (0.009165)	0.503357 (0.028571)	0.542141 (0.016257)
	40	0.531427 (0.007821)	0.504855 (0.025000)	0.538560 (0.013976)
	50	0.524947 (0.005979)	0.503949 (0.020000)	0.530502 (0.010909)
1.0	10	1.270088 (0.253769)	1.016070 (0.100000)	1.357781 (0.175386)
	20	1.129065 (0.078688)	1.016159 (0.050000)	1.161798 (0.065466)
	35	1.071738 (0.036919)	1.010496 (0.028571)	1.088355 (0.033234)
	40	1.064445 (0.031392)	1.011223 (0.025000)	1.078733 (0.028577)
	50	1.054733 (0.024143)	1.012544 (0.020000)	1.065895 (0.022323)
1.5	10	1.900357 (0.565738)	1.520285 (0.100000)	2.031567 (0.262420)
	20	1.698752 (0.178223)	1.528877 (0.050000)	1.748001 (0.098498)
	35	1.613077 (0.083599)	1.520901 (0.028571)	1.638087 (0.050021)
	40	1.611101 (0.071899)	1.530546 (0.025000)	1.632727 (0.043253)
	50	1.585802 (0.054568)	1.522369 (0.020000)	1.602583 (0.033563)
2.0	10	2.511191 (0.988524)	2.008953 (0.100000)	2.684576 (0.346770)
	20	2.227257 (0.306221)	2.004531 (0.050000)	2.291828 (0.129143)
	35	2.135355 (0.146505)	2.013335 (0.028571)	2.168463 (0.066216)
	40	2.113051 (0.123653)	2.007398 (0.025000)	2.141415 (0.056728)
	50	2.091700 (0.094916)	2.008032 (0.020000)	2.113835 (0.044270)
		Inverted Gamma Prior		
θ	n	SELF	QLF	PLF
0.5	10	0.577418 (0.042485)	0.476997 (0.086956)	0.610439 (0.066043)
	20	0.536997 (0.016281)	0.487045 (0.046512)	0.551320 (0.028645)
	35	0.522087 (0.008351)	0.493479 (0.027397)	0.529822 (0.015470)
	40	0.518337 (0.007143)	0.493357 (0.024096)	0.525026 (0.013377)
	50	0.513238 (0.005536)	0.493306 (0.019417)	0.518502 (0.010528)
1.0	10	0.577418 (0.042485)	0.476997 (0.086957)	0.610439 (0.066043)
	20	0.536998 (0.016281)	0.487045 (0.046512)	0.551320 (0.028645)
	35	0.522087 (0.008351)	0.493479 (0.027397)	0.529822 (0.015470)
	40	0.518337 (0.007143)	0.493357 (0.024096)	0.525026 (0.013377)
	50	0.513238 (0.005536)	0.493306 (0.019417)	0.518502 (0.010528)
1.5	10	0.755752 (0.073145)	0.624317 (0.086956)	0.798972 (0.086440)
	20	0.715454 (0.028941)	0.648901 (0.046512)	0.734537 (0.038164)
	35	0.693336 (0.014743)	0.655346 (0.027397)	0.703608 (0.020544)
	40	0.690731 (0.012689)	0.657442 (0.024096)	0.024096 (0.017826)
	50	0.687263 (0.009932)	0.660574 (0.019417)	0.694313 (0.014098)
2.0	10	0.579361 (0.042703)	0.478603 (0.086956)	0.612494 (0.066265)
	20	0.537892 (0.016351)	0.487852 (0.046512)	0.552238 (0.028693)
	35	0.521209 (0.008327)	0.492650 (0.027397)	0.528932 (0.015444)
	40	0.517716 (0.007125)	0.492765 (0.024096)	0.524396 (0.013361)
	50	0.513656 (0.005545)	0.493709 (0.019417)	0.518925 (0.010537)

Table 6 Bayes estimator (posterior risk) for half normal distribution ($a = 0.5, b = c = 2$)

		Uniform Prior		
θ	n	SELF	QLF	PLF
0.5	10	1.073432 (0.094389)	0.954162 (0.053934)	1.112330 (0.089864)
	20	1.022171 (0.032949)	0.968373 (0.025961)	1.037305 (0.048385)
	35	1.002159 (0.016344)	0.972684 (0.014596)	1.010018 (0.032816)
	40	1.000531 (0.014014)	0.974876 (0.012737)	1.007313 (0.030240)
	50	0.994868 (0.010834)	0.974564 (0.010152)	1.000173 (0.026329)
1.0	10	2.200836 (0.397029)	1.956299 (0.053934)	2.280587 (0.377996)
	20	2.114273 (0.141233)	2.002996 (0.025961)	2.145577 (0.207394)
	35	2.064155 (0.069378)	2.003444 (0.014596)	2.080341 (0.139298)
	40	2.065371 (0.059807)	2.012413 (0.012737)	2.079372 (0.129055)
	50	2.050667 (0.046079)	2.008817 (0.010151)	2.061603 (0.111985)
1.5	10	3.323679 (0.903468)	2.954381 (0.053933)	3.444118 (0.860157)
	20	3.162555 (0.315722)	2.996104 (0.025961)	3.209379 (0.463623)
	35	3.116065 (0.158107)	3.024416 (0.014596)	3.140501 (0.317450)
	40	3.093174 (0.133948)	3.013862 (0.012738)	3.114142 (0.289040)
	50	3.078275 (0.103819)	3.015453 (0.010151)	3.094691 (0.252309)
2.0	10	4.385883 (0.073584)	3.898563 (0.053933)	4.544813 (0.062984)
	20	4.164003 (0.030513)	3.944845 (0.025961)	4.225655 (0.027987)
	35	4.096360 (0.015996)	3.975878 (0.014596)	4.128482 (0.015217)
	40	4.075651 (0.013768)	3.971147 (0.012738)	4.103279 (0.013190)
	50	4.064293 (0.010836)	3.981348 (0.010151)	4.085967 (0.010456)
		Inverted Gamma Prior		
θ	n	SELF	QLF	PLF
0.5	10	1.024350 (0.069039)	0.926793 (0.046435)	1.054855 (0.061009)
	20	0.992198 (0.028350)	0.943798 (0.024086)	1.005690 (0.026985)
	35	0.976269 (0.014783)	0.948769 (0.013984)	0.983581 (0.014624)
	40	0.976240 (0.012798)	0.952135 (0.012269)	0.982599 (0.012718)
	50	0.974807 (0.010063)	0.955503 (0.009851)	0.979844 (0.010075)
1.0	10	1.998831 (0.266664)	1.808466 (0.046435)	2.058355 (0.119049)
	20	1.981259 (0.113307)	1.884612 (0.024086)	2.008201 (0.053885)
	35	1.969730 (0.060194)	1.914245 (0.013984)	1.984482 (0.029505)
	40	1.971454 (0.052176)	1.922776 (0.012268)	1.984296 (0.025683)
	50	1.973194 (0.041236)	1.934121 (0.009852)	1.983390 (0.020393)
1.5	10	1.384587 (0.127602)	1.252722 (0.046435)	1.425820 (0.082465)
	20	1.361242 (0.053567)	1.294840 (0.024085)	1.379753 (0.037022)
	35	1.346935 (0.028175)	1.308994 (0.013984)	1.357023 (0.020175)
	40	1.351357 (0.024556)	1.317990 (0.012268)	1.360160 (0.017605)
	50	1.351143 (0.019352)	1.324388 (0.009851)	1.358125 (0.013964)
2.0	10	1.051952 (0.072826)	0.951765 (0.046435)	1.083279 (0.062654)
	20	1.018381 (0.029846)	0.968704 (0.024085)	1.032230 (0.027697)
	35	1.004142 (0.015635)	0.975856 (0.013984)	1.011663 (0.015041)
	40	1.007066 (0.013621)	0.982200 (0.012269)	1.013626 (0.013119)
	50	1.003789 (0.010673)	0.983912 (0.009852)	1.008977 (0.010374)

Table 7 Bayes estimator (posterior risk) for Rayleigh distribution ($a = 1, b = 1, c = 2$)

			Uniform Prior	
θ	n	SELF	QLF	PLF
0.5	10	0.721679 (0.018394)	0.681586 (0.027381)	0.733036 (0.026217)
	20	0.707218 (0.007393)	0.688607 (0.013070)	0.712146 (0.015739)
	35	0.699225 (0.003841)	0.688942 (0.007325)	0.701878 (0.011055)
	40	0.700078 (0.003332)	0.691102 (0.006389)	0.702384 (0.010288)
	50	0.697341 (0.002598)	0.690225 (0.005088)	0.699159 (0.009014)
1.0	10	1.494524 (0.078921)	1.411495 (0.027381)	1.518044 (0.112481)
	20	1.463518 (0.031692)	1.425004 (0.013070)	1.473715 (0.067462)
	35	1.446449 (0.016462)	1.425177 (0.007325)	1.451938 (0.047373)
	40	1.441123 (0.014120)	1.422647 (0.006389)	1.445871 (0.043594)
	50	1.443232 (0.011143)	1.428505 (0.005088)	1.446996 (0.038651)
1.5	10	2.238633 (0.176700)	2.114265 (0.027381)	2.273863 (0.251842)
	20	2.191208 (0.071031)	2.133544 (0.013070)	2.206476 (0.151202)
	35	2.169087 (0.036993)	2.137189 (0.007325)	2.177318 (0.106455)
	40	2.168755 (0.031968)	2.140951 (0.006389)	2.175901 (0.098697)
	50	2.164156 (0.025040)	2.142073 (0.005088)	2.169799 (0.086854)
2.0	10	2.972134 (0.311048)	2.807015 (0.027382)	3.018908 (0.443323)
	20	2.886929 (0.122903)	2.810957 (0.013070)	2.907044 (0.261622)
	35	2.858461 (0.064157)	2.816425 (0.007325)	2.869309 (0.184622)
	40	2.848495 (0.055123)	2.811976 (0.006389)	2.857880 (0.170181)
	50	2.846554 (0.043292)	2.817507 (0.005089)	2.853976 (0.150165)
			Inverted Gamma Prior	
θ	n	SELF	QLF	PLF
0.5	10	0.696578 (0.014050)	0.663408 (0.023519)	0.705800 (0.018443)
	20	0.684935 (0.006358)	0.668229 (0.012119)	0.689339 (0.008808)
	35	0.682998 (0.003496)	0.673378 (0.007017)	0.685474 (0.004958)
	40	0.683962 (0.003052)	0.675518 (0.006153)	0.686131 (0.004335)
	50	0.684149 (0.002422)	0.677375 (0.004938)	0.685878 (0.003459)
1.0	10	1.361886 (0.054515)	1.297035 (0.023519)	1.379916 (0.036059)
	20	1.374044 (0.025662)	1.340531 (0.012119)	1.382879 (0.017670)
	35	1.380158 (0.014286)	1.360719 (0.007017)	1.385167 (0.010018)
	40	1.382307 (0.012472)	1.365242 (0.006153)	1.386689 (0.008762)
	50	1.384185 (0.009912)	1.370480 (0.004938)	1.387685 (0.006999)
1.5	10	0.940943 (0.025952)	0.896136 (0.023519)	0.953405 (0.024913)
	20	0.941939 (0.012086)	0.918965 (0.012119)	0.947991 (0.012113)
	35	0.942955 (0.006680)	0.929674 (0.007017)	0.946378 (0.006845)
	40	0.947493 (0.005866)	0.935796 (0.006153)	0.950496 (0.006006)
	50	0.946113 (0.004636)	0.936745 (0.004938)	0.948505 (0.004784)
2.0	10	0.715841 (0.014894)	0.681753 (0.023519)	0.725317 (0.018953)
	20	0.706655 (0.006775)	0.689419 (0.012119)	0.711198 (0.009087)
	35	0.706401 (0.003745)	0.696452 (0.007017)	0.708965 (0.005127)
	40	0.706102 (0.003252)	0.697384 (0.006153)	0.708339 (0.004475)
	50	0.703006 (0.002558)	0.696045 (0.004938)	0.704783 (0.003554)

Table 8 Bayes estimator (posterior risk) for chi-square distribution ($a = 0.5, b = 1, c = 2$)

		Uniform Prior		
θ	n	SELF	QLF	PLF
0.5	10	1.150475 (0.127288)	1.006665 (0.060437)	1.199199 (0.112520)
	20	1.049248 (0.036951)	0.990956 (0.027382)	1.065760 (0.052666)
	35	1.015573 (0.017334)	0.984798 (0.015035)	1.023795 (0.034261)
	40	1.017079 (0.014875)	0.990313 (0.013070)	1.024165 (0.031664)
	50	1.006147 (0.011321)	0.985185 (0.010361)	1.011629 (0.027221)
1.0	10	2.375528 (0.543602)	2.078587 (0.060437)	2.476136 (0.480536)
	20	2.167816 (0.157839)	2.047382 (0.027382)	2.201932 (0.224961)
	35	2.106973 (0.074612)	2.043126 (0.015035)	2.124031 (0.147466)
	40	2.094754 (0.063211)	2.039629 (0.013070)	2.109349 (0.134558)
	50	2.075192 (0.048202)	2.031959 (0.010361)	2.086501 (0.115899)
1.5	10	3.587984 (1.237248)	3.139486 (0.060436)	3.739942 (1.093707)
	20	3.261786 (0.357243)	3.080576 (0.027381)	3.313118 (0.509163)
	35	3.150174 (0.166696)	3.054714 (0.015035)	3.175677 (0.329465)
	40	3.134777 (0.141433)	3.052283 (0.013070)	3.156619 (0.301067)
	50	3.118812 (0.108882)	3.053836 (0.010361)	3.135806 (0.261800)
2.0	10	4.725959 (2.143402)	4.135214 (0.060437)	4.926113 (1.894733)
	20	4.294922 (0.618901)	4.056315 (0.027381)	4.362513 (0.882092)
	35	4.157921 (0.290307)	4.031923 (0.015035)	4.191582 (0.573773)
	40	4.133677 (0.245742)	4.024896 (0.013071)	4.162479 (0.523107)
	50	4.096936 (0.187645)	4.011583 (0.010362)	4.119261 (0.451180)
		Inverted Gamma Prior		
θ	n	SELF	QLF	PLF
0.5	10	0.995774 (0.061534)	0.905249 (0.044377)	1.023765 (0.055981)
	20	0.978673 (0.026840)	0.932069 (0.023519)	0.991629 (0.025913)
	35	0.970899 (0.014405)	0.943930 (0.013791)	0.978063 (0.014328)
	40	0.970313 (0.012485)	0.946647 (0.012119)	0.976553 (0.012478)
	50	0.968854 (0.009841)	0.949857 (0.009755)	0.973810 (0.009911)
1.0	10	1.943938 (0.237183)	1.767216 (0.044377)	1.998580 (0.109285)
	20	1.957754 (0.107572)	1.864527 (0.023515)	1.983672 (0.051836)
	35	1.956865 (0.058506)	1.902508 (0.013791)	1.971305 (0.028879)
	40	1.957759 (0.050825)	1.910009 (0.012119)	1.970348 (0.025177)
	50	1.962409 (0.040354)	1.923931 (0.009755)	1.972447 (0.020074)
1.5	10	1.345106 (0.113378)	1.222824 (0.044377)	1.382916 (0.075619)
	20	1.335297 (0.050126)	1.271711 (0.023519)	1.352975 (0.035355)
	35	1.342544 (0.027571)	1.305251 (0.013791)	1.352450 (0.019813)
	40	1.343104 (0.023942)	1.310345 (0.012119)	1.351740 (0.017272)
	50	1.338646 (0.018791)	1.312398 (0.009755)	1.345493 (0.013693)
2.0	10	1.029682 (0.065831)	0.936074 (0.044377)	1.058625 (0.057887)
	20	1.006171 (0.028363)	0.958258 (0.023519)	1.019491 (0.026640)
	35	1.000004 (0.015283)	0.972226 (0.013791)	1.007383 (0.014758)
	40	0.999284 (0.013245)	0.974911 (0.012119)	1.005710 (0.012851)
	50	0.996819 (0.010416)	0.977274 (0.009755)	1.001918 (0.010196)

Table 9 Bayes estimator (posterior risk) for Maxwells failure distribution ($a = 1.5, b = 1, c = 2$)

		Uniform Prior		
θ	n	SELF	QLF	PLF
0.5	10	0.571435 (0.007049)	0.551026 (0.017693)	0.576954 (0.012769)
	20	0.568476 (0.003070)	0.558675 (0.008583)	0.571020 (0.008141)
	35	0.567385 (0.001649)	0.561876 (0.004842)	0.568791 (0.005868)
	40	0.567322 (0.001430)	0.562510 (0.004228)	0.568546 (0.005454)
	50	0.566442 (0.001126)	0.562515 (0.003372)	0.567413 (0.004816)
1.0	10	1.175675 (0.029861)	1.133687 (0.017695)	1.187031 (0.054092)
	20	1.171068 (0.013015)	1.150878 (0.008583)	1.176308 (0.034514)
	35	1.173134 (0.007071)	1.161744 (0.004842)	1.176041 (0.025156)
	40	1.169028 (0.006083)	1.159121 (0.004228)	1.171551 (0.023187)
	50	1.172273 (0.004830)	1.164352 (0.003373)	1.174282 (0.020652)
1.5	10	1.769124 (0.067526)	1.705941 (0.017694)	1.786212 (0.122319)
	20	1.761770 (0.029504)	1.731395 (0.008583)	1.769653 (0.078237)
	35	1.751380 (0.015737)	1.734376 (0.004842)	1.755720 (0.055986)
	40	1.758235 (0.013762)	1.743335 (0.004228)	1.762028 (0.052463)
	50	1.756130 (0.010830)	1.744265 (0.003372)	1.759140 (0.046306)
2.0	10	2.343734 (0.118174)	2.260029 (0.017694)	2.366373 (0.214066)
	20	2.316804 (0.050913)	2.276860 (0.008583)	2.327170 (0.135006)
	35	2.311144 (0.027385)	2.288705 (0.004843)	2.316871 (0.097420)
	40	2.313503 (0.023771)	2.293897 (0.004228)	2.318494 (0.090610)
	50	2.306405 (0.018671)	2.290821 (0.003372)	2.310357 (0.079830)
		Inverted Gamma Prior		
θ	n	SELF	QLF	PLF
0.5	10	0.564553 (0.006031)	0.546341 (0.015990)	0.569440 (0.009773)
	20	0.561251 (0.002815)	0.552050 (0.008162)	0.563634 (0.004766)
	35	0.557401 (0.001542)	0.552142 (0.004705)	0.558742 (0.002682)
	40	0.555047 (0.001332)	0.550460 (0.004123)	0.556214 (0.002334)
	50	0.556666 (0.001064)	0.552980 (0.003305)	0.557601 (0.001869)
1.0	10	1.102664 (0.023307)	1.067095 (0.015996)	1.112209 (0.019089)
	20	1.120704 (0.011269)	1.102332 (0.008165)	1.125463 (0.009517)
	35	1.126075 (0.006304)	1.115452 (0.004706)	1.128786 (0.005420)
	40	1.128644 (0.005512)	1.119317 (0.004123)	1.131018 (0.004747)
	50	1.128929 (0.004377)	1.121453 (0.003306)	1.130825 (0.003791)
1.5	10	0.765775 (0.011220)	0.741073 (0.015997)	0.772404 (0.013257)
	20	0.768474 (0.005302)	0.755876 (0.008162)	0.771737 (0.006521)
	35	0.768564 (0.002938)	0.761314 (0.004705)	0.770414 (0.003699)
	40	0.770561 (0.002572)	0.764193 (0.004123)	0.772182 (0.003241)
	50	0.772728 (0.002053)	0.767610 (0.003305)	0.774025 (0.002595)
2.0	10	0.582261 (0.006416)	0.563478 (0.015997)	0.587301 (0.010080)
	20	0.576829 (0.002979)	0.567373 (0.008163)	0.579279 (0.004899)
	35	0.575355 (0.001645)	0.569927 (0.004706)	0.576740 (0.002769)
	40	0.574294 (0.001426)	0.569548 (0.004124)	0.575502 (0.002415)
	50	0.574748 (0.001134)	0.570942 (0.003306)	0.575714 (0.001930)

Table 10 Minimum and maximum values of the posterior risk for different loss function in case of uniform prior

Distribution	$\theta = 0.5$ min	$\theta = 0.5$ max	$\theta = 1.0$ min	$\theta = 1.0$ max	$\theta = 1.5$ min	$\theta = 1.5$ max	$\theta = 2.0$ min	$\theta = 2.0$ max
One-parameter exponential	SELF	QLF	QLF	SELF	QLF	SELF	QLF	SELF
Gamma	SELF	QLF	QLF	SELF	QLF	SELF	QLF	SELF
Generalised gamma	SELF	QLF	QLF	SELF	QLF	SELF	QLF	SELF
Erlang	SELF	QLF	QLF	SELF	QLF	SELF	QLF	SELF
Weibull	SELF	QLF	QLF	SELF	QLF	SELF	QLF	SELF
Half normal	SELF	QLF	QLF	SELF	QLF	SELF	QLF	SELF
Rayleigh	SELF	QLF	QLF	PLF	QLF	PLF	QLF	PLF
Chi-square	QLF	SELF	QLF	SELF	QLF	SELF	QLF	SELF
Maxwell Failure	SELF	QLF	QLF	PLF	QLF	PLF	QLF	PLF

Table 11 Minimum and maximum values of the posterior risk for different loss function in case of inverted gamma prior

Distribution	$\theta = 0.5$ min	$\theta = 0.5$ max	$\theta = 1.0$ min	$\theta = 1.0$ max	$\theta = 1.5$ min	$\theta = 1.5$ max	$\theta = 2.0$ min	$\theta = 2.0$ max
One-parameter exponential	SELF	QLF	QLF	SELF	SELF	QLF	SELF	QLF
Gamma	SELF	QLF	QLF	SELF	SELF	PLF	SELF	QLF
Generalized gamma	SELF	QLF	QLF	SELF	SELF	QLF	SELF	QLF
Erlang	SELF	QLF	QLF	SELF	SELF	QLF	SELF	QLF
Weibull	SELF	QLF	SELF	QLF	SELF	QLF	SELF	QLF
Half normal	QLF	SELF	QLF	SELF	QLF	SELF	QLF	SELF
Rayleigh	SELF	QLF	QLF	SELF	QLF	SELF	SELF	QLF
Chi-square	QLF	SELF	QLF	SELF	QLF	SELF	QLF	SELF
Maxwells Failure	SELF	QLF	QLF	SELF	SELF	QLF	SELF	QLF

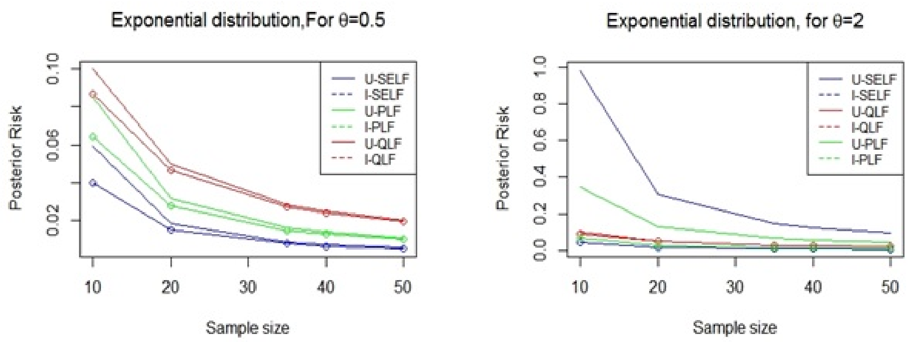

Figure 2 Exponential distribution

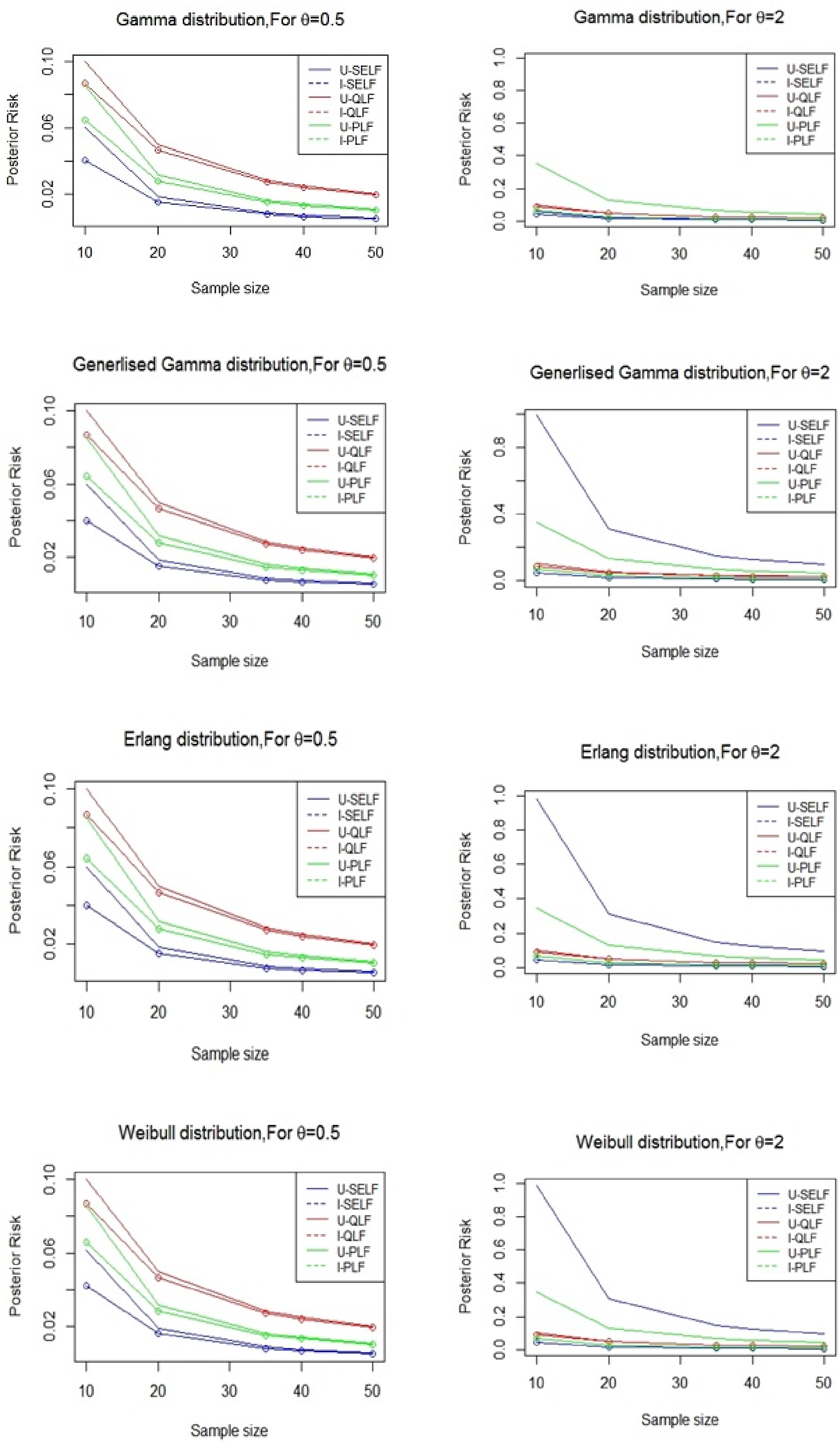

Figure 3 Gamma, generlised gamma, Erlang and Weibull distributions

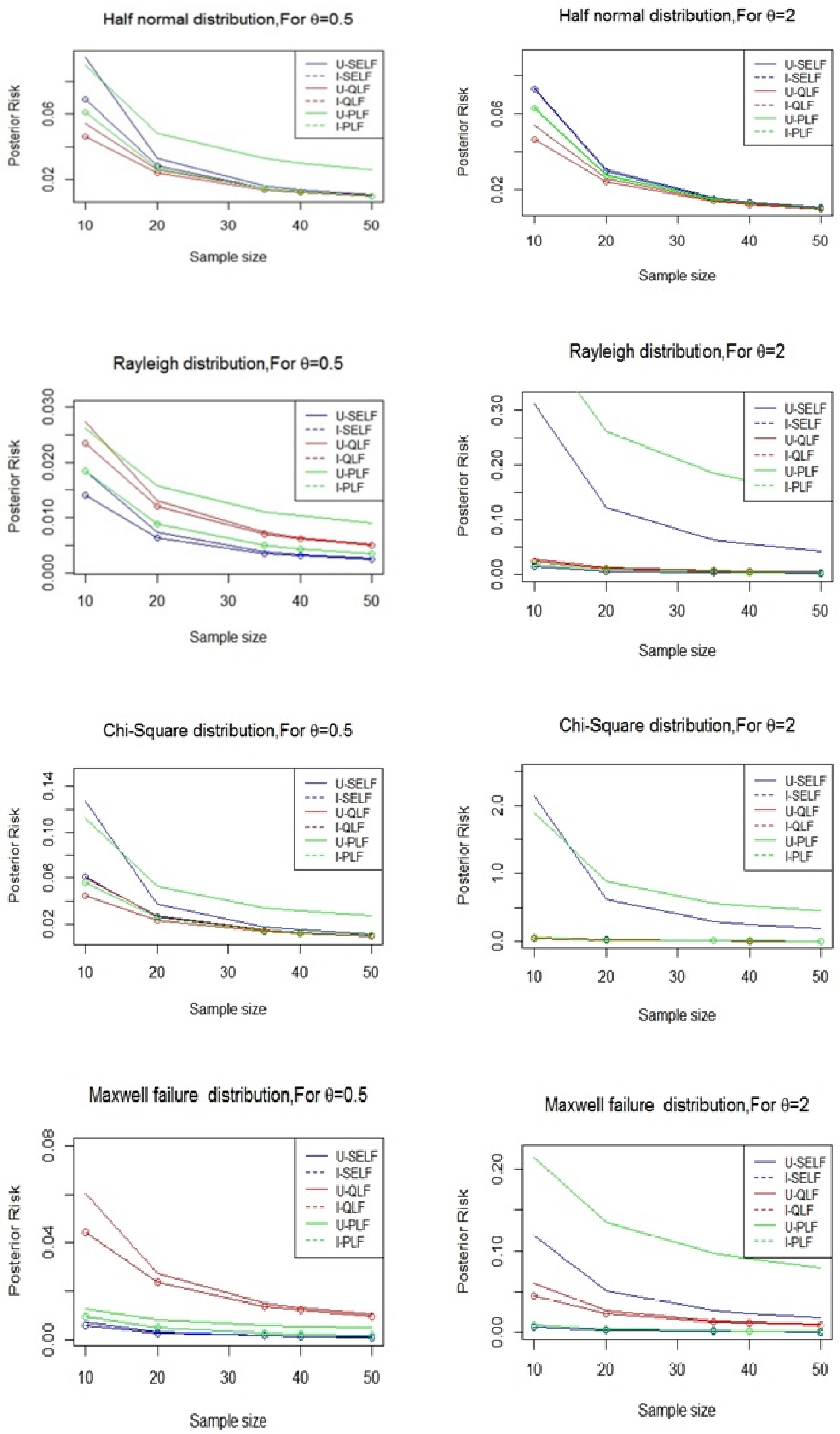

Figure 4 Half normal, Rayleigh, chi-square and Maxwell failure distributions

6. Conclusions

In case of uniform prior, the distributions considered in the family of lifetime distributions i.e. one-parameter exponential distribution, gamma, generalised gamma, Erlang, Weibull, Rayleigh, chi-square and Maxwell distribution, here an increase in the values of θ brings about an increase in the posterior risk for SELF and PLF. In case of inverted gamma prior, for $\theta = 0.5$ and $\theta = 1$, the value of the posterior risk increases and decreases afterwards i.e. for $\theta = 1$ to $\theta = 2$ for SELF and PLF.

In half normal distribution, for the uniform prior, the values of posterior risk increases for $\theta = 0.5$ to $\theta = 1.5$ and it decreases afterwards i.e. $\theta = 1.5$ to $\theta = 2$ for SELF and PLF. In case of inverted gamma prior, for $\theta = 0.5$ to $\theta = 2$, the posterior risk decreases for SELF and PLF.

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