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Class of Ratio-Cum-Product Type Estimator under Double Sampling: A Simulation Study

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Abstract

In the present paper, a class of ratio-cum-product type estimator has been developed for estimating the population mean of the study variable using an auxiliary variable under two-phase sampling. The expressions for the biases and mean squared errors (MSE's) have been obtained to the first-order of approximation. Theoretical and empirical study has been done to compare the performance of the proposed estimator with the existing estimators that utilize auxiliary information for finite population mean. Both the theoretical and empirical study concludes that the proposed estimator for estimation of mean perform better than the usual unbiased estimators and other existing estimators.

Keywords: Bias, mean square error, auxiliary variable, two-phase sampling.

1. Introduction

To estimate any parameter, an efficient estimator is a corresponding statistic. Like if we are interested in estimating the population mean, we suppose sample mean as an appropriate estimator. We proposed an estimator that may be a bit biased but must have a smaller mean square error than the mean square error for the sample mean. This can be attained if we have more and more information about our study variable. So, in our study, we use the auxiliary variable that has a positive and negative correlation with the study variable. The information about the auxiliary variable is used to enhance the efficiency of the estimators.

Cochran (1940) estimated the population mean using auxiliary information of the positively correlated variables. Robson (1952) and Murthy (1964) proposed traditional product estimator using negatively correlated auxiliary variables. Kawathekar and Ajagaonkar (1984) proposed a ratio estimator based on the coefficient of variation in double sampling. Pandey and Dubey (1989) suggested the modified ratio estimator using auxiliary information in two-phase sampling. Kumar and Bahl (2006) suggested dual to ratio estimator of population mean in two-phase sampling. Singh and Vishwakarma (2007) suggested exponential type ratio and product-type estimators of population mean in two-phase sampling. Malik and Tailor (2013) suggested ratio type estimator for estimating the population mean in double sampling. Kumar and Vishwakarma (2017) studied exponential type ratio and product estimators for finite population mean in double sampling using multi-auxiliary information. Some other authors Singh et al. (2012), Kung'u and Nderitu (2016) and Vadlamudi et al. (2017) studied estimation under two-phase sampling.

Let us consider a finite population of N units, Y and X are the study and auxiliary variable, respectively. In two-phase sampling, $\bar{X}$ of x is not known, thus a first phase sample of size n' is drawn from the population N on which only auxiliary characteristics are studied to make a good estimate of the population mean of auxiliary variable. After this, a second phase sample of size $n(n<n')$ is drawn on which both the auxiliary as well as study variables are measured.

The usual ratio and product estimator in double sampling are given as

$$\bar{y}_{rd}=\bar{y}_n\left(\frac{\bar{x}_{n'}}{\bar{x}_n}\right) \text{ and } \bar{y}_{pd}=\bar{y}_n\left(\frac{\bar{x}_n}{\bar{x}_{n'}}\right),$$

where $\bar{x}_n=\frac{1}{n'}\sum_{i=1}^{n'}x_i$ is an unbiased estimator of population mean $\bar{X}$ based on sample of size n', $\bar{x}_n=\frac{1}{n}\sum_{i=1}^{n}x_i$ is an unbiased estimator of population mean $\bar{X}$ based on sample of size n' and $\bar{x}_n=\frac{1}{n}\sum_{i=1}^{n}x_i$ is an unbiased estimator of population mean $\bar{Y}$ based on sample of size n.

The bias and mean square error of $\bar{y}_{rd}$ and $\bar{y}_{pd}$ are given by

$$B(\bar{y}_{rd})=\theta^*\bar{Y}\left(C_x^2-\rho C_yC_x\right),\ MSE(\bar{y}_{rd})=\bar{Y}^2\left(\theta C_y^2+\theta^*\left(C_x^2-2\rho C_yC_x\right)\right),$$

$$B(\bar{y}_{pd})=\theta^*\bar{Y}\left(C_x^2+\rho C_yC_x\right),\ MSE(\bar{y}_{pd})=\bar{Y}^2\left(\theta C_y^2+\theta^*\left(C_x^2+2\rho C_yC_x\right)\right),$$

where $\theta^*=(\theta-\theta')=\frac{1}{n}-\frac{1}{n'},\ \theta'=\frac{1}{n'}-\frac{1}{N},\ \theta=\frac{1}{n}-\frac{1}{N},$

$$C_y^2=\frac{S_y^2}{\bar{Y}^2},\ C_x^2=\frac{S_x^2}{\bar{X}^2},\ C_{yx}=\frac{S_{yx}}{\bar{Y}\bar{X}},\ S_{yx}=\frac{(y_i-\bar{Y})(x_i-\bar{X})}{N-1},\ S_y^2=\frac{(y_i-\bar{Y})^2}{N-1},\ S_x^2=\frac{(x_i-\bar{X})^2}{N-1},\ \rho=\frac{S_{yx}}{S_yS_x}.$$

Ratio estimator by Kwathekar and Ajagonkar (1984) using information on coefficient of variation "C_x" auxiliary variable x, is given by

$$\bar{y}_{KA}=\bar{y}_n\left(\frac{\bar{x}_{n'}+C_x}{\bar{x}_n+C_x}\right).$$

The bias and MSE of $\bar{y}_{KA}$ are given by

$$B(\bar{y}_{KA})=\bar{Y}\theta^*\left\{C_y^2+\left(C_x''^2-\rho C_yC_x''\right)\right\} \text{ and } MSE(\bar{y}_{KA})=\bar{Y}^2\left\{\theta C_y^2+\theta^*\left(C_x''^2-2\rho C_yC_x''\right)\right\},$$

where $C_x''=\frac{\bar{X}C_x}{\bar{X}+C_x}$.

Pandey and Dubey (1989) developed a modified ratio estimator by using coefficient of variation as

$$\bar{y}_{md}=\bar{y}_n\left\{\frac{\bar{x}_{n'}(1+C_x)}{\bar{x}_n+\bar{x}_{n'}C_x}\right\}.$$

The bias and MSE of $\bar{y}_{md}$ are given by

$$B(\bar{y}_{md})=\bar{Y}\theta^*\left(C_x'^2-\rho C_yC_x'\right) \text{ and } MSE(\bar{y}_{md})=\bar{Y}^2\left\{\theta C_y^2+\theta^*\left(C_x'^2-2\rho C_yC_x'\right)\right\},$$

where $C_x' = \dfrac{\overline{X}C_x}{1+C_x}$.

Singh and Vishwakarma (2007) proposed the exponential ratio-type and product-type estimator in two-phase sampling as

$$\overline{y}_{SVr} = \overline{y}_n \exp\left(\frac{\overline{x}_{n'} - \overline{x}_n}{\overline{x}_{n'} + \overline{x}_n}\right) \text{ and } \overline{y}_{SVp} = \overline{y}_n \exp\left(\frac{\overline{x}_n - \overline{x}_{n'}}{\overline{x}_{n'} + \overline{x}_n}\right).$$

According to Singh and Vishwakarma (2007), the estimators $\overline{y}_{SVr}$ and $\overline{y}_{SVp}$ are biased estimators, but their bias being of the order n^{-1}, can be assumed negligible in large samples. Thus, we assumed that the sample size n is large enough so that the biases of the estimators $\overline{y}_{SVr}$ and $\overline{y}_{SVp}$ are negligible and the variances of these estimators are obtained up to the terms of order n^{-1} as

$$V\left(\overline{y}_{SVr}\right) = \overline{Y}^2 C_y^2\left[\theta + (\alpha/4)\theta^*(\alpha - 4\rho)\right] \text{ and } V\left(\overline{y}_{SVp}\right) = \overline{Y}^2 C_y^2\left[\theta + (\alpha/4)\theta^*(\alpha + 4\rho)\right].$$

Malik and Tailor (2013) suggested an estimator using information on correlation coefficient as

$$\overline{y}_{MT} = \overline{y}_n\left(\frac{\overline{x}_{n'} + \rho}{\overline{x}_n + \rho}\right).$$

The bias and mean square error of $\overline{y}_{MT}$ are given by

$$B\left(\overline{y}_{MT}\right) = \theta'\overline{Y}C_x^2\left(\theta^* - 2\rho\frac{C_y}{C_x}\right) \text{ and } MSE\left(\overline{y}_{MT}\right) = \overline{Y}^2\left\{\theta C_y^2 + \theta'\left(\theta^* - 2\rho\frac{C_y}{C_x}\right)\right\}.$$

In the next section, we proposed a class of ratio-cum-product estimator for estimating the population mean of study variable by using auxiliary variable under two-phase sampling.

2. The Proposed Estimator

In a situation, when an auxiliary information is used for estimating population mean of the study variable and population mean of auxiliary variable is unknown, there exist various ratio and product type estimators by using two-phase sampling. By reviewing these estimators, we define a class of ratio-cum-product type estimator as

$$\overline{y}_v = \overline{y}_n\left(\frac{\overline{x}_{n'} + A}{\overline{x}_n + A}\right)\left(\frac{\overline{x}_n + B}{\overline{x}_{n'} + B}\right), \tag{1}$$

where $\overline{x}_{n'} = \dfrac{1}{n'}\sum_{i=1}^{n'} x_i$ is an unbiased estimator of population mean $\overline{X}$ based on sample of size n'. Also the constants A and B can take any real as well as parametric values viz., $A, B = 0, C_x, \sigma_x, \rho_{yx}, \beta_1, \beta_2,$ etc. Following are the notations used to obtain the bias and MSE of the estimator.

Let $e_0 = \dfrac{\overline{y}_n - \overline{Y}}{\overline{Y}}, e_1 = \dfrac{\overline{x}_n - \overline{X}}{\overline{X}}, e_2 = \dfrac{\overline{x}_{n'} - \overline{X}}{\overline{X}}$, with $E(e_0) = E(e_1) = E(e_2) = 0$ and $E(e_0^2) = \lambda_n C_y^2$, $E(e_1^2) = \lambda_n C_x^2$, $E(e_2^2) = \lambda_{n'} C_x^2$, $\lambda_{n'} = \dfrac{1}{n'} - \dfrac{1}{N}$, $\lambda_n = \dfrac{1}{n} - \dfrac{1}{N}$.

3. Bias and Mean Square Error of Proposed Estimator

To get the bias and mean square error of the proposed estimator we expand the right hand side of (1), and neglecting the terms of e_i's greater than second degree, one can obtain

$$\bar{y}_v = \bar{Y}\left(1+e_0\right)\left(\frac{1+\alpha e_2}{1+\alpha e_1}\right)\left(\frac{1+\gamma e_1}{1+\gamma e_2}\right),$$

where $\alpha = \frac{\bar{X}}{\bar{X}+A}$ and $\gamma = \frac{\bar{X}}{\bar{X}+B}$.

Solving the above equation further, we can have

$$\bar{y}_v - \bar{Y} = \bar{Y}\{e_0 + \gamma(e_1 - e_2 + e_0e_1 - e_0e_2) + \alpha(e_2 - e_1 + e_0e_2 - e_0e_1) + \gamma^2(e_2^2 - e_1e_2) + \alpha^2(e_1^2 - e_1e_2) - \alpha\gamma(e_1^2 + e_2^2 - 2e_1e_2)\}. \quad (2)$$

Taking expectation both side of (2), we get the bias of the estimator "$\bar{y}_v$"

$$B(\bar{y}_v) = \theta^*\bar{Y}\left\{(\gamma-\alpha)C_{yx} + (\alpha^2 - \alpha\gamma)C_x^2\right\}. \quad (3)$$

Squaring both sides of (2) to the first degree of approximation, we get

$$(\bar{y}_v - \bar{Y})^2 = \bar{Y}^2\{e_0^2 + \gamma^2(e_1^2 + e_2^2 - 2e_1e_2) + \alpha^2(e_1^2 + e_2^2 - 2e_1e_2) + 2\gamma(e_0e_1 - e_0e_2) + 2\gamma(e_0e_1 - e_0e_2) - 2\alpha\gamma(e_1^2 + e_2^2 - 2e_1e_2)\}. \quad (4)$$

Taking expectations on both sides of Equation (4) we get the MSE's of "$\bar{y}_v$", which is given by

$$MSE(\bar{y}_v) = \bar{Y}^2\{\theta C_y^2 + \gamma^2(\theta-\theta')C_x^2 + \alpha^2(\theta-\theta')C_x^2 + 2\gamma(\theta-\theta')C_{yx} + 2\alpha(\theta-\theta')C_{yx} - 2\alpha\gamma(\theta-\theta')C_x^2\}$$

$$= \bar{Y}^2\{\theta C_y^2 + (\theta-\theta')(\gamma^2 + \alpha^2 - 2\alpha\gamma)C_x^2 + (\theta-\theta')(2\gamma - 2\alpha)C_{yx}\}$$

$$= \bar{Y}^2\{\theta C_y^2 + \theta^*(\gamma-\alpha)^2C_x^2 + \theta^*(2\gamma-2\alpha)C_{yx}\}. \quad (5)$$

Members of the proposed class of estimator with bias and MSE are presented in Table 1.

4. Efficiency Comparison

We have compared our proposed estimator with the usual mean estimators and some other already existed estimators in the literature and developed the following conditions,

$$\bar{y}_v = \bar{Y}\left(1+e_0\right)\left(\frac{1+\alpha e_2}{1+\alpha e_1}\right)\left(\frac{1+\gamma e_1}{1+\gamma e_2}\right).$$

i) $MSE(\bar{y}_v) - V(\bar{y}) \le 0$, if $\eta \le \frac{C_{yx}}{C_x^2}$, where $\eta = \frac{\alpha}{2} + \frac{\beta^2}{2(\alpha-\beta)}$. (6)

ii) $MSE(\bar{y}_v) - MSE(\bar{y}_{rd}) \le 0$, if $\frac{\alpha(\alpha-\beta)+\beta^2-1}{1+\alpha-\beta} \le \frac{2C_{yx}}{C_x^2}$. (7)

iii) $MSE(\bar{y}_v) - MSE(\bar{y}_{pd}) \le 0$, if $\frac{\alpha(\alpha-\beta)+\beta^2-1}{\alpha-\beta-1} \le \frac{2C_{yx}}{C_x^2}$. (8)

iv) $MSE(\bar{y}_v) - MSE(\bar{y}_{KA}) \le 0$, if $\eta \le (2+K_1)\frac{C_{yx}}{C_x^2}$, where $K_1 = \frac{C_x''(1+2\rho C_y)}{C_{yx}(\alpha-\beta)}$. (9)

v) $MSE(\bar{y}_v) - MSE(\bar{y}_{md}) \le 0$, if $\eta \le (2+K_2)\frac{C_{yx}}{C_x^2}$, where $K_2 = \frac{C_x'^2(1+2\rho C_y)}{C_{yx}(\alpha-\beta)}$. (10)

vi) $MSE(\bar{y}_v) - MSE(\bar{y}_{MT}) \le 0$, if $\frac{\alpha^2+\beta^2-\alpha\beta-\theta^*}{1+\beta-\alpha} \le -\frac{2C_{yx}}{C_x^2}$. (11)

Table 1 Members of estimator for different values of A and B

A	B	Estimators	Bias	MSE
1	β_2	$\bar{y}_1=\bar{y}_n\left(\frac{\bar{x}_{n'}+1}{\bar{x}_n+1}\right)\left(\frac{\bar{x}_{n'}+\beta_2}{\bar{x}_n+\beta_2}\right)$	$B(\bar{y}_1)=\theta^*\bar{Y}\{(\gamma_1-\alpha_1)C_{yx}+(\alpha_1^2-\alpha_1\gamma_1)C_x^2\}$	$MSE(\bar{y}_1)=\bar{Y}^2[\theta C_y^2+\theta^*\{(\gamma_1-\alpha_1)^2C_x^2+2(\gamma_1-\alpha_1)C_{yx}\}]$, where $\alpha_1=\frac{\bar{X}}{\bar{X}+1}$ and $\gamma_1=\frac{\bar{X}}{\bar{X}+\beta_2}$.
β_1	1	$\bar{y}_2=\bar{y}_n\left(\frac{\bar{x}_{n'}+\beta_1}{\bar{x}_n+\beta_1}\right)\left(\frac{\bar{x}_{n'}+1}{\bar{x}_n+1}\right)$	$B(\bar{y}_2)=\theta^*\bar{Y}\{(\gamma_2-\alpha_2)C_{yx}+(\alpha_2^2-\alpha_2\gamma_2)C_x^2\}$	$MSE(\bar{y}_2)=\bar{Y}^2[\theta C_y^2+\theta^*\{(\gamma_2-\alpha_2)^2C_x^2+2(\gamma_2-\alpha_2)C_{yx}\}]$, where $\alpha_2=\frac{\bar{X}}{\bar{X}+\beta_1}$ and $\gamma_2=\frac{\bar{X}}{\bar{X}+1}$.
σ_x	β_2	$\bar{y}_3=\bar{y}_n\left(\frac{\bar{x}_{n'}+\sigma_x}{\bar{x}_n+\sigma_x}\right)\left(\frac{\bar{x}_{n'}+\beta_2}{\bar{x}_n+\beta_2}\right)$	$B(\bar{y}_3)=\theta^*\bar{Y}\{(\gamma_3-\alpha_3)C_{yx}+(\alpha_3^2-\alpha_3\gamma_3)C_x^2\}$.	$MSE(\bar{y}_3)=\bar{Y}^2[\theta C_y^2+\theta^*\{(\gamma_3-\alpha_3)^2C_x^2+2(\gamma_3-\alpha_3)C_{yx}\}]$ where $\alpha_3=\frac{\bar{X}}{\bar{X}+\sigma_x}$ and $\gamma_3=\frac{\bar{X}}{\bar{X}+\beta_2}$.
C_x	β_2	$\bar{y}_4=\bar{y}_n\left(\frac{\bar{x}_{n'}+C_x}{\bar{x}_n+C_x}\right)\left(\frac{\bar{x}_{n'}+\beta_2}{\bar{x}_n+\beta_2}\right)$	$B(\bar{y}_4)=\theta^*\bar{Y}\{(\gamma_4-\alpha_4)C_{yx}+(\alpha_4^2-\alpha_4\gamma_4)C_x^2\}$	$MSE(\bar{y}_4)=\bar{Y}^2[\theta C_y^2+\theta^*\{(\gamma_4-\alpha_4)^2C_x^2+2(\gamma_4-\alpha_4)C_{yx}\}]$, where $\alpha_4=\frac{\bar{X}}{\bar{X}+C_x}$ and $\gamma_4=\frac{\bar{X}}{\bar{X}+\beta_2}$.
β_1	ρ_{yx}	$\bar{y}_5=\bar{y}_n\left(\frac{\bar{x}_{n'}+\beta_1}{\bar{x}_n+\beta_1}\right)\left(\frac{\bar{x}_{n'}+\rho_{yx}}{\bar{x}_n+\rho_{yx}}\right)$	$B(\bar{y}_5)=\theta^*\bar{Y}\{(\gamma_5-\alpha_5)C_{yx}+(\alpha_5^2-\alpha_5\gamma_5)C_x^2\}$	$MSE(\bar{y}_5)=\bar{Y}^2[\theta C_y^2+\theta^*\{(\gamma_5-\alpha_5)^2C_x^2+2(\gamma_5-\alpha_5)C_{yx}\}]$, where $\alpha_5=\frac{\bar{X}}{\bar{X}+\beta_1}$ and $\gamma_5=\frac{\bar{X}}{\bar{X}+\rho_{yx}}$.
C_x	σ_x	$\bar{y}_6=\bar{y}_n\left(\frac{\bar{x}_{n'}+C_x}{\bar{x}_n+C_x}\right)\left(\frac{\bar{x}_{n'}+\sigma_x}{\bar{x}_n+\sigma_x}\right)$	$B(\bar{y}_6)=\theta^*\bar{Y}\{(\gamma_6-\alpha_6)C_{yx}+(\alpha_6^2-\alpha_6\gamma_6)C_x^2\}$	$MSE(\bar{y}_6)=\bar{Y}^2[\theta C_y^2+\theta^*\{(\gamma_6-\alpha_6)^2C_x^2+2(\gamma_6-\alpha_6)C_{yx}\}]$, where $\alpha_6=\frac{\bar{X}}{\bar{X}+C_x}$ and $\gamma_6=\frac{\bar{X}}{\bar{X}+\sigma_x}$.
ρ_{yx}	β_2	$\bar{y}_7=\bar{y}_n\left(\frac{\bar{x}_{n'}+\rho_{yx}}{\bar{x}_n+\rho_{yx}}\right)\left(\frac{\bar{x}_{n'}+\beta_2}{\bar{x}_n+\beta_2}\right)$	$B(\bar{y}_7)=\theta^*\bar{Y}\{(\gamma_7-\alpha_7)C_{yx}+(\alpha_7^2-\alpha_7\gamma_7)C_x^2\}$	$MSE(\bar{y}_7)=\bar{Y}^2[\theta C_y^2+\theta^*\{(\gamma_7-\alpha_7)^2C_x^2+2(\gamma_7-\alpha_7)C_{yx}\}]$, where $\alpha_7=\frac{\bar{X}}{\bar{X}+\rho_{yx}}$ and $\gamma_7=\frac{\bar{X}}{\bar{X}+\beta_2}$.

Table 1 (continued)

A	B	Estimators	Bias	MSE
ρ_{yx}	σ_x	$\bar{y}_8 = \bar{y}_n\left(\frac{\bar{x}_{n'}+\rho_{yx}}{\bar{x}_n+\rho_{yx}}\right)\left(\frac{\bar{x}_{n'}+\sigma_x}{\bar{x}_n+\sigma_x}\right)$	$B(\bar{y}_8)=\theta^*\bar{Y}\{(\gamma_8-\alpha_8)C_{yx}$ $+(\alpha_8^2-\alpha_8\gamma_8)C_x^2\}$	$MSE(\bar{y}_8)=\bar{Y}^2[\theta C_y^2+\theta^*\{(\gamma_8-\alpha_8)^2C_x^2$ $+2(\gamma_8-\alpha_8)C_{yx}\}]$, where $\alpha_8=\frac{\bar{X}}{\bar{X}+\rho_{yx}}$ and $\gamma_8=\frac{\bar{X}}{\bar{X}+\sigma_x}$.
β_1	C_x	$\bar{y}_9 = \bar{y}_n\left(\frac{\bar{x}_{n'}+\beta_1}{\bar{x}_n+\beta_1}\right)\left(\frac{\bar{x}_{n'}+C_x}{\bar{x}_n+C_x}\right)$	$B(\bar{y}_9)=\theta^*\bar{Y}\{(\gamma_9-\alpha_9)C_{yx}$ $+(\alpha_9^2-\alpha_9\gamma_9)C_x^2\}$	$MSE(\bar{y}_9)=\bar{Y}^2[\theta C_y^2+\theta^*\{(\gamma_9-\alpha_9)^2C_x^2$ $+2(\gamma_9-\alpha_9)C_{yx}\}]$, where $\alpha_9=\frac{\bar{X}}{\bar{X}+\beta_1}$ and $\gamma_9=\frac{\bar{X}}{\bar{X}+C_x}$.
1	σ_x	$\bar{y}_{10} = \bar{y}_n\left(\frac{\bar{x}_{n'}+1}{\bar{x}_n+1}\right)\left(\frac{\bar{x}_{n'}+\sigma_x}{\bar{x}_n+\sigma_x}\right)$	$B(\bar{y}_{10})=\theta^*\bar{Y}\{(\gamma_{10}-\alpha_{10})C_{yx}$ $+(\alpha_{10}^2-\alpha_{10}\gamma_{10})C_x^2\}$	$MSE(\bar{y}_{10})=\bar{Y}^2[\theta C_y^2+\theta^*\{(\gamma_{10}-\alpha_{10})^2C_x^2$ $+2(\gamma_{10}-\alpha_{10})C_{yx}\}]$, where $\alpha_{10}=\frac{\bar{X}}{\bar{X}+1}$ and $\gamma_{10}=\frac{\bar{X}}{\bar{X}+\sigma_x}$.
0	σ_x	$\bar{y}_{11} = \bar{y}_n\left(\frac{\bar{x}_{n'}}{\bar{x}_n}\right)\left(\frac{\bar{x}_{n'}+\sigma_x}{\bar{x}_n+\sigma_x}\right)$	$B(\bar{y}_{11})=\theta^*\bar{Y}\{(\gamma_{11}-\alpha_{11})C_{yx}$ $+(\alpha_{11}^2-\alpha_{11}\gamma_{11})C_x^2\}$	$MSE(\bar{y}_{11})=\bar{Y}^2[\theta C_y^2+\theta^*\{(\gamma_{11}-\alpha_{11})^2C_x^2$ $+2(\gamma_{11}-\alpha_{11})C_{yx}\}]$, where $\alpha_{11}=1$ and $\gamma_{11}=\frac{\bar{X}}{\bar{X}+\sigma_x}$.
β_1	σ_x	$\bar{y}_{12} = \bar{y}_n\left(\frac{\bar{x}_{n'}+\beta_1}{\bar{x}_n+\beta_1}\right)\left(\frac{\bar{x}_{n'}+\sigma_x}{\bar{x}_n+\sigma_x}\right)$	$B(\bar{y}_{12})=\theta^*\bar{Y}\{(\gamma_{12}-\alpha_{12})C_{yx}$ $+(\alpha_{12}^2-\alpha_{12}\gamma_{12})C_x^2\}$	$MSE(\bar{y}_{12})=\bar{Y}^2[\theta C_y^2+\theta^*\{(\gamma_{12}-\alpha_{12})^2C_x^2$ $+2(\gamma_{12}-\alpha_{12})C_{yx}\}]$, where $\alpha_{12}=\frac{\bar{X}}{\bar{X}+\beta_1}$ and $\gamma_{12}=\frac{\bar{X}}{\bar{X}+\sigma_x}$.

vii) $MSE(\bar{y}_v)-MSE(\bar{y}_{SVr})\le 0,$ if $\eta \le K_3\frac{C_{yx}}{C_x^2}$, where $K_3=1+\frac{\alpha}{4}(\alpha-4\rho)$. (12)

viii) $MSE(\bar{y}_v)-MSE(\bar{y}_{SVp})\le 0,$ if $\eta \le K_4\frac{C_{yx}}{C_x^2}$, where $K_4=1+\frac{\alpha}{4}(\alpha+4\rho)$. (13)

If the above conditions (6)-(13) holds true, then the proposed estimator $\bar{y}_v$ will perform efficiently. As above conditions are dependent on the values of A and B so we conduct the simulation study in next section to see for what values of A and B our proposed estimator performs better than the existing estimators.

5. Simulation Study

To support the theoretical findings in Section 4, we have performed simulation. To generate the population, we define the study variable y by using the model $y = x + \text{rnorm}(N,\text{mean},\text{SD.}) + e$ which follows normal distribution and error term e which follows standard normal distribution i.e., $N(0,1)$. In this study, we have mean $= 2.5$, and SD. $= 1.5$ and population size $N = 1{,}000$. Further, we select a random sample of size $n' = 450$ for the first phase sample from population N and again we select a random sample of size $n = 250$ for second phase, from the first phase sample.

To evaluate the percent relative efficiency (PRE) of the proposed estimator, we have calculated the variance for mean per unit, mean square error and variances of the estimators which are considered for comparison. The PRE of each estimator with respect to variance of mean per unit is calculated. After that we did the comparison of PRE of each considered estimator and proposed estimator, the one with the highest PRE is the most efficient estimator. We replicated our findings for 60 times to obtain more precise results. The PRE of all existing estimators with respect to the usual unbiased estimator are presented in Table 2 and PRE of the proposed estimator for different values of A and B with respect to " $\bar{y}$ " are presented in Table 3, respectively.

Table 2 Percent relative efficiency (PRE) of existing estimators with respect to mean per unit estimator

Existing Estimators	PRE with respect to $\bar{y}_n$
$\text{PRE}(\bar{y}, \bar{y})$	100.00
$\text{PRE}(\bar{y}_{rd}, \bar{y})$	583.21
$\text{PRE}(\bar{y}_{pd}, \bar{y})$	329.86
$\text{PRE}(\bar{y}_{KA}, \bar{y})$	766.48
$\text{PRE}(\bar{y}_{md}, \bar{y})$	889.04
$\text{PRE}(\bar{y}_{SVr}, \bar{y})$	901.61
$\text{PRE}(\bar{y}_{SVp}, \bar{y})$	563.62
$\text{PRE}(\bar{y}_{MT}, \bar{y})$	719.25

It is noted from Table 2 that for the defined population, (Singh and Vishwakarma 2007) ratio-type estimator $\bar{y}_{SVr}$ performs efficiently in terms of PRE with respect to $\bar{y}$ as compared to other estimators. Also, it is envisaged that all the considered estimators performed efficiently with respect to $\bar{y}$.

From the tabular values of Table 3, we conclude that for $A = \beta_1$ and $B = C_x$, our proposed ratio-cum-product estimator is most efficient than the other values of A and B and other existing estimators mentioned in Table 2. For all different combinations of A and B, the proposed estimator " $\bar{y}_v$ " performs efficiently than the usual unbiased estimator " $\bar{y}$ ", the usual ratio " $\bar{y}_{rd}$ " and product " $\bar{y}_{pd}$ " estimators, (Kwathekar and Ajagonkar 1984) estimator " $\bar{y}_{KA}$ ", (Pandey and Dubey 1989) estimator " $\bar{y}_{md}$ ", (Singh and Vishwakarma 2007) exponential ratio-type " $\bar{y}_{SVr}$ " and exponential product-type " $\bar{y}_{SVp}$ " estimators and (Malik and Tailor 2013) " $\bar{y}_{MT}$ " estimator.

6. Conclusions

This paper considers the problem of estimating the population mean of the study variable using auxiliary information under double sampling. We have proposed a class of ratio-cum-product type estimator using auxiliary information when population mean " $\bar{X}$ " of auxiliary variable " X " is unknown. The results for bias and mean square error have been obtained to the first order of approximation. The conditions have been obtained under which the proposed estimator performs better than the considered estimators. We have done simulation for validating the theoretical findings. Comparing the values of percent relative efficiency of already existing estimators with percent relative efficiency of the proposed estimator, we can envisage that the proposed estimator in double sampling is most efficient than the already existing estimators in a similar situation. Thus, our recommendation is to use the proposed estimator for estimating the population mean of the study variable by using the auxiliary information under double sampling.

Table 3 Percent relative efficiency of proposed class of estimator with respect to mean per unit estimator for different value of constants A and B

A	B	PRE $(\bar{y}_p, \bar{y})$
1	β_2	960.46
β_1	1	961.76
σ_x	β_2	944.17
C_x	β_2	953.83
β_1	ρ_{yx}	961.10
C_x	σ_x	926.33
ρ_{yx}	β_2	952.40
ρ_{yx}	σ_x	951.56
β_1	C_x	968.79
1	σ_x	930.22
0	σ_x	949.65
β_1	σ_x	951.30

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