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A Performance of Kumaraswamy Transmuted Rayleigh Distribution

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Abstract

This study derived a four-parameter Kumaraswamy transmuted generalized form of Rayleigh distribution called Kumaraswamy transmuted Rayleigh distribution (KWTR) by including two-shape parameters and a transmutation parameter to the single parameter baseline Rayleigh distribution which will take care of the spread and also give a generalised distribution. The statistical properties of the proposed distribution including the shape of the distribution, hazard rate and survival function were derived. These properties were compared with the existing baseline distributions, that is, exponentiated transmuted generalized Rayleigh distribution (ETGR), transmuted generalized Rayleigh (TGR), generalized Rayleigh (GR) and Rayleigh (R) distributions. Estimates of KWTR parameters were derived by using maximum likelihood method. The performances of the distributions were compared using Akaike information criterion (AIC), Bayesian information criterion (BIC) and goodness of fit statistic contained in "fitdistrplus" of R. Two different datasets were considered.

Keywords: Kumaraswamy, lifetime, AIC, exponentiated.

1. Introduction

In applied science, it is very important to model and analyze lifetime data and the probability model determines the pattern or steps used in the statistical modelling. As a result of this, past and present researchers have done tremendous work on improving standard of probability distributions with relevant statistical methodologies. Nevertheless, there are still instances where classical or standard probability models does not follow real data set. Hence, the need for developing more flexible statistical distributions is required. This research is aimed to provide a more flexible distribution that can efficiently model robust lifetime datasets.

Rayleigh distribution is commonly used in lifetime analysis, it is applicable to modeling and analyzing several aspect of statistics. The distribution also has a special form of Weibull distribution when the shape parameter is 2. Literatures shows that researchers have given much attention to studying Rayleigh distribution in the last couple of decades. Maximum likelihood and Bayes approaches were applied by Abushal (2011) in estimating the properties of Rayleigh distribution based on progressive first-failure censored data. The empirical Bayes estimates for parameter and reliability function associated to compound Rayleigh distribution under record data were obtained by Shajaee et al. (2012). Also, the maximum likelihood and Bayes estimates of the reliability parameters corresponding to compound Rayleigh distribution under a progressive type-II censored data was compared by Barot and Patal (2015). The compound Rayleigh distribution with constant partially accelerated life tests under an adaptive type-II progressive hybrid censored data was studied by Abd-Elmougod and Mahmoud (2016).

The $f(x)$ which is the probability density function of the Rayleigh distribution is given by

$$f(x)=\frac{x}{\alpha^2}\exp\left(-\frac{x}{2\alpha^2}\right),\ \text{for}\ x\in[0,\infty). \qquad (1)$$

The corresponding cumulative function, for Rayleigh is given by

$$F(x)=1-\exp\left(-\frac{x}{2\alpha^2}\right), \qquad (2)$$

where α represent the distribution scale parameter.

One of the limitations of the classical Rayleigh distribution is that it cannot provide adequate fit to real datasets, because it has only the scale parameter but no shape parameter. The rationale behind this research is that the proposed distribution will be more flexible as two shape parameters and a transmuted parameter will be added to the baseline distribution. The pdf plots can be left or right skewed. The proposed Kumuraswamy transmuted Rayleigh distribution will be efficient in the modeling of lifetime and reliability datasets. The proposed distribution will provide statistical model that has a wide variety of application in different areas and an advantage among other distributions in its ability in context of lifetime data.

In this paper, we improve on the existing distribution of Ahmed et al. (2016) where the authors introduced Kumaraswamy transmuted generalized family of distribution. This distribution has its CDF given as

$$F(x)=1-\left\{1-\left[(1+\lambda)H(x,\alpha)-\lambda H(x,\alpha)\right]^{a}\right\}^{b}. \qquad (3)$$

And the corresponding pdf to (3) is given by

$$f(x)=abh(x,\alpha)\{1+\lambda-2\lambda H(x,\alpha)\}\{H(x,\alpha)\}\left[1+\lambda-2\lambda H(x,\alpha)\right]^{a-1}$$
$$\times\left\{1-\left[(1+\lambda)H(x,\alpha)-\lambda H(x,\alpha)^2\right]^{a}\right\}^{b-1}, \qquad (4)$$

where $h(x,\alpha)$ and $H(x,\alpha)$ are the probability density function (PDF) and cumulative density function (CDF) of an arbitrary baseline distribution respectively.

The main motivations of this research are to derive a robust distribution that will efficiently model lifetime datasets. The classical Rayleigh distribution does not provide adequate fit to real data as it has only the scale parameter but no shape parameter. This proposed distribution is more flexible as three additional shape parameters are added to the base distribution. The PDF plots can be left or right skewed. Kumuraswamy transmuted Rayleigh distribution provides a statistical model which has a wide variety of application in different areas and the main advantage is its ability to model lifetime data among better than other distributions considered in this study.

2. The Proposed Distribution

This paper aimed to introduce a novel distribution known as the Kumaraswamy transmuted Rayleigh distribution (KWTR) by inserting the Rayleigh distribution into the generalized family of

Kumaraswamy transmuted distribution of Ahmed et al. (2016). The PDF and the CDF of the proposed distribution are given in (6) and (7), respectively.

Let X be a random variable with Rayleigh distribution as specified in (1) and (2). Letting $k=\exp\left(-\frac{x}{2\alpha^2}\right)$. Then Equation (2) above becomes

$$H(x,\alpha)=1-k. \quad (5)$$

Inserting (1) and (5) into (4) to have the derived PDF as

$$f(x,a,b,\alpha,\lambda)=ab\frac{x}{\alpha^2}\exp\left(-\frac{x^2}{2\alpha^2}\right)\left\{1-\lambda+2\lambda\exp\left(-\frac{x^2}{2\alpha^2}\right)\right\}$$
$$\times\left\{1+\lambda\exp\left(-\frac{x}{2\alpha^2}\right)-\exp\left(-\frac{x}{2\alpha^2}\right)-\lambda\left(\exp\left(-\frac{x}{2\alpha^2}\right)\right)^2\right\}^{a-1}$$
$$\times\left\{1-\left[1+\lambda\exp\left(-\frac{x}{2\alpha^2}\right)-\exp\left(-\frac{x}{2\alpha^2}\right)-\lambda\left(\exp\left(-\frac{x}{2\alpha^2}\right)\right)^2\right]^a\right\}^{b-1}. \quad (6)$$

Verification of the true PDF,

$$f(x)=ab\int_0^\infty h(x,\alpha)\{1+\lambda-2\lambda H(x,\alpha)\}\{H(x,\alpha)\}[1+\lambda-\lambda H(x,\alpha)]^{a-1}$$
$$\times\left\{1-\left[(1+\lambda)H(x,\alpha)-\lambda H(x,\alpha)^2\right]^a\right\}^{b-1}dx.$$

Let $P=H(x,\alpha),\ \frac{dP}{dx}=h(x,\alpha),\ dx=\frac{dP}{h(x,\alpha)},$

$$f(x)=ab\int_0^1 h(x,\alpha)\{1+\lambda-2\lambda P\}\{P\}[1+\lambda-\lambda P]^{a-1}\times\left\{1-\left[(1+\lambda)P-\lambda P^2\right]^a\right\}^{b-1}\frac{dP}{h(x,\alpha)}.$$

Let

$$n=(1+\lambda)-\lambda P^2,\ \frac{dn}{dP}=(1+\lambda)P-\lambda P,\ dP=\frac{dn}{(1+\lambda)P-2\lambda P},$$

$$f(x)=ab\int_0^1\{1+\lambda-2\lambda P\}n^{a-1}(1-n^a)^{b-1}\frac{dn}{(1+\lambda)-2\lambda P}=ab\int_0^1 n^{a-1}(1-n^a)^{b-1}dn.$$

Let $g=n^a,\ \frac{dg}{dn}=an^{a-1},\ n=g^{\frac{1}{a}},\ dn=\frac{dg}{an^{a-1}},$

$$f(x)=ab\int_0^1 n^{a-1}(1-g)^{b-1}\frac{dg}{an^{a-1}}=b\int_0^1(1-g)^{b-1}dg=b\frac{(1-g)^{b-1}}{-b}\Bigg|_0^1=-(1-g)^{b-1}\Big|_0^1=1.$$

Since the derivation of the KWTR is equal to 1, this shows that it is a true PDF.

The corresponding cumulative density function of the novel distribution can be expressed as

$$F(x,a,b,\alpha,\lambda)=1-\left\{1-\left[1+\lambda\exp\left(-\frac{x}{2\alpha^2}\right)-\exp\left(-\frac{x}{2\alpha^2}\right)-\lambda\left(\exp\left(-\frac{x}{2\alpha^2}\right)\right)^2\right]^a\right\}^{b-1}. \quad (7)$$

It can be shown that $\lim_{x\to 0}F(x)=0$ and $\lim_{x\to\infty}F(x)=1,$

$$\lim_{x\to 0} F(x) = 1-\left\{1-\left[(1+\lambda)\left(1-\exp(-0)\right)-\lambda\left(1-\exp(-0)\right)^2\right]^a\right\}^b$$
$$= 1-\left\{1-\left[(1+\lambda)(1-1)-\lambda(1-1)^2\right]^a\right\}^b = 0,$$
$$\lim_{x\to \infty} F(x) = 1-\left\{1-\left[(1+\lambda)\left(1-\exp(-\infty)\right)-\lambda\left(1-\exp(-\infty)\right)^2\right]^a\right\}^b$$
$$= 1-\left\{1-\left[(1+\lambda)(1-0)-\lambda(1-0)^2\right]^a\right\}^b = 1.$$

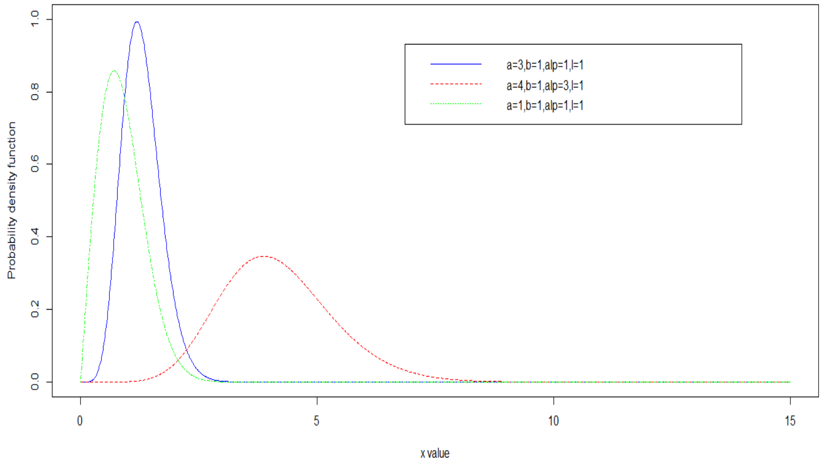

Figure 1 The Probability density function plot of the proposed chart

3. Properties of Proposed Kumaraswamy Transmuted Rayleigh Distribution

The properties of the proposed Kumaraswamy transmuted Rayleigh distribution including the survival function, hazard rate function, mixture representation, quantiles function and the maximum likelihood shall be investigated and discussed in the subsections below

3.1. Survival function

$$S(x;\alpha,a,b,\lambda) = 1-F(x;\alpha,a,b,\lambda)$$
$$= 1-\left(1-\left\{1-\left[1+\lambda \mathrm{k}-\mathrm{k}-\lambda k^2)\right]^a\right\}^b\right) = \left\{1-\left[1+\lambda k-k-\lambda k^2)\right]^a\right\}^b.$$

$$S(x,a,b,\alpha,\lambda) = \left\{1-\left[1+\lambda\exp\left(-\frac{x^2}{2\alpha^2}\right)-\exp\left(-\frac{x^2}{2\alpha^2}\right)-\lambda\left(\exp\left(-\frac{x^2}{2\alpha^2}\right)\right)^2\right]^a\right\}^{b-1}. \tag{8}$$

3.2. Hazard rate function

The hazard rate function of the proposed distribution in (6) is given by

$$t(x) = \frac{f(x;\alpha,a,b,\lambda)}{S(x;\alpha,a,b,\lambda)}$$

$$=\frac{ab\frac{x}{\alpha^2}\exp\left(-\frac{x^2}{2\alpha^2}\right)\left\{1-\lambda+2\lambda\exp\left(-\frac{x^2}{2\alpha^2}\right)\right\}\left\{1+\lambda\exp\left(-\frac{x^2}{2\alpha^2}\right)-\exp\left(-\frac{x^2}{2\alpha^2}\right)-\lambda\left(\exp\left(-\frac{x^2}{2\alpha^2}\right)\right)^2\right\}^{a-1}}{1-\left[1+\lambda\exp\left(-\frac{x^2}{2\alpha^2}\right)-\exp\left(-\frac{x^2}{2\alpha^2}\right)-\lambda\left(\exp\left(-\frac{x^2}{2\alpha^2}\right)\right)^2\right]^a} \quad (9)$$

Figure 2 shows that hazard rate (failure rate) could be monotonically increasing or decreasing which makes it suitable for failure times datasets.

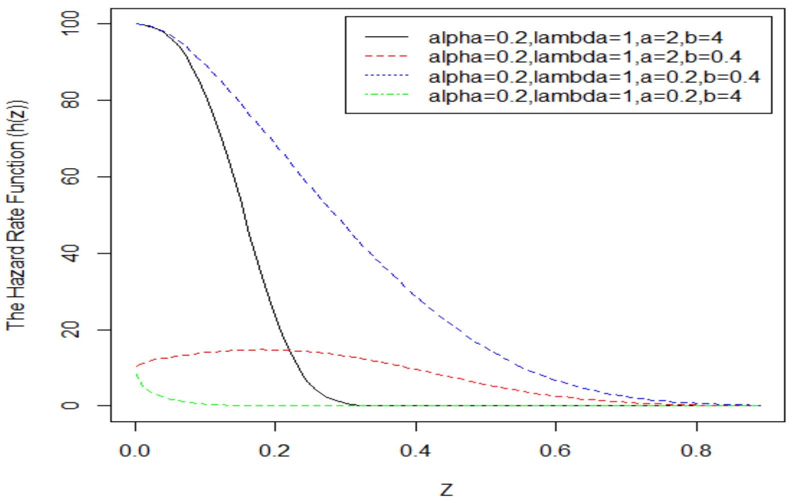

Figure 2 The hazard rate function plot

3.3. Mixture representation

The expansion of the power series of the novel distribution to be powers of $(a-1)$ then $(b-1)$ is given as

$$(1-z)^{b-1}=\sum_{k=0}^{\infty}\frac{(-1)^q\,\Gamma(b)}{q!\,\Gamma(b-q)}z^q. \quad (10)$$

For any real non-integer which holds for $|z|\prec 1$ and $b\succ 0$, expanding the term with power $a-1$ in (6) after applying the power series of (10) to (6), we have

$$\left\{1+\lambda k-k-\lambda k^2\right\}^{a-1}=\left\{1+\lambda k-k-\lambda k^2\right\}^{a-1}.$$

Then applying (10) to the last term of (6) i.e., the term with power $b-1$

$$\sum_{k=0}^{\infty}\frac{(-1)^q\,\Gamma(b)}{q!\,\Gamma(b-q)}\left\{\left[1+\lambda k-k-\lambda k^2\right]^a\right\}^q=\sum_{k=0}^{\infty}\frac{(-1)^q\,\Gamma(b)}{q!\,\Gamma(b-q)}\left[1+\lambda k-k-\lambda k^2\right]^{aq}. \quad (11)$$

Then the pdf of Kw-TR becomes

$$f(x)=\underbrace{\frac{x}{\alpha^2}k\left\{1+\lambda k-k-\lambda k^2\right\}}_{g(x)}\underbrace{\sum_{q=0}^{\infty}\frac{(-1)^q\,ab\Gamma(b)}{q!\,\Gamma(b-q)}\left[1+\lambda k-k-\lambda k^2\right]^{(q+1)a-1}}_{G(x)^{(q+1)a-1}}. \quad (12)$$

This can be re-written as

$$f(x)=\sum_{q=0}^{\infty} v_q g(x) G(x)^{(q+1)a-1} \tag{13}$$

where $v_q=\dfrac{(-1)^q ab\Gamma(b)}{q!\Gamma(b-q)}$, $g(x)=\dfrac{x}{\alpha^2}k\{1+\lambda k-k-\lambda k^2\}$ and $G(x)=\{1+\lambda k-k-\lambda k^2\}$

where $g(x)$ and $G(x)$ are the PDF and CDF of transmuted Rayleigh distribution, respectively.

3.4. Quantile function

Quantile function is defined as the inverse function of the CDF of a random variable X. The quantile function of a distribution is the real solution of $F(xq)=q$ for $0\le q\le 1$. And the quantiles (xq) of KWTR distribution is expressed in the form given below as

$$x_q=\alpha\sqrt{-\ln\left[1-\left[1-(1-q)^{\frac{1}{b}}\right]^{\frac{1}{a}}\right]}. \tag{14}$$

3.5. Maximum likelihood estimation of the proposed distribution

Maximum likelihood estimation (MLE) is generally known for its estimation of unknown parameters. Considering a random sample $x_1,x_2,...,x_n$ from the Kumaraswamy transmuted Rayleigh (Kw-TR) distribution. Then, the likelihood function is given by

$$l=n\log a+n\log b+\sum_{i=1}^{n}\log\left(k_i\frac{x}{\alpha^2}\right)+\sum_{i=1}^{n}\log(1-\lambda+2\lambda k_i)+(a-1)\sum_{i=1}^{n}\log(1-k_i)$$
$$+(b-1)\sum_{i=1}^{n}\left[1-\left(1-k_i+\lambda k_i-\lambda k_i^2\right)^a\right] \tag{15}$$

where $k_i=\exp\left(\dfrac{-x_i^{\,2}}{2\alpha^2}\right)$.

Let $j_i=1-\lambda+2\lambda k_i$, $z_i=1+\lambda k_i$ and $r_i=z_i(1-k_i)$

$$l=n\log a+n\log b+\sum_{i=1}^{n}\log\left(\frac{x_i}{\alpha^2}\right)-\sum_{i=1}^{n}\left(\frac{x_i}{2\alpha^2}\right)+\sum_{i=1}^{n}\log j_i+(a-1)\sum_{i=1}^{n}\log r_i$$
$$+(b-1)\sum_{i=1}^{n}\log\left(1-r_i^a\right) \tag{16}$$

$$\frac{\partial l}{\partial a}=\frac{n}{a}+\sum_{i=1}^{n}\log r_i-(b-1)\sum_{i=1}^{n}\frac{r_i^a\log r_i}{1-r_i^a} \tag{17}$$

$$\frac{\partial l}{\partial b}=\frac{n}{b}+\sum_{i=1}^{n}\log\left(1-r_i^a\right) \tag{18}$$

$$\frac{\partial l}{\partial \alpha}=\frac{2n}{\alpha}+\sum_{i=1}^{n}\frac{x_i^2}{\alpha^3}+2\lambda\sum_{i=1}^{n}\frac{k_i}{\alpha^3}\bigg/j_i-(a-1)\sum_{i=1}^{n}\left[\frac{k_i}{\alpha^3}\left[z_i-\lambda(1-k_i)\right]\Big/r_i\right]$$
$$+(b-1)\sum_{i=1}^{n}\left[\frac{k_i}{\alpha^3}\left[z_i-\lambda(1-k_i)\right]\Big/r_i\right]-\left\{(b-1)\sum_{i=1}^{n}\left[\frac{k_ix_i^2}{\alpha^3}\left[z_i-\lambda(1-k_i)\right]\Big/(1-r_i)\right]\right\} \tag{19}$$

$$\frac{\partial l}{\partial \lambda}=\sum_{i=1}^{n}\frac{2k_i}{j_i}+(a-1)\sum_{i=1}^{n}\frac{k_i(1-k_i)}{r_i}-(b-1)\sum_{i=1}^{n}\frac{a\left[k_i(1-k_i)\right]}{1-r_i^{a}}. \tag{20}$$

4. Simulation Study

The Monte Carlo Simulation approach will be used to evaluate the maximum likelihood estimations (MLEs) of the KWTR distribution for varying values of parameters of the proposed distribution. A random samples of size N will be selected in the estimation. This approach was also implemented by Cakmakyapan and Ozel (2018). The plots of the several sample sizes is presented in Figures 3-5.

Table 1 Maximum likelihood estimates of the proposed distribution for varying values

Parameter	α	λ	a	b
Actual value	1.00	0.80	3.00	2.00
Estimate (N = 10,000)	0.90	0.69	3.92	1.64
Estimate (N = 1,000)	1.07	0.74	2.84	1.96
Estimate (N = 100)	0.95	0.53	4.26	2.19

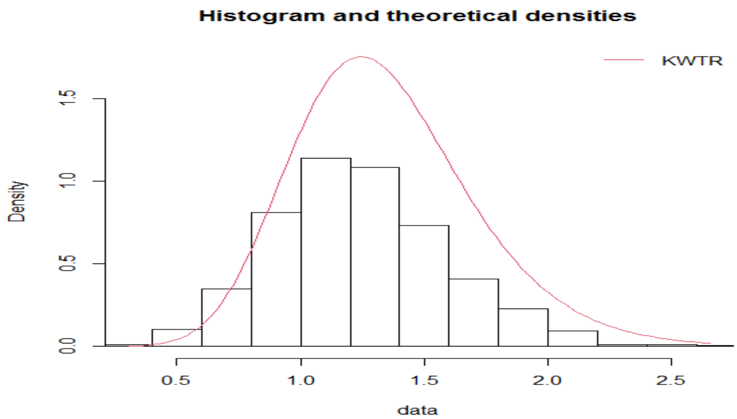

Figure 3 The density plot when N = 10,000

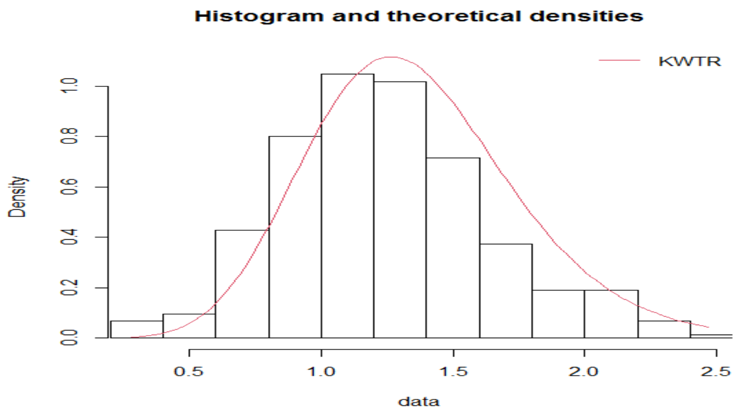

Figure 4 The density plot when N = 1,000

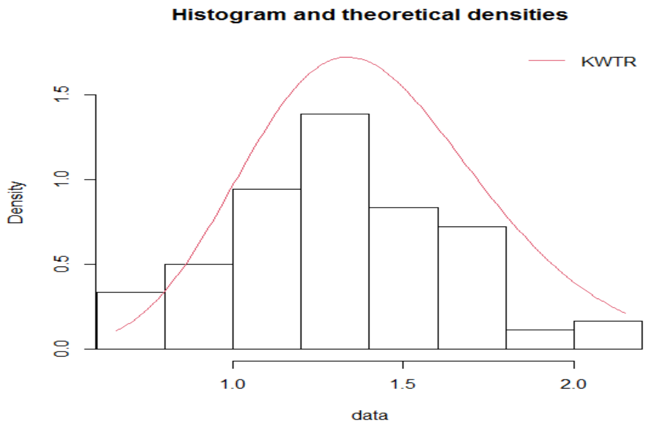

Figure 5 The density plot when N = 100

4.1. Applications

In order to investigate the robustness of the proposed distribution, two different datasets were used in this section. The proposed KWTR distribution will be compared with the other existing distributions including the exponentiated transmuted generalized Rayleigh distribution (ETGR) (Ahmed et al. 2015). The goodness of fit statistic and the goodness of fit plot were provided in order to check the models that best fit the data among the models for each dataset used for this research. The first and second datasets considered were single fibers tested under tension at gauge lengths 20 mm (data set 1 with sample size of 74 observations) and 10 mm (data set 2 with sample size of 63 observations) obtained from Kundu and Raqab (2009).

Table 2 Maximum likelihood estimates for 20 mm. data set

Model	Parameters			
	σ	A	B	Λ
KWTR	12.58	3.51	2.05	−0.33
	(33.81)	(3.35)	(1.50)	(2.05)
ETGR	0.70	2.12	7.79	0.32
	(0.04)	(0.32)	(1.73)	(0.23)
TGR	0.62	5.51	0.36	-
	(0.02)	(0.78)	(0.25)	-
GR	0.64	7.78	-	-
	(1.49)	(0.05)	-	-
R	0.40	-	-	-
	(0.02)	-	-	-

Table 3 Log-likelihood, AIC and BIC for the fitted distributions for 20 mm. data set

MODEL	Log-likelihood	AIC	BIC
KWTR	51.132	110.264	119.480
ETGR	113.400	121.352	130.600
TGR	123.610	129.610	136.500
GR	135.202	139.202	143.811
R	188.302	190.302	192.606

Table 4 Goodness of fit statistic for 20 mm. data set

MODEL	Goodness of fit statistic		
	Kolmogorov-Smirnov statistic	Cramér-von Mises statistic	Anderson-Darling statistic
KWTR	0.05621079	0.02566735	0.20015125
ETGR	0.07381433	0.12020216	0.78642796
TGR	0.61522390	8.94058030	-
GR	0.07360201	0.10969873	0.71394327
R	0.33929870	2.65435530	13.31257390

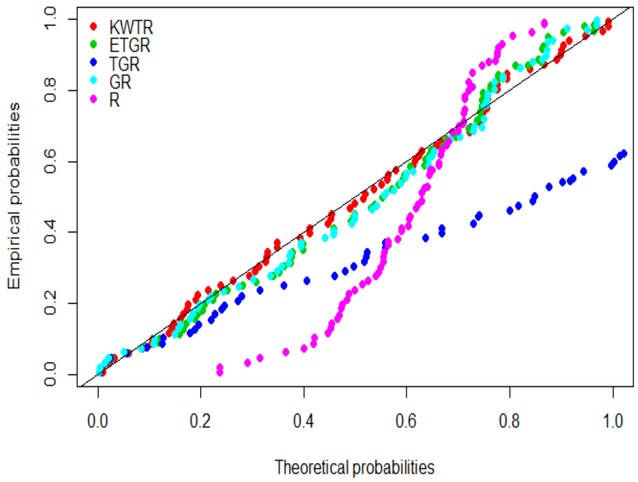

Figure 6 The P-P plots of gauge lengths of 20 mm. data

Table 2 shows the MLE of the parameters and their standard errors in parenthesis also Table 3 shows the log-likelihood, Akaike information criterion (AIC) and Bayesian information criterion (BIC) values for the fitted Kumaraswamy transmuted Rayleigh (KWTR), exponentiated transmuted generalized Rayleigh (ETGR), transmuted generalized Rayleigh (TGR), generalized Rayleigh (GR) and Rayleigh (R) distributions for the first data set of 20 mm. gauge length. Also, the goodness of fit statistics is presented in Table 4. The probability plot of the datasets for the distributions used in this study is presented in Figure 6. The histogram of the data and the estimated density functions is also shown in Figure 7. Also Figure 8 shows the empirical cumulative function of the data and the estimates.

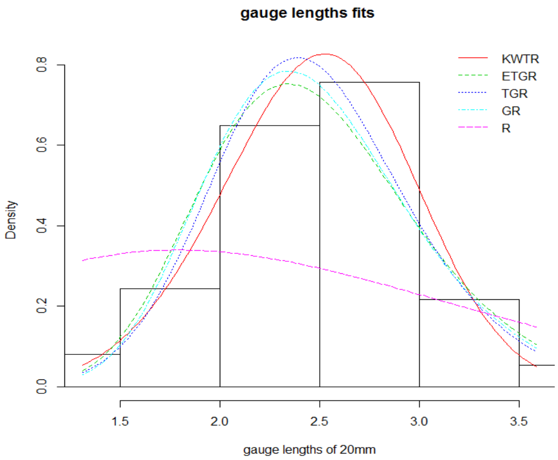

Figure 7 The histogram of the fitted and estimated distributions of gauge lengths of 20 mm. data

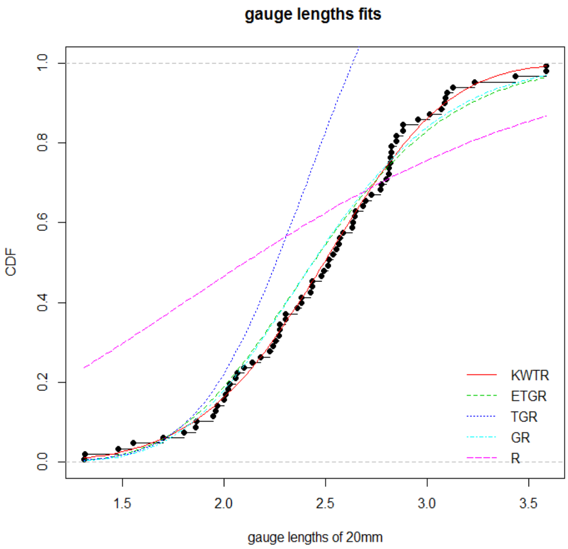

Figure 8 The CDF of the fitted and estimated distributions of gauge lengths of 20 mm. data

Table 5 Maximum likelihood estimates for 10 mm. data set

Model	Parameters			
	Σ	Α	*b*	Λ
KWTR	1.6366 (1.6961)	2.1061 (5.9399)	1.7517 (1.8407)	−0.7344 (1.8224)
ETGR	0.5347 (0.0510)	2.4051 (0.8800)	0.5830 (0.2070)	6.9503 (1.4880)
TGR	0.5021 (0.0110)	6.2143 (1.2160)	0.1207 (0.3690)	- -
GR	0.5145 (0.0240)	6.2130 (0.9660)	- -	- -
R	0.3200 (0.0240)	- -	- -	- -

Table 6 Log-likelihood, AIC and BIC for the fitted distributions for 10 mm. data set

Model	Log-likelihood	AIC	BIC
KWTR	55.95	119.91	128.49
ETGR	115.00	122.97	131.50
TGR	118.64	124.63	131.06
GR	122.60	126.62	130.91
R	187.04	189.04	191.18

Table 7 Goodness of fit statistic for 10 mm. data set

Model	Goodness of fit statistic		
	Kolmogorov-Smirnov	Cramér-von Mises	Anderson-Darling
KWTR	0.07173262	0.04429238	0.26657112
ETGR	0.08261972	0.06183080	0.33526336
TGR	0.12452320	0.20166460	1.51058230
GR	0.08737195	0.06418944	0.35980264
R	0.36071560	2.19116620	11.0175148

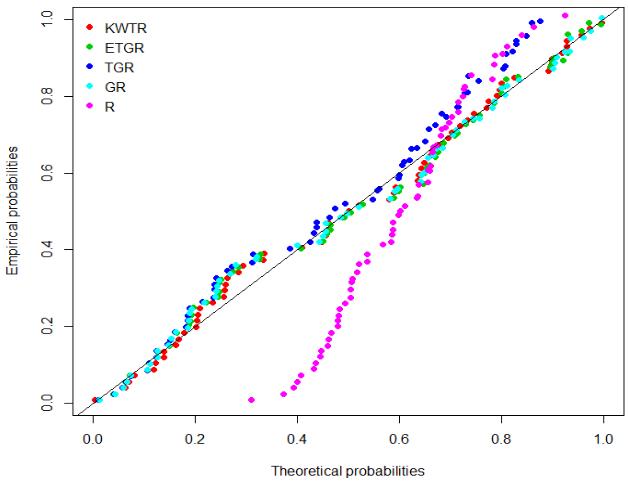

Figure 9 The P-P plots of gauge lengths of 10 mm. data

In Table 5, the maximum likelihood estimation (MLEs) of the parameters and their standard errors in parenthesis is presented and Table 6 shows the log-likelihood, AIC and BIC values for the Proposed distribution and its counterparts considered in this study for the second data set of 10 mm. gauge length. Figure 9 shows the probability of the data and the estimated ones. Figure 10 shows the histogram of the data and the estimated density functions. Figure 11 shows the empirical cumulative function of the data and the estimated ones.

The numerical results in Tables 3 and 6 shows that the log-likelihood, AIC and BIC of the KWTR distribution are lower than the other four allied fitted distributions. Likewise, the goodness of fit statistic in Tables 4 and 7 shows that KWTR has the lowest Kolmogorov-Smirnov, Cramér-von Mises and Anderson-Darling statistics which served as baseline of comparison. Figures 6-11 show that

KWTR distribution provides a good fit as its points were closer to the expected line. Therefore, KWTR distribution can be chosen as the best model for lifetime dataset.

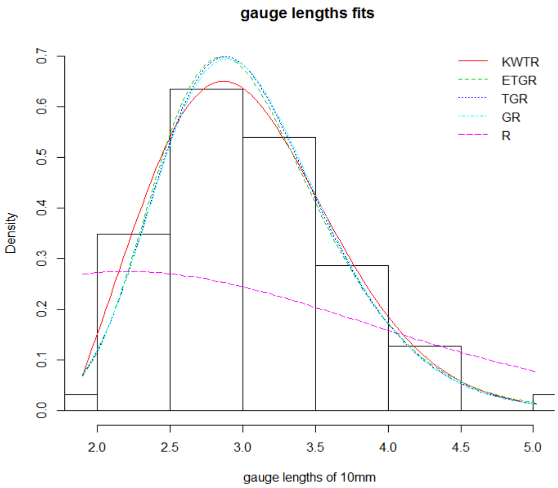

Figure 10 The histogram of the fitted and estimated distributions of gauge lengths of 10 mm. data

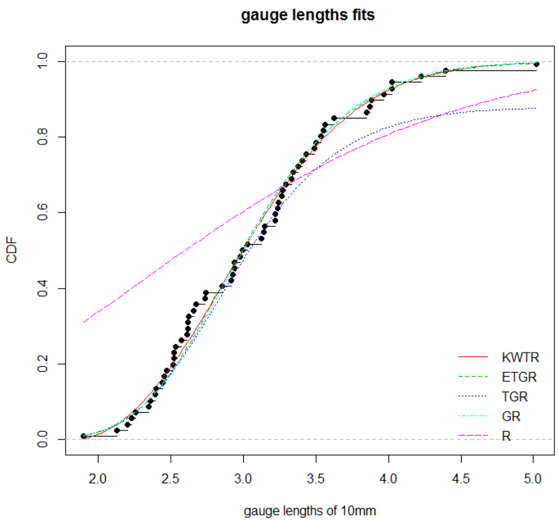

Figure 11 The CDF of the fitted and estimated distributions of gauge lengths of 10 mm. data

5. Conclusions

We proposed a new family of Rayleigh distribution, named the Kumaraswamy transmuted Rayleigh distribution. The KWTR distribution provides better results than the ETGR, TGR, GR and R distributions. In this model, the new distribution provides more flexibility in modeling reliability data. We were able to derive the density and distribution function, survival function, hazard function and quantile function of the proposed distribution. We discussed the maximum likelihood estimation and also obtained the standard error of each parameter estimate as well as the information criteria used for comparison. Application to real dataset using MLE, AIC, BIC and goodness of fit statistic to show the usefulness of the new distribution was illustrated. It is evident KWTR could be chosen as the best model over other models in comparison from the application to real life dataset. We hope that the proposed KWTR model may attract and find wider application in the analysis of reliability data, insurance, engineering, etc.

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