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Modified Searls Predictive Estimation of Population Mean Using Known Auxiliary Character

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Abstract

This paper suggests an improved Searls predictive estimation of population mean of the variable under study through predictive approach using known auxiliary information. The statistical sampling properties of the sampling distribution of the suggested class are studied up to the approximation of degree one. The optimum values of the Searls type characterizing constants are obtained which minimizes the mean squared error (MSE) of the proposed estimator. The minimum value of the MSE of the suggested class of estimators is also obtained for these optimum values of the characterizing constants. Through the theoretical comparison, the conditions under which, the suggested estimator performs better than the existing competing estimators are obtained. To verify the theoretical findings, a numerical illustration is also presented using some natural populations. The theoretical conditions are justified through the improvement of suggested family over competing estimators as it has least MSE among the class of all competing estimators for simple random sampling scheme under predictive approach.

Keywords: Predictive approach, Searls type estimators, auxiliary variable, bias, mean squared error, percentage relative efficiency.

1. Introduction

Sampling is a very good alternative of complete enumeration, whenever the population is very large, as it is a time and cost effective technique of data collection and provides valid inferences about the population parameters under consideration. As sample is only a part of a large population, prediction or extrapolation may cause the errors while our purpose is to reduce the errors by searching for such estimators whose sampling distributions are more and more concentrated around the true population parameter under consideration. This is achieved through the use of auxiliary information, supplied by the auxiliary variable which has a high positive or negative correlation with the study variable. It is well established from the survey literature that there are various sampling techniques including design based and model based which enhances the estimation of population parameters through more efficient estimators. These estimators make use of known auxiliary or supplementary information for improved estimation of population parameters under consideration for predictive approach.

Under the predictive approach of survey sampling, a specific model is formulated for the population values under consideration and through this model the prediction of the non-sampled values is done. As for as the improved estimation of population mean of study variable is concerned, Basu (1971) suggested prediction approach for the estimation of finite population mean of main variable using known information on auxiliary variable. Under this approach, he has shown the compromise between the prediction of the mean of the unobserved units and the mean of the sampled units of the population under consideration. He has given the methodology for improved estimation of the population mean of primary variable utilizing information on secondary variable under predictive approach of survey sampling design.

Following Basu (1971) prediction approach, various authors suggested different modified and improved estimators of ratio, product, difference and regression type for improved estimation of population mean of study variable using known information on auxiliary variable under this approach. The known auxiliary information has been utilized in the form of its parameters, which includes population mean, coefficient of variation (CV), coefficient of skewness and kurtosis, coefficient of correlation, median, quartiles, deciles and their functions and many other traditional and non-traditional parameters of auxiliary variable. The contributions by the authors Srivastava (1983), Biradar and Singh (1998), Agrawal and Roy (1999), Sahoo and Panda (1999), Sahoo and Sahoo (2001), Ahmed (2004), Sahoo et al. (2009), Nayak and Sahoo (2012), Saini (2013), Singh et al. (2014), Yadav and Mishra (2015), Singh and Singh (2014), Singh et al. (2019), and Singh and Vishwakarma (2019) are remarkable for improved estimation of population mean under simple random sampling technique under predictive approach.

In the present investigation, a new general class of estimators for enhanced estimation of finite population mean under predictive approach is suggested and its sampling properties are studied up to the approximation of order one. The whole manuscript is presented in various sections which includes Introduction, review of existing estimators, proposed estimator, efficiency comparison, numerical illustration, results and discussion, Conclusion and the paper completes with the references.

2. Review of Existing Estimators

Let the population U under investigation be finite, consisting of N distinct and identifiable units $U_1, U_2, ..., U_N$. Let the study variable and the auxiliary variables be denoted by Y and X respectively. The i^{th} observations for the population under study on the characteristics (Y, X) be denoted by $(y_i, x_i),\ i = 1, 2, ..., N.$ Further let a required sample of size n be taken from the above population using simple random sampling with replacement technique and $(\bar{Y}, \bar{X})$ and $(\bar{y}, \bar{x})$ are the population means and the sample means for primary and secondary variables, respectively. It is assumed that we have full information about the auxiliary variable that is $\bar{X}$ is known. Let S be the collection of all possible samples taken from the population U with $s \in S$ as the sample of size n and let $\vartheta(s)$ be the effective sample size representing the number of distinct units of the sample s. Let $\bar{s}$ be the set of all units of the population U which do not belong to s, thus $\bar{s} \in \bar{S} = U - S.$

Now it is defined in predictive approach of sampling that, $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i,$ the population mean of study variable $y,$ to be estimated, $\bar{Y}_s = \frac{1}{\vartheta(s)}\sum_{i\in s} y_i,$ mean of the sampled units, $\bar{Y}_{\bar{s}} = \frac{1}{[N-\vartheta(s)]}\sum_{i\in\bar{s}} y_i,$ mean of the non-sampled units. For any observed $s \in S,$ the population mean may be written as, $\bar{Y} = \left[\frac{\vartheta(s)}{N}\bar{Y}_s + \frac{[N-\vartheta(s)]}{N}\bar{Y}_{\bar{s}}\right].$

In the above setup of the population mean $\bar{Y},$ the mean of the sampled units $\bar{Y}_s$ is known as it is computed for the observed values of y in the sample $s.$ Therefore, our attempt is to predict $\bar{Y}_s,$ the mean of the non-observed units of the population U on the basis of the sampled units of the sample $s.$ Many decision-theorists might not be willing for the choice of predictive estimators but Basu (1971), still taken into account such approach for representing the "heart of the matter" for estimating the finite population mean (see Cassel 1977, p.110).

Now for simple random sampling technique with sample size n $(\vartheta(s) = n)$ and thus the sample mean is given by

$$\bar{Y}_s = \bar{y} = \frac{1}{n}\sum_{i\in\bar{s}} y_i.$$

The population mean of the main variable may be written as,

$$\bar{Y} = \left[\frac{n}{N}\bar{Y}_s + \frac{[N-n]}{N}\bar{Y}_{\bar{s}}\right]. \quad (1)$$

Now any estimator t of the population mean $\bar{Y}$ may be given as,

$$t = \left[\frac{n}{N}\bar{y} + \frac{[N-n]}{N}T\right], \quad (2)$$

where T is taken as the predictor of $\bar{Y}_{\bar{s}}.$

Table 1 represents estimators of population mean $\bar{Y}$ using auxiliary information for different choices of $T,$ the predictor of $\bar{Y}_{\bar{s}}$ under predictive approach as, where $\theta = (1-f)^{-1},$ $f = (n/N),$ $C_y^2 = (S_y^2/\bar{Y}^2),$ $C_x^2 = (S_x^2/\bar{X}^2),$ $C = \rho(C_y/C_x),$ $\rho = (S_{yx}/S_y S_x),$ $S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2,$ $S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \bar{X})^2,$ $S_{yx} = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X}),$ $\bar{x} = \frac{1}{n}\sum_{i\in s} x_i,$ $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i,$ $\bar{X}_{\bar{s}} = \frac{1}{N-n}\sum_{i\in\bar{s}} x_i = \frac{N\bar{X} - n\bar{x}}{N-n}$ and $L = \frac{1}{8}\left[\frac{4C + 1 + 7f + 4f^2}{(1-f)} - C^2\right].$

Singh et al. (2019) suggested the Searls (1964) type estimators of the above estimators presented in Table 1 except Yadav and Mishra (2015) estimator, which are presented in Table 2 along with their biases and mean squared errors up to the approximation of order one.

Table 1 Estimators of $\bar{Y}$ and $\bar{Y}_s$ along with the bias and MSE of estimators of $\bar{Y}$

S. No.	Estimator of $\bar{Y}$	Estimator T for predictor $\bar{Y}_s$	Bias	Variance/MSE
1.	$t_1=\left[\frac{n}{N}\bar{y}+\frac{[N-n]}{N}T_1\right]$ Srivastava (1983) Mean Per Unit Estimator	$T_1=\frac{1}{n}\sum_{i\in\bar{s}}y_i=\bar{y}$	0	$\theta\bar{Y}^2C_y^2$
2.	$t_2=\left[\frac{n}{N}\bar{y}+\frac{[N-n]}{N}T_2\right]$ Srivastava (1983) Regression Estimator	$T_2=\bar{y}+b(\bar{X}_{\bar{s}}-\bar{x})$	0	$\theta\bar{Y}^2[C_y^2-CC_x^2]$
3.	$t_3=\left[\frac{n}{N}\bar{y}+\frac{[N-n]}{N}T_3\right]$ Srivastava (1983) Ratio Estimator	$T_3=\bar{y}\left(\frac{\bar{X}_{\bar{s}}}{\bar{x}}\right)$	$\theta\bar{Y}C_x^2(1-C)$	$\theta\bar{Y}^2[C_y^2+(1-2C)C_x^2$
4.	$t_4=\left[\frac{n}{N}\bar{y}+\frac{[N-n]}{N}T_4\right]$ Srivastava (1983) Product type Estimator	$T_4=\bar{y}\left(\frac{\bar{x}}{\bar{X}_{\bar{s}}}\right)$	$\theta\bar{Y}C_x^2[C+f(1-f)^{-1}]$	$\theta\bar{Y}^2[C_y^2+(1+2C)C_x^2$
5.	$t_5=\left[\frac{n}{N}\bar{y}+\frac{[N-n]}{N}T_5\right]$ Srivastava (1983) Product Estimator	$T_5=\bar{y}\left(\frac{\bar{x}}{\bar{X}_s}\right)$	$\theta\bar{Y}CC_x^2$	$\theta\bar{Y}^2[C_y^2+(1+2C)C_x^2$
6.	$t_6=\left[\frac{n}{N}\bar{y}+\frac{[N-n]}{N}T_6\right]$ Singh et al. (2014)	$T_6=\bar{y}\exp\left(\frac{\bar{X}_{\bar{s}}-\bar{x}}{\bar{X}_{\bar{s}}+\bar{x}}\right)$	$\frac{\theta}{8}\bar{Y}C_x^2[3-4(C+f)]$	$\theta\bar{Y}^2[C_y^2+(1-4C)\frac{C_x^2}{4}]$
7.	$t_7=\left[\frac{n}{N}\bar{y}+\frac{[N-n]}{N}T_7\right]$ Singh et al. (2014)	$T_7=\bar{y}\exp\left(\frac{\bar{x}-\bar{X}_{\bar{s}}}{\bar{x}+\bar{X}_{\bar{s}}}\right)$	$\frac{\theta}{8}\bar{Y}C_x^2[4C-\frac{1}{(1-f)}]$	$\theta\bar{Y}^2[C_y^2+(1+4C)\frac{C_x^2}{4}]$
8.	$t_8=\left[\alpha t_6+(1-\alpha)t_7\right]$ Yadav and Mishra (2015)	T_6 and T_7	$\bar{Y}L\theta C_x^2$	$\theta\bar{Y}^2[C_y^2-CC_x^2]$

Table 2 Estimators, bias and minimum MSE of Singh et al. (2019) estimators

S.No.	Estimator of $\bar{Y}$	Bias	Minimum MSE
1	$t_1' = \omega_1 t_1$ Singh et al. (2019)	0	$\bar{Y}^2\left[\frac{MSE(t_1)}{\bar{Y}^2 + MSE(t_1)}\right]$
2	$t_2' = \omega_2 t_2$ Singh et al. (2019)	$\omega_2\bar{Y}\left[1+(3-4f)\frac{\theta}{8}C_x^2 - \frac{1}{2}\theta C_{yx}\right]$	$\bar{Y}^2\left[\frac{MSE(t_2)-\{Bias(t_2)\}^2}{\bar{Y}^2 + MSE(t_2) - 2\bar{Y}Bias(t_2)}\right]$
3	$t_3' = \omega_3 t_3$ Singh et al. (2019)	$\omega_3\bar{Y}\left[1+(3-4f)\frac{\theta}{8}C_x^2 - \frac{1}{2}\theta C_{yx}\right]$	$\bar{Y}^2\left[\frac{MSE(t_3)-\{Bias(t_2)\}^2}{\bar{Y}^2 + MSE(t_3) - 2\bar{Y}Bias(t_3)}\right]$
4	$t_4' = \omega_4 t_4$ Singh et al. (2019)	$\omega_4\bar{Y}\left[1+(3-4f)\frac{\theta}{8}C_x^2 - \frac{1}{2}\theta C_{yx}\right]$	$\bar{Y}^2\left[\frac{MSE(t_4)-\{Bias(t_4)\}^2}{\bar{Y}^2 + MSE(t_4) - 2\bar{Y}Bias(t_4)}\right]$
5	$t_5' = \omega_5 t_5$ Singh et al. (2019)	$\omega_5\bar{Y}\left[1+(3-4f)\frac{\theta}{8}C_x^2 - \frac{1}{2}\theta C_{yx}\right]$	$\bar{Y}^2\left[\frac{MSE(t_5)-\{Bias(t_5)\}^2}{\bar{Y}^2 + MSE(t_5) - 2\bar{Y}Bias(t_5)}\right]$
6	$t_6' = \omega_6 t_6$ Singh et al. (2019)	$\omega_6\bar{Y}\left[1+(3-4f)\frac{\theta}{8}C_x^2 - \frac{1}{2}\theta C_{yx}\right]$	$\bar{Y}^2\left[\frac{MSE(t_6)-\{Bias(t_6)\}^2}{\bar{Y}^2 + MSE(t_6) - 2\bar{Y}Bias(t_6)}\right]$
7	$t_7' = \omega_7 t_7$ Singh et al. (2019)	$\omega_7\bar{Y}\left[1+(3-4f)\frac{\theta}{8}C_x^2 - \frac{1}{2}\theta C_{yx}\right]$	$\bar{Y}^2\left[\frac{MSE(t_7)-\{Bias(t_7)\}^2}{\bar{Y}^2 + MSE(t_7) - 2\bar{Y}Bias(t_7)}\right]$

The optimum values of the characterizing scalars $\omega_i, (i = 2,3,...,7),$ respectively are

$$\omega_{i(opt)} = \left[\frac{\bar{Y}^2 + \bar{Y}Bias(t_i)}{\bar{Y}^2 + MSE(t_i) + 2\bar{Y}Bias(t_i)}\right], \ (i = 2,3,...,7).$$

3. Proposed Estimator

In the present section, we suggest a new class of estimators by combining any two estimators of Singh et al. (2019) estimators as,

$$t_p = \omega_i t_i + \omega_j t_j; \ i = 2,3,...,7, j = 2,3,...,7 \text{ and } i \neq j, \quad (3)$$

where ω_i and ω_j are the characterizing scalars to be determined such that the MSE of the proposed estimator is minimum and $\omega_i + \omega_j \neq 1.$

The following are worthy observations that:

(1) The suggested class of estimators reduces to the estimators of Table 1 other than Yadav and Mishra (2015) estimator if we put and $\omega_i = 0$ and $\omega_j = 1$ or vice-versa.

(2) If $\omega_i + \omega_j = 1,$ then the suggested class reduces to Yadav and Mishra (2015) estimator for t_6 and t_7, respectively.

(3) If either ω_i or ω_j is zero, then the suggested class of estimators reduces to Singh et al. (2019) class of estimators.

Following Table 3 presents some members of the suggested family as,

Table 3 Some members of the proposed family of estimators

S.No.	Estimator
1	$t_{p1} = \omega_2 t_2 + \omega_3 t_3$
2	$t_{p2} = \omega_2 t_2 + \omega_4 t_4$
3	$t_{p3} = \omega_3 t_3 + \omega_5 t_5$
4	$t_{p4} = \omega_3 t_3 + \omega_6 t_6$
5	$t_{p5} = \omega_4 t_4 + \omega_5 t_5$
6	$t_{p6} = \omega_4 t_4 + \omega_6 t_6$
7	$t_{p7} = \omega_5 t_5 + \omega_6 t_6$
8	$t_{p8} = \omega_5 t_5 + \omega_7 t_7$
9	$t_{p9} = \omega_6 t_6 + \omega_7 t_7$
10	$t_{p10} = \omega_7 t_7 + \omega_6 t_6$

Many more members of this family may be obtained for different values of i and j defined above such that $i \neq j.$ As it is evident from literature that the exponential ratio and product type estimators are more efficient than the traditional ratio and product type estimator, we are considering the case of exponential estimators as

$$t_p = \omega_6 t_6 + \omega_7 t_7. \tag{4}$$

To study the large sampling properties of the suggested class of estimators $t_p,$ we use the following standard approximations as $\bar{y} = \bar{Y}(1+e_0)$ and $\bar{x} = \bar{X}(1+e_1)$ such that $E(e_0) = E(e_1) = 0$ and $E(e_0^2) = \theta C_y^2,\ E(e_1^2) = \theta C_x^2,\ E(e_0 e_1) = \theta C C_x^2.$

To find the bias and mean squared error of the proposed estimator, we express the suggested estimator in terms of above approximations and we have

$$t_p = \omega_6 \bar{Y}[1+e_0 - \frac{e_1}{2} - \frac{e_0 e_1}{2} + \frac{e_1^2}{8}(3-4f)] + \omega_7 \bar{Y}[1+e_0 + \frac{e_1}{2} + \frac{e_0 e_1}{2} + \frac{e_1^2}{8}(4f-1)]$$

$$= \omega_6 \bar{Y}[1+e_0 - \frac{e_1}{2} - \frac{e_0 e_1}{2} + \frac{e_1^2}{8} A_1] + \omega_7 \bar{Y}[1+e_0 + \frac{e_1}{2} + \frac{e_0 e_1}{2} + \frac{e_1^2}{8} A_2],$$

where $A_1 = (3-4f)$ and $A_2 = (4f-1).$ Now, we have

$$t_p = \bar{Y}\left[\omega_6\left(1+e_0 - \frac{e_1}{2} - \frac{e_0 e_1}{2} + \frac{e_1^2}{8} A_1\right) + \omega_7\left(1+e_0 + \frac{e_1}{2} + \frac{e_0 e_1}{2} + \frac{e_1^2}{8} A_2\right)\right]. \tag{5}$$

Subtracting $\bar{Y}$ on both sides of (5), we have

$$t_p - \bar{Y} = \bar{Y}\left[\omega_6\left(1+e_0 - \frac{e_1}{2} - \frac{e_0 e_1}{2} + \frac{e_1^2}{8} A_1\right) + \omega_7\left(1+e_0 + \frac{e_1}{2} + \frac{e_0 e_1}{2} + \frac{e_1^2}{8} A_2\right) - 1\right]. \tag{6}$$

Taking expectations on both sides of (6) and putting values of different expectations and simplifying, we get the bias of t_p up to approximation of order one as

$$Bias(t_p) = \bar{Y}\left[(\omega_6 + \omega_7) + \left(\frac{A_1\omega_6 + A_2\omega_7}{8}\right)\theta C_x^2 - \left(\frac{\omega_6 - \omega_7}{2}\right)\theta C_{yx} - 1\right]. \quad (7)$$

Squaring on both sides of (6), simplifying and taking expectations on both sides, we get the MSE of the suggested estimator as,

$$MSE\ (t_p) =$$

$$\bar{Y}^2 \left\{ \begin{array}{l} 1 + \omega_6^2\left[1 + \theta C_y^2 + \frac{(A_1+1)}{4}\theta C_x^2 - 2\theta C_{yx}\right] + \omega_7^2\left[1 + \theta C_y^2 + \frac{(A_2+1)}{4}\theta C_x^2 - 2\theta C_{yx}\right] \\ -2\omega_6\left[1 + \frac{A_1}{4}\theta C_x^2 - \frac{1}{2}\theta C_{yx}\right] - 2\omega_7\left[1 + \frac{A_2}{4}\theta C_x^2 + \frac{1}{2}\theta C_{yx}\right] + 2\omega_6\omega_7\left[1 + \theta C_y^2 + \frac{A_3}{8}\theta C_x^2\right] \end{array} \right\}, \quad (8)$$

where, $A_3 = (A_1 + A_2 - 2)$.

Now,

$$MSE(t_p) = \bar{Y}^2[1 + \omega_6^2 D_1 + \omega_7^2 D_2 - 2\omega_6 D_3 - 2\omega_7 D_4 + 2\omega_6\omega_7 D_5], \quad (9)$$

where, $D_1 = \left\{1 + \theta C_y^2 + \frac{(A_1+1)}{4}\theta C_x^2 - 2\theta C_{yx}\right\}$, $D_2 = \left\{1 + \theta C_y^2 + \frac{(A_2+1)}{4}\theta C_x^2 - 2\theta C_{yx}\right\}$,

$D_3 = \left\{1 + \frac{A_1}{4}\theta C_x^2 - \frac{1}{2}\theta C_{yx}\right\}$, $D_4 = \left\{1 + \frac{A_2}{4}\theta C_x^2 + \frac{1}{2}\theta C_{yx}\right\}$ and $D_5 = \left\{1 + \theta C_y^2 + \frac{A_3}{8}\theta C_x^2\right\}$.

The optimum values of ω_6 and ω_7 in (9) are obtained by partially differentiating equation (9) with respect to ω_6 and ω_7 respectively and equating them to zero. The optimum values of ω_6 and ω_7 respectively are,

$$\omega_{6(opt)} = \frac{D_2D_3 - D_4D_5}{D_1D_2 - D_5^2}, \text{ and } \omega_{7(opt)} = \frac{D_1D_4 - D_3D_5}{D_1D_2 - D_5^2}.$$

The minimum MSE of the suggested estimator for these optimum values of characterizing scalars is obtained as,

$$MSE(t_p) = \bar{Y}^2 \left\{ \begin{array}{l} 1 + \left(\frac{D_2D_3 - D_4D_5}{D_1D_2 - D_5^2}\right)^2 D_1 + \left(\frac{D_1D_4 - D_3D_5}{D_1D_2 - D_5^2}\right)^2 D_2 - 2\left(\frac{D_2D_3 - D_4D_5}{D_1D_2 - D_5^2}\right)D_3 \\ -2\left(\frac{D_1D_4 - D_3D_5}{D_1D_2 - D_5^2}\right)D_4 + 2\left(\frac{D_2D_3 - D_4D_5}{D_1D_2 - D_5^2}\right)\left(\frac{D_1D_4 - D_3D_5}{D_1D_2 - D_5^2}\right)D_5 \end{array} \right\},$$

$$= \bar{Y}^2 \left\{ 1 + \frac{\left[\begin{array}{l} (D_2D_3 - D_4D_5)^2 D_1 + (D_1D_4 - D_3D_5)^2 D_2 - 2(D_2D_3 - D_4D_5)(D_1D_2 - D_5^2)D_3 \\ -2(D_1D_4 - D_3D_5)(D_1D_2 - D_5^2)D_4 + 2(D_2D_3 - D_4D_5)(D_1D_4 - D_3D_5)D_5 \end{array}\right]}{(D_1D_2 - D_5^2)^2} \right\},$$

$$MSE(t_p) = \bar{Y}^2\left[1 + \frac{H}{M^2}\right], \quad (10)$$

where $H = \left\{ \begin{array}{l} (D_2D_3 - D_4D_5)^2 D_1 + (D_1D_4 - D_3D_5)^2 D_2 - 2(D_2D_3 - D_4D_5)(D_1D_2 - D_5^2)D_3 \\ -2(D_1D_4 - D_3D_5)(D_1D_2 - D_5^2)D_4 + 2(D_2D_3 - D_4D_5)(D_1D_4 - D_3D_5)D_5 \end{array} \right\}$ and

$M = (D_1D_2 - D_5^2)$.

4. Relative Efficiency Comparison of Competing Estimators

Under this section the theoretical relative efficiencies of the suggested estimator over competing estimators of population mean has been presented. The efficiency conditions under which the proposed estimator performs better than the competing estimators are also given in Table 4.

Table 4 Relative efficiency of proposed estimator over competing estimators

S.No.	Efficiency comparison	Condition to be more efficient
1	$V(t_1)-MSE(t_p)>0$	$\theta C_y^2-\left[1+\frac{H}{M^2}\right]>0$
2	$V(t_2)-MSE(t_p)>0$	$\theta[C_y^2-CC_x^2]-\left[1+\frac{H}{M^2}\right]>0$
3	$MSE(t_3)-MSE(t_p)>0$	$\theta[C_y^2+(1-2C)C_x^2-\left[1+\frac{H}{M^2}\right]>0$
4	$MSE(t_4)-MSE(t_p)>0$	$\theta[C_y^2+(1+2C)C_x^2-\left[1+\frac{H}{M^2}\right]>0$
5	$MSE(t_5)-MSE(t_p)>0$	$\theta[C_y^2+(1+2C)C_x^2-\left[1+\frac{H}{M^2}\right]>0$
6	$MSE(t_6)-MSE(t_p)>0$	$\theta[C_y^2+(1-4C)\frac{C_x^2}{4}]-\left[1+\frac{H}{M^2}\right]>0$
7	$MSE(t_7)-MSE(t_p)>0$	$\theta[C_y^2+(1+4C)\frac{C_x^2}{4}]-\left[1+\frac{H}{M^2}\right]>0$
8	$MSE(t_8)-MSE(t_p)>0$	$\theta[C_y^2-CC_x^2]-\left[1+\frac{H}{M^2}\right]>0$
9	$MSE(t_i')-MSE(t_p)>0$ $(i=2,3,\ldots,7)$	$\left[\frac{MSE(t_i)-\{Bias(t_i)\}^2}{\bar{Y}^2+MSE(t_i)-2\bar{Y}Bias(t_i)}\right]-\left[1+\frac{H}{M^2}\right]>0$

5. Empirical Study

To verify the theoretical conditions for practical applicability, seven natural populations have been taken into account. Out of these seven populations, first four populations are positively correlated while rest three populations have negative correlation in the two variables. The descriptions of the variables as well as the values of different parameters are presented in the following Table 5.

Table 5 Descriptions of various populations under considerations

S. No.	Population	Variables	Parameters
1	I: Steel and Torrie (1996)	Y: Log of leaf burn in sec. X : Chlorine percentage	$N=30$, $n=30$, $C_x=0.7493$, $C_y=0.7000$, $\rho_{yx}=0.4996$
2	II: Cochran (1977)	Y: Placebo children X : Number of paralytic polio cases in the placebo group	$N=34$, $n=10$, $C_x=1.0720$, $C_y=1.0123$, $\rho_{yx}=0.6837$
3	III: Das (1988)	Y: Number of agricultural labourers for 1961 X : Number of agricultural labourers for 1971	$N=278$, $n=60$, $C_x=1.6198$, $C_y=1.4451$, $\rho_{yx}=0.7213$
4	IV: Cochran (1977)	Y: Number of persons per block X : Number of rooms per block	$N=20$, $n=8$, $C_x=0.1281$, $C_y=0.1445$, $\rho_{yx}=0.6500$
5	V: Johnston (1972)	Y: Date of flowering of a particular species (no. of days starting from first January) X : Percentage of hives affected by disease	$N=10$, $n=4$, $C_x=0.1304$, $C_y=0.1562$, $\rho_{yx}=-0.940$
6	VI: Johnston (1972)	Y: Date of flowering of a particular species (no. of days starting from first January) X : Average of temperatures in January	$N=10$, $n=4$, $C_x=0.130$, $C_y=0.156$, $\rho_{yx}=-0.730$

Table 5 (Continuesd)

S. No.	Population	Variables	Parameters
7	VII: Maddala (1977)	Y: Consumption per capita X : Deflated prices of veal	$N=30$, $n=6$, $C_x=0.0986$, $C_y=0.2278$, $\rho_{yx}=-0.682$

Table 6 Percentage relative efficiency of t - class of estimators with respect to mean per unit estimator t_1

Estimator	Population						
	I	II	III	IV	V	VI	VII
t_1	100.00	100.00	100.00	100.00	100.00	100.00	100.00
t_2	133.30	187.80	208.40	173.20	859.20	214.00	187.00
t_3	92.92	148.50	156.40	157.90	30.62	34.30	56.25
t_4	31.10	28.02	25.82	34.03	784.60	209.00	167.50
t_5	31.10	28.02	25.82	34.03	784.60	209.00	167.50
t_6	133.00	179.70	197.80	161.20	51.05	56.10	74.51
t_7	54.91	49.89	47.11	56.41	256.80	177.00	133.00
t_8	133.30	187.80	208.40	173.20	859.20	214.00	187.00

Table 7 Percentage relative efficiency of t' - class of estimators with respect to mean per unit estimator t_1

Estimator	Population						
	I	II	III	IV	V	VI	VII
t_1'	106.53	139.8	109.34	82.57	82.57	140.41	80.55
t_2'	107.23	195.01	167.11	71.08	71.08	184.48	77.46
t_3'	102.73	211.18	164.36	70.78	70.78	199.82	79.52
t_4'	100.16	173.32	158.15	67.83	67.83	161.32	74.97
t_5'	100.37	859.47	46.78	785.39	785.39	39.45	256.75
t_6'	100.37	214.45	56.35	209.34	209.34	58.72	177.16
t_7'	100.69	187.65	63.46	167.75	167.75	74.34	133.48

Table 8 Percentage relative efficiency of proposed estimator t_P over t - class of estimators

Estimator	Population						
	I	II	III	IV	V	VI	VII
t_1	203.37	439.38	420.61	395.46	940.81	409.24	550.88
t_2	152.61	233.99	201.78	228.38	109.51	191.15	294.65
t_3	218.87	295.87	268.94	250.50	3,073.1	1,191.3	979.32
t_4	653.90	1,568.40	1,629.20	1,162.00	119.92	195.52	328.85
t_5	653.90	1,568.40	1,629.20	1,162.00	119.92	195.52	328.85
t_6	152.86	244.44	212.66	245.28	1,843.00	729.24	739.30
t_7	370.38	880.69	892.78	701.03	366.44	231.33	414.06
t_8	152.61	233.99	201.78	228.38	109.51	191.15	294.65

Table 9 Percentage relative efficiency of proposed estimator t_P over t' - class of estimators

Estimator	Population						
	I	II	III	IV	V	VI	VII
t_1'	190.90	314.29	384.68	478.94	1,139.40	291.44	683.81
t_2'	142.32	119.99	120.75	321.30	154.07	103.61	380.34
t_3'	213.06	140.10	163.63	353.91	4341.7	596.20	1231.50
t_4'	652.86	904.89	1,030.20	1,713.10	176.79	121.20	438.59
t_5'	651.49	182.48	3,482.70	147.95	152.69	495.63	128.08
t_6'	152.30	113.99	377.39	117.17	880.40	1,241.90	417.30
t_7'	366.86	469.32	1,406.80	417.90	218.45	311.14	310.21

Figures 1 and 2 represent the Percentage relative efficiency of the suggested estimator over the t - class and the t' - class of estimators in the pictorial form.

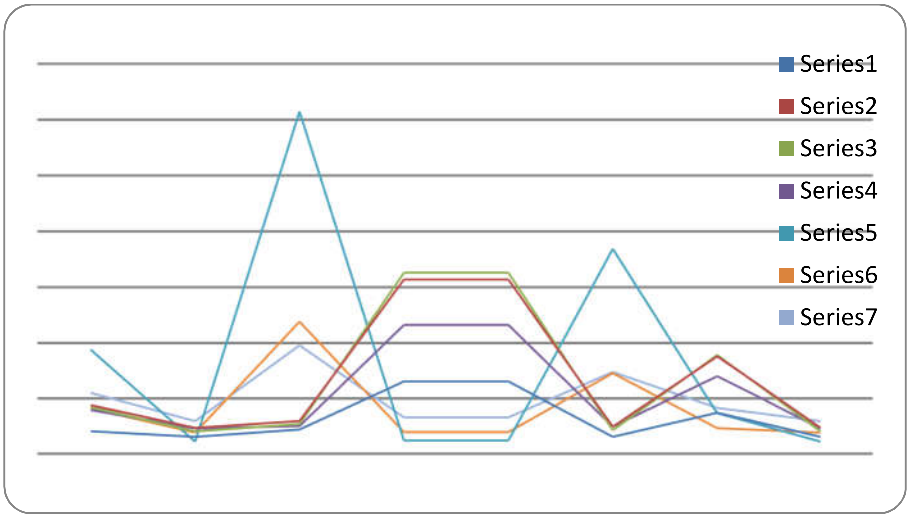

Figure 1 Percentage relative efficiency of suggested estimator over t - class of estimators

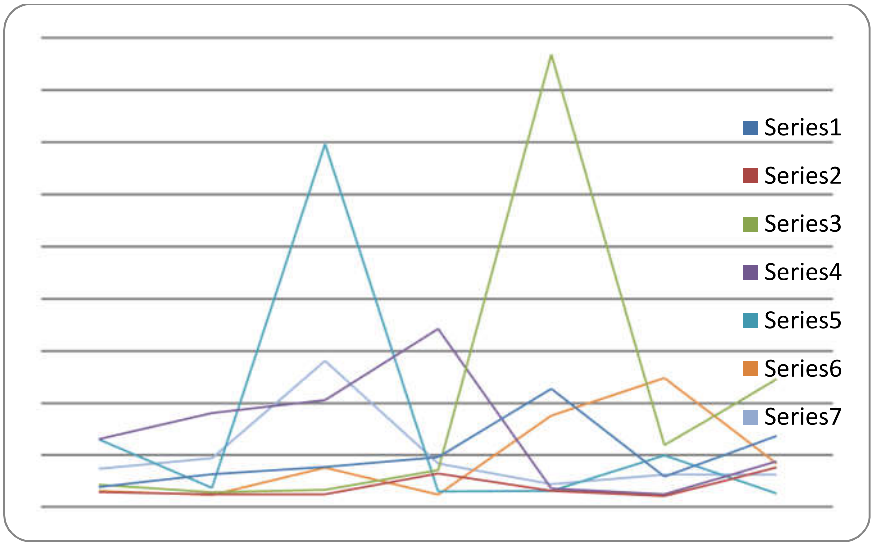

Figure 2 Percentage relative efficiency of suggested estimator over t' - class of estimators

The series 1 to series 7 in Figures 1 and 2 represent the curves of the percentage relative efficiencies (PRE) of the suggested estimator over the t - class and t' - class of competing estimators for population 1 to population 7, respectively.

6. Results and Conclusion

In this scripture, an improved class of ratio-cum-product type estimators is suggested for elevated estimation of population mean using known information on auxiliary variable. The sampling properties bias and the mean squared errors have been derived up to the approximation of degree one. The optimum values of the characterizing scalars which minimize the MSE of the suggested estimator have been obtained. The minimum value of the MSE of the suggested estimator for these values of the characterizing scalars has also been obtained. The suggested estimators have been compared theoretically with the two different classes of estimators of population mean. The conditions for the proposed estimator to perform more efficiently than the competing estimators have been obtained. These theoretical conditions are verified using seven natural populations presented in Table 5. The PRE of various estimators of t - class with respect to the mean per unit estimator are presented in Table 6. Table 7 represents the PRE of various members of t' - class of estimators with respect to the mean per unit estimator. The PRE of the suggested estimator over the t - class of estimators are presented in Table 8. Table 9 shows the PRE of the proposed estimator over t' - class of estimators. From Tables 8 and 9, it is evident that the suggested estimator is better than both t - class as well as t' - class of estimators. Thus, the suggested class of estimators may be utilized for practical applicability for improved estimation of population mean using auxiliary variable under predictive approach of simple random sampling scheme.

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