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An Efficient Class on Linear Combination of Ratio Type Estimators for Estimating the Population Mean

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Abstract

This paper presents a class of estimators for population mean of a study variable using information on an auxiliary variable in simple random sampling without replacement. The properties of suggested class of estimators have been investigated under large sample approximation. A particular case of the proposed class of estimators has been identified with its mean square error formula. It has been shown that the suggested class of estimator is more efficient than some existing estimators such as usual unbiased estimator, usual ratio estimator, usual product estimator, usual regression estimator, dual to ratio estimator due to Srivenkataramana (1980) and other classes estimators. An empirical study carried out to throw light on the performance of the suggested class of estimators over existing estimators.

Keywords: Auxiliary variable, study variable, ratio estimator, mean squared error.

1. Introduction

The use of auxiliary information in survey sampling at the estimation stage has an eminent role to increase the precision of the estimators available in the literature. In this context ratio, product and regression methods of estimation are good example (see Cochran 1977). When the population mean $\bar{X}$ of the auxiliary variable x is known, a large number of the estimator has been suggested on mean $\bar{Y}$ of the study variable y by several researchers, for instance, see Robson (1957), Murthy (1964, 1967), Sukhatme et al. (1984), Singh (1986), Singh and Vishwakarma (2008), Vishwakarma et al. (2014) the references therein. It is to be pointed out that the estimators reported in the literature have the same mean square error (MSE) / minimum MSE (to the first degree of approximation) as that of the usual regression estimator. In this paper we suggested a class of estimators for population mean in simple random sampling using auxiliary information on an auxiliary variable with its bias and mean square error formulae under large sample approximation. We have shown that suggested class

of estimators is more efficient than usual unbiased, usual ratio, usual product, usual linear regression, Srivastava (1967), Srivenkataramana (1980) dual to ratio, Srivastava (1974), Prasad (1989) and Sharma and Tailor (2010) ratio-cum-dual to ratio estimators/class of estimators. In the support of the theoretically results we give an empirical study. The objective of this paper is to develop a class of ratio type estimator of the population mean of the study variable y using auxiliary information on, auxiliary variable x under simple random sampling without replacement (SRSWOR).

Consider a finite population $U=(U_1,U_2,...,U_N)$ containing N units. Let y and x denote the study variable and the auxiliary variable taking values y_i and x_i, respectively on the unit U_i $(i=1,2,...,N)$ of population U. Let s denote the typical sample of size n which was chosen by simple random sampling without replacement (SRSWOR) from the population U. We assume a simple random sample for convenience, since in many problems of this type will often be more purposeful see Meeden (1995, p.71). It is well known that the sample mean $\bar{y}=n^{-1}\sum_{i=1}^{n}y_i$ is an unbiased estimator of the population mean $\bar{Y}$ of the study variable y whose variance is

$$MSE\left(\bar{y}\right)=Var\left(\bar{y}\right)=\frac{\left(1-f\right)}{n}S_y^2=\frac{\left(1-f\right)}{n}\bar{Y}^2C_y^2, \tag{1}$$

where $f=n/N, S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^2$ and $C_y^2=S_y^2/\bar{Y}^2$. When the population mean $\bar{X}$ of the auxiliary variable x is known and the two variables y and x are positively correlated, the classical ratio estimator for the population mean $\bar{Y}$ is defined by

$$t_R=\bar{y}\frac{\bar{X}}{\bar{x}}, \tag{2}$$

where $\bar{x}=n^{-1}\sum_{i=1}^{n}x_i$. In the same situation, Srivenkataramana (1980) suggested a dual to ratio estimator for the population mean $\bar{Y}$ is

$$t_d=\bar{y}\frac{x^*}{\bar{X}}, \tag{3}$$

where $x^*=\frac{\left(N\bar{X}-n\bar{x}\right)}{\left(N-n\right)}$. To the first degree of approximation, the mean squared errors of the estimator t_R and t_d are respectively given by

$$MSE\left(t_R\right)=\frac{\left(1-f\right)}{n}\bar{Y}^2\left[C_y^2+C_x^2\left(1-2K\right)\right], \tag{4}$$

$$MSE\left(t_d\right)=\frac{\left(1-f\right)}{n}\bar{Y}^2\left[C_y^2+g\left(g-2K\right)C_x^2\right], \tag{5}$$

where $g=n/\left(N-n\right), C_x^2=\left(S_x^2/\bar{X}^2\right), S_x^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2, K=\rho\left(C_y/C_x\right), \quad \rho=\frac{S_{xy}}{\left(S_xS_y\right)}$ and $S_{xy}=\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)\left(y_i-\bar{Y}\right)$. We note that the estimators $\bar{y}$, t_R and t_d has larger variance compare to the usual regression estimator defined as

$$\bar{y}_{lr}=\bar{y}+\hat{\beta}\left(\bar{X}-\bar{x}\right), \tag{6}$$

where $\hat{\beta} = \left(s_{xy} / s_x^2\right)$, $s_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)$ and $s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$.

Then mean square error of the regression estimator $\bar{y}_{lr}$ to the first degree of approximation is given by

$$MSE\left(\bar{y}_{lr}\right) = \frac{\left(1-f\right)}{n} S_y^2\left(1-\rho\right) = \frac{\left(1-f\right)}{n}\bar{Y}^2 C_y^2\left(1-\rho\right). \tag{7}$$

2. The Suggested Class of Estimators and Properties

We suggest the following class of estimators for population mean of the study variable as

$$t = \bar{y}\left\{\lambda_1\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha} + \lambda_2\left(\frac{x^*}{\bar{X}}\right)\right\}, \tag{8}$$

where α being scalar takes values (0, 1, −1) for designing the different estimators and (λ_1, λ_2) are suitably chosen constants to be determined such that MSE of t is minimum.

To obtain the bias and MSE of suggested class of estimators t we write $\bar{y} = \bar{Y}(1+e_0)$ and $\bar{x} = \bar{X}(1+e_1)$ such that $E(e_0) = E(e_1) = 0$ and

$$\left.\begin{aligned} E\left(e_0^2\right) &= \frac{\left(1-f\right)}{n} C_y^2, \\ E\left(e_1^2\right) &= \frac{\left(1-f\right)}{n} C_x^2, \\ E\left(e_0 e_1\right) &= \frac{\left(1-f\right)}{n} K C_x^2. \end{aligned}\right\} \tag{9}$$

Now we express suggested class of estimators t defined at (8) in terms of e's as

$$t = \bar{Y}\left\{\lambda_1\left(1+e_0\right)\left(1+e_1\right)^{-\alpha} + \lambda_2\left(1+e_0\right)\left(1-ge_1\right)\right\}. \tag{10}$$

Now we express $\left|e_1\right| < 1$ so that the term $\left(1+e_1\right)^{-\alpha}$ is expandable in term of power series. Thus, by expanding the right-hand side of (10) multiplying out and neglecting terms of e's having power greater than two we have

$$t \cong \bar{Y}\left[\lambda_1\left\{1 + e_0 - \alpha e_1 - \alpha e_0 e_1 + \frac{\alpha\left(\alpha+1\right)}{2} e_1^2\right\} + \lambda_2\left\{1 + e_0 - ge_1 - ge_0 e_1\right\}\right],$$

$$\left(t - \bar{Y}\right) \cong \bar{Y}\left[\lambda_1\left\{1 + e_0 - \alpha e_1 - \alpha e_0 e_1 + \frac{\alpha\left(\alpha+1\right)}{2} e_1^2\right\} + \lambda_2\left\{1 + e_0 - ge_1 - ge_0 e_1\right\} - 1\right]. \tag{11}$$

Taking expectation of both sides of (11) we get the bias of t to the first degree of approximation as

$$B\left(t\right) = \bar{Y}\left[\left(\lambda_1 + \lambda_2 - 1\right) + \frac{\left(1-f\right)}{n} C_x^2\left\{\frac{\lambda_1 \alpha}{2}\left(\alpha - 2K + 1\right) - \lambda_2 gK\right\}\right]. \tag{12}$$

Squaring both sides of (11) and neglecting terms of e's having power greater than two we have

$$\begin{aligned}\left(t-\bar{Y}\right)^2 = \bar{Y}^2\Big[&1+\lambda_1^2\left\{1+2e_0-2\alpha e_1+e_0^2-4\alpha e_0e_1+\alpha\left(2\alpha+1\right)e_1^2\right\}\\&+\lambda_2^2\left\{1+2e_0-2ge_1+e_0^2-4ge_0e_1+g^2e_1^2\right\}\\&+2\lambda_1\lambda_2\left\{1+2e_0-\left(g+\alpha\right)\left(e_1+2e_0e_1\right)+e_0^2+\frac{\alpha}{2}\left(\alpha+2g+1\right)e_1^2\right\}\\&-2\lambda_1\left\{1+e_0-\alpha e_1-\alpha e_0e_1+\frac{\alpha\left(\alpha+1\right)}{2}e_1^2\right\}-2\lambda_2\left\{1+e_0-ge_1-ge_0e_1\right\}\Big].\end{aligned} \tag{13}$$

Taking expectation of the both sides of (13) we get the MSE of suggested class of estimators t to the first degree of approximation as

$$MSE\left(t\right)=\bar{Y}^2\left[1+\lambda_1^2A+\lambda_2^2B+2\lambda_1\lambda_2C-2\lambda_1D-2\lambda_2E\right], \tag{14}$$

where

$$A=\left[1+\frac{\left(1-f\right)}{n}\left\{C_y^2+\alpha C_x^2\left(2\alpha-4K+1\right)\right\}\right],\quad B=\left[1+\frac{\left(1-f\right)}{n}\left\{C_y^2+gC_x^2\left(g-4K\right)\right\}\right],$$

$$C=\left[1+\frac{\left(1-f\right)}{n}\left\{C_y^2+\left(\frac{\alpha}{2}\right)C_x^2\left(\alpha-4\frac{\left(\alpha+g\right)}{\alpha}K+2g+1\right)\right\}\right],$$

$$D=\left[1+\frac{\left(1-f\right)}{n}\left\{\left(\frac{\alpha}{2}\right)C_x^2\left(\alpha-2K+1\right)\right\}\right],\quad E=\left[1-\frac{\left(1-f\right)}{n}gKC_x^2\right].$$

Differentiating (14) with respect to λ_1 and λ_2 equating to zero we have

$$\begin{bmatrix}A & C\\ C & B\end{bmatrix}\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix}=\begin{bmatrix}D\\ E\end{bmatrix}. \tag{15}$$

After simplification of (15) we get the optimum values of λ_1 and λ_2 as

$$\left.\begin{aligned}\lambda_1&=\frac{BD-CE}{AB-C^2}=\lambda_{10},\\ \lambda_2&=\frac{AE-CD}{AB-C^2}=\lambda_{20}.\end{aligned}\right\} \tag{16}$$

Thus, the resulting minimum MSE of suggested class of estimators t is obtained as

$$MSE_{\min}\left(t\right)=\left[1-\frac{\left(BD^2+AE^2-2CDE\right)}{\left(AB-C^2\right)}\right]. \tag{17}$$

3. Particular Case $(\lambda_1+\lambda_2=1)$

If we set $(\lambda_1+\lambda_2=1)$ in (8), we get another class of estimators for the population mean $\bar{Y}$ as

$$T_1=\bar{y}\left\{\lambda_1\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha}+\left(1-\lambda_1\right)\left(\frac{x^*}{\bar{X}}\right)\right\}. \tag{18}$$

Putting $(\lambda_1+\lambda_2=1)$ in (12) and (14), we get the bias and MSE of T_1, respectively as

$$B\left(T_1\right)=\bar{Y}\left[\frac{\left(1-f\right)}{n}\left\{\frac{\lambda_1\alpha}{2}\left(\alpha-2K+1\right)-\left(1-\lambda_1\right)gK\right\}\right], \tag{19}$$

$$MSE\left(T_1\right)=\left[1+B-2E+\lambda_1^2\left(A+B-2C\right)-2\lambda_1\left(B-C+D-E\right)\right]. \tag{20}$$

The bias and MSE of T_1 are minimized for

$$\lambda_1 = \frac{B-C+D-E}{A+B-2C} = \lambda_1^*. \tag{21}$$

Thus, the resulting bias and MSE of T_1 to the first degree of approximation are respectively given by

$$B_{\min}\left(T_1\right) = \bar{Y}\left[\frac{(1-f)}{n}\left\{\frac{(B-C+D-E)}{2(A+B-2C)}\left(\alpha^2 - 2\alpha K + \alpha - 2gK\right)\right\}\right], \tag{22}$$

$$MSE_{\min}\left(T_1\right) = \bar{Y}^2\left[1+B-2E-\frac{(B-C+D-E)^2}{A+B-2C}\right] = \frac{(1-f)}{n}S_y^2\left(1-\rho^2\right) = MSE\left(\bar{y}_{lr}\right). \tag{23}$$

4. Efficiency Comparisons

From (1), (4), (5) and (7), we have

$$MSE\left(\bar{y}\right) - MSE\left(\bar{y}_{lr}\right) = \frac{(1-f)}{n}S_y^2\rho^2 \geq 0, \tag{24}$$

$$MSE\left(t_R\right) - MSE\left(\bar{y}_{lr}\right) = \frac{(1-f)}{n}\bar{Y}^2\left\{(1-2K)C_x^2 + C_y^2\rho^2\right\} \geq 0, \tag{25}$$

$$MSE\left(t_d\right) - MSE\left(\bar{y}_{lr}\right) = \frac{(1-f)}{n}\bar{Y}^2\left\{g(g-2K)C_x^2 + C_y^2\rho^2\right\} \geq 0. \tag{26}$$

It follows from (24) to (26) that the regression estimator $\bar{y}_{lr}$ is more efficient than $\bar{y}$, t_R and t_d. We note that class of estimators t reduces to the following class of estimators of the population mean $\bar{Y}$ as

$$t_1 = \lambda_1\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha} \text{ for } \left(\lambda_1,\lambda_2,\alpha\right) = \left(\lambda_1, 0, \alpha\right), \tag{27}$$

$$t_1^{(1)} = \lambda_1\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \text{ for } \left(\lambda_1,\lambda_2,\alpha\right) = \left(\lambda_1, 0, 1\right), \tag{28}$$

$$t_2 = \bar{y}\lambda_2\left(\frac{x^*}{\bar{X}}\right) \text{ for } \left(\lambda_1,\lambda_2,\alpha\right) = \left(0, \lambda_2, \alpha\right), \tag{29}$$

$$t_{(\alpha)} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha} \text{ for } \left(\lambda_1,\lambda_2,\alpha\right) = \left(1, 0, \alpha\right), \tag{30}$$

$$t_{ST} = \bar{y}\left\{\lambda_1\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha} + \left(1-\lambda_1\right)\left(\frac{\bar{x}^*}{\bar{X}}\right)\right\} \text{ for } \left(\lambda_1,\lambda_2,\alpha\right) = \left(\lambda_1, 1-\lambda_1, 0\right). \tag{31}$$

It is to be mentioned that the class of estimators t_1, $t_{(\alpha)}$ and t_{ST} are due to Srivastava (1974) and revisited by Prasad (1989), Srivastava (1967) and Sharma and Tailor (2010), respectively. To the first degree of approximation, the MSE to t_1, $t_1^{(1)}$, t_2, $t_{(\alpha)}$ and t_{ST} are respectively given by

$$MSE\left(t_1\right) = \bar{Y}^2\left[1+\lambda_1^2A - 2\lambda_1D\right], \tag{32}$$

$$MSE\left(t_1^{(1)}\right) = \bar{Y}^2\left[1+\lambda_1^2A^* - 2\lambda_1D^*\right], \tag{33}$$

$$MSE\left(t_2\right) = \bar{Y}^2\left[1+\lambda_2^2B - 2\lambda_2E\right], \tag{34}$$

$$MSE\left(t_{(\alpha)}\right) = \bar{Y}^2\left[1 + A - 2D\right], \tag{35}$$

$$MSE\left(t_{ST}\right) = \bar{Y}^2 \frac{(1-f)}{n}\left[C_y^2 + \left\{(1-g)\lambda_1 + g\right\}^2 C_x^2 \left\{(1-g)\lambda_1 + g - 2K\right\}\right], \tag{36}$$

which are respectively minimized for

$$\lambda_1 = \frac{D}{A} = \lambda_{10}^*, \tag{37}$$

$$\lambda_1 = \frac{D^*}{A^*} = \lambda_{10}^{(1)}, \tag{38}$$

$$\lambda_2 = \frac{E}{B} = \lambda_{20}^*, \tag{39}$$

$$\lambda_1 = \frac{K-g}{1-g} = \lambda_{ST}, \tag{40}$$

where $A^* = \left[1 + \frac{(1-f)}{n}\left\{C_y^2 + C_x^2\left(3-4K\right)\right\}\right]$, $D^* = \left[1 + C_x^2\left(1-K\right)\right]$ and $\alpha = K$.

Thus, the resulting minimum MSEs to t_1, $t_1^{(1)}$, t_2, $t_{(\alpha)}$ and t_{ST} are respectively given by

$$MSE_{\min}\left(t_1\right) = \bar{Y}^2\left[1 - \frac{D^2}{A}\right], \tag{41}$$

$$MSE_{\min}\left(t_1^{(1)}\right) = \bar{Y}^2\left[1 - \frac{D^{*2}}{A^*}\right], \tag{42}$$

$$MSE_{\min}\left(t_2\right) = \bar{Y}^2\left[1 - \frac{E^2}{B}\right], \tag{43}$$

$$MSE_{\min}\left(t_{(\alpha)}\right) = \bar{Y}^2 \frac{(1-f)}{n} C_y^2\left[1-\rho^2\right] = MSE\left(\bar{y}_{lr}\right), \tag{44}$$

$$MSE_{\min}\left(t_{ST}\right) = \bar{Y}^2 \frac{(1-f)}{n} C_y^2\left[1-\rho^2\right] = MSE\left(\bar{y}_{lr}\right). \tag{45}$$

Form (7), (17), (23), (44) and (45) we have

$$\begin{aligned}
&\left(MSE\left(\bar{y}_{lr}\right) \text{ or } MSE_{\min}\left(T_1\right) \text{ or } MSE_{\min}\left(t_{(\alpha)}\right) \text{ or } MSE_{\min}\left(t_{ST}\right) - MSE_{\min}\left(t\right)\right) \\
&= \bar{Y}^2\left[B - 2E - \frac{\left(B - C + D - E\right)}{\left(A + B - 2C\right)} + \frac{\left(BD^2 + AE^2 - 2CDE\right)}{\left(AB - C^2\right)}\right], \\
&= \bar{Y}^2 \frac{\left[B\left(D-A\right) + C\left(C-D\right) + E\left(A-C\right)\right]^2}{\left(A+B-2C\right)\left(AB-C^2\right)} \geq 0 \ \ if\ \left(A+B-2C\right)\left(AB-C^2\right) \neq 0.
\end{aligned} \tag{46}$$

Which shows that proposed class of estimators t is more efficient than the usual regression estimators $\bar{y}_{lr}$ (or T_1 or $t_{(\alpha)}$ or t_{ST}) and hence it is more efficient than $\bar{y}$, t_R and t_d.

Now from (5), (35), (41) and (43) we have

$$MSE\left(t_{(\alpha)}\right) - MSE_{\min}\left(t_1\right) = \bar{Y}^2 \frac{\left(A-D\right)^2}{A} \geq 0 \ \ if\ A \neq 0, \tag{47}$$

$$MSE\left(t_d\right) - MSE_{\min}\left(t_2\right) = \bar{Y}^2 \frac{\left(B-E\right)^2}{B} \geq 0 \ \ if\ B \neq 0. \tag{48}$$

From (47) and (48), it is observed that the estimators t_1 and t_2 are better than $t_{(\alpha)}$ (Srivastava 1967) class of estimators) and t_d ((Srivenkataramana 1980) dual to ratio estimator) respectively. Also, we find here, the estimator t_1' is better than $t_{(\alpha)}$ for all the cases of α. If $\alpha = 1$ then $t_{(\alpha)}$ is the usual ratio estimator t_R and for $\alpha = -1$, $t_{(\alpha)}$ is the usual product estimator $t_p = \bar{y}\left(\bar{x} / \bar{X}\right)$.

From (17), (41) and (43) we have

$$MSE_{\min}\left(t_1\right) - MSE_{\min}\left(t\right) = \bar{Y}^2 \frac{\left(AE - CD\right)^2}{A\left(AB - C^2\right)} \geq 0 \ \ if \ A\left(AB - C^2\right) \neq 0, \tag{49}$$

$$MSE_{\min}\left(t_2\right) - MSE_{\min}\left(t\right) = \bar{Y}^2 \frac{\left(BD - CE\right)^2}{B\left(AB - C^2\right)} \geq 0 \ \ if \ B\left(AB - C^2\right) \neq 0. \tag{50}$$

From (49) and (50) we note that the proposed class of estimators t is more efficient than the classes of estimators t_1 and t_2. It is to be mentioned that for $\left(\lambda_1, \alpha\right) = \left(\lambda_1, 1\right)$ the class of estimators t_1 and $t_1^{(1)}$, as the proposed class of estimators t is more efficient than t_1' and hence it is more efficient than the estimator $t_1^{(1)}$.

5. Empirical Study

To judge the merits of the proposed class of estimators t over $\bar{y}$, t_R, t_d, t_1, $t_1^{(1)}$, t_2, T_1, $t_{(\alpha)}$, t_{ST} and $\bar{y}_{lr}$, we consider the five population data sets whose description is given below:

Population I: (Kadılar and Çıngı 2006)

Data concerning the level of apple production (as the study variable) and the number of apple trees (as the auxiliary variable) (1 unit=1,000 trees) in 104 villages in the East Anatolia region of Turkey in 1999 (Source: Institute of Statistics, Republic of Turkey). The variables considered are:

y: level of apple population, x: the number of apple trees (1 unit =1,000 trees),

$N = 104,\ n = 20,\ \bar{Y} = 625.37,\ C_y = 1.866,\ C_x = 1.653,\ \rho = 0.865.$

Population II: (Cochran 1977)

Data concerning for 256 commercial peach orchards in North Carolina in June 1946. The purpose was to determine the most efficient sampling procedure for estimating commercial peach production in this area. Information was obtained on the number of peach trees and estimated total peach of production in each orchard. The variables considered are:

y: production in bushels of peaches, x: peach trees in an orchard,

$N = 256,\ n = 100,\ \bar{Y} = 56.47,\ C_y = 1.42,\ C_x = 1.40,\ \rho = 0.89.$

Population III: (Koyuncu and Kadılar 2009)

Data concerning primary and secondary schools for 923 districts of Turkey in 2007 (Source: Ministry of Education, Republic of Turkey), taking the number of teachers as study variable and the number of students as auxiliary variable in both primary and secondary schools. A sample of size n = 180 was selected from the dataset and the correlations observed between auxiliary and study variables are positive. The variables considered are:

y: number of teachers, x: number of students,

$N = 923,\ n = 180,\ \bar{Y} = 436.4345,\ C_y = 1.72,\ C_x = 1.86,\ \rho = 0.95.$

Population IV: (Murthy 1967)

It consists of 128 villages in a Tehsil, the character x being geographical area given by 1951census village-wise data. In this case, we take variable as

y: $(x-\bar{X})^2$, x: geographical area,

$N=128,\ n=30,\ \bar{Y}=11.9076,\ \bar{X}=5.5923,\ C_y=1.5288,\ C_x=0.6171,\ \rho=0.5782.$

Population V: (Das 1988)

It consists of 278 villages or town/wards under Gajole Police Station of Malda district of West Bengal, India (in fact, only those villages or towns/ wards have been considered which are shown as inhabited and which are common to both census 1961 and census 1971, lists). The variables considered are:

y: number of agricultural labourers for 1961, x: number of agricultural labourers for 1971,

$N=278,\ n=30,\ \bar{Y}=39.0680,\ C_y=1.4451,\ C_x=1.6198,\ \rho=0.7213.$

We have computed the percentage relative efficiencies (PREs) of different estimators (τ) with respect to usual unbiased estimator by using following formula

$$PRE(\tau,\bar{y})=\frac{MSE(\bar{y})}{MSE_{\min}(\tau)}\times 100. \tag{51}$$

where $\tau=\bar{y},\ t_d,\ t_2,\ t_R,\ t_1^{(1)},\ \bar{y}_{lr},\ t$ and findings are summarized in Table 1.

Table 1 PREs of different estimators with respect to $\bar{y}$

Population Estimator	PRE				
	I	II	III	IV	V
$\bar{y}$	100.00	100.00	100.00	100.00	100.00
t_d	147.15	364.34	176.25	114.63	121.54
t_2	154.72	365.72	176.66	119.66	126.54
t_R	396.50	460.62	939.71	143.65	156.40
$t_1^{(1)}$	412.70	463.15	946.92	148.47	174.71
$\bar{y}_{lr}$, (or T_1 or $t_{(\alpha)}$ or t_{ST})	397.18	481.00	1,119.68	150.22	208.45
$t\to t_{(\alpha)}$ for $(\alpha=1)$	412.84	481.46	1,124.41	**155.00**	**222.27**
$t\to t_{(\alpha)}$ for $(\alpha=-1)$	**491.10**	**491.18**	**1,248.73**	151.74	208.50

To highlighting the properties of proposed estimator, we have carried five population data set. The results in terms of PREs of the proposed estimator and estimators from the proposed estimator are shown in Table 1 for all the population data set. Table 1 clearly exhibits that the estimator $\bar{y}_{lr}$ have greater PRE than the estimators $\bar{y},\ t_d,\ t_2,\ t_R$ and $t_1^{(1)}$ but have same PRE as estimators $T_1,\ t_{(\alpha)}$ and t_{ST}. Theoretically, we find that the proposed estimator has shown supremacy over all the estimators extracted from the proposed estimator for all the values of α. Here, in Table 1, we have obtained PRE of the proposed estimator for $\alpha=1$ and $\alpha=-1,$ and it is clear from Table 1 that our proposed estimator has greater PRE than to all extracted estimators from the proposed estimator, i.e.

our proposed estimator have shown superiority over all the estimator derived from the proposed estimator. Thus, suggested class of estimators is to be preferred in practice.

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