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On the Precision of Estimators in Sampling Surveys by Multi-Parametric Calibration Weightings

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Abstract

Calibration weighting is the formulation of calibration constraints with respect to a given distance measure to obtain expression of calibration weights in order to improve the efficiency of the study variable. In this paper, the effect of multi-parametric calibration weightings on the precision of estimators of mean under the stratified random sampling is examined. The results showed that at the same optimum conditions, calibration estimators with more parametric auxiliary information are more precise and highly efficient than calibration estimators with less parametric auxiliary information.

Keywords: Calibration constraints, calibration weights, large sample approximation, highly efficient, optimum conditions, percentage relative efficiency.

1. Introduction

Calibration estimation is a method that uses auxiliary variable(s) to adjust the original design weights to improve the precision of survey estimates of population or subpopulation parameters. The process of adjustment is called calibration (Deville and Sarndal 1992).

In calibration estimation theory, calibration weights are chosen to minimize a given distance measure (or loss function) while satisfying constraints related auxiliary variable information. The calibration constraint(s) provide(s) additional information which help to increase the efficiency of the estimation of the population parameter(s) of interest. Calibration estimators in sampling theory often used population information of the auxiliary variable such as the total, mean and variance to formulate the constraint(s). The aim is to obtain optimum calibration weights that would improve the precision of survey estimates of the population parameter(s) of interest.

The formulation of the calibration constraints with respect to a given distance measure to obtain expression of calibration weights in order to improve the efficiency of the study variable is called calibration weighting. Deville and Sarndal (1992) introduced the concept of calibration estimation in survey sampling. They used both univariate and multivariate auxiliary information to derive weighting system (calibration weights) with the aid of a distance measure and a set of calibration equations (calibration constraints). They noted that for every distance measure there is a corresponding set of calibrated weights and a calibration estimator. Hence, the efficiency of the resulting calibration estimator depends on the strength of the formulated calibration constraints. Many authors have defined some modified calibration estimators in survey sampling using univariate auxiliary information (univariate calibration weightings). A few key references include (Singh et al. 1998, Kim et al. 2007, Koyuncu and Kadilar 2013, Clement et al. 2014, 2015, Clement and Enang 2015a, 2017, Clement 2015, 2017a, 2017b and Enang and Clement 2020).

Equally, many authors have used multivariate auxiliary information (multivariate calibration weightings), to propose improved calibration estimators. Work in this aspect include (Rao et al. 2012, Clement and Enang 2015b) among others. Multivariate calibration weightings is the use of the same parameter of two or more auxiliary variables in formulating calibration constraints.

In the progression to improve calibration estimation, Tracy et al. (2003) introduced the concept of multi-parametric calibration weightings in calibration estimation. Multi-parametric calibration weightings is the formulation of calibration constraints with respect to a given distance measure to obtain expression of calibration weights using information from two or more parameters of the same auxiliary variable. Work in this aspect include (Tracy et al. 2003, Koyuncu and Kadilar 2014, 2016, and Clement 2018).

Tracy et al. (2003) proposed improved calibration estimator of population mean using two parameters (mean and variance) of the same auxiliary variable to formulate calibration constraints. Koyuncu and Kadilar (2016) advocated a modification to the Tracy et al. (2003) calibration estimator by introducing new constraints involving three parameters (mean, variance and a combination of the design and calibration weights) of the same auxiliary variable and proposed a more efficient calibration estimator than the Tracy et al. (2003) calibration estimator.

It has been observed that the use of parameters of the same auxiliary variable to formulate calibration constraints gives more precise and efficient calibration estimators than the use of different auxiliary variables or different parameters of different auxiliary variables. In the present study, a new improved calibration estimator of population mean is proposed under new calibration constraints using auxiliary information of four parameters (mean, variance, coefficient of kurtosis and a combination of the design and calibration weights) of the same auxiliary variable. The choice is obvious; coefficient of Kurtosis and its functions are unaffected by extreme values or the presence of outliers. Further, it always has strong correlation with other population parameters like the mean and variance.

2. Basic Definitions and Notations

Consider a finite population $U=\left(U_1,U_2,\ldots,U_N\right)$ of size N. Let X and Y denote the auxiliary and study variables taking values X_i and Y_i, respectively on the i^{th} unit $U_i\left(i=1,2,\ldots,N\right)$ of the population. It is assumed that $\left(x_i,y_i\right)\geq 0$, (since survey variables are generally non-negative) and information on the population mean $(\bar{X})$ of the auxiliary variable X is known. Let a sample of size n be drawn by simple random sampling without replacement (SRSWOR) based on which we obtain the means $(\bar{x})$ and $(\bar{y})$ for the auxiliary variable X and the study variable Y.

Let the population ($U=\left(U_1,U_2,\ldots,U_N\right)$ of size (N)) be divided into H strata with N_h units in the h^{th} stratum from which a simple random sample of size n_h is taken without replacement. The total population size be $N=\sum_{h=1}^{H}N_h$ and the sample size $n=\sum_{h=1}^{H}n_h$, respectively. Associated with the i^{th} element of the h^{th} stratum are y_{hi} and x_{hi} with $x_{hi}>0$ being the covariate; where y_{hi} is the y

value of the i^{th} element in stratum h, and x_{hi} is the x value of the i^{th} element in stratum h, $h = 1, 2, \ldots, H$ and $i = 1, 2, \ldots, N_h$.

For the h^{th} stratum, let $W_h = \frac{N_h}{N}$ be the stratum weights and $f_h = \frac{n_h}{N_h}$ the sample fraction. Let the h^{th} stratum means of the study variable Y and auxiliary variable X, $\left(\bar{y}_h = \sum_{i=1}^{n_h} \frac{y_{hi}}{n_h}; \bar{x}_h = \sum_{i=1}^{N_h} \frac{x_{hi}}{n_h} \right)$ be the unbiased estimator of the population mean, $\left(\bar{Y}_h = \sum_{i=1}^{N_h} \frac{y_{hi}}{N_h}; \bar{X}_h = \sum_{i=1}^{N_h} \frac{x_{hi}}{N_h} \right)$ of Y and X respectively, based on n_h observations.

$$S_{hx_i}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(x_{hi} - \bar{X}_{hi} \right)^2, \; S_{hy}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(\bar{y}_h - \bar{Y}_h \right)^2, \; \bar{y}_{st} = \sum_{h=1}^{H} W_h \bar{y}_h$$

$$S_{hx_i y} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(x_{hi} - \bar{X}_{hi} \right) \left(y_h - \bar{Y}_h \right), \; S_{hx_i x_j} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(x_{hi} - \bar{X}_{hi} \right) \left(x_{hj} - \bar{X}_{hj} \right)$$

and $\bar{x}_{i,st} = \sum_{h=1}^{H} W_h \bar{x}_{hi}$. Let $\bar{x}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} x_{hi}$, $S_{hx}^2 = \frac{1}{n_h - 1} \sum_{i=1}^{n_h} \left(x_{hi} - \bar{x}_h \right)^2$, $\bar{y}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{hi}$, $S_{hy}^2 = \frac{1}{n_h - 1} \sum_{i=1}^{n_h} \left(y_{hi} - \bar{y}_h \right)^2$ denote the sample means and variances for the auxiliary variable and study variable, respectively.

3. Review of Some Existing Estimators

This section gives a summary of some existing estimators under the stratified random sampling design with their variance expressions that would be considered for modifications in the study.

3.1. Regression estimator

The conventional regression estimator of population mean in stratified random sampling design is defined by Cochran (1977) as

$$\phi_1 = \bar{y}_{st} + b\left(\bar{X} - \bar{x}_{st} \right), \tag{1}$$

where $\bar{y}_{st} = \sum_{h=1}^{H} W_h \bar{y}_h$ and $\bar{x}_{st} = \sum_{h=1}^{H} w_h \bar{x}_h$ are the Horvitz-Thompson-type estimators, $b = \frac{\sum_{h=1}^{H} w_h \bar{x}_h \bar{y}_h}{\sum_{h=1}^{H} w_h \bar{x}_h^2}$ is the regression coefficient and w_h are design weights. In calibration estimation, this estimator is modified as:

$$\phi_1^* = \bar{y}_{st}^* + B_h^* \mu_{10}, \tag{2}$$

where $\bar{y}_{st} = \sum_{h=1}^{H} W_h \bar{y}_h$ and $\bar{x}_{st} = \sum_{h=1}^{H} w_h \bar{x}_h$ are the Horvitz-Thompson-type calibration estimators, $B_h^* = \frac{\sum_{h=1}^{H} w_h^* \bar{x}_h \bar{y}_h}{\sum_{h=1}^{H} w_h^* \bar{x}_h^2}$ is the regression coefficient, w_h^* are calibration weights and $\mu_{10} = \sum_{h=1}^{H} w_h^* \left(\bar{X}_h - \bar{x}_h \right)$.

3.2. Koyuncu and Kadilar estimator

Koyuncu and Kadilar (2016) considered the following calibration estimator in stratified random sampling:

$$\phi_1 = \sum_{h=1}^{H} \Omega_h \bar{y}_h. \tag{3}$$

Using the chi-square loss functions of the form,

$$L\left(, \Omega_h W_h\right) = \sum_{h=1}^{H} \frac{\left(\Omega_h - W_h\right)^2}{W_h Q_h}. \tag{4}$$

Subject to the calibration constraints defined by

$$\sum_{h=1}^{H} \Omega_h \bar{x}_h = \sum_{h=1}^{H} W_h \bar{X}_h, \tag{5}$$

$$\sum_{h=1}^{H} \Omega_h s_{hx}^2 = \sum_{h=1}^{H} W_h S_{hx}^2, \tag{6}$$

$$\sum_{h=1}^{H} \Omega_h = \sum_{h=1}^{H} W_h. \tag{7}$$

The calibration weights are obtained as

$$\Omega_h = W_h + W_h Q_h \left(\lambda_1 \bar{x}_h + \lambda_2 s_{hx}^2 + \lambda_3 \right). \tag{8}$$

Substituting (8) in (5), (6) and (7), respectively and solving the resulting system of equations gives the values of the lambdas. On substituting the lambdas in (8) and the resulting equation in (3), Koyuncu and Kadilar (2016) obtained their calibration regression estimator as:

$$\phi_1 = \bar{y}_{st} + \mathrm{B}_{1h} \mu_{10} + \mathrm{B}_{2h} \mu_{20}, \tag{9}$$

where $\bar{y}_{st} = \sum_{h=1}^{H} W_h \bar{y}_h$ is the Horvitz-Thompson-type estimator; B_{1h} and B_{2h} are coefficients of regression and are given by

$$\mathrm{B}_{1h} = \frac{\tau_{22}\tau_{13} - \tau_{12}\tau_{23}}{\tau_{11}\tau_{22} - \tau_{12}^2}, \quad \mathrm{B}_{2h} = \frac{\tau_{11}\tau_{23} - \tau_{12}\tau_{13}}{\tau_{11}\tau_{22} - \tau_{12}^2}$$

where $\tau_{11} = \sum_{h=1}^{H} w_h \bar{x}_h^2, \tau_{22} = \sum_{h=1}^{H} w_h s_{hx}^4, \ \tau_{13} = \sum_{h=1}^{H} w_h \bar{x}_h \bar{y}_h, \tau_{12} = \sum_{h=1}^{H} w_h \bar{x}_h s_{hx}^2,$

$\tau_{23} = \sum_{h=1}^{H} w_h s_{hx}^2 \bar{y}_h, \mu_{10} = \sum_{h=1}^{H} w_h \left(\bar{X}_h - \bar{x}_h \right), \mu_{20} = \sum_{h=1}^{H} w_h \left(S_{hx}^2 - s_{hx}^2 \right)$ (See Koyuncu and Kadilar (2016) for detail).

4. Proposed Estimator

This paper considers a new calibration estimator of population mean in stratified random sampling design as

$$\phi_2 = \sum_{h=1}^{H} W_h^* \bar{y}_h, \tag{10}$$

where W_h^* are calibration weights, using the chi-square loss functions

$$L\left(W_h^*, W_h\right) = \sum_{h=1}^{H} \frac{\left(W_h^* - W_h\right)^2}{W_h Q_h},$$

and subject to the following calibration constraints

$$\sum_{h=1}^{H} W_h^* \bar{x}_h = \sum_{h=1}^{H} W_h \bar{X}_h. \tag{12}$$

$$\sum_{h=1}^{H} W_h^* s_{hx}^2 = \sum_{h=1}^{H} W_h S_{hx}^2. \tag{13}$$

$$\sum_{h=1}^{H} W_h^* \beta_{2h}(x) = \sum_{h=1}^{H} W_h B_{2h}(x). \tag{14}$$

$$\sum_{h=1}^{H} W_h^* = \sum_{h=1}^{H} W_h. \tag{15}$$

The Lagrange function is given by

$$\Omega = \sum_{h=1}^{H} \frac{\left(W_h^* - W_h\right)^2}{W_h Q_h} - 2\lambda_1^* \left[\sum_{h=`}^{H} W_h^* \bar{x}_h - \sum_{h=1}^{H} W_h \bar{X}_h\right] - 2\lambda_2^* \left[\sum_{h=1}^{H} W_h^* s_{hx}^2 - \sum_{h=1}^{H} W_h S_{hx}^2\right]$$
$$-2\lambda_3^* \left[\sum_{h=1}^{H} W_h^* \beta_{2h}(x) - \sum_{h=1}^{H} W_h B_{2h}(x)\right] - 2\lambda_4^* \left[\sum_{h=1}^{H} W_h^* - \sum_{h=1}^{H} W_h\right] \tag{16}$$

Minimizing the chi-square loss functions (11) subject to the calibration constraints ((12), (13), (14), (15)) gives the calibration weights for stratified random sampling as follows

$$W_h^* = W_h + W_h Q_h \left(\lambda_1^* \bar{x}_h + \lambda_2^* s_{hx}^2 + \lambda_3^* \beta_{2h}(x) + \lambda_4^*\right). \tag{17}$$

Substituting (17) into (12), (13), (14), and (15), respectively gives the following system of equations

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{bmatrix} \begin{bmatrix} \lambda_1^* \\ \lambda_2^* \\ \lambda_3^* \\ \lambda_4^* \end{bmatrix} = \begin{bmatrix} \mu_{10} \\ \mu_{20} \\ \mu_{30} \\ \mu_{40} \end{bmatrix}. \tag{18}$$

Solving the system of equations in (18) for λ^*'s gives

$$\lambda_1^* = \frac{(\varphi_1\varphi_2 - \varphi_3\varphi_4)(\varphi_2\varphi_5 - \varphi_4\varphi_6) - (\varphi_1\varphi_5 - \varphi_4\varphi_9)(\varphi_2^2 - \varphi_4\varphi_7)}{(\varphi_2\varphi_5 - \varphi_4\varphi_6)^2 - (\varphi_5^2 - \varphi_4\varphi_8)(\varphi_2^2 - \varphi_4\varphi_7)},$$

$$\lambda_2^* = \frac{(\varphi_1\varphi_5 - \varphi_4\varphi_9)(\varphi_2\varphi_6 - \varphi_5\varphi_7) - (\varphi_2\varphi_3 - \varphi_1\varphi_7)(\varphi_5^2 - \varphi_4\varphi_8)}{(\varphi_2\varphi_5 - \varphi_4\varphi_6)^2 - (\varphi_5^2 - \varphi_4\varphi_8)(\varphi_2^2 - \varphi_4\varphi_7)},$$

$$\lambda_3^* = \frac{(\varphi_1\varphi_5 - \varphi_4\varphi_9)(\varphi_2\varphi_5 - \varphi_4\varphi_6) - (\varphi_1\varphi_2 - \varphi_3\varphi_4)(\varphi_5^2 - \varphi_4\varphi_8)}{(\varphi_2\varphi_5 - \varphi_4\varphi_6)^2 - (\varphi_5^2 - \varphi_4\varphi_8)(\varphi_2^2 - \varphi_4\varphi_7)},$$

$$\lambda_4^* = \frac{(\varphi_1\varphi_2 - \varphi_3\varphi_4)(\varphi_1\varphi_5 - \varphi_4\varphi_9) - (\varphi_2\varphi_6 - \varphi_5\varphi_7)(\varphi_2^2 - \varphi_4\varphi_7)}{(\varphi_2\varphi_5 - \varphi_4\varphi_6)^2 - (\varphi_5^2 - \varphi_4\varphi_8)(\varphi_2^2 - \varphi_4\varphi_7)}$$

where

$\varphi_1 = \sigma_{12}\mu_{10} - \sigma_{11}\mu_{20}$, $\varphi_2 = \sigma_{12}\sigma_{13} - \sigma_{11}\sigma_{23}$, $\varphi_3 = \sigma_{13}\mu_{10} - \sigma_{11}\mu_{30}$, $\varphi_4 = \sigma_{12}^2 - \sigma_{11}\sigma_{22}$

$\varphi_5 = \sigma_{12}\sigma_{14} - \sigma_{11}\sigma_{24}$, $\varphi_6 = \sigma_{13}\sigma_{14} - \sigma_{11}\sigma_{34}$, $\varphi_7 = \sigma_{13}^2 - \sigma_{11}\sigma_{33}$, $\varphi_8 = \sigma_{14}^2 - \sigma_{11}\sigma_{44}$ and $\varphi_9 = \sigma_{11}\mu_{10}$

and

$$\sigma_{11} = \sum_{h=1}^{H} W_h Q_h \bar{x}_h^2,\ \sigma_{22} = \sum_{h=1}^{H} W_h Q_h s_{hx}^4,\ \sigma_{33} = \sum_{h=1}^{H} W_h Q_h \beta_{2h}^2(x),, \sigma_{44} = \sum_{h=1}^{H} W_h Q_h$$

$$\sigma_{12} = \sum_{h=1}^{H} W_h Q_h \bar{x}_h s_{hx}^2, \sigma_{13} = \sum_{h=1}^{H} W_h Q_h \bar{x}_h \beta_{2h}(x),\ \sigma_{14} = \sum_{h=1}^{H} W_h Q_h \bar{x}_h,\ \sigma_{23} = \sum_{h=1}^{H} W_h Q_h s_{hx}^2 \beta_{2h}(x),$$

$$\sigma_{24} = \sum_{h=1}^{H} W_h Q_h s_{hx}^2,\ \sigma_{34} = \sum_{h=1}^{H} W_h Q_h \beta_{2h}(x),\ \mu_{10} = \sum_{h=1}^{H} W_h(\bar{X}_h - \bar{x}_h),\ \mu_{20} = \sum_{h=1}^{H} W_h(S_{hx}^2 - s_{hx}^2),$$

$$\mu_{30} = \sum_{h=1}^{H} W_h\left[B_{2h}(x) - \beta_{2h}(x)\right] \text{ and } \mu_{40} = 0.$$

Substituting the λ^* s in (16) and the resulting equation in (10) while setting, $Q_h = 1$, gives the proposed calibration regression estimator of population mean in stratified random sampling as follows

$$\phi_2 = \bar{y}_{st} + B_{1h}^*\mu_{10} + B_{2h}^*\mu_{20} + B_{3h}^*\mu_{30} \tag{19}$$

where $\bar{y}_{st} = \sum_{h=1}^{H} W_h \bar{y}_h$ is the Horvitz-Thompson-type estimator B_{1h}^*, B_{2h}^* and B_{3h}^* are the coefficients of regression and are given by

$$B_{1h}^* = \frac{\tau_{12}\left[\tau_{14}(\tau_{22}\tau_{33} - \tau_{23}^2) + \tau_{24}(\tau_{13}\tau_{23} - \tau_{12}\tau_{23}) + \tau_{34}(\tau_{12}\tau_{23} - \tau_{13}\tau_{22})\right]}{(\tau_{12}^2 - \tau_{11}\tau_{22})(\tau_{13}\tau_{23} - \tau_{12}^2) - (\tau_{13}\tau_{22} - \tau_{12}\tau_{23})(\tau_{12}\tau_{13} - \tau_{11}\tau_{23})},$$

$$B_{2h}^* = \frac{\tau_{12}\left[\tau_{14}(\tau_{13}\tau_{23} - \tau_{12}\tau_{33}) + \tau_{24}(\tau_{11}\tau_{33} - \tau_{13}^2) + \tau_{34}(\tau_{12}\tau_{13} - \tau_{11}\tau_{23})\right]}{(\tau_{12}^2 - \tau_{11}\tau_{22})(\tau_{13}\tau_{23} - \tau_{12}^2) - (\tau_{13}\tau_{22} - \tau_{12}\tau_{23})(\tau_{12}\tau_{13} - \tau_{11}\tau_{23})},$$

$$B_{3h}^* = \frac{\tau_{12}\left[\tau_{14}(\tau_{13}\tau_{22} - \tau_{12}\tau_{23}) + \tau_{24}(\tau_{12}\tau_{13} - \tau_{11}\tau_{23}) + \tau_{34}(\tau_{11}\tau_{22} - \tau_{12}^2)\right]}{(\tau_{12}^2 - \tau_{11}\tau_{22})(\tau_{13}\tau_{23} - \tau_{12}^2) - (\tau_{13}\tau_{22} - \tau_{12}\tau_{23})(\tau_{12}\tau_{13} - \tau_{11}\tau_{23})},$$

where $\tau_{11} = \sum_{h=1}^{H} W_h \bar{x}_h^2$, $\tau_{22} = \sum_{h=1}^{H} W_h s_{hx}^4$, $\tau_{33} = \sum_{h=1}^{H} W_h \beta_{2h}^2(x)$, $\tau_{12} = \sum_{h=1}^{H} W_h \bar{x}_h s_{hx}^2$, $\tau_{13} = \sum_{h=1}^{H} W_h \bar{x}_h \beta_{2h}(x)$,

$\tau_{14} = \sum_{h=1}^{H} W_h \bar{x}_h \bar{y}_h$, $\tau_{23} = \sum_{h=1}^{H} W_h s_{hx}^2 \beta_{2h}(x)$, $\tau_{24} = \sum_{h=1}^{H} W_h s_{hx}^2 \bar{y}_h$, and $\tau_{34} = \sum_{h=1}^{H} W_h \beta_{2h}(x) \bar{y}_h$.

5. Sample Design and Estimation

5.1. Estimation of variance

This section derives the estimator of variance for the Koyuncu and Kadilar (2016) calibration estimator and the proposed calibration estimator using the large sample approximation (LASAP) method. Let consider the following equations;

$$e_{hy}=\left(\frac{\bar{y}_h-\bar{Y}_h}{\bar{Y}_h}\right) \text{ so that } \bar{y}_h=\bar{Y}_h\left(1+e_{hy}\right),\ e_{hx}=\left(\frac{\bar{x}_h-\bar{X}_h}{\bar{X}_h}\right) \text{ so that } \bar{x}_h=\bar{X}_h\left(1+e_{hy}\right),$$

$$e_{hs}=\left(\frac{s_{hx}^2-S_{hx}^2}{S_{hx}^2}\right) \text{ so that } s_{hx}^2=S_{hx}^2\left(1+e_{hs}\right),$$

$$e_{h\beta}=\left(\frac{\beta_{2h}(x)-\mathrm{B}_{2h}(x)}{\mathrm{B}_{2h}(x)}\right) \text{ so that } \beta_{2h}(x)=\mathrm{B}_{2h}(x)\left(1+e_{h\beta}\right),$$

$$E\left(e_{hy}^2\right)=\gamma_h C_{hy}^2, E\left(e_{hx}^2\right)=\gamma_h C_{hx}^2, E\left(e_{hs}^2\right)=\gamma_h C_{hs}^2, E\left(e_{h\beta}^2\right)=\gamma_h C_{h\beta}^2,$$

$$E\left(e_{hy}e_{hx}\right)=\gamma_h\rho_{hyx}C_{hy}C_{hx},\ E\left(e_{hy}e_{hs}\right)=\gamma_h\rho_{hys}C_{hy}C_{hs},\ E\left(e_{hy}e_{h\beta}\right)=\gamma_h\rho_{hy\beta}C_{hy}C_{h\beta},$$

$$E\left(e_{hx}e_{hs}\right)=\gamma_h\rho_{hxs}C_{hx}C_{hs},\ E\left(e_{hx}e_{h\beta}\right)=\gamma_h\rho_{hx\beta}C_{hx}C_{h\beta}, E\left(e_{hs}e_{h\beta}\right)=\gamma_h\rho_{hs\beta}C_{hs}C_{h\beta},$$

$$m_h=\frac{\bar{X}_h}{\bar{Y}_h},\theta_h=\frac{S_{hx}^2}{\bar{Y}_h},\xi_h=\frac{B_{2h}(x)}{\bar{Y}_h} \text{ and } \gamma_h=\left(\frac{1}{n_h}-\frac{1}{N_h}\right).$$

The parameters are defined wherever they appear as follows:

$\bar{y}_h$ is the sample stratum mean of the study variable.

$\bar{Y}_h$ is the population stratum mean of the study variable.

$\bar{x}_h$ is the sample stratum mean of the auxiliary variable.

$\bar{X}_h$ is the population stratum mean of the auxiliary variable.

s_{hx}^2 is the sample stratum variance of the auxiliary variable.

S_{hx}^2 is the population stratum variance of the auxiliary variable.

$\beta_{2h}(x)$ is the sample coefficient of kurtosis of the auxiliary variable.

$\mathrm{B}_{2h}(x)$ is the population coefficient of kurtosis of the auxiliary variable.

C_{hx}^2 is the coefficient of variation of the auxiliary variable.

C_{hy}^2 is the coefficient of variation of the auxiliary variable.

ρ_{hxy} is the correlation coefficient between the auxiliary variable and the study variable.

ρ_{hxs} is the correlation coefficient between the mean and variance of the auxiliary variable.

ρ_{hys} is the correlation coefficient between the mean of the study variable and variance of the auxiliary variable.

$\rho_{hx\beta}$ is the correlation coefficient between the mean and coefficient of kurtosis of the auxiliary variable.

$\rho_{hy\beta}$ is the correlation coefficient between the mean of the study variable and coefficient of kurtosis of the auxiliary variable.

$\rho_{hs\beta}$ is the correlation coefficient between the variance and coefficient of kurtosis of the auxiliary variable.

5.1.1. Regression estimator

Expressing (2) in terms of the e's gives

$$\phi_1^* = \sum_{h=1}^{H} w_h \left[\bar{Y}_h \left(1+e_{hy}\right) - B_h^* \bar{X}_h e_{hx} \right].$$

So that

$$\left[\phi_1^* - \bar{Y} \right] = \sum w_h \left[\bar{Y}_h e_{hy} - B_h^* \bar{X}_h e_{hx} \right]. \tag{20}$$

Squaring both sides of (20) gives

$$\left[\phi_1^* - \bar{Y} \right]^2 = \sum_{h=1}^{H} w_h^2 \left[\bar{Y}_h^2 e_{hy}^2 + \mathrm{B}_h^{*2} \bar{X}_h^2 e_{hx}^2 - 2\mathrm{B}_h^* \bar{Y}_h \bar{X}_h e_{hy} e_{hx} \right]. \tag{21}$$

Taking expectation of both sides of (21) gives

$$\hat{V}\left[\phi_1^* \right] = \sum_{h=1}^{H} w_h^2 \bar{Y}_h^2 \gamma_h \left[C_{hy}^2 + \mathrm{B}_h^{*2} m_h^2 C_{hx}^2 - 2\mathrm{B}_h^* m_h \rho_{hxy} C_{hx} C_{hy} \right]. \tag{22}$$

5.1.2. Koyuncu and Kadilar calibration estimator

Expressing (9) in terms of the e's gives

$$\phi_2^* = \sum_{h=1}^{H} w_h \left[\bar{Y}_h \left(1+e_{hy}\right) - \mathrm{B}_{1h} \bar{X}_h e_{hx} - \mathrm{B}_{2h} S_{hx}^2 e_{hs} \right].$$

So that

$$\left[\phi_2^* - \bar{Y} \right] = \sum_{h=1}^{H} w_h \left[\bar{Y}_h e_{hy} - \mathrm{B}_{1h} \bar{X}_h e_{hx} - \mathrm{B}_{2h} S_{hx}^2 e_{hs} \right]. \tag{23}$$

Squaring both sides of (23) gives

$$\left[\phi_2^* - \bar{Y} \right]^2 = \sum_{h=1}^{H} w_h^2 \begin{bmatrix} \bar{Y}_h^2 e_{hy}^2 + \mathrm{B}_{1h}^2 \bar{X}_h^2 e_{hx}^2 + \mathrm{B}_{2h}^2 S_{hx}^4 e_{hs}^2 - 2\bar{Y}_h \mathrm{B}_{1h} \bar{X}_h e_{hy} e_{hx} - 2\bar{Y}_h \mathrm{B}_{2h} S_{hx}^2 e_{hy} e_{hs} \\ +2\mathrm{B}_{1h} \mathrm{B}_{2h} \bar{X}_h S_{hx}^2 e_{hx} e_{hs} \end{bmatrix}. \tag{24}$$

Taking expectation of both sides of (24) gives

$$\hat{V}\left[\phi_2^* \right] = \sum_{h=1}^{H} w_h^2 \bar{Y}_h^2 \gamma_h \begin{bmatrix} C_{hy}^2 + \mathrm{B}_{1h}^2 m_h^2 C_{hx}^2 + \mathrm{B}_{2h}^2 \theta_h^2 C_{hs}^2 - 2\mathrm{B}_{1h} m_h \rho_{hxy} C_{hx} C_{hy} - 2\mathrm{B}_{2h} \theta_h \rho_{hsy} C_{hy} C_{hs} \\ +2\mathrm{B}_{1h} \mathrm{B}_{2h} m_h \theta_h \rho_{hxs} C_{hx} C_{hs} \end{bmatrix}. \tag{25}$$

5.1.3. Proposed calibration estimator

Expressing (19) in terms of the e's gives

$$\phi_3^* = \sum_{h=1}^{H} w_h \left[\bar{Y}_h (1+e_{hy}) - B_{1h}^* e_{hx} - B_{2h}^* S_{hx}^2 e_{hs} - B_{3h}^* \mathrm{B}_{2h}\left(x\right) e_{h\beta} \right].$$

So that

$$\left[\phi_3^* - \bar{Y} \right] = \sum_{h=1}^{H} w_h \left[\bar{Y}_h e_{hy} - B_{1h}^* e_{hx} - B_{2h}^* S_{hx}^2 e_{hs} - B_{3h}^* \mathrm{B}_{2h}\left(x\right) e_{h\beta} \right]. \tag{26}$$

Squaring both sides of (26) gives

$$\left[\phi_3^* - \bar{Y} \right]^2 = \sum_{h=1}^{H} w_h^2 \begin{bmatrix} \bar{Y}_h^2 e_{hy}^2 + \mathrm{B}_{1h}^{*2} \bar{X}_h^2 e_{hx}^2 + \mathrm{B}_{2h}^{*2} S_{hx}^4 e_{hs}^2 + \mathrm{B}_{3h}^{*2} \mathrm{B}_{2h}^2\left(x\right) e_{h\beta}^2 - 2\bar{Y}_h \bar{X}_h \mathrm{B}_{1h}^* e_{hy} e_{hx} \\ -2\mathrm{B}_{2h}^* \bar{Y}_h S_{hx}^2 e_{hy} e_{hs} - 2\mathrm{B}_{3h}^* \bar{Y}_h \mathrm{B}_{2h}\left(x\right) e_{hy} e_{h\beta} + 2\mathrm{B}_{1h}^* \mathrm{B}_{2h}^* \bar{X}_h S_{hx}^2 e_{hy} e_{hs} \\ +2\mathrm{B}_{1h}^* \mathrm{B}_{3h}^* \bar{X}_h \mathrm{B}_{2h}\left(x\right) e_{hx} e_{h\beta} + 2\mathrm{B}_{2h}^* \mathrm{B}_{3h}^* S_{hx}^2 \mathrm{B}_{2h}\left(x\right) e_{hs} e_{h\beta} \end{bmatrix}. \tag{27}$$

Taking expectation of both sides of (27) gives

$$\hat{V}\left[\phi_3^*\right]=\sum_{h=1}^{H}w_h^2\bar{Y}_h^2\gamma_h\begin{bmatrix}C_{hy}^2+\mathrm{B}_{1h}^{*2}m_h^2C_{hx}^2+\mathrm{B}_{2h}^{*2}\theta_h^2C_{hs}^2+\mathrm{B}_{3h}^{*2}\xi_h^2C_{h\beta}^2-2m_h\mathrm{B}_{1h}^*\rho_{hxy}C_{hy}C_{hx}-2\mathrm{B}_{2h}^*\theta_h\rho_{hsy}C_{hy}C_{hs}\\-2\mathrm{B}_{3h}^*\xi_h\mathrm{B}_{2h}\left(x\right)\rho_{h\beta y}C_{hy}C_{h\beta}+2\mathrm{B}_{1h}^*\mathrm{B}_{2h}^*m_h\theta_h\rho_{hxs}C_{hx}C_{hs}+2\mathrm{B}_{1h}^*\mathrm{B}_{3h}^*m_h\xi_h\rho_{hx\beta}C_{hx}C_{h\beta}\\+2\mathrm{B}_{2h}^*\mathrm{B}_{3h}^*\theta_h\xi_h\rho_{hs\beta}C_{hs}C_{h\beta}\end{bmatrix} \tag{28}$$

5.2. Optimality conditions

This section deduced the optimality conditions that would guarantee optimum performance of the Koyuncu and Kadilar (2016) calibration estimator and the proposed calibration estimator on satisfaction.

5.2.1. Regression estimator

Setting $\dfrac{\partial\hat{V}\left[\phi_1^*\right]}{\partial\mathrm{B}_h^*}=0$ so that

$$\mathrm{B}_h^*m_h^2C_{hx}^2=m_h\rho_{hyx}C_{hy}C_{hx}. \tag{29}$$

The $\hat{V}\left[\phi_1^*\right]$ in (22) is minimized when

$$\mathrm{B}_h^*=\frac{\rho_{hyx}C_{hy}}{m_hC_{hx}}=\mathrm{B}_{h,opt}^*\left(say\right). \tag{30}$$

Substituting the value of $\mathrm{B}_{h,opt}^*$ in (30) in (2), gives the regression calibration asymptotically optimum estimator (CAOE) for population mean in stratified random sampling as

$$\phi_1^*=\bar{y}_{st}+\frac{\rho_{hyx}C_{hy}}{m_hC_{hx}}\mu_{10}. \tag{31}$$

Similarly, substituting the value of $\mathrm{B}_{h,opt}^*$ in (30) for B_h^* in (22), gives the variance of regression calibration asymptotically optimum estimator (CAOE) $\phi_{1,opt}^*$ (or minimum variance of ϕ_1^*) as

$$\hat{V}\left[\phi_{1,opt}^*\right]=\sum_{h=1}^{H}w_h^2\gamma_h\vec{Y}_h^2C_{hy}^2\left(1-\rho_{hxy}^2\right). \tag{32}$$

5.2.2. Koyuncu and Kadilar (2016) calibration estimator

Setting $\dfrac{\partial\hat{V}\left[\phi_1\right]}{\partial\mathrm{B}_{1h}}=0,\ \dfrac{\partial\hat{V}\left[\phi_1\right]}{\partial\mathrm{B}_{2h}}=0$ so that

$$\mathrm{B}_{1h}m_h^2C_{hx}^2+\mathrm{B}_{2h}m_h\theta_h\rho_{hxs}C_{hx}C_{hs}=m_h\rho_{hyx}C_{hy}C_{hx}, \tag{33}$$

$$\mathrm{B}_{1h}m_h\theta_h\rho_{hxs}C_{hx}C_{hs}+\mathrm{B}_{2h}\theta_h^2C_{hs}^2=\theta_h\rho_{hys}C_{hy}C_{hs}. \tag{34}$$

The $\hat{V}\left[\phi_2\phi_1^*\right]$ in (25) is minimized when

$$\mathrm{B}_{1h}=\frac{C_{hy}\left(\rho_{hyx}-\rho_{hys}\rho_{hxs}\right)}{m_hC_{hx}\left(1-\rho_{hxs}^2\right)}=\mathrm{B}_{1h,opt}\left(say\right). \tag{35}$$

and

$$\mathrm{B}_{2h}=\frac{C_{hy}\left(\rho_{hys}-\rho_{hyx}\rho_{hxs}\right)}{\theta_hC_{hs}\left(1-\rho_{hxs}^2\right)}=\mathrm{B}_{2h,opt}\left(say\right). \tag{36}$$

Substituting the value of $\mathrm{B}_{1h,opt}$ in (35) and $\mathrm{B}_{2h,opt}$ in (36) for B_{1h} and B_{2h} in (9), gives the Koyuncu and Kadilar (2016) calibration asymptotically optimum estimator (CAOE) for population mean in stratified sampling as

$$\phi_2^* = \bar{y}_{st} + \frac{C_{hy}\left(\rho_{hyx} - \rho_{hys}\rho_{hxs}\right)}{m_h C_{hx}\left(1-\rho_{hxs}^2\right)}\mu_{10} + \frac{C_{hy}\left(\rho_{hys} - \rho_{hyx}\rho_{hxs}\right)}{\theta_h C_{hs}\left(1-\rho_{hxs}^2\right)}\mu_{20}. \tag{37}$$

Similarly, substituting the value of $\mathrm{B}_{1h,opt}$ in (35) and $\mathrm{B}_{2h,opt}$ in (36) for B_{1h} and B_{2h} in (25), gives the variance of Koyuncu and Kadilar (2016) calibration asymptotically optimum estimator (CAOE) $\phi_{2,opt}$ (or minimum variance of ϕ_2) as

$$\hat{V}\left[\phi_{2,opt}^*\right] = \sum_{h=1}^{H} w_h^2 \gamma_h \bar{Y}_h^2 C_{hy}^2 \left[\frac{1+2\rho_{hyx}\rho_{hys}\rho_{hxs} - \rho_{hyx}^2 - \rho_{hys}^2 - \rho_{hxs}^2}{\left(1-\rho_{hxs}^2\right)}\right]. \tag{38}$$

5.2.3. Proposed calibration estimator

Setting $\frac{\partial\hat{v}\left[\phi_3^*\right]}{\partial \mathrm{B}_{1h}^*} = 0, \frac{\partial\hat{v}\left[\phi_3^*\right]}{\partial \mathrm{B}_{2h}^*} = 0, \frac{\partial\hat{v}\left[\phi_3^*\right]}{\partial \mathrm{B}_{3h}^*} = 0,$ respectively, gives the following system of equations

$$\begin{bmatrix} m_h^2 C_{hx}^2 & m_h\theta_h\rho_{hxs}C_{hx}C_{hs} & m_h\xi_h\rho_{hx\beta}C_{hx}C_{h\beta} \\ m_h\theta_h\rho_{hxs}C_{hx}C_{hs} & \theta_h^2 C_{hs}^2 & \theta_h\xi_h\rho_{hs\beta}C_{hs}C_{h\beta} \\ m_h\xi_h\rho_{hx\beta}C_{hx}C_{h\beta} & \theta_h\xi_h\rho_{hs\beta}C_{hs}C_{h\beta} & \xi_h^2 C_{h\beta}^2 \end{bmatrix}\begin{bmatrix} \mathrm{B}_{1h}^* \\ \mathrm{B}_{2h}^* \\ \mathrm{B}_{3h}^* \end{bmatrix} = \begin{bmatrix} m_h\rho_{hyx}C_{hy}C_{hx} \\ \theta_h\rho_{hys}C_{hy}C_{hs} \\ \xi_h\rho_{hy\beta}C_{hy}C_{h\beta} \end{bmatrix}. \tag{39}$$

The $\hat{v}\left[\phi_3^*\right]$ in (28) is minimized when

$$\mathrm{B}_{1h}^* = \frac{C_{hy}\begin{bmatrix}\rho_{hyx} + \rho_{hx\beta}\rho_{hs\beta}\rho_{hys} + \rho_{hxs}\rho_{hs\beta}\rho_{hy\beta} \\ -\rho_{hs\beta}^2\rho_{hyx} - \rho_{hxs}\rho_{hys} - \rho_{hx\beta}\rho_{hy\beta}\end{bmatrix}}{m_h C_{hx}\left[1+2\rho_{hxs}\rho_{hx\beta}\rho_{hs\beta} - \rho_{hxs}^2 - \rho_{hx\beta}^2 - \rho_{hs\beta}^2\right]} = \mathrm{B}_{1h,opt}^*\,(say), \tag{40}$$

$$\mathrm{B}_{2h}^* = \frac{C_{hy}\begin{bmatrix}\rho_{hys} + \rho_{hxs}\rho_{hx\beta}\rho_{hy\beta} + \rho_{hx\beta}\rho_{hs\beta}\rho_{hyx} \\ -\rho_{hx\beta}^2\rho_{hys} - \rho_{hs\beta}\rho_{hy\beta} - \rho_{hxs}\rho_{hyx}\end{bmatrix}}{\theta_h C_{hs}\left[1+2\rho_{hxs}\rho_{hx\beta}\rho_{hs\beta} - \rho_{hxs}^2 - \rho_{hx\beta}^2 - \rho_{hs\beta}^2\right]} = \mathrm{B}_{2h,opt}^*\,(say), \tag{41}$$

and

$$\mathrm{B}_{3h}^* = \frac{C_{hy}\begin{bmatrix}\rho_{hy\beta} + \rho_{hxs}\rho_{hs\beta}\rho_{hyx} + \rho_{hxs}\rho_{hx\beta}\rho_{hys} \\ -\rho_{hxs}^2\rho_{hy\beta} - \rho_{hx\beta}\rho_{hyx} - \rho_{hs\beta}\rho_{hys}\end{bmatrix}}{\varphi_3 C_{h\beta}\left[1+2\rho_{hxs}\rho_{hx\beta}\rho_{hs\beta} - \rho_{hxs}^2 - \rho_{hx\beta}^2 - \rho_{hs\beta}^2\right]} = \mathrm{B}_{3h,opt}^*\,(say). \tag{42}$$

Substituting the value of $\mathrm{B}_{1h,opt}^*$ in (40); $\mathrm{B}_{2h,opt}^*$ in (41) and $\mathrm{B}_{3h,opt}^*$ in (42) for B_{1h}^*, B_{2h}^* and B_{3h}^* in (19), gives the proposed calibration asymptotically optimum estimator (CAOE) for population mean in stratified sampling as

$$\phi_{3,opt}^{*}=\bar{y}_{st}+\frac{C_{hy}\begin{bmatrix}\rho_{hyx}+\rho_{hx\beta}\rho_{hs\beta}\rho_{hys}+\rho_{hxs}\rho_{hs\beta}\rho_{hy\beta}\\ -\rho_{hs\beta}^{2}\rho_{hyx}-\rho_{hxs}\rho_{hys}-\rho_{hs\beta}\rho_{hx\beta}\end{bmatrix}}{m_{h}C_{hx}\left[1+2\rho_{hxs}\rho_{hx\beta}\rho_{hs\beta}-\rho_{hxs}^{2}-\rho_{hx\beta}^{2}-\rho_{hs\beta}^{2}\right]}\mu_{10}$$

$$+\frac{C_{hy}\begin{bmatrix}\rho_{hys}+\rho_{hx\beta}\rho_{hs\beta}\rho_{hyx}+\rho_{hxs}\rho_{hx\beta}\rho_{hy\beta}\\ -\rho_{hx\beta}^{2}\rho_{hys}-\rho_{hs\beta}\rho_{hy\beta}-\rho_{hxs}\rho_{hyx}\end{bmatrix}}{\theta_{h}C_{hs}\left[1+2\rho_{hxs}\rho_{hx\beta}\rho_{hs\beta}-\rho_{hxs}^{2}-\rho_{hx\beta}^{2}-\rho_{hs\beta}^{2}\right]}\mu_{20}$$

$$+\frac{C_{hy}\begin{bmatrix}\rho_{hy\beta}+\rho_{hxs}\rho_{hs\beta}\rho_{hyx}+\rho_{hxs}\rho_{hx\beta}\rho_{hys}\\ -\rho_{hxs}^{2}\rho_{hy\beta}-\rho_{hx\beta}\rho_{hyx}-\rho_{hs\beta}\rho_{hys}\end{bmatrix}}{\xi_{h}C_{h\beta}\left[1+2\rho_{hxs}\rho_{hx\beta}\rho_{hs\beta}-\rho_{hxs}^{2}-\rho_{hx\beta}^{2}-\rho_{hs\beta}^{2}\right]}\mu_{30}. \qquad (43)$$

Similarly, substituting the value of $B_{1h,opt}^{*}$ in (40); $B_{2h,opt}^{*}$ in (41) and $B_{3h,opt}^{*}$ in (42) for B_{1h}^{*}, B_{2h}^{*} and B_{3h}^{*} in (28), gives the variance of the proposed calibration asymptotically optimum estimator (CAOE) $\phi_{3,opt}$ (or minimum variance of ϕ_{3}^{*}) as

$$\hat{V}\left[\phi_{3,opt}^{*}\right]=\sum_{h=1}^{H}w_{h}^{2}Y_{h}^{2}\gamma_{h}C_{hy}^{2}\begin{bmatrix}1+b_{1h,opt}^{*2}+b_{2h,opt}^{*2}+b_{3h,opt}^{*2}-2b_{1h,opt}^{*}\rho_{hxy}-2b_{2h,opt}^{*}\rho_{hsy}-2b_{3h,opt}^{*}\rho_{h\beta y}\\ +2b_{1h,opt}^{*}b_{2h,opt}^{*}\rho_{hxs}+2b_{1h,opt}^{*}b_{3h,opt}^{*}\rho_{hx\beta}+2b_{2h,opt}^{*}b_{3h,opt}^{*}\rho_{hs\beta}\end{bmatrix}. \qquad (44)$$

where

$$b_{1h,opt}^{*}=\frac{\begin{bmatrix}\rho_{hyx}+\rho_{hx\beta}\rho_{hs\beta}\rho_{hys}+\rho_{hxs}\rho_{hs\beta}\rho_{hy\beta}\\ -\rho_{hs\beta}^{2}\rho_{hyx}-\rho_{hxs}\rho_{hys}-\rho_{hs\beta}\rho_{hx\beta}\end{bmatrix}}{\left[1+2\rho_{hxs}\rho_{hx\beta}\rho_{hs\beta}-\rho_{hxs}^{2}-\rho_{hx\beta}^{2}-\rho_{hs\beta}^{2}\right]},\quad b_{2h,opt}^{*}=\frac{\begin{bmatrix}\left[\rho_{hys}+\rho_{hx\beta}\rho_{hs\beta}\rho_{hyx}+\rho_{hxs}\rho_{hx\beta}\rho_{hy\beta}\right.\\ -\rho_{hx\beta}^{2}\rho_{hys}-\rho_{hs\beta}\rho_{hy\beta}-\rho_{hxs}\rho_{hyx}\end{bmatrix}}{\left[1+2\rho_{hxs}\rho_{hx\beta}\rho_{hs\beta}-\rho_{hxs}^{2}-\rho_{hx\beta}^{2}-\rho_{hs\beta}^{2}\right]},$$

$$b_{3h,opt}^{*}=\frac{\begin{bmatrix}\rho_{hy\beta}+\rho_{hxs}\rho_{hs\beta}\rho_{hyx}+\rho_{hxs}\rho_{hx\beta}\rho_{hys}\\ -\rho_{hxs}^{2}\rho_{hy\beta}-\rho_{hx\beta}\rho_{hyx}-\rho_{hs\beta}\rho_{hys}\end{bmatrix}}{\left[1+2\rho_{hxs}\rho_{hx\beta}\rho_{hs\beta}-\rho_{hxs}^{2}-\rho_{hx\beta}^{2}-\rho_{hs\beta}^{2}\right]}.$$

6. Empirical Study

To judge the relative performances of the proposed calibration estimator over members of its class in stratified sampling, the data set in Table 1 was considered. Two measuring criteria; variance and percent relative efficiency (PRE) were used to compare the performance of each estimator.

The percent relative efficiency (PRE) of an estimator (say (α)) with respect to the conventional regression estimator in stratified sampling $\left(\phi_{1}^{*}\right)$ is defined by

$$PRE\left[\alpha,\phi_{1}^{*}\right]=\frac{\hat{V}\left[\phi_{1}^{*}\right]}{\hat{V}\left[\alpha\right]}\times 100,\ \hat{V}\left[\phi_{1,opt}^{*}\right]=3883.9883,\ \hat{V}\left[\phi_{2,opt}^{*}\right]=3050.5208,\ \text{and}$$

$$\hat{V}\left[\phi_{3,opt}^{*}\right]=2420.3462.$$

The percent relative efficiency of the conventional calibration regression estimator in stratified random sampling $\left(\phi_{1}^{*}\right)$, Koyuncu and Kadilar (2016) calibration regression estimator in stratified sampling $\left(\phi_2^*\right)$ and the proposed calibration regression estimator in stratified random sampling $\left(\phi_3^*\right)$ with respect to $\left(\phi_1^*\right)$ is presented in Table 2.

$\rho_{hs\beta}$

Table 1 Data Statistics

Parameter	Stratum 1	Stratum 2	Stratum 3
N_h	52.0000	76.0000	82.0000
n_h	15.0000	20.0000	28.0000
$\bar{X}_h$	6.8130	10.1200	7.9670
$\bar{Y}_h$	417.3300	503.3750	340.0000
S_{hx}^2	15.9712	132.6600	38.4380
S_{hy}^2	74775.4670	259113.7000	65885.6000
S_{hxy}	1007.6547	5709.1629	1404.7100
γ_h	0.2308	0.1868	0.1307
ρ_{hyx}	0.7030	0.7380	0.8050
ρ_{hys}	0.8020	0.7610	0.8260
$\rho_{hy\beta}$	0.8600	0.7640	0.7260
ρ_{hxs}	0.7140	0.8120	0.7420
$\rho_{hx\beta}$	0.8200	0.8030	0.7820
$\rho_{hs\beta}$	0.8360	0.8460	0.8120

Table 2 Performance of estimators

Estimator	Variance	$PRE\left(\alpha,\phi_1^*\right)$
ϕ_1^*	3883.9883	100.0000
ϕ_2^*	3050.5208	127.3221
ϕ_3^*	2420.3462	160.4724

Numerical results for the percent relative efficiency (PREs) in Table 2 reveals that the proposed calibration estimator $\left(\phi_3^*\right)$ has 60 percent gains in efficiency while the Koyuncu and Kadilar (2016) calibration estimator $\left(\phi_2^*\right)$ has 27 percent gains in efficiency; this shows that the proposed calibration estimator $\left(\phi_3^*\right)$ is 33 percent more efficient than the Koyuncu and Kadilar (2016) calibration estimator $\left(\phi_2^*\right)$. This means that in using our proposed calibration estimator $\left(\phi_3^*\right)$ (and by extension the new calibration weights) one will have 33 percent efficiency gain over the Koyuncu and Kadilar (2016) calibration estimator $\left(\phi_2^*\right)$ This result proves the robustness of the newly introduced calibration constraint.

7. Conclusions

This study derives new calibration weights in stratified random sampling using information from four parameters of the same auxiliary variable and proposes a more improved calibration estimator. Theoretical variance expression under large sample approximation is derived for the proposed estimator and the Koyuncu-Kadilar (2016) calibration estimator. Calibration asymptotic optimum estimator (CAOE) and its approximate variance estimator are derived for the proposed calibration estimator and the Koyuncu and Kadilar (2016) calibration estimator.

Numerical results showed that at the same optimum conditions the proposed estimator $\left(\phi_3^*\right)$ is highly efficient than the Koyuncu and Kadilar (2016) calibration estimator $\left(\phi_2^*\right)$ and by extension the Tracy et al. (2003) calibration estimator in stratified random sampling [since the Koyuncu and Kadilar (2016) calibration estimator $\left(\phi_2^*\right)$ is always more efficient than the Tracy et al. (2003) calibration estimator [see Koyuncu and Kadilar (2016) for detail]. It is observed that the new calibration estimator $\left(\phi_3^*\right)$ is very attractive and should be preferred in practice as it provides consistent and more precise parameter estimates than existing calibration estimators in stratified random sampling.

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