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Objective Bayesian Analysis for the Power Function II Distribution under Doubly Type II Censored Data

Tabassum Naz Sindhu* [a], and Zawar Hussain [b]
[a] Department of Statistics, Quaid-i-Azam University, Islamabad, Pakistan.
[b] Department of Statistics, Faculty of Computing, The Islamia University of Bahawalpur, Pakistan.
*Corresponding author; e-mail: sindhuqau@gmail.com

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Abstract

Trimmed samples are widely utilized in several areas of statistical practice, especially when some sample values at either or both extremes might have been adulterated. In this paper, the problem of estimating the parameter of power function II distribution based on trimmed samples under informative and non-informative priors has been addressed. The problem discussed using Bayesian approach to estimate the parameter of power function II distribution. The explicit expressions for estimator and risk are developed under all loss functions. Elicitation of hyperparameter through prior predictive approach is also discussed. Posterior predictive distributions along with posterior predictive intervals and credible intervals are also derived under different priors. A comparison is made using the Monte Carlo simulation. The influence of parametric value on the estimate and risk is also discussed.

Keywords: Inverse transformation method, doubly censored samples, loss functions, posterior predictive distributions, credible intervals, predictive intervals.

1. Introduction

The power function distribution is often used to study the electrical component reliability. The distribution function of power function II distribution is given by

$$F(x) = 1-(1-x)^{\lambda},\ \lambda > 0,\ 0 < x < 1. \tag{1}$$

And the corresponding PDF of power function II distribution has the following form

$$f(x) = \lambda(1-x)^{\lambda-1},\ \lambda > 0,\ 0 < x < 1, \tag{2}$$

where λ is the shape parameter. Trimmed samples are widely employed in several areas of statistical practice, especially when some sample values at either or both extremes might have been contaminated. The problem of estimating the parameters of power function distribution based on a trimmed sample and prior information has been considered in this paper. There are a few works available in literature on the Bayesian analysis of the power function distribution and its mixture. Meniconi and Barry (1996) discussed the electrical component reliability using the power function distribution. With the rapid progression of VLSI products, electrostatic discharge, accelerated life, electromagnetic pulse, radiation monitoring, and subsequent modelling of failure are regarded by both producers and consumers as an important issue in the quality control and assurance of semiconductors. Most distributions and their established relevant explanation of physical failure phenomena are used to estimate system reliability due to the variety and complexity of determining failure modes and causes in semiconductor devices. So, the power function distribution taken into account when estimating the device reliability of electronic components due to its applicability, simplicity and therefore attractiveness to reliability engineers.

Ahsanullah and Kabir (1974) gave a brief characterization of the power function distribution. Al-Hussaini and Jaheen (1992). Wingo (1993) derived the theory for the maximum likelihood (ML) point estimation of the parameters of the Burr distribution when type II singly censored sample is at hand. Akhter and Hirai (2009) studied the scale parameter from the Rayleigh distribution from type II singly and doubly censored data.

Sindhu et al. (2019a, 2019b) studied the Bayesian estimation approach for mixture models. Fernández (2000) studied a Bayesian approach to inference in reliability studies based on type II doubly censored data from a Rayleigh distribution. Sindhu and Hussain (2022) studied predictive inference and parameter estimation from the half-normal distribution for the left censored data. They also studied the problem of predicting an independent future sample from the same distribution in a Bayesian setting.

Saleem and Aslam (2009) investigated the Rayleigh distributed survival time in conjunction with the Rayleigh distributed censor time in order to derive the maximum likelihood and Bayes estimators for the unknown parameters and their corresponding variances. To find the Bayes estimators under the squared error loss function, they assumed informative and noninformative priors. In their study, they derived and evaluated the posterior predictive distribution of future observations, the predictive intervals, the credible intervals, and the highest posterior intervals. Saleem et al. (2010) used a two-component mixture of the power function distribution to model a heterogeneous population. They investigated a comprehensive simulation scheme with a large number of parameter points to highlight the properties and behavior of the estimates in terms of sample size, censoring rate, parameter size, and the proportion of the mixture’s components.

Feroze and Aslam (2012) studied Bayesian analysis of Gumbel type II distribution under doubly censored samples using different loss functions. Sindhu et al. (2013a) studied Bayes estimation of the parameters of the inverse Rayleigh distribution for left censored data. Sindhu et al. (2013b) discussed the Bayesian analysis of the Kumaraswamy distribution under failure censoring sampling scheme. Anum et al. (2021, 2022) used modified Kies generalized transformation and the new power function to suggest a unique statistical models. Sindhu et al. (2022) discussed different estimation method of mixture distribution and modeling the COVID-19 pandemic. Sindhu and Atangana (2021) suggested the model of reliability formed on inverse power law and generalized inverse Weibull model. Sindhu et al. (2021a, 2021b) studied new family of Gumbel type II distribution.

Doubly censoring of type II is used to indicate that a specified number of observations at both ends is missing in an ordered sample of size n while the number of censored observations is a random variable in type I censoring and the time of analysis is fixed. A number of extreme sample values, especially in reliability analysis or biomedical studies, due to negligence of inexperienced observers or other factors. Therefore excluding these findings from the original data set may be appropriate. The remaining sample is often referred as to the type II doubly censored sample, where the lowest and the highest values have been censored or discarded.

The present study investigated the prominent features of the unknown parameter of the power function II distribution based on doubly censored type II. The analysis of this type has not been studied earlier in literature through Bayesian structure to the best of our knowledge.

The rest of paper is organized as follows. In Section 2, the posterior distributions have been derived under non-informative and informative priors. Estimation of parameter has been discussed in Section 3. Credible intervals have been derived in Section 4. Method of elicitation of the hyper-parameters via prior predictive approach has been discussed in Section 5. Posterior predictive distribution and posterior predictive intervals are developed in Section 6. Simulation study and graphical comparison have been performed in Section 7. The conclusions regarding the study have been presented in Section 8.

2. Prior and Posterior Distributions

Consider a random sample of size n from the power function II distribution with parameter λ. Some data may not be observed, a known number of observation in an ordered sample are missing at both ends in failure censored experiments, the observations which are the smallest r and the largest s are random then data collected will be $x_{(r)} \le x_{(r+1)} \le ... \le x_{(s)}$, while the $r-1$ smallest observations and $n-s$ largest observations have been censored. The likelihood function for λ of the given type II double censored sample $\mathbf{x} = \left(x_r, ..., x_s\right)$ is then:

$$L(x,\lambda) = \frac{n!}{(r-1)!(n-s)!}\prod_{i=r}^{s} f\left(x_{(i)},\lambda\right)\left\{F\left(x_{(r)},\lambda\right)\right\}^{r-1}\left\{1-F\left(x_{(s)},\lambda\right)\right\}^{n-s},$$

$$L(x,\lambda) = \frac{n!}{(r-1)!(n-s)!}\prod_{i=r}^{s}\left\{\lambda(1-x_{(i)})^{(\lambda-1)}\right\}\left\{1-(1-x_{(r)})^{\lambda}\right\}^{r-1}\left\{(1-x_{(s)})^{\lambda}\right\}^{n-s},$$

$$L(x,\lambda) \propto \sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\lambda^{s-r+1}\exp\left[-\lambda\left\{\sum_{i=r}^{s}\ln\left(1-x_{(i)}\right)^{-1} - k\ln\left(1-x_{(r)}\right) - (n-s)\ln\left(1-x_{(s)}\right)\right\}\right],$$

$$L(x,\lambda) \propto \sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\lambda^{\tau}\exp\left[-\lambda\Im_{irs}\left(x_{(i)}\right)\right], \tag{3}$$

where $\tau = s-r+1,$ and $\Im_{irs}\left(x_{(i)}\right) = \left\{\sum_{i=r}^{s}\ln\left(1-x_{(i)}\right)^{-1} - k\ln\left(1-x_{(r)}\right) - (n-s)\ln\left(1-x_{(s)}\right)\right\}.$

2.1. Posterior distribution under non-informative prior

The uniform and Jeffreys prior are the example of non-informative priors which materializes the use of the Bayesian estimation methods when no prior information is available. The posterior distribution under the assumption of uniform and Jeffreys priors have been derived and presented in the following.

The uniform distribution is assumed to be

$$\pi_U(\lambda) \propto k, \lambda > 0. \tag{4}$$

The Jeffreys prior has been derived to be

$$\pi_J(\lambda) \propto \sqrt{I(\lambda)}, \lambda > 0, \tag{5}$$

where $I(\lambda) = -E\left(\frac{\partial^2 \ln L(\lambda, x)}{\partial\lambda^2}\right)$ is the Fisher's information matrix. For the model (1) $I(\lambda) = \frac{n}{\lambda^2},$

hence, $\pi_J(\lambda) \propto \frac{1}{\lambda}$. The general posterior distribution under the assumption of noninformative prior is

$$p(\lambda \mid \mathbf{x}) = \frac{\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} \lambda^{\tau-\varsigma} \exp\left[-\lambda\left\{\Im_{irs}\left(x_{(i)}\right)\right\}\right]}{\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} \frac{\Gamma(\tau+\upsilon)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\upsilon+\upsilon}}}, \lambda > 0. \quad (6)$$

The posterior distribution under the uniform prior is obtained when $\zeta = 0$ and $\upsilon = 1$ while under the Jeffreys prior it is obtained under the conditions $\zeta = 1$ and $\upsilon = 0$.

2.2. Posterior distribution under informative prior

In case of informative prior, the use of prior information is equivalent to add a number of observations to the given sample size and hence leads to a reduction of posterior risks of the Bayes estimates based on the said informative prior. Bolstad (2004) studied a method to evaluate the worth of prior information in terms of the number of additional observations supposed to be added to the given sample size.

The informative prior for the parameter λ is assumed to be exponential distribution:

$$\pi_{\exp}(\lambda) = me^{-\lambda m}, \lambda > 0. \quad (7)$$

The informative prior for the parameter λ is assumed to be gamma distribution:

$$\pi_{\text{gam}}(\lambda) = \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda}, a, b, \lambda > 0. \quad (8)$$

The informative prior for the parameter λ is assumed to be inverse Levy distribution:

$$\pi_{\text{In-Lev}}(\lambda) = \sqrt{\frac{c}{2\pi}} \lambda^{-\frac{1}{2}} \exp\left[-\left(\frac{c\lambda}{2}\right)\right], c > 0. \quad (9)$$

The posterior distribution under the assumption of gamma prior is:

$$p(\lambda \mid \mathbf{x}) = \frac{\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} \lambda^{\tau+a-1} \exp\left[-\lambda\left\{b+\Im_{irs}\left(x_{(i)}\right)\right\}\right]}{\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} \frac{\Gamma(\tau+a)}{\left\{b+\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+a}}}, a, b, \lambda > 0. \quad (10)$$

The expressions for the posterior distribution under exponential prior and inverse Levy prior are obtained when $a = 1, b = m$ and $a = 1/2, b = c/2$ respectively. It is observed that the posterior density function under non-informative and informative prior is recognized as the mixture of gamma density functions.

3. Bayes Estimators and Posterior Risks under Different Loss Functions

The theory of the decision implies that a loss function must be defined and used to measure the risk associated with each of the potential estimates in order to select the best estimator. Since there is no definite analytical process to identify the proper loss function that should be used. This section enlightens the derivation of the Bayes estimator (BE) and corresponding posterior risks (PR) under different loss functions. The Bayes estimators are evaluated under squared error loss function (SELF), precautionary loss function (PLF), weighted squared error loss function (WSELF), quasi-quadratic loss function (QQLF), squared-log error loss function (SLELF), and entropy loss function (ELF). The Bayes estimator (BE) and corresponding posterior risks (PR) under different loss functions are given in the following table.

Table 1 Bayes estimator and posterior risks under different loss functions

Loss Function= $L(\lambda, \hat{\lambda})$	Bayes Estimator	Posterior Risk
SELF: $(\lambda-\hat{\lambda})^2$	$E(\lambda \mid \mathbf{x})$	$Var(\lambda \mid \mathbf{x})$
PLF: $\frac{(\lambda-\hat{\lambda})^2}{\hat{\lambda}}$	$\sqrt{E(\lambda^2 \mid \mathbf{x})}$	$2\left\{\sqrt{E(\lambda^2 \mid \mathbf{x})}-E(\lambda \mid \mathbf{x})\right\}$
WSELF: $\frac{(\lambda-\hat{\lambda})^2}{\lambda}$	$\{E(\lambda \mid \mathbf{x})\}^{-1}$	$E(\lambda \mid \mathbf{x})-\{E(\lambda \mid \mathbf{x})\}^{-1}$
QQLF: $\left(e^{-c\hat{\lambda}}-e^{-c\lambda}\right)^2$	$\frac{-1}{c}\ln\left\{E\left(e^{-c\lambda} \mid \mathbf{x}\right)\right\}$	$E\left(e^{-c\lambda}\right)-\left\{E\left(e^{-c\lambda}\right)\right\}^2$
SLELF: $(\ln\hat{\lambda}-\ln\lambda)^2$	$\exp\{E(\ln\lambda \mid \mathbf{x})\}$	$E\{(\ln\lambda \mid \mathbf{x})\}^2-\{E(\ln\lambda \mid \mathbf{x})\}^2$
ELF: $b\left\{\left(\frac{\hat{\lambda}}{\lambda}\right)-\ln\left(\frac{\hat{\lambda}}{\lambda}\right)-1\right\}$	$\left\{E\left(\lambda^{-1} \mid \mathbf{x}\right)\right\}^{-1}$	$\ln\left\{E\left(\lambda^{-1} \mid \mathbf{x}\right)\right\}+E(\ln\lambda)$

The Bayes estimators and posterior risks under uniform prior are

$$\hat{\lambda}_{\substack{SELF(\zeta=1,\gamma=1,\delta=2)\\ PLF(\zeta=0.5,\gamma=1,\delta=3)\\ WSELF(\zeta=1,\gamma=0,\delta=1)}}=\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+\delta)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+\delta}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+\gamma)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+\gamma}}}\right\}^{\zeta},$$

$$\rho(\hat{\lambda})_{\substack{SELF(\zeta=1,\delta=3,\gamma=1,\psi=2)\\ PLF(\zeta=0.5,\delta=3,\gamma=1,\psi=1)\\ WSELF(\zeta=1,\delta=2,\gamma=0,\psi=1)}}=\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+\delta)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+\delta}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}\right\}^{\zeta}-\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+2)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+2}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+\gamma)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+\gamma}}}\right\}^{\psi}.$$

$$\hat{\lambda}_{QQLF}=\log\left(\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{1+\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}\right)^{-1},$$

$$\rho\left(\hat{\lambda}_{QQLF}\right)=\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{2+\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}\right\}-\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{1+\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}\right\}^2.$$

$$\hat{\lambda}_{SLELF}=\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)\exp(\psi(\tau+1))}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+2}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}},\ \rho\left(\hat{\lambda}_{SLELF}\right)=\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)\psi'(\tau+1)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}\right\}.$$

$$\hat{\lambda}_{ELF}=\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau}}}\right\},$$

$$\rho\left(\hat{\lambda}_{ELF}\right)=\ln\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{\Gamma(\tau+1)}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}\right\}+\psi(\tau+1)-\ln\left\{\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+2}}}\right\}.$$

The Bayes estimators and posterior risks under the rest of priors can be obtained in a similar manner.

4. Bayesian Credible Intervals for the Doubly Type II Censored Data

The Bayesian credible intervals for the doubly type II censored data under informative and non-informative priors, as discussed by Saleem and Aslam (2009) are presented in the following. The credible intervals for doubly type II censored data under uninformative priors are

$$\frac{\chi^2_{2(\tau+1)\left(\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+\gamma}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+\delta}}}<\lambda_{U/J}<\frac{\chi^2_{2(\tau+1)\left(1-\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+\gamma}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+\delta}}}.$$

The credible intervals are obtained assuming uniform and Jeffreys priors under the conditions $\gamma=2,\delta=1$ and $\gamma=1,\delta=0$ respectively. The credible intervals for doubly type II censored data under informative priors are

$$\frac{\chi^2_{2(\tau+a)\left(\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{b+\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+a+1}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{b+\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+a}}}<\lambda_{Gamma}<\frac{\chi^2_{2(\tau+a)\left(1-\frac{\alpha}{2}\right)}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{b+\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+a+1}}}{2\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{b+\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+a}}}.$$

The credible interval assuming exponential prior is obtained when $a=1, b=w$ and for inverse Levy $a=1/2, b=c/2.$

5. Elicitation

Bayesian analysis elicitation of opinion is a crucial step. It helps to make it easy for us to understand what the experts believe in and what their opinions are. In statistical inference, the characteristics of a certain predictive distribution proposed by an expert determine the hyperparameters of a prior distribution.

In this paper, we focus on a probability elicitation method known as prior predictive elicitation. Predictive elicitation is a method for estimating hyperparameters of prior distributions by inverting corresponding prior predictive distributions. Elicitation of hyperparameter from the prior $p(\lambda)$ is conceptually difficult task because we first have to identify prior distribution and then its hyperparameters. The prior predictive distribution is used for the elicitation of the hyperparameters which is compared with the experts' judgment about this distribution and then the hyperparameters are chosen in such a way so as to make the judgment agree closely as possible with the given distribution (reader desires more detail see Grimshaw et al. (2001), Kadane (1980), O'Hagan et al. (2006), Kadane et al. (1996), Jenkinson (2005) and Leon et al. (2003)). According to Aslam (2003), the method of assessment is to compare the predictive distribution with experts' assessment about this distribution and then to choose the hyperparameters that make the assessment agree closely with the member of the family. He discusses three important methods to elicit the hyperparameters: (i) via the prior predictive probabilities (ii) via elicitation of the confidence levels (iii) via the predictive mode and confidence level.

5.1. Prior predictive distribution

The prior predictive distribution is

$$g(y)=\int_0^\infty p(y\mid\lambda)p(\lambda)\,d\lambda. \tag{11}$$

The predictive distribution under gamma prior is

$$g(y)=\frac{ab^a}{(1-y)\left\{b-\ln(1-y)\right\}^{a+1}},\ 0<y<1. \tag{12}$$

The expressions for the predictive distribution under exponential prior and Inverse Levy prior are obtained when $a=1, b=m$ and $a=1/2, b=c/2$ respectively. By using the method of elicitation defined by Aslam (2003), we obtain the following hyperparameters $m=0.146341$, $a=4.898331$, $b=1.098839$ and $c=1.098839$.

6. Predictive Distribution

The predictive distribution contains the information about the independent future random observation given preceding observations. The reader desires more details can see Bolstad (2004) and Bansal (2007).

6.1. Posterior predictive distribution and predictive interval

The posterior predictive distribution of the future observation $y = x_{n+1}$ is

$$p(y \mid \mathbf{x}) = \int_0^\infty p(\lambda \mid \mathbf{x}) p(y \mid \lambda) d\lambda, \tag{13}$$

where $p(y \mid \lambda) = \lambda (1-y)^{(\lambda-1)}$ is the future observation density and $p(\lambda \mid \mathbf{x})$ is the posterior distribution obtained by incorporating the likelihood with the respective prior distributions.

A $(1-\alpha)100\%$ Bayesian interval (L, U) can be obtained by solving the following two equations simultaneously

$$\int_{-\infty}^{L} p(y \mid \mathbf{x}) dy = \frac{k}{2} = \int_{U}^{\infty} p(y \mid \mathbf{x}) dy. \tag{14}$$

The posterior predictive distribution of the future observation $y = x_{n+1}$ under uniform or Jeffreys priors are obtained under these conditions $\rho = \upsilon = 1$ and $\rho = \upsilon = 0$ in following expression.

$$p(y \mid \mathbf{x}) = \frac{\sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \dfrac{\tau+\rho}{(1-y)\left\{\Im_{irs}\left(x_{(i)}\right) - \ln(1-y)\right\}^{\tau+\upsilon+1}}}{\sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \dfrac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+\rho}}}, 0 < y < 1. \tag{15}$$

The posterior predictive distribution of the future observation $y = x_{n+1}$ under gamma prior is

$$p(y \mid \mathbf{x}) = \frac{\sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \dfrac{(\tau+a)}{(1-y)\left\{b + \Im_{irs}\left(x_{(i)}\right) - \ln(1-y)\right\}^{\tau+a+1}}}{\sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \dfrac{1}{\left\{b + \Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+a}}}, 0 < y < 1. \tag{16}$$

The expressions for the posterior predictive distribution of the future observation $y = x_{n+1}$ under exponential prior and inverse Levy prior are obtained when $a = 1, b = m$ and $a = 1/2, b = c/2$, respectively. Posterior predictive interval under uniform prior is

$$\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\left[\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}-\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)-\ln\left(1-L\right)\right\}^{\tau+1}}\right]}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}=\frac{k}{2},$$

$$\frac{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\left[\frac{1}{\left\{\Im_{irs}\left(x_{(i)}\right)-\ln\left(1-U\right)\right\}^{\tau+1}}\right]}{\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\frac{1}{\left\{\xi_{irs}\left(x_{(i)}\right)\right\}^{\tau+1}}}=\frac{k}{2}.$$

Posterior predictive intervals for the rest of priors can be obtained by little modifications.

7. Simulation Study

Monte Carlo simulation techniques are widely used in statistical research. Since real-world data sets can often be radically non-normal, it is essential that statisticians have a variety of techniques available for univariate or multivariate non-normal data generation. This section shows how simulation can be helpful and illuminating way to approach problems in Bayesian analysis. Bayesian problems of updating estimates can be handled easily and straight forwardly with simulation. Here, the inverse transformation method of simulation is used to compare the performance of different estimators. The study has been carried out for different values of $(n, r$ and $s)$ using $\lambda \in (3.5, 7$ and $10)$. Censoring rate is assumed to be 20%. The estimation has been done under 10% left and 10% right censored samples. Sample size is varied to observe the effect of small and large samples on the estimators. Changes in the estimators and their risks have been determined when changing the loss function and the prior distribution of λ while keeping the sample size fixed. All these results are obtained from 5,000 Monte Carlo replications. In the Tables, the estimators for the parameter and the risk, is averaged over the total number of repetitions. Mathematica 8.0 has been used to carry out the results. All the results are summarized in the Tables 2-21 and Figures 1-18.

Table 2 Bayes estimates and the posterior risks (given in parentheses) under SELF

n	Uniform Prior			Jeffreys Prior		
	$\lambda=3.5$	$\lambda=7$	$\lambda=10$	$\lambda=3.5$	$\lambda=7$	$\lambda=10$
20	3.92533	7.77441	11.08181	3.71341	7.44810	10.5978
$r=3, n-s=18$	(0.86122)	(3.38166)	(6.86337)	(0.81246)	(3.28642)	(6.61851)
40	3.70702	7.29598	10.53040	3.61550	7.14997	10.2546
$r=5, n-s=36$	(0.38246)	(1.47990)	(3.08415)	(0.37401)	(1.45009)	(3.00765)
60	3.63335	7.25471	10.41370	3.55732	7.12365	10.21040
$r=7, n-s=54$	(0.24463)	(0.97518)	(2.00985)	(0.23882)	(0.95754)	(1.97036)
80	3.59448	7.19386	10.27320	3.53655	7.09076	10.08170
$r=9, n-s=72$	(0.17901)	(0.70797)	(1.47419)	(0.17709)	(0.70660)	(1.42546)

Table 3 Bayes estimates and the posterior risks (given in parentheses) under SELF

n	Exponential Prior			Gamma Prior		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.81379	7.35705	10.23330	3.82389	6.40303	8.12309
$r = 3, n - s = 18$	(0.80961)	(2.99378)	(5.79352)	(0.66588)	(1.83888)	(2.94527)
40	3.62664	7.16263	10.05310	3.65350	6.688747	8.86411
$r = 5, n - s = 36$	(0.36587)	(1.42462)	(2.80108)	(0.33374)	(1.11268)	(1.95458)
60	3.58728	7.139440	10.03860	3.60160	6.82085	9.12666
$r = 7, n - s = 54$	(0.23834)	(0.94397)	(1.86996)	(0.22372)	(0.80199)	(1.43107)
80	3.55628	7.07613	10.03343	3.59740	6.85248	9.37437
$r = 9, n - s = 72$	(0.17548)	(0.69823)	(1.40224)	(0.17059)	(0.61694)	(1.13663)

Table 4 Bayes estimates and the posterior risks (given in parentheses) under In-Levy

n	SELF			PLF		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.39827	6.19757	8.28772	3.52882	6.41695	8.48414
$r = 3, n - s = 18$	(0.66891)	(2.15804)	(3.84658)	(0.18298)	(0.33347)	(0.44089)
40	3.46435	6.69062	8.95265	3.52232	6.74176	9.19329
$r = 5, n - s = 36$	(0.33805)	(1.25501)	(2.24817)	(0.09458)	(0.18102)	(0.24685)
60	3.49571	6.71834	9.34759	3.50652	6.78337	9.46309
$r = 7, n - s = 54$	(0.22846)	(0.84183)	(1.62749)	(0.06348)	(0.12280)	(0.17130)
80	3.49620	6.80950	9.52627	3.50003	6.84533	9.57812
$r = 9, n - s = 72$	(0.17875)	(0.64503)	(1.26232)	(0.04758)	(0.09459)	(0.13104)

Table 5 Bayes estimates and the posterior risks (given in parentheses) under PLF

n	Uniform Prior			Jeffreys Prior		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.98717	8.02401	11.47080	3.82573	7.63340	11.16410
$r = 3, n - s = 18$	(0.20198)	(0.40647)	(0.58107)	(0.20413)	(0.40731)	(0.59571)
40	3.76031	7.47589	10.53900	3.66187	7.31773	10.40680
$r = 5, n - s = 36$	(0.09964)	(0.19808)	(0.27925)	(0.09967)	(0.19917)	(0.28325)
60	3.67819	7.34681	10.43240	3.59795	7.18242	10.25790
$r = 7, n - s = 54$	(0.06598)	(0.13181)	(0.18716)	(0.06600)	(0.13195)	(0.18739)
80	3.62149	7.24346	10.30200	3.58433	7.17615	10.20560
$r = 9, n - s = 72$	(0.05087)	(0.09829)	(0.13972)	(0.051149)	(0.09868)	(0.14024)

Table 6 Bayes estimates and the posterior risks (given in parentheses) under PLF

n	Exponential Prior			Gamma Prior		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.86466	7.54919	10.65220	3.91809	6.59282	8.28236
$r = 3, n - s = 18$	(0.19577)	(0.38241)	(0.53958)	(0.16574)	(0.27887)	(0.35032)
40	3.72165	7.27841	10.29801	3.71449	6.71447	8.95066
$r = 5, n - s = 36$	(0.09861)	(0.19285)	(0.27286)	(0.08921)	(0.16125)	(0.21495)
60	3.60543	7.18868	10.16510	3.66105	6.85698	9.28930
$r = 7, n - s = 54$	(0.06468)	(0.12897)	(0.18236)	(0.06139)	(0.114972)	(0.15576)
80	3.60403	7.14848	10.16210	3.61198	6.90519	9.47683
$r = 9, n - s = 72$	(0.04878)	(0.09704)	(0.13758)	(0.04652)	(0.09022)	(0.119412)

Table 7 Bayes estimates and the posterior risks (given in parentheses) under WSELF

n	Uniform Prior			Jeffreys Prior		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.72527	7.45755	10.61520	3.46476	7.04198	10.05660
$r = 3, n - s = 18$	(0.20070)	(0.41444)	(0.589911)	(0.20387)	(0.41435)	(0.59174)
40	3.57036	7.18436	10.35280	3.49387	7.05023	10.13230
$r = 5, n - s = 36$	(0.09920)	(0.19960)	(0.28763)	(0.09984)	(0.20147)	(0.28955)
60	3.54435	7.12553	10.17870	3.52771	7.00775	10.16510
$r = 7, n - s = 54$	(0.06565)	(0.13198)	(0.18853)	(0.06657)	(0.13224)	(0.18994)
80	3.52103	7.07107	10.09340	3.51561	6.97971	10.2771
$r = 9, n - s = 72$	(0.04909)	(0.09740)	(0.14014)	(0.04993)	(0.09844)	(0.13938)

Table 8 Bayes estimates and the posterior risks (given in parentheses) under WSELF

n	Exponential Prior			Gamma Prior		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.58268	7.01863	9.77231	3.64830	6.11131	7.75482
$r = 3, n - s = 18$	(0.19910)	(0.39003)	(0.54306)	(0.16664)	(0.27913)	(0.35418)
40	3.54268	6.99132	9.78016	3.58615	6.52838	8.59472
$r = 5, n - s = 36$	(0.09843)	(0.19368)	(0.27172)	(0.08990)	(0.16365)	(0.21549)
60	3.50695	6.98862	9.87215	3.53039	6.67008	9.05560
$r = 7, n - s = 54$	(0.06495)	(0.12962)	(0.18285)	(0.06098)	(0.11522)	(0.15642)
80	3.50438	6.98104	9.88729	3.53003	6.75080	9.23674
$r = 9, n - s = 72$	(0.04873)	(0.09718)	(0.13935)	(0.04655)	(0.08654)	(0.12220)

Table 9 Bayes estimates and the posterior risks (given in parentheses) under In-Levy

n	WSELF			QQLF		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.25806	5.86659	7.93722	3.11082	5.38011	6.80766
$r = 3, n - s = 18$	(0.18622)	(0.33530)	(0.45363)	(0.00141)	(0.00010)	(0.00002)
40	3.37253	6.45195	8.84808	3.32100	6.07971	8.11384
$r = 5, n - s = 36$	(0.09502)	(0.18177)	(0.24928)	(0.00050)	(0.00002)	(9.47×10^{-7})
60	3.41603	6.67175	9.16089	3.38948	6.38621	8.64783
$r = 7, n - s = 54$	(0.06386)	(0.12472)	(0.17125)	(0.00029)	(5.47×10^{-6})	(2.12×10^{-7})
80	3.43855	6.70822	9.41587	3.45129	6.50295	9.26928
$r = 9, n - s = 72$	(0.04815)	(0.09326)	(0.13195)	(0.00020)	(2.82×10^{-6})	(8.38×10^{-8})

Table 10 Bayes estimates and the posterior risks (given in parentheses) under QQLF

n	Uniform Prior			Jeffreys Prior		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.53766	6.48044	8.67685	3.34816	6.14469	8.28434
$r = 3, n - s = 18$	(0.00091)	(0.00003)	(2.99×10^{-6})	(0.00119)	(0.00005)	(4.90×10^{-6})
40	3.52699	6.67729	9.14271	3.43872	6.51616	8.99632
$r = 5, n - s = 36$	(0.00040)	(7.90×10^{-6})	(3.08×10^{-7})	(0.00046)	(9.27×10^{-6})	(4.54×10^{-6})
60	3.52069	6.83900	9.47975	3.46397	6.69697	9.29961
$r = 7, n - s = 54$	(0.00025)	(2.82×10^{-6})	(7.18×10^{-8})	(0.00027)	(3.58×10^{-6})	(1.04×10^{-6})
80	3.49447	6.84470	9.61220	3.47716	6.80313	9.51691
$r = 9, n - s = 72$	(0.00018)	(1.77×10^{-6})	(3.04×10^{-8})	(0.00019)	(1.92×10^{-6})	(3.44×10^{-6})

Table 11 Bayes estimates and the posterior risks (given in parentheses) under QQLF

n	Exponential Prior			Gamma Prior		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.43177	6.14714	8.14054	3.52196	5.64855	6.88925
$r = 3, n - s = 18$	(0.00103)	(0.00005)	(4.31×10^{-6})	(0.00068)	(0.00005)	(8.30×10^{-6})
40	3.47050	6.54578	8.98536	3.52907	6.15483	7.97230
$r = 5, n - s = 36$	(0.00041)	(8.85×10^{-6})	(3.71×10^{-7})	(0.00035)	(0.00001)	(7.81×10^{-7})
60	3.48098	6.64879	9.26670	3.50222	6.38192	8.54705
$r = 7, n - s = 54$	(0.00026)	(3.69×10^{-6})	(1.05×10^{-7})	(0.00023)	(4.28×10^{-6})	(1.84×10^{-7})
80	3.49523	6.69979	9.45261	3.50186	6.54692	8.80563
$r = 9, n - s = 72$	(0.00018)	(2.05×10^{-6})	(4.11×10^{-8})	(0.00017)	(2.40×10^{-6})	(8.00×10^{-8})

Table 12 Bayes estimates and the posterior risks (given in parentheses) under SLELF

n	Uniform Prior			Jeffreys Prior		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.82828	7.69723	10.83940	3.58403	7.25302	10.66760
$r = 3, n - s = 18$	(0.06059)	(0.06059)	(0.06059)	(0.06449)	(0.06449)	(0.06449)
40	3.66733	7.32027	10.44560	3.54135	7.13360	10.17720
$r = 5, n - s = 36$	(0.03077)	(0.03077)	(0.03077)	(0.03174)	(0.03174)	(0.03174)
60	3.59536	7.13222	10.37320	3.52742	7.01056	10.07440
$r = 7, n - s = 54$	(0.02062)	(0.02062)	(0.02062)	(0.02105)	(0.02105)	(0.02105)
80	3.55622	7.12479	10.17620	3.52631	7.00789	10.05920
$r = 9, n - s = 72$	(0.01550)	(0.01550)	(0.01550)	(0.01575)	(0.01575)	(0.01575)

Table 13 Bayes estimates and the posterior risks (given in parentheses) under SLELF

n	Exponential Prior			Gamma Prior		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.68864	7.28203	9.87553	3.71928	6.24026	7.84896
$r = 3, n - s = 18$	(0.06059)	(0.06059)	(0.06059)	(0.04901)	(0.04901)	(0.04901)
40	3.61541	7.10907	9.95057	3.65825	6.57596	8.74732
$r = 5, n - s = 36$	(0.03077)	(0.03077)	(0.03077)	(0.02747)	(0.02747)	(0.02747)
60	3.52799	7.07746	9.97436	3.58032	6.69454	9.13093
$r = 7, n - s = 54$	(0.02062)	(0.02062)	(0.02062)	(0.01908)	(0.01908)	(0.01908)
80	3.51374	7.03647	10.00910	3.57626	6.81407	9.30911
$r = 9, n - s = 72$	(0.01550)	(0.01550)	(0.01550)	(0.01462)	(0.01462)	(0.01462)

Table 14 Bayes estimates and the posterior risks (given in parentheses) under In-Levy

n	SLELF			ELF		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.40260	6.09065	8.09419	3.24454	5.89633	7.85509
$r = 3, n - s = 18$	(0.06248)	(0.06248)	(0.06248)	(0.02497)	(0.02497)	(0.02497)
40	3.41286	6.51539	8.91392	3.39342	6.44534	8.86326
$r = 5, n - s = 36$	(0.03125)	(0.03125)	(0.03125)	(0.01232)	(0.01232)	(0.01232)
60	3.46201	6.65328	9.22523	3.42523	6.63678	9.27257
$r = 7, n - s = 54$	(0.02083)	(0.02083)	(0.02083)	(0.00818)	(0.00818)	(0.00818)
80	3.46537	6.73060	9.44915	3.45478	6.70293	9.40219
$r = 9, n - s = 72$	(0.01562)	(0.01562)	(0.01562)	(0.00608)	(0.00608)	(0.00608)

Table 15 Bayes estimates and the posterior risks (given in parentheses) under ELF

n	Uniform Prior			Jeffreys Prior		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.68275	7.40959	10.58120	3.52264	6.90079	9.97814
$r = 3, n-s = 18$	(0.02438)	(0.02438)	(0.02438)	(0.02560)	(0.02560)	(0.02560)
40	3.59664	7.18686	10.29190	3.51985	6.94760	9.98612
$r = 5, n-s = 36$	(0.01218)	(0.01218)	(0.01218)	(0.01247)	(0.01247)	(0.01247)
60	3.57060	7.11202	10.18220	3.49462	6.99606	9.99775
$r = 7, n-s = 54$	(0.00811)	(0.00811)	(0.00811)	(0.00824)	(0.00824)	(0.00824)
80	3.55195	7.10718	10.16270	3.49167	7.04029	10.06120
$r = 9, n-s = 72$	(0.00618)	(0.00618)	(0.00618)	(0.00625)	(0.00625)	(0.00625)

Table 16 Bayes estimates and the posterior risks (given in parentheses) under ELF

n	Exponential Prior			Gamma Prior		
	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 3.5$	$\lambda = 7$	$\lambda = 10$
20	3.59567	6.98372	9.65015	3.24454	5.89633	7.85509
$r = 3, n-s = 18$	(0.02438)	(0.02438)	(0.02438)	(0.02055)	(0.02055)	(0.02055)
40	3.55142	6.99283	9.91212	3.39342	6.44534	8.86326
$r = 5, n-s = 36$	(0.01218)	(0.01218)	(0.01218)	(0.01111)	(0.01111)	(0.01111)
60	3.54733	7.03105	9.97753	3.42523	6.63368	9.27257
$r = 7, n-s = 54$	(0.00811)	(0.00811)	(0.00811)	(0.00765)	(0.00765)	(0.00765)
80	3.51064	6.99718	9.97861	3.45478	6.70293	9.40219
$r = 9, n-s = 72$	(0.00618)	(0.00618)	(0.00618)	(0.00501)	(0.00501)	(0.00501)

Table 17 The lower (LL), the upper (UL) and the width of the 95% credible intervals under uniform prior

$r, n, n-s$	$\lambda = 3.5$		Width	$\lambda = 7$		Width	$\lambda = 10$		Width
	LL	UL		LL	UL		LL	UL	
3, 20, 18	2.4662	6.4707	4.0045	4.3813	11.4952	7.1139	6.0188	15.7915	9.7727
5, 40, 36	2.3591	4.6915	2.3324	4.4702	8.8899	4.4197	7.3400	14.5970	7.2570
7, 60, 54	2.5709	4.5134	1.9425	5.2772	9.2646	3.9874	7.6086	13.3576	5.7490
9, 80, 72	2.6399	4.3008	1.6609	5.8521	9.5339	3.6818	7.6884	12.5255	4.8371

Table 18 The lower (LL), the upper (UL) and the width of the 95% credible intervals under jeffreys prior

$r, n, n-s$	$\lambda = 3.5$		Width	$\lambda = 7$		Width	$\lambda = 10$		Width
	LL	UL		LL	UL		LL	UL	
3, 20, 18	2.2925	6.2017	3.9092	4.0725	11.0169	6.9444	5.5947	15.1348	9.5401
5, 40, 36	2.2808	4.5851	2.3043	4.3219	8.6884	4.3665	7.0964	14.2661	7.1697
7, 60, 54	2.5157	4.4426	1.9269	5.1638	9.1191	3.9553	7.4452	13.1479	5.7027
9, 80, 72	2.5982	4.2490	1.6508	5.7596	9.4192	3.6596	7.5668	12.3746	4.8078

Table 19 The lower (LL), the upper (UL) and the width of the 95% credible intervals under exponential prior

$r, n, n-s$	$\lambda = 3.5$		Width	$\lambda = 7$		Width	$\lambda = 10$		Width
	LL	UL		LL	UL		LL	UL	
3, 20, 18	2.3884	6.2664	3.8780	4.1412	10.8652	6.7241	5.5749	14.6270	9.0521
5, 40, 36	2.3371	4.6477	2.3107	4.3583	8.6672	4.3090	7.0429	14.0062	6.9633
7, 60, 54	2.5473	4.4721	1.9247	5.1789	9.0920	3.9131	7.7124	13.0018	5.2894
9, 80, 72	2.6219	4.2715	1.6496	5.7645	9.3912	3.6267	7.5378	12.2801	4.7423

Table 20 The lower (LL), the upper (UL) and the width of the 95% credible intervals under gamma prior

$r,n,n-s$	$\lambda=3.5$		Width	$\lambda=7$		Width	$\lambda=10$		Width
	LL	UL		LL	UL		LL	UL	
3, 20, 18	2.5338	6.0337	3.4999	3.9046	9.2978	5.3933	4.8182	11.4734	6.6552
5, 40, 36	2.4196	4.6331	2.2135	4.2348	8.1089	3.8742	6.2994	12.0624	5.7630
7, 60, 54	2.6058	4.4782	1.8724	5.0067	8.6041	3.5975	6.8416	11.7575	4.9159
9, 80, 72	2.6657	4.2819	1.6162	5.5772	8.9587	3.3815	7.0986	11.4025	4.3039

Table 21 The lower (LL), the upper (UL) and the width of the 95% credible intervals under inverse levy prior

$r,n,n-s$	$\lambda=3.5$		Width	$\lambda=7$		Width	$\lambda=10$		Width
	LL	UL		LL	UL		LL	UL	
3, 20, 18	2.1401	5.6996	3.5595	3.5264	9.3914	5.8650	4.5623	12.1502	7.5880
5, 40, 36	2.2170	4.4325	2.2155	4.0405	8.0784	4.0379	6.3069	12.6097	6.3028
7, 60, 54	2.4652	4.3405	1.8753	4.9018	8.6306	3.7289	6.8818	12.1169	5.2352
9, 80, 72	2.5589	4.1768	1.6178	5.5185	9.0074	3.4890	7.1390	11.6526	4.5136

7.1. Graphical comparison

The risks profile at different values of parameter under different priors is given in this section.

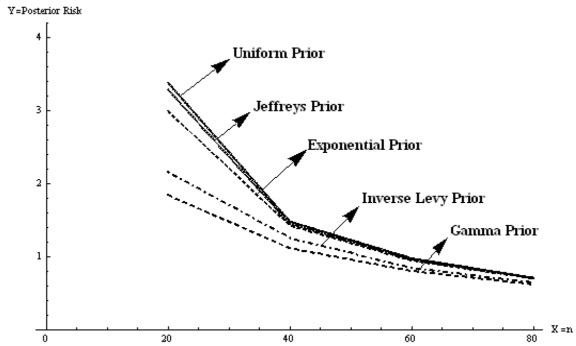

Figure 1 Risks of estimators of $\lambda = 3.5$ for different sample sizes under SELF

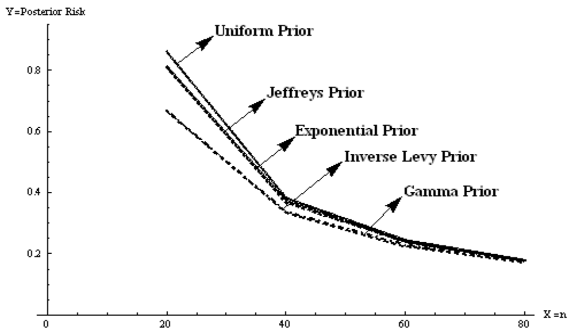

Figure 2 Risks of estimators of $\lambda = 7$ for different sample sizes under SELF

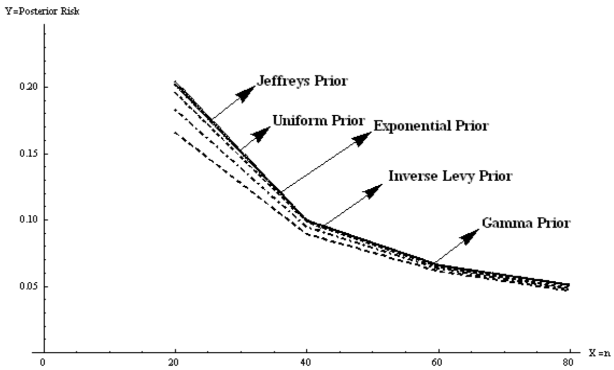

Figure 3 Risks of estimators of $\lambda = 10$ for different sample sizes under SELF

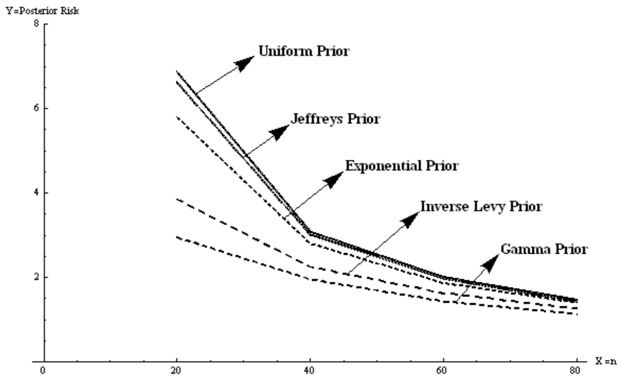

Figure 4 Risks of estimators of $\lambda = 3.5$ for different sample sizes under PLF

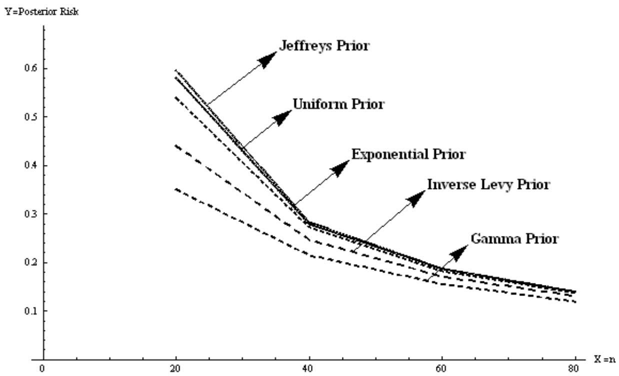

Figure 5 Risks of estimators of $\lambda = 7$ for different sample sizes under PLF

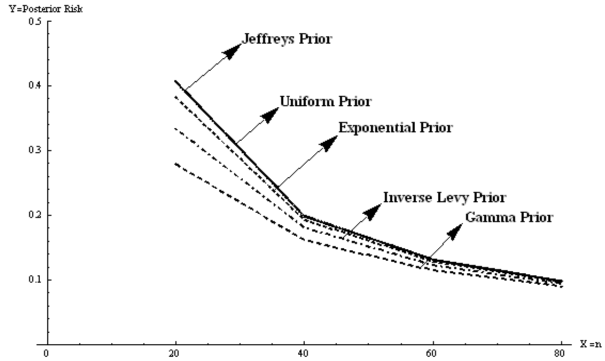

Figure 6 Risks of estimators of $\lambda = 10$ for different sample sizes under PLF

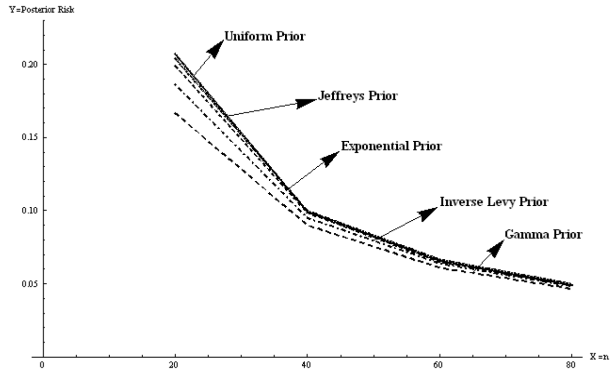

Figure 7 Risks of estimators of $\lambda = 3.5$ for different sample sizes under WSELF

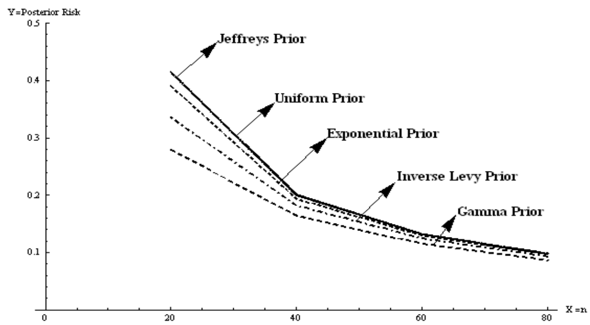

Figure 8 Risks of estimators of $\lambda = 7$ for different sample sizes under WSELF

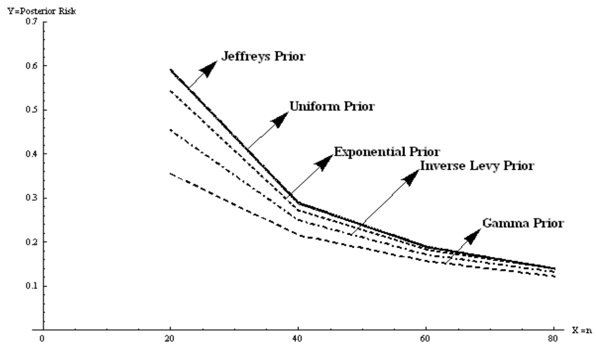

Figure 9 Risks of estimators of $\lambda = 10$ for different sample sizes under WSELF

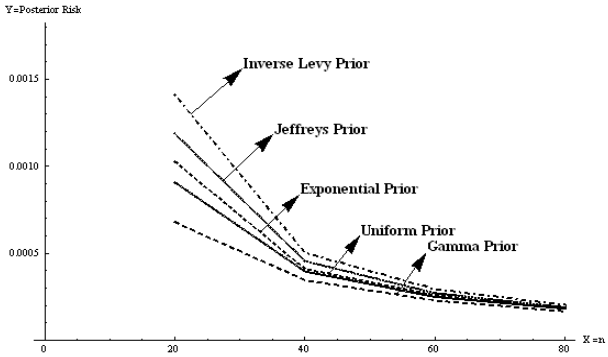

Figure 10 Risks of estimators of $\lambda = 3.5$ for different sample sizes under QQLF

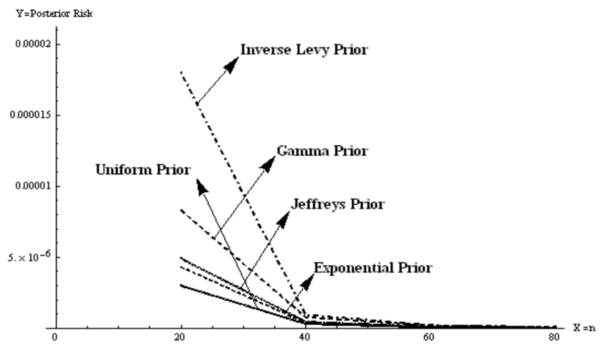

Figure 11 Risks of estimators of $\lambda = 7$ for different sample sizes under QQLF

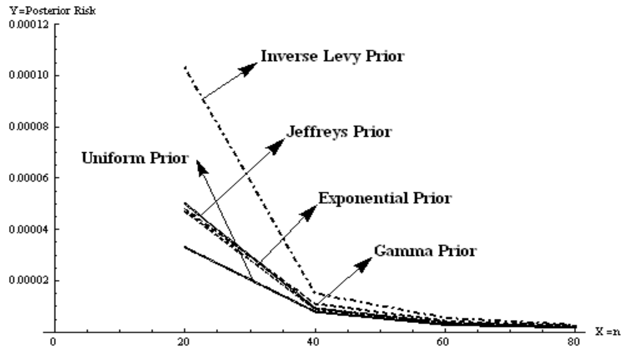

Figure 12 Risks of estimators of $\lambda = 10$ for different sample sizes under QQLF

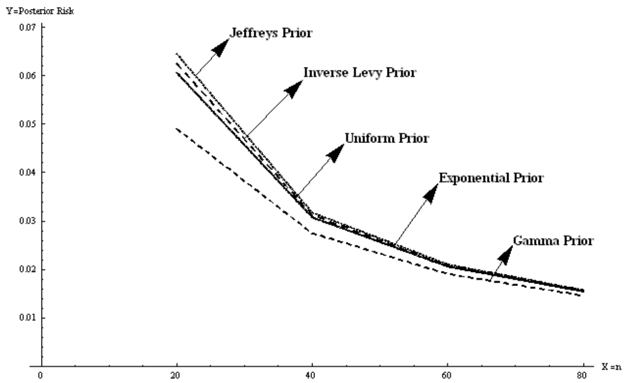

Figure 13 Risks of estimators of $\lambda = 3.5$ for different sample sizes under SLELF

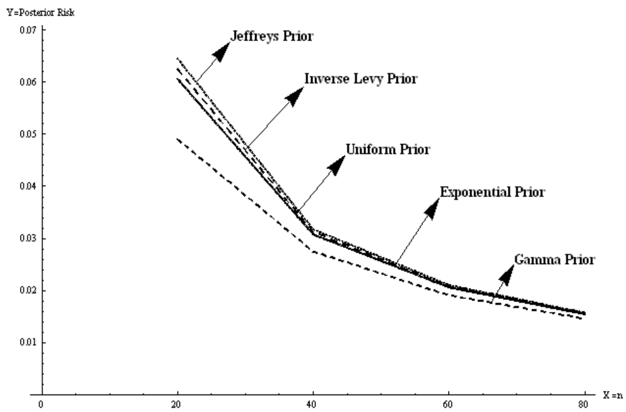

Figure 14 Risks of estimators of $\lambda = 7$ for different sample sizes under SLELF

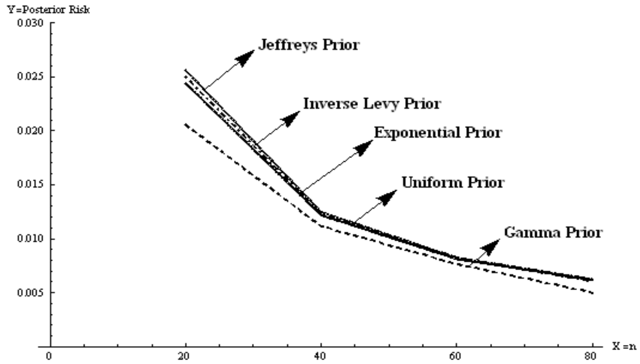

Figure 15 Risks of estimators of $\lambda = 10$ for different sample sizes under SLELF

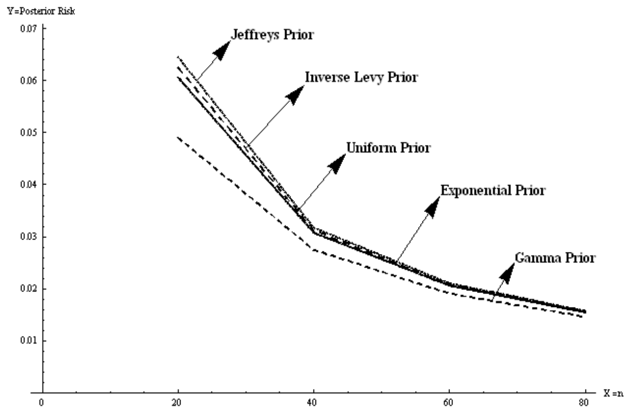

Figure 16 Risks of estimators of $\lambda = 3.5$ for different sample sizes under ELF

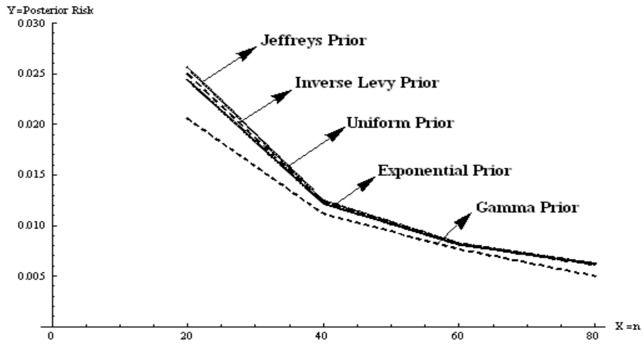

Figure 17 Risks of estimators of $\lambda = 7$ for different sample sizes under ELF

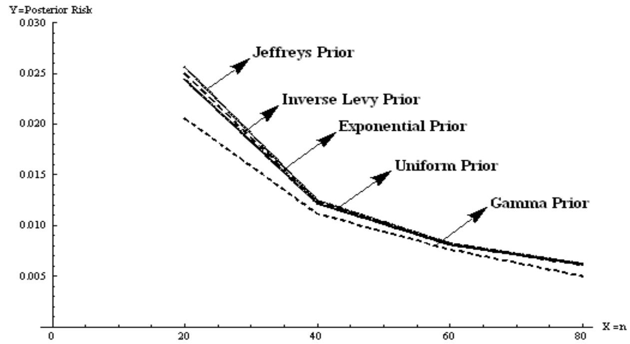

Figure 18 Risks of estimators of $\lambda = 10$ for different sample sizes under ELF

8. Conclusions

The simulation study has displayed some interesting properties of the Bayes estimates. After an extensive study of results, conclusions are drawn regarding the behavior of the estimators. The risks of the estimates seem to be large in case when the value of the parameter is large and small for relative smaller value of the parameter except under quasi-quadratic loss function. However, the risks under said loss functions are reduced as the sample size increases. Another interesting remark concerning the risks of the estimates is that increasing (decreasing) the value of the parameter reduces (increases) the risks of the estimates under quasi-quadratic loss function. The performance of squared-log error loss function and entropy loss function is independent of choice of parametric value. The above study depicts that the estimated value of the parameter converges to the true value of the parameter by increasing the sample size. The greater values of the parameter impose a negative impact on convergence and performance of the estimates. The effect of the increasing values of the parameter is in the form of underestimation assuming each informative prior. The patterns of the estimates discussed above, are almost similar under uniform and Jeffreys priors. However, the performance of the uniform prior is better for estimates under SLELF, ELF, PLF and QQLF. While for estimates, under SELF and WSELF, the performance of the Jeffreys prior is better than uniform prior. In comparison of informative priors, the gamma prior provides the better estimates as the corresponding risks are least under said loss functions with few exceptions. While the exponential prior turns out to perform better under QQLF for larger values of the parameter, therefore it produces more efficient estimates as compared to other informative priors. After an extensive study of the results, thus obtained, we observed that the risks of the estimators under doubly type II censored data assuming uniform behave similarly to the risks of the estimators under exponential prior under SLELF and ELF. In addition, estimates under quasi-quadratic loss function give the minimum risks among all loss functions for each prior. The Credible interval are in accordance with the point estimates, that is, the width of credible interval is inversely proportional to sample size while, it is directly proportional to the parametric value. From Tables 17-21, appended above, reveal that the effect of the parametric values in the form of larger width of interval. The Credible interval assuming gamma prior is much narrower than the credible intervals assuming informative and non-informative priors. It is the use of prior information that makes a difference in terms of gain in precision. To see the effects of the posterior risks assuming different priors under different loss functions with various values of the parameter λ, Figures 1-18 are prepared. It is observed from all the figures that posterior risk decreases with the increase in sample size under said loss functions. It is evident from Figures 13-18 that behaviors of posterior risks are similar in all aspects under $\lambda = 3.5,\ 7$ and 10. Results of above graphs clearly show that gamma prior has least posterior risk as compared to its competitors prior under all loss function with few exceptions.

So result of graphical study and simulation study are similar in all aspect. In the future, this study can be conducted under different informative priors, other loss functions and under different censoring schemes and also for mixture distribution.

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