Thailand Statistician
October 2022; 20(4): 744-753
http://statassoc.or.th
Contributed paper

Sequential Analysis and Robustness Study for the Parameters of Positive Exponential Family of Distributions with Known Coefficient of Variation

Surinder Kumar, Prem Lata Gautam* and Vaidehi Singh
Department of Statistics, School for Physical and Decision Sciences
Babasaheb Bhimrao Ambedkar University, Lucknow, India
*Corresponding author; e-mail: premgautam61@gmail.com

Received: 30 August 2020
Revised: 21 November 2020
Accepted: 18 January 2021

Abstract

In the present paper, we consider the positive exponential family of distributions, which has three parameters θ, ν and ρ. On assigning different values to these parameters, it covers many distributions. For the testing of hypotheses regarding the parameters θ and ν, a sequential testing procedures and robustness of the Operating characteristics and Average sample number functions are derived. The robustness of the Sequential probability ratio test for the parameter θ, when the coefficient of variation is known are also obtained. The acceptance and rejection regions for the null hypotheses H_0 against the alternative hypotheses H_1 are also studied and presented through a graph. The numerical values of operating characteristics and Average sample number function are presented in the tables and graphs.

Keywords: PEFD, sequential probability ratio test, operating characteristics, average sample number, acceptance and rejection regions, null hypotheses, alternative hypotheses, type I error and type II error.

1. Introduction

The sequential probability ratio test (SPRT) for two simple hypotheses is first developed by Wald (1947). He derived the theoretical expressions for the operating characteristics (OC) and average sample number (ASN) functions for the performance of SPRTS. The SPRT has been applied by various authors for testing the statistical hypotheses, for references, one may refer to Oakland (1950), Epstein and Sobel (1955), Johnson (1966), Phatarford (1971), Bain and Engelhardt (1982), Chaturvedi et al.(2000), Sevil and Demirhan (2008), etc.

The robustness of the SPRT in respect of OC and ASN functions has been studied by several researchers. For references, Harter and Moore (1976) gives the robustness of the exponential SPRT, when the underlying distribution is a Weibull distribution. Montagne and Singpurwalla (1985) investigated the robustness of the sequential life-testing procedure with respect to the risks and the expected sample sizes for the exponential distribution when the life length is not exponential. Hubbard and Allen (1991) applied SPRT on the mean of the negative binomial distribution when the dispersion parameter is known and the robustness of the test to the misspecification of dispersion parameter is studied. Chaturvedi et al. (1998) considered a family of life-testing models and studied the robustness of the SPRTS for various parameters involved in the model and also generalised the results of Montagne and Singpurwalla (1985).

For testing a simple hypothesis (against a simple alternative) for the mean of an inverse Gaussian distribution, Joshi and Shah (1990) developed SPRT, assuming the coefficient of variation (CV) to be known. They obtained theoretical expressions for the OC and the ASN functions. Kumar et al. (2018) derived the robustness of the OC and the ASN functions for the rate and the scale parameters of the Erlang distribution. They also derived the robustness with respect to OC and ASN functions with known coefficient of variation for the scale parameter.

2. Positive Exponential Family of Distribution

Liang (2008) proposed a positive exponential family of distribution (PEFD). Let us consider a random variable (r.v.) X follows the PEFD presented by the probability density function (pdf)

$$f(x;\theta,\nu,\rho) = \frac{\rho x^{\rho\nu-1} e^{\left(\frac{-x^{\rho}}{\theta}\right)}}{\Gamma(\nu)\theta^{\nu}}; \quad x > 0, \theta, \nu, \rho > 0 \tag{1}$$

where, θ is assumed to be unknown and ρ, ν are known constants. When $\rho = \nu = 1$; we get one-parameter exponential distribution, for $\rho = 1$; we get gamma distribution, for ν=1; we get Weibull distribution, for $\nu > 0$, $\rho = 1$; we get Erlang distribution, for $\nu > 1/2, \rho = 2$; we get half - normal distribution, for $\nu > m/2$, $\rho = 2$; we get chi-distribution, for $\nu = 1$, $\rho = 2$; we get Rayleigh distribution and for $\nu = p + 1$, $\rho = 2$; we get Generalized Rayleigh distribution.

In the model (1), for testing the simple null hypothesis against the simple alternative, the SPRTS and the robustness of the SPRTS in respect of OC and ASN functions are developed in the Section 3, 4, 5 and 6. In Section 7, the robustness of the SPRT for a misspecified coefficient of variation is also studied. In Section 8, the acceptance and rejection regions for H_0 vs H_1 for θ are derived and plotted in Figure 8.1. Finally, in Sections 9 and 10, the results and findings are presented through tables and graphs, respectively.

3. SPRT for Testing the Hypothesis Regarding 'θ'

For a given sequence of observations X_1, X_2, X_3, ... from (1), the problem of testing the simple null hypothesis $H_0 : \theta = \theta_0$ against the simple alternative hypothesis $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$ is considered. The SPRT for testing is defined as follows

$$Z_i = \ln\left(\frac{\theta_0}{\theta_1}\right)^{\nu} - \left[x_i^{\rho}\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}.\right)\right] \tag{2}$$

Wald (1947), suggested two constants A and B depending on α and β such that $0 < B < 1 < A$, where α and β are Type I and Type II errors, respectively. The approximate values of A and B are given by

$$A \approx \frac{1-\beta}{\alpha} \text{ and } B \approx \frac{\beta}{1-\alpha}.$$

The OC function $L(\theta)$ is given by

$$L(\theta) = \frac{A^h - 1}{A^h - B^h}$$

where 'h' is the non-zero solution of

$$E[e^{Z_i}]^h = 1.$$

From (1) and (2), we have

$$E\left[e^{Z_i}\right]^h = \frac{\left(\frac{\theta_0}{\theta_1}\right)^{h\nu}}{\left[1 + h\theta^{\nu}\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)\right]}. \tag{3}$$

On taking logarithm and using the expression $\ln(1+x),\ -1 < x < 1$ in (3). After retaining the terms up to third degree in 'h' and on simplifying, we obtain the following quadratic equation in 'h'-

$$\frac{h^2}{3}\theta^3\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)^3 - \frac{h}{2}\theta^2\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)^2 + \theta\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right) - \ln\left(\frac{\theta_0}{\theta_1}\right) = 0. \tag{4}$$

On solving (4), we get the numerical values of OC function. The ASN function is approximately given by

$$E(N|\theta) = \frac{L(\theta)\ln B + [1 - L(\theta)]\ln A}{E(Z)} \tag{5}$$

provided that $E(Z) \neq 0$, where

$$E(Z) = \nu\left[\ln\left(\frac{\theta_0}{\theta_1}\right) - \theta\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)\right].$$

From Equation(5), the ASN function under H_0 and H_1 is given by

$$E_0(N) = \frac{(1-\alpha)\ln B + \alpha\ln A}{\nu\left[\ln\left(\frac{\theta_0}{\theta_1}\right) - \theta\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)\right]}$$

and

$$E_1(N) = \frac{\beta\ln B + (1-\beta)\ln A}{\nu\left[\ln\left(\frac{\theta_0}{\theta_1}\right) - \theta\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)\right]}.$$

4. SPRT for Testing the Hypothesis Regarding 'ν'

Now, we derive the OC and ASN function for the parameter 'ν' under the simple null hypothesis $H_0 : \nu = \nu_0$ against the simple alternative hypothesis $H_1 : \nu = \nu_1 (\nu_1 > \nu_0)$. The value of Z_i is

$$Z_i = \frac{\Gamma(\nu_0)}{\Gamma(\nu_1)}\theta^{\nu_0 - \nu_1} x_i^{\rho(\nu_1 - \nu_0)}. \tag{6}$$

Using (1) and (6) for the OC function, we have

$$\left(\frac{\Gamma(\nu_0)}{\Gamma(\nu_1)}\right)^h \frac{1}{\Gamma(\nu)}\Gamma[h(\nu_1 - \nu_0) + \nu] = 1. \tag{7}$$

Again, on taking the logarithm of both sides of (7), with $\ln(1+x);\ -1 < x < 1$ and using the approximation

$$\ln\Gamma(x) = \ln\sqrt{2\pi} - x + \left(x - \frac{1}{2}\right)\ln x \tag{8}$$

we get,

$$\frac{h^2}{6}\left(\frac{\nu_1-\nu_0}{\nu}\right)^3(\nu+1)-\frac{h}{4}\left(\frac{\nu_1-\nu_0}{\nu}\right)^2(2\nu+1)-\left(\nu_0-\frac{1}{2}\right)\ln\nu_0+\left(\nu_1-\frac{1}{2}\right)\ln\nu_1$$
$$-\left(1+\ln\nu-\frac{1}{2\nu}\right)(\nu_1-\nu_0)=0 \tag{9}$$

which is a quadratic equation in 'h'. The numerical values of OC function is now obtained from Equation (9). We get the values of ASN function by replacing the denominator of (5) by

$$E(Z_i|\nu)=\ln\Gamma(\nu_0)-\ln\Gamma(\nu_1)+(\nu_1-\nu_0)\ln\lambda+(\nu_1-\nu_0)E(\ln x_i).$$

Using the result of Gradshteyn and Ryzhik(1965, p.576, 4.352(1)) that

$$\psi(x)=\ln x-\frac{1}{2x} \tag{10}$$

$$E(Z_i|\nu)=\left(\nu_0-\frac{1}{2}\right)\ln\nu_0-\left(\nu_1-\frac{1}{2}\right)\ln\nu_1+\left(1+\ln\nu-\frac{1}{2\nu}\right)(\nu_1-\nu_0).$$

5. Robustness of the SPRT for 'θ' when 'ν' has Undergone a Change.

Let us consider that the parameter 'ν' has undergone a change to ν^* and then probability distribution (1) becomes $f(x_i;\theta,\nu^*,\rho)$. For the robustness of SPRT developed in Section 3 with respect to OC function, the values of 'h' are obtained by Equations (1) and (2) as

$$\left(\frac{\theta_0}{\theta_1}\right)^{h\nu}\left[1+h\theta\left(\frac{1}{\theta_1}-\frac{1}{\theta_0}\right)\right]^{-\nu^*}=1. \tag{11}$$

Taking logarithm on both sides of Equation (11) and using the expansion of $\ln(1+x)$, $-1<x<1$, we have

$$\frac{Qh^2}{3}\theta^3\left(\frac{1}{\theta_1}-\frac{1}{\theta_0}\right)^3-\frac{Qh}{2}\theta^2\left(\frac{1}{\theta_1}-\frac{1}{\theta_0}\right)^2+Q\theta\left(\frac{1}{\theta_1}-\frac{1}{\theta_0}\right)-\ln\left(\frac{\theta_0}{\theta_1}\right)=0 \tag{12}$$

which is a quadratic equation in 'h'. On solving (12), we get the real roots of 'h'. The robustness of the SPRT with respect to ASN is studied by replacing the denominator of (5) by

$$E(Z|\theta)=\ln\left(\frac{\theta_0}{\theta_1}\right)-Q\theta\left(\frac{1}{\theta_1}-\frac{1}{\theta_0}\right)$$

where $Q=\frac{\nu^*}{\nu}$.

6. Robustness of the SPRT for 'ν' when 'θ' has Undergone a Change.

Let us suppose that the parameter 'θ' has undergone a change to θ^* and then probability distribution (1) becomes $f(x_i;\theta^*,\nu,\rho)$. For OC function, value of 'h' are obtained by the equation

$$\left(\frac{\Gamma(\nu_0)}{\Gamma(\nu_1)}\right)^h\frac{1}{\Gamma(\nu)}\theta^{h(\nu_1-\nu_0)}\frac{\rho}{(\theta^*)^\nu}\int_0^\infty x^{h(\nu_1-\nu_0)+\nu-1}e^{\left(-\frac{x^\rho}{\theta^*}\right)}dx=1$$

$$\left(\frac{\Gamma(\nu_0)}{\Gamma(\nu_1)}\right)^h\frac{1}{\Gamma(\nu)}\phi^{h(\nu_1-\nu_0)}\Gamma[h(\nu_1-\nu_0)+\nu]=1 \tag{13}$$

where $\phi = \frac{\theta^*}{\theta}$.

Taking logarithm on both sides of Equation (13) and using the approximation (8), we get the roots of 'h' from the following equation

$$\frac{h^2}{6}\left(\frac{\nu_1-\nu_0}{\nu}\right)^3(\nu+1)-\frac{h}{4}\left(\frac{\nu_1-\nu_0}{\nu}\right)^2(2\nu+1)-\left(\nu_0-\frac{1}{2}\right)\ln\nu_0+\left(\nu_1-\frac{1}{2}\right)\ln\nu_1$$
$$-(\nu_1-\nu_0)\ln\phi-\left(1+\ln\nu-\frac{1}{2\nu}\right)(\nu_1-\nu_0)=0.$$

The robustness of the SPRT with respect to ASN function is studied by replacing the denominator of (5) by

$$E(Z_i|\nu)=\ln\Gamma(\nu_0)-\ln\Gamma(\nu_1)+(\nu_1-\nu_0)\ln\lambda+(\nu_1-\nu_0)E(\ln x_i).$$

Using the result (10), we get

$$E(Z_i|\nu)=\left(\nu_0-\frac{1}{2}\right)\ln\nu_0+\left(\nu_1-\frac{1}{2}\right)\ln\nu_1+\left(1+\ln\phi+\ln\nu-\frac{1}{2\nu}\right)(\nu_1-\nu_0).$$

7. Robustness of the SPRT for 'θ' with Known Coefficient of Variation(CV)

For PEFD, the mean and variance are $(\theta\nu)$ and $(\theta^2\nu)$ for $\rho=1$, then coefficient of variation $(CV)=\frac{1}{\sqrt{\nu}}$. Let us suppose that coefficient of variation changes from c to c^*, so that, the pdf (1) shifts to $f(x_i;\theta,c^*,\rho)$. Then, from (2) and (1), the OC and ASN functions are

$$\left(\frac{\theta_0}{\theta_1}\right)^{\frac{h}{c^2}}\left[1+h\theta\left(\frac{1}{\theta_1}-\frac{1}{\theta_0}\right)\right]^{\left(\frac{-1}{c^*}\right)^2}=1. \quad (14)$$

Taking logarithm on both sides of Equation (14) and using the expansion of $\ln(1+x)$, $-1<x<1$ and retaining the terms upto third degree in 'h', we get the following quadratic equation

$$\frac{\psi h^2}{3}\theta^3\left(\frac{1}{\theta_1}-\frac{1}{\theta_0}\right)^3-\frac{\psi h}{2}\theta^2\left(\frac{1}{\theta_1}-\frac{1}{\theta_0}\right)^2+\psi\theta\left(\frac{1}{\theta_1}-\frac{1}{\theta_0}\right)-\ln\left(\frac{\theta_0}{\theta_1}\right)=0. \quad (15)$$

On solving (15), we get the real roots of 'h'. The robustness of the SPRT with respect to ASN function is studied by replacing the denominator of (10) by

$$E(Z|\lambda)=\ln\left(\frac{\theta_0}{\theta_1}\right)-\psi\theta\left(\frac{1}{\theta_1}-\frac{1}{\theta_0}\right)$$

where $\psi=\left(\frac{c}{c^*}\right)^2$.

8. Acceptance and Rejection Region of PEFD

We wish to test the simple null hypothesis $H_0:\theta=\theta_0$ against the simple alternative $H_1:\theta=\theta_1(\theta_1>\theta_0)$ having pre-assigned $0<\alpha,\beta<1$. Z_i is defined as

$$Z_i=\nu\ln\left(\frac{\theta_0}{\theta_1}\right)+x_i\left(\frac{1}{\theta_0}-\frac{1}{\theta_1}\right).$$

Let us define, $Y(n) = \sum_{i=1}^{n} X_i$ and $N =$ first integer $n(\geq 1)$, for which the inequality $Y(n) \leq c_1 + dn$ or $Y(n) \geq c_2 + dn$ holds with the constants

$$c_1 = \frac{\ln B}{\left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right)}, c_2 = \frac{\ln A}{\left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right)} \text{ and } d = \frac{\nu \ln\left(\frac{\theta_0}{\theta_1}\right)}{\left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right)}.$$

9. Results and Findings

I. The numerical values of the OC and ASN functions for the parameters θ and ν are derived in Table 1 and the curves are represented in Figures 1 and 2. The Table and Curves gives the satisfactory results.

II. From Table 2, the values of OC and ASN curves are plotted in Figure 3 for various values of 'Q'. The OC curve shifts to the right(left) and ASN curve shifts to the right upward (left downward) for $Q < 1(Q > 1)$. From both the curves, it is evident that the SPRT is highly sensitive for any change in 'ν'.

III. From Table 3, the values of OC and ASN curves are plotted in Figure 4 for various values of 'ϕ'. The OC curve shifts to the right(left) and ASN curve shifts to the right upward (left downward) for $\phi < 1(\phi > 1)$. From both the curves, it is evident that the SPRT is highly sensitive for any change in 'θ'.

IV. From Table 4, the values of OC and ASN curves are plotted in Figure 5 for various values of 'ψ'. The OC curve shifts to the right (left) and ASN curve shifts to the right upward (left downward) for $\psi < 1(\psi > 1)$. From both the curves, it is evident that the SPRT is highly sensitive for any change in 'ψ'.

V. Figure 6, shows the acceptance and rejection regions for H_0 under the case when $H_0 : \theta_0 = 13$ vs $H_1 : \theta_1 = 15$ for $\alpha = \beta = 0.05$ and $\nu = 2$. The values of constants $c_1 = -287.0828$, $c_2 = 287.0828$ and $d = -27.90466$, respectively. Thus, if $Y(N) \leq -27.90466N + 287.0828$, we accept H_0 and if $Y(N) \geq -27.90466N - 287.0828$, we accept H_1. At the intermediate stages, we continue sampling.

10. Tables and Figures

Table 1 OC and ASN Functions under $\alpha = \beta = 0.05$

$H_0 : \theta_0 = 13, H_1 : \theta_1 = 15$						$H_0 : \nu_0 = 13, H_1 : \nu_1 = 15$					
θ	$L(\theta)$	$E(N)$	θ	$L(\theta)$	$E(N)$	ν	$L(\nu)$	$E(N)$	ν	$L(\nu)$	$E(N)$
12.0	0.9986	146.6379	14.0	0.4649	421.7277	12.0	0.9978	9.2084	14.0	0.4909	29.2767
12.2	0.9969	162.8450	14.2	0.3268	401.3977	12.2	0.9958	10.2831	14.2	0.3489	28.5099
12.4	0.9936	182.6031	14.4	0.2148	365.6746	12.4	0.9921	11.5862	14.4	0.2299	26.4396
12.6	0.9903	206.8220	14.6	0.1344	324.0585	12.6	0.9852	13.1771	14.6	0.1429	23.7140
12.8	0.9936	236.4311	14.8	0.0813	283.5593	12.8	0.9727	15.1207	14.8	0.0852	20.9075
13.0	**0.9511**	272.0115	15.0	**0.0481**	247.6194	13.0	**0.9506**	17.4707	15.0	**0.0496**	18.3477
13.2	0.9745	312.9813	15.2	0.0280	217.1700	13.2	0.9128	20.2269	15.2	0.0284	16.1527
13.4	0.9101	356.2505	15.4	0.0161	191.8948	13.4	0.8514	23.2560	15.4	0.0161	14.3239
13.6	0.8427	394.9805	15.6	0.0091	171.0361	13.6	0.7591	26.1920	15.6	0.0091	12.8154
13.8	0.7423	419.4413	15.8	0.0051	153.7820	13.8	0.6350	28.4159	15.8	0.0051	11.5706

Table 2 OC and ASN Functions $H_0 : \theta_0 = 13, H_1 : \theta_1 = 15, \alpha = \beta = 0.05$

	$Q = 0.96$		$Q = 0.98$		$Q = 1$		$Q = 1.02$		$Q = 1.04$	
θ	$L(\theta)$	$E(N)$	$L(\theta)$	$E(N)$	$L(\theta)$	$E(N)$	$L(\theta)$	$E(N)$	$L(\theta)$	$E(N)$
12.0	0.9997	117.9642	0.9994	130.7879	0.9986	146.6379	0.9969	166.6099	0.993	192.2676
12.2	0.9994	127.9919	0.9987	143.4212	0.9970	162.8450	0.9933	187.7760	0.9851	220.2996
12.4	0.9987	139.8035	0.9972	158.5618	0.9937	182.6031	0.9860	213.933	0.9692	255.0499
12.6	0.9974	153.8510	0.9942	176.8701	0.9871	206.8220	0.9717	246.1303	0.9389	297.0357
12.8	0.9948	170.6990	0.9884	199.1305	0.9745	236.4311	0.9448	284.9204	0.8844	344.6670
13.0	0.9898	191.0183	0.9775	226.1628	0.9512	272.0115	0.8970	329.26	0.7952	392.0871
13.2	0.9806	215.5267	0.9577	258.5597	0.9101	312.9813	0.8188	374.7806	0.6673	428.1692
13.4	0.9640	244.8080	0.9231	296.1020	0.8427	356.2505	0.7043	412.5785	0.5125	440.5583
13.6	0.9354	278.9059	0.8662	336.7466	0.7424	394.9805	0.5604	431.4178	0.3583	424.5372
13.8	0.8884	316.5908	0.7801	375.4905	0.6113	419.4413	0.4086	424.5431	0.2306	387.0856
14.0	0.8162	354.4084	0.6636	404.3614	0.4650	421.7277	0.2745	395.0849	0.1395	340.6018
14.2	0.7155	386.2119	0.5263	415.2478	0.3269	401.3977	0.1729	353.212	0.0811	294.8779
14.4	0.5904	404.5178	0.3874	404.8214	0.2149	365.6746	0.1042	308.9473	0.0460	254.6703
14.6	0.4552	404.1337	0.2665	376.9041	0.1345	324.0585	0.0609	268.2821	0.0256	221.1477
14.8	0.3283	385.5071	0.1738	339.5334	0.0814	283.5593	0.0350	233.4569	0.0141	193.7957
15.0	0.2238	354.2942	0.1090	300.2335	0.0482	247.6194	0.0198	204.5853	0.0076	171.5783
15.2	0.1460	317.7116	0.0667	263.6472	0.0281	217.1700	0.0111	180.9222	0.0041	153.4506
15.4	0.0925	281.3539	0.0401	231.7132	0.0162	191.8948	0.0061	161.5276	0.0021	138.5275
15.6	0.0573	248.2856	0.0238	204.7226	0.0092	171.0361	0.0033	145.5348	0.0011	126.1096

Table 3 OC and ASN Functions $H_0 : \nu_0 = 13, H_1 : \nu_1 = 15, \alpha = \beta = 0.05$

	$\phi = 0.96$		$\phi = 0.98$		$\phi = 1$		$\phi = 1.02$		$\phi = 1.04$	
ν	$L(\nu)$	$E(N)$	$L(\nu)$	$E(N)$	$L(\nu)$	$E(N)$	$L(\nu)$	$E(N)$	$L(\nu)$	$E(N)$
12.2	0.9994	8.0434	0.9983	9.0470	0.9958	10.2831	0.9903	11.8171	0.9787	13.7175
12.4	0.9987	8.8527	0.9967	10.0687	0.9921	11.5862	0.9820	13.4827	0.9611	15.8179
12.6	0.9975	9.8159	0.9938	11.3033	0.9852	13.1771	0.9669	15.5140	0.9305	18.3199
12.8	0.9952	10.9748	0.9882	12.8073	0.9727	15.1207	0.9405	17.9528	0.8796	21.1584
13.0	0.9910	12.3819	0.9782	14.6459	0.9506	17.4707	0.8960	20.7670	0.8003	24.0814
13.2	0.9833	14.1009	0.9604	16.8816	0.9128	20.2269	0.8251	23.7618	0.6884	26.5821
13.4	0.9693	16.1989	0.9294	19.5421	0.8514	23.2560	0.7221	26.4916	0.5501	28.0034
13.6	0.9448	18.7256	0.8779	22.5548	0.7591	26.1920	0.5896	28.3015	0.4044	27.8953
13.8	0.9033	21.6623	0.7979	25.6463	0.6350	28.4159	0.4435	28.6312	0.2743	26.3483
14.0	0.8366	24.8326	0.6852	28.2725	0.4909	29.2767	0.3072	27.4095	0.1741	23.9095
14.2	0.7382	27.8024	0.5465	29.7362	0.3489	28.5099	0.1982	25.1018	0.1054	21.1947
14.4	0.6091	29.8941	0.4008	29.5709	0.2299	26.4396	0.1213	22.3555	0.0618	18.6177
14.6	0.4635	30.4711	0.2713	27.8882	0.1429	23.7140	0.0717	19.6606	0.0356	16.3631
14.8	0.3245	29.3709	0.1720	25.2770	0.0852	20.9075	0.0414	17.2626	0.0203	14.4676
15.0	0.2112	27.0272	0.1040	22.3887	0.0496	18.3477	0.0237	15.2307	0.0115	12.9001
15.2	0.1300	24.1288	0.0610	19.6563	0.0284	16.1527	0.0134	13.5457	0.0065	11.6077
15.4	0.0771	21.2315	0.0351	17.2702	0.0161	14.3239	0.0076	12.1568	0.0036	10.5380
15.6	0.0447	18.6290	0.0200	15.2662	0.0091	12.8154	0.0043	11.0089	0.0020	9.6455

Table 4 OC and ASN Functions $H_0 : \theta_0 = 13, H_1 : \theta_1 = 15, \alpha = \beta = 0.05$

	$\phi = 0.98$		$\phi = 1$		$\phi = 1.02$	
θ	$L(\theta)$	$E(N)$	$L(\theta)$	$E(N)$	$L(\theta)$	$E(N)$
12.2	0.9994	128.2688	0.9970	162.8450	0.9849	221.0448
12.4	0.9987	140.1371	0.9937	182.6031	0.9687	255.9894
12.6	0.9974	154.2568	0.9871	206.8220	0.9379	298.1778
12.8	0.9947	171.1966	0.9745	236.4311	0.8827	345.9449
13.0	0.9897	191.6313	0.9512	272.0115	0.7926	393.2965
13.2	0.9803	216.2803	0.9101	312.9813	0.6637	428.9734
13.4	0.9635	245.7220	0.8427	356.2505	0.5084	440.6648
13.6	0.9344	247.3173	0.7424	171.0361	0.3545	125.7705
13.8	0.8868	279.9782	0.6113	394.9805	0.2276	423.9333
14.0	0.8138	280.2862	0.4650	191.8948	0.1375	138.1281
14.2	0.7122	316.6156	0.3269	217.1700	0.0798	152.9754
14.4	0.5864	317.7702	0.2149	419.4413	0.0452	386.0318
14.6	0.4511	353.3112	0.1345	247.6194	0.0251	171.0077
14.8	0.3247	355.5624	0.0814	421.7277	0.0138	339.4071
15.0	0.2208	384.8421	0.0482	283.5593	0.0075	193.1064
15.2	0.1439	387.1238	0.0281	401.3977	0.0040	293.7435
15.4	0.0910	403.9848	0.0162	324.0585	0.0021	220.3159
15.6	0.0564	404.9570	0.0092	365.6746	0.0010	253.6803

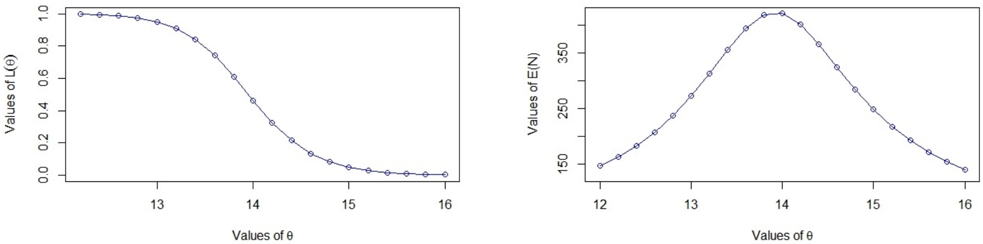

Figure 1 Value of θ

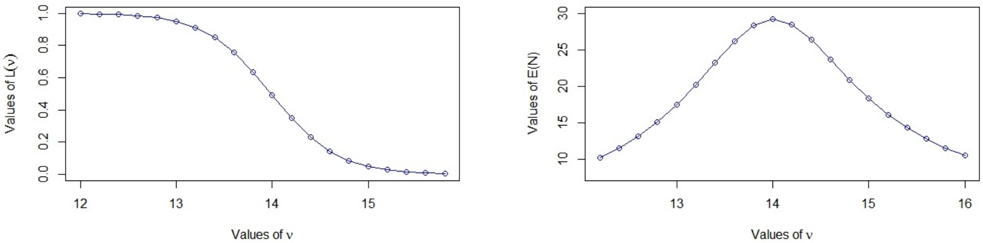

Figure 2 Value of ν

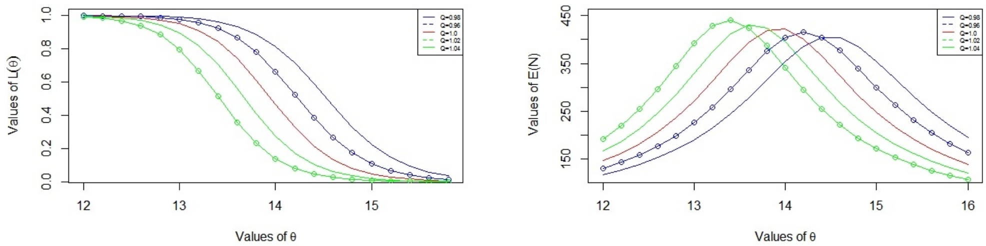

Figure 3 Value of θ

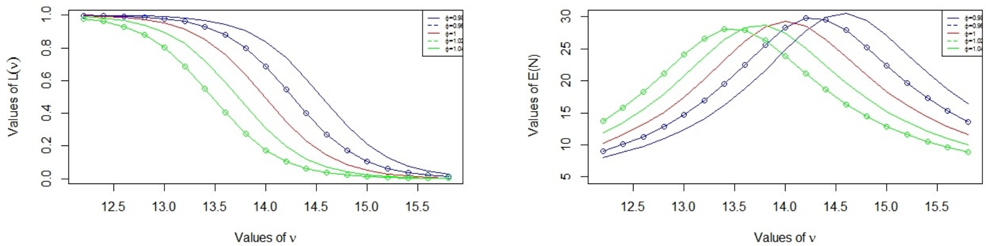

Figure 4 Value of ν

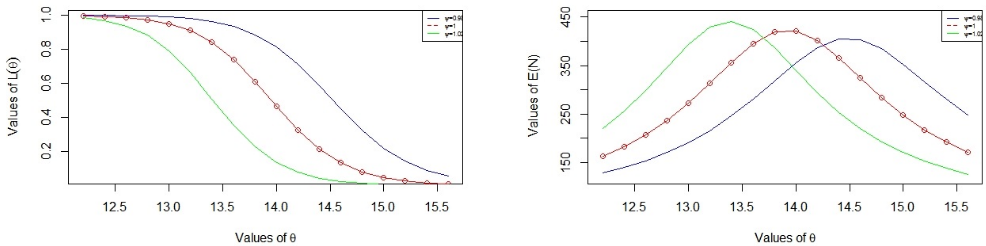

Figure 5 Value of θ

References

Bacanli S, Demirhan YP. A group sequential test for the inverse gaussian mean. Stat Pap. 2008; 49: 337-386.

Bain LJ, Engelhardt M. Sequential probability ratio tests for the shape parameter of NHPP. IEEE Trans Reliab. 1982; R-31: 79-83.

Chaturvedi A, Kumar A, Surinder K. Robustness of the sequential procedures for a family of life-testing models. Metron. 1998; 56: 117-137.

Chaturvedi A, Kumar A, Surinder K. Sequential testing procedures for a class of distributions representing various life testing models. Stat Pap. 2000; 41: 65-84.

Chaturvedi A, Tiwari N, Tomer S. Robustness of the sequential testing procedure for the generalized life distribution. Braz J Prob Stat. 2002; 16: 7-24.

Epstein B, Sobel M. Sequential life tests in exponential case. Ann Math Stat. 1955; 26: 82-93.

Gradshteyn IS, Ryzhik IM. Tables of integrals, series of products. New York: Academic Press; 2007.

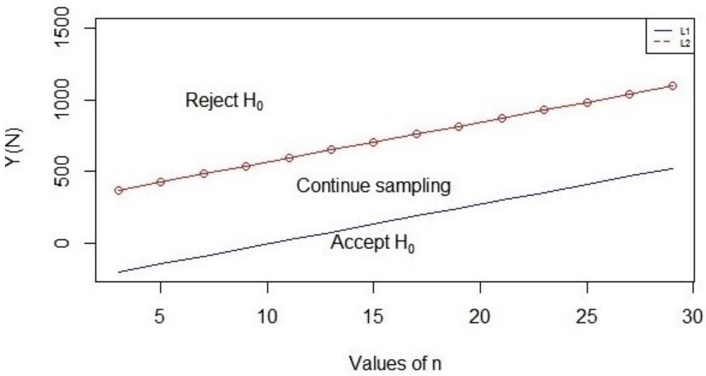

Figure 6 Value of n

Harter L, Moore AH. An evaluation of the exponential and Weibull test plans. IEEE Trans Reliab. 1976; 25: 100-104.

Hubbard DJ, Allen OB. Robustness of the SPRT for a negative binomial to misspecification of the dispersion parameter. Biometrics. 1991; 47: 419-427.

Johnson NL. Cumulative sum control chart and the Weibull distribution. Technometrics. 1966; 8: 481-491.

Kendall M, Stuart A. The advanced theory of statistics; Inference and relationship. New York: Macmillan Company, 1979.

Joshi S, Shah M. (1990): Sequential analysis applied to testing the mean of an inverse Gaussian distribution with known coefficient of variation. Commun Stat-Theory Methods. 1990; 19(4): 1457-1466.

Kumar S, Singh V, Gautam PL. Sequential testing procedure and their robustness study for Erlang distribution. Int Res J Nat Appl Sci. 2018; 5(3): 223-244.

Montagne ER, Singpurwalla ND. Robustness of sequential exponential life-testing procedures. J Am Stat Assoc. 1985; 391: 715-719.

Oakland GB. An application of sequential analysis to whitefish sampling. Biometrics. 1950; 6: 59-67.

Phatarfod RM. A sequential test for gamma distribution. J Am Stat Assoc. 1971; 66: 876-878.

Liang T. Empirical Bayes estimation of reliability in a positive exponential family. Commun Stat-Theory Methods. 2008; 37(13): 2052-2065.

Wald A. Sequential analysis. New York: John Wiley and Sons; 1947.