Thailand Statistician
October 2022; 20(4): 842-859
http://statassoc.or.th
Contributed paper

Modified Two Parameter Regression Estimator for Solving the Multicollinearity

Issam Dawoud
Department of Mathematics, Al-Aqsa University, Gaza, Palestine.
*Corresponding author; e-mail: isamdawoud@gmail.com

Received: 17 September 2020
Revised: 23 January 2021
Accepted: 7 February 2021

Abstract

When the multicollinearity problem appears in the multiple linear regression model, the performance of the unbiased estimator which is the ordinary least squares (OLS) is ineﬃcient. To solve the above-mentioned problem, several biased and almost unbiased regression estimators are introduced. In this study, as an alternative to the OLS estimator, a modified two-parameter regression estimator called the Dawoud biased regression (DBR) estimator is proposed. Moreover, we theoretically compare the performance of the DBR estimator with the OLS and some existing estimators by the criterion of the mean squares error. Furthermore, a Monte Carlo simulation study and real-life data are given to evaluate the performance of the DBR estimator. The main finding is that the DBR estimator performs better than other regression estimators under determined conditions.

Keywords: Dawoud biased estimator, Liu estimator, Monte Carlo simulation, multicollinearity, ridge estimator.

1. Introduction

The multiple linear regression model is known as

$$y = X\beta + \varepsilon, \tag{1}$$

where y is an $n\times 1$ vector of the known response variable, X is an $n\times p$ full rank matrix of the known explanatory variables, β is an $p\times 1$ vector of the unknown regression parameters, and ε is defined as an $n\times 1$ vector of disturbances such that $Cov(\varepsilon)=\sigma^2 I_n$. The unbiased ordinary least squares (OLS) estimator of β in (1) is defined by

$$\hat{\beta} = S^{-1}X'y, \tag{2}$$

where $S = X'X$.

The instability and inefficiency of the OLS estimator in the appearance of the multicollinearity problem for the multiple linear regression model encourage authors to introduce many biased and almost unbiased estimators to cope with this problem (Hoerl and Kennard (1970)), to mention a few of the related studies, Stein (1956), Massy (1965), Hoerl and Kennard (1970), Mayer and Willke (1973), Swindel (1976), Liu (1993), Akdeniz and Kaçiranlar (1995), Ozkale and Kaçiranlar (2007), Sakallıoglu and Kaçıranlar (2008), Yang and Chang (2010), and recently Roozbeh (2018), Akdeniz and Roozbeh (2019) and Lukman et al. (2019a, 2019b) among others. The proposed biased and almost unbiased regression estimators in the previous-mentioned studies and others have biasing parameters. The determination of these parameters plays a useful role in giving a good view of these estimators' performances, to mention some of the studies related to obtaining the biasing parameters problem in regression models using an efficient criterion, Amini and Roozbeh (2015), Akdeniz and Roozbeh (2017), Roozbeh et al. (2020) and Roozbeh and Hamzah (2020), among others. The paper objective is to propose a new kind of two-parameter estimator for the regression parameter in the appearance of the multicollinearity problem and then to compare the performance of the newly introduced estimator with the OLS, the ridge of Hoerl and Kennard (1970), the Liu of Liu (1993), the two-parameter of Ozkale and Kaçiranlar (2007), and the modified ridge type of Lukman et al. (2019a) estimators.

1.1. Some alternative biased regression estimators

The ordinary ridge regression (ORR) estimator is known as follows:

$$\hat{\beta}_k = WS\hat{\beta}, \quad k \geq 0, \tag{3}$$

where $W = (S + k\mathrm{I}_p)^{-1}$ and k is known as the biasing parameter (Hoerl and Kennard 1970).

The Liu estimator is known as follows:

$$\hat{\beta}_d = F\hat{\beta}, \quad 0 < d < 1, \tag{4}$$

where $F = (S + \mathrm{I}_p)^{-1}(S + d\mathrm{I}_p)$ and d is known as the biasing parameter (Liu 1993).

The two-parameter (TP) estimator is known as follows:

$$\hat{\beta}_{TP} = M\hat{\beta}, \quad k \geq 0, \ 0 < d < 1, \tag{5}$$

where $M = (S + k\mathrm{I}_p)^{-1}(S + kd\mathrm{I}_p)$ (Ozkale and Kaçiranlar 2007).

The modified ridge type (MRT) estimator is known as follows:

$$\hat{\beta}_{MRT} = RS\hat{\beta}, \quad k \geq 0, \ 0 < d < 1, \tag{6}$$

where $R = (S + k(1+d)\mathrm{I}_p)^{-1}$ (Lukman et al. 2019a).

1.2. The proposed modified two-parameter regression estimator

Following the same method that used by Liu (1993), Kaciranlar et al. (1999) and Yang and Chang (2010), we propose a new kind of two-parameter regression estimator for β by replacing $\hat{\beta}$ with $\hat{\beta}_{MRT}$ in $\hat{\beta}_{TP}$ as follows:

$$\hat{\beta}_{DBR} = MRS\hat{\beta}. \tag{7}$$

This estimator is going to be called the Dawoud biased regression (DBR) estimator.

Properties of the DBR estimator:

$$E(\hat{\beta}_{DBR}) = MRSE(\hat{\beta}) = MRS\beta. \tag{8}$$

The bias and the covariance of the DBR estimator are given respectively,

$$B(\hat{\beta}_{DBR}) = [MRS - I_p]\beta, \tag{9}$$

$$D(\hat{\beta}_{DBR}) = \sigma^2 MRSR'M', \tag{10}$$

and the mean square error matrix (MSEM) is calculated as

$$MSEM(\hat{\beta}_{DBR}) = \sigma^2 MRSR'M' + [MRS - I_p]\beta\beta'[MRS - I_p]'. \tag{11}$$

Writing Equation (1) in the canonical form for comparing the DBR estimator performance with the mentioned estimators as follows:

$$y = Z\alpha + \varepsilon, \tag{12}$$

where $Z = X\mathrm{N}$ and $\alpha = \mathrm{N}'\beta.$ Here, N is an orthogonal matrix such that $Z'Z = \mathrm{N}'X'X\mathrm{N} = T = diag(t_1, t_2, \ldots, t_p).$ The OLS estimator of α is known as

$$\hat{\alpha} = T^{-1}Z'y, \tag{13}$$

$$MSEM(\hat{\alpha}) = \sigma^2 T^{-1}. \tag{14}$$

The ORR of α (Hoerl and Kennard 1970) is known as

$$\hat{\alpha}_k = WT\hat{\alpha}, \tag{15}$$

where $W = [T + kI_p]^{-1}$ and the MSEM is known as

$$MSEM(\hat{\alpha}_k) = \sigma^2 WTW' + (WT - I_p)\alpha\alpha'(WT - I_p)'. \tag{16}$$

Hoerl et al. (1975) gave the harmonic-mean of the biasing parameter for the ORR estimator as follows:

$$\hat{k}_{HM} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\hat{\alpha}_i^2}. \tag{17}$$

The Liu estimator of α (Liu 1993) is known as

$$\hat{\alpha}_d = F\hat{\alpha}, \tag{18}$$

where $F = [T + I_p]^{-1}[T + dI_p],$ d is known as the biasing parameter of the Liu estimator and given as

$$\hat{d}_{opt} = 1 - \hat{\sigma}^2\left[\frac{\sum_{i=1}^{p}(1/(t_i(t_i+1)))}{\sum_{i=1}^{p}(\hat{\alpha}_i^2/(t_i+1)^2)}\right], \tag{19}$$

and the MSEM of this estimator is known as

$$MSEM(\hat{\alpha}_d) = \sigma^2 FT^{-1}F' + (1-d)^2(T+I_p)^{-1}\alpha\alpha'(T+I_p)^{-1}. \tag{20}$$

In case of $\hat{d}_{opt}$ becomes negative, Ozkale and Kaçiranlar (2007) gave the following alternative for the biasing parameter:

$$\hat{d}_{alt} = \min\left[\frac{\hat{\alpha}_i^2}{(\hat{\sigma}^2/t_i) + \hat{\alpha}_i^2}\right]_{i=1}^{p}. \tag{21}$$

The TP estimator of α (Ozkale and Kaçiranlar 2007) is known as

$$\hat{\alpha}_{TP} = M\hat{\alpha}, \tag{22}$$

where $M = (T + k\mathrm{I}_p)^{-1}(T + kd\mathrm{I}_p),$ the biasing parameters k and d of the TP estimator are known as

$$\hat{k}_{\min}(TP) = \min\left[\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2 - d((\hat{\sigma}^2/t_i) + \hat{\alpha}_i^2)}\right], \tag{23}$$

$$\hat{d}_{\min}(TP) = \min\left[\frac{t_i(k\hat{\alpha}_i^2 - \hat{\sigma}^2)}{k(\hat{\sigma}^2 + \hat{\alpha}_i^2 t_i)}\right], \tag{24}$$

and the MSEM of this estimator is known as

$$MSEM(\hat{\alpha}_{TP}) = \sigma^2 MT^{-1}M' + [M - I_p]\alpha\alpha'[M - I_p]'. \tag{25}$$

In case of $\hat{d}_{\min}(TP)$ becomes negative, we can use $\hat{d}_{alt}$ that was derived by Ozkale and Kaçiranlar (2007). The MRT estimator of α (Lukman et al. 2019a) is known as

$$\hat{\alpha}_{MRT} = RT\hat{\alpha}, \tag{26}$$

where $R = (T + k(1+d)\mathrm{I}_p)^{-1}$, the biasing parameters k and d of the MRT estimator are known as

$$\hat{k}_{\min}(MRT) = \min\left[\frac{\hat{\sigma}^2}{(1+d)\hat{\alpha}_i^2}\right], \tag{27}$$

$$\hat{d}_{\min}(MRT) = \min\left[\frac{\hat{\sigma}^2}{k\hat{\alpha}_i^2} - 1\right], \tag{28}$$

and the MSEM of this estimator is known as

$$MSEM(\hat{\alpha}_{MRT}) = \sigma^2 RTR' + [RT - I_p]\alpha\alpha'[RT - I_p]'. \tag{29}$$

The proposed DBR estimator of α is given by

$$\hat{\alpha}_{DBR} = MRT\hat{\alpha}. \tag{30}$$

The MSEM of the proposed DBR estimator of α is going to be

$$MSEM(\hat{\alpha}_{DBR}) = \sigma^2 MRTR'M' + [MRT - I_p]\alpha\alpha'[MRT - I_p]'. \tag{31}$$

The lemmas that are useful in the theoretical comparisons are stated in the section below**.**

Lemma 1. (Farebrother 1976) *Let G be an $n \times n$ positive definite (pd) matrix, which is $G > 0$ and α be the vector; then $G - \alpha\alpha' > 0$ iff $\alpha' G^{-1}\alpha < 1$.*

Lemma 2. (Trenkler and Toutenburg 1990) *Let $\alpha_i = C_i y,\ i = 1,2$ be the two linear estimators of α. Suppose that $Difference = Cov(\hat{\alpha}_1) - Cov(\hat{\alpha}_2) > 0$, such that $Cov(\hat{\alpha}_i)\, i = 1,2$ be the covariance matrix of $\hat{\alpha}_i$ and $b_i = Bias(\hat{\alpha}_i) = (C_i X - I)\alpha,\ i = 1,2.$ Then,*

$$\Delta(\hat{\alpha}_1 - \hat{\alpha}_2) = MSEM(\hat{\alpha}_1) - MSEM(\hat{\alpha}_2) = \sigma^2 Difference + b_1 b_1' - b_2 b_2' > 0 \tag{32}$$

iff $b_2'[\sigma^2 Difference + b_1' b_1]^{-1} b_2 < 1$ where $MSEM(\hat{\alpha}_i) = Cov(\hat{\alpha}_i) + b_i b_i'$.

The paper remaining part is as follows: in Section 2, the proposed DBR estimator is compared theoretically with each mentioned estimator and then the optimal biasing parameters k and d of the DBR estimator are found. Then, a Monte Carlo simulation study is completed in Section 3. Real-life chemical data is used in Section 4. Finally, concluding remarks are stated in Section 5.

2. Comparisons among the Estimators

1. Comparison between $\hat{\alpha}$ and $\hat{\alpha}_{DBR}$. The difference between $MSEM(\hat{\alpha})$ and $MSEM(\hat{\alpha}_{DBR})$ is given as

$$MSEM(\hat{\alpha}) - MSEM(\hat{\alpha}_{DBR}) = \sigma^2(T^{-1} - MRTR'M') - [MRT - I_p]\alpha\alpha'[MRT - I_p]' \tag{33}$$

Theorem 1. *$\hat{\alpha}_{DBR}$ is superior to $\hat{\alpha}$ iff*

$$\alpha'[MRT-I_p]'[\sigma^2(T^{-1}-MRTR'M')][MRT-I_p]\alpha<1 \tag{34}$$

Proof: The difference of the dispersion matrices is given as

$$Difference=\sigma^2(T^{-1}-MRTR'M')=\sigma^2 diag\left\{\frac{1}{t_i}-\frac{t_i(t_i+kd)^2}{(t_i+k(1+d))^2(t_i+k)^2}\right\}_{i=1}^{p} \tag{35}$$

where $T^{-1}-MRTR'M'$ is pd iff $(t_i+k(1+d))^2(t_i+k)^2-t_i^2(t_i+kd)^2>0$ or $(t_i+k(1+d))(t_i+k)-t_i(t_i+kd)>0.$ So, for $k>0$ and $0<d<1,$ $(t_i+k(1+d))(t_i+k)-t_i(t_i+kd)=2t_ik+k^2(1+d)>0.$ Therefore, $T^{-1}-MRTR'M'$ is pd.

2. Comparison between $\hat{\alpha}_k$ and $\hat{\alpha}_{DBR}$. The difference between $MSEM(\hat{\alpha}_k)$ and $MSEM(\hat{\alpha}_{DBR})$ is given as

$$MSEM(\hat{\alpha}_k)-MSEM(\hat{\alpha}_{DBR})=\sigma^2(WTW'-MRTR'M')+[WT-I_p]\alpha\alpha'[WT-I_p]' -[MRT-I_p]\alpha\alpha'[MRT-I_p]' \tag{36}$$

Theorem 2. *$\hat{\alpha}_{DBR}$ is superior to $\hat{\alpha}_k$ iff*

$$\alpha'[MRT-I_p]'[V_1+[WT-I_p]\alpha\alpha'[WT-I_p]'][MRT-I_p]\alpha<1, \tag{37}$$

where $V_1=\sigma^2(WTW'-MRTR'M')$.

Proof: The difference of the dispersion matrices is given as

$$V_1=\sigma^2(WTW'-MRTR'M')=\sigma^2 diag\left\{\frac{t_i}{(t_i+k)^2}-\frac{t_i(t_i+kd)^2}{(t_i+k(1+d))^2(t_i+k)^2}\right\}_{i=1}^{p}, \tag{38}$$

where $WTW'-MRTR'M'$ is pd iff $(t_i+k(1+d))^2-(t_i+kd)^2>0$ or $(t_i+k(1+d))-(t_i+kd)>0.$ So, for $k>0$ and $0<d<1,$ $(t_i+k(1+d))-(t_i+kd)=k>0.$ Therefore, $WTW'-MRTR'M'$ is pd.

3. Comparison between $\hat{\alpha}_d$ and $\hat{\alpha}_{DBR}$. The difference between $MSEM(\hat{\alpha}_d)$ and $MSEM(\hat{\alpha}_{DBR})$ is given as

$$MSEM(\hat{\alpha}_d)-MSEM(\hat{\alpha}_{DBR})=\sigma^2(FT^{-1}F'-MRTR'M')+(1-d)^2(T+I_p)^{-1}\alpha\alpha'(T+I_p)^{-1} -[MRT-I_p]\alpha\alpha'[MRT-I_p]' \tag{39}$$

Theorem 3. *$\hat{\alpha}_{DBR}$ is superior to $\hat{\alpha}_d$ iff*

$$\alpha'[MRT-I_p]'[V_2+(1-d)^2(T+I_p)^{-1}\alpha\alpha'(T+I_p)^{-1}][MRT-I_p]\alpha<1, \tag{40}$$

where $V_2=\sigma^2(FT^{-1}F'-MRTR'M')$.

Proof: The difference of the dispersion matrices is given as

$$V_2=\sigma^2(FT^{-1}F'-MRTR'M')=\sigma^2 diag\left\{\frac{(t_i+d)^2}{t_i(t_i+1)^2}-\frac{t_i(t_i+kd)^2}{(t_i+k(1+d))^2(t_i+k)^2}\right\}_{i=1}^{p}, \tag{41}$$

where $FT^{-1}F' - MRTR'M'$ is pd iff $(t_i + k(1+d))^2 (t_i + d)^2 (t_i + k)^2 - t_i^2 (t_i + kd)^2 (t_i + 1)^2 > 0$ or $(t_i + k(1+d))(t_i + d)(t_i + k) - t_i(t_i + kd)(t_i + 1) > 0.$ So, if $k > 0$ and $0 < d < 1,$ $(t_i + k(1+d))(t_i + d)(t_i + k) - t_i(t_i + kd)(t_i + 1) = t_i^2(2k + d - 1) + t_i k(1+d)(k+d) + k^2 d(1+d) > 0.$ Therefore, $FT^{-1}F' - MRTR'M'$ is pd.

4. Comparison between $\hat{\alpha}_{TP}$ and $\hat{\alpha}_{DBR}$. The difference between $MSEM(\hat{\alpha}_{TP})$ and $MSEM(\hat{\alpha}_{DBR})$ is given as

$$MSEM(\hat{\alpha}_{TP}) - MSEM(\hat{\alpha}_{DBR}) = \sigma^2(MT^{-1}M' - MRTR'M') + [M - I_p]\alpha\alpha'[M - I_p]' - [MRT - I_p]\alpha\alpha'[MRT - I_p]'. \quad (42)$$

Theorem 4. *$\hat{\alpha}_{DBR}$ is superior to $\hat{\alpha}_{TP}$ iff*

$$\alpha'[MRT - I_p]'[V_3 + [M - I_p]\alpha\alpha'[M - I_p]'][MRT - I_p]\alpha < 1 \quad (43)$$

where $V_3 = \sigma^2(MT^{-1}M' - MRTR'M').$

Proof: The difference of the dispersion matrices is given as

$$V_3 = \sigma^2(MT^{-1}M' - MRTR'M') = \sigma^2 diag\left\{ \frac{(t_i + kd)^2}{t_i(t_i + k)^2} - \frac{t_i(t_i + kd)^2}{(t_i + k(1+d))^2 (t_i + k)^2} \right\}_{i=1}^{p}, \quad (44)$$

where $MT^{-1}M' - MRTR'M'$ is pd iff $(t_i + k(1+d))^2 - t_i^2 > 0$ or $(t_i + k(1+d)) - t_i > 0.$ Obviously, for $k > 0$ and $0 < d < 1,$ $(t_i + k(1+d)) - t_i = k(1+d) > 0.$ Therefore, $MT^{-1}M' - MRTR'M'$ is pd.

5. Comparison between $\hat{\alpha}_{MRT}$ and $\hat{\alpha}_{DBR}$. The difference between $MSEM(\hat{\alpha}_{MRT})$ and $MSEM(\hat{\alpha}_{DBR})$ is given as

$$MSEM(\hat{\alpha}_{MRT}) - MSEM(\hat{\alpha}_{DBR}) = \sigma^2(RTR' - MRTR'M') + [RT - I_p]\alpha\alpha'[RT - I_p]' - [MRT - I_p]\alpha\alpha'[MRT - I_p]'. \quad (45)$$

Theorem 5. *$\hat{\alpha}_{DBR}$ is superior to $\hat{\alpha}_{MRT}$ iff*

$$\alpha'[MRT - I_p]'[V_4 + [RT - I_p]\alpha\alpha'[RT - I_p]'][MRT - I_p]\alpha < 1, \quad (46)$$

where $V_4 = \sigma^2(RTR' - MRTR'M').$

Proof: The difference of the dispersion matrices is given as

$$V_4 = \sigma^2(RTR' - MRTR'M') = \sigma^2 diag\left\{ \frac{t_i}{(t_i + k(1+d))^2} - \frac{t_i(t_i + kd)^2}{(t_i + k(1+d))^2 (t_i + k)^2} \right\}_{i=1}^{p}, \quad (47)$$

where $RTR' - MRTR'M'$ is pd iff $(t_i + k)^2 - (t_i + kd)^2 > 0$ or $(t_i + k) - (t_i + kd) > 0.$ Obviously, for $k > 0$ and $0 < d < 1,$ $(t_i + k) - (t_i + kd) = k(1-d) > 0.$ Therefore, $RTR' - MRTR'M'$ is pd.

6. The proposed DBR estimator parameters determination k and d

Here, we discuss getting both of the biasing parameters k and d which are unknown and should be estimated from the known data. The optimal biasing parameter k in the ORR estimator and the optimal d in the Liu estimator are given by Hoerl and Kennard (1970) and Liu (1993), respectively. Several proposed estimators for both of the biasing parameters k and d are given and illustrated in lots of previous studies, for example, Hoerl et al. (1975), Kibria (2003), Kibria and Banik (2016), Lukman and Ayinde (2017), Månsson et al. (2015) and Khalaf and Shukur (2005), among others.

This time, we discuss getting the optimal values of both k and d for the proposed DBR estimator. At first, in case of d is fixed, the optimal value of k is gotten by minimizing

$$MSEM(\hat{\alpha}_{DBR}) = E((\hat{\alpha}_{DBR}-\alpha)'(\hat{\alpha}_{DBR}-\alpha)),$$

$$MSE(k,d) = tr(MSEM(\hat{\alpha}_{DBR})),$$

$$MSE(k,d) = \sigma^2\sum_{i=1}^{p}\frac{t_i \ (t_i+kd)^2}{(t_i+k)^2(t_i+k(1+d))^2}+\sum_{i=1}^{p}\frac{(k^2(1+d)+2t_ik)^2\alpha_i^2}{(t_i+k)^2(t_i+k(1+d))^2}. \tag{48}$$

After setting $(\partial MSE(k,d)/\partial k)=0,$ we obtain the optimal value of k as follows

$$k=\frac{(-2t_i\alpha_i^2+d\sigma^2)\pm\sqrt{4t_i^2(\alpha_i^2)^2+4t_i\sigma^2\alpha_i^2+d^2(\sigma^2)^2}}{2(d\alpha_i^2+\alpha_i^2)}. \tag{49}$$

Since $k>0,$ so

$$k=\frac{(-2t_i\alpha_i^2+d\sigma^2)+\sqrt{4t_i^2(\alpha_i^2)^2+4t_i\sigma^2\alpha_i^2+d^2(\sigma^2)^2}}{2(d\alpha_i^2+\alpha_i^2)}. \tag{50}$$

After setting $(\partial MSE(k,d)/\partial d)=0$, we obtain the optimal value of d as follows

$$d=\frac{\alpha_i^2k^2+2t_i\alpha_i^2k-\sigma^2t_i}{k(\sigma^2-\alpha_i^2k)}. \tag{51}$$

Following the similar technique used in Akdeniz and Roozbeh (2017), we find the values of k that make the given value of d is between 0 and 1 using Equation (51). Since $0<d<1,$ that means $0<\frac{\alpha_i^2k^2+2t_i\alpha_i^2k-\sigma^2t_i}{k(\sigma^2-\alpha_i^2k)}<1,$ so after examining the signs of the previous inequality, the value of k should be as follows

$$\frac{\sqrt{\alpha_i^2t_i(\sigma^2+\alpha_i^2t_i)}-\alpha_i^2t_i}{\alpha_i^2}<k<\frac{\sigma^2}{2\alpha_i^2}. \tag{52}$$

So, there are many values of k we can choose according to Equation (52) which are positive and make d values are between 0 and 1. To restrict the chosen value of the parameter $k,$ we take the midpoint of the interval in Equation (52) as follows

$$k=0.5\left(\frac{\sqrt{\alpha_i^2t_i \ (\sigma^2+\alpha_i^2t_i)}-\alpha_i^2t_i}{\alpha_i^2}+\frac{\sigma^2}{2\alpha_i^2}\right). \tag{53}$$

We change the unknown parameters in Equations (52) and (53) by their unbiased estimators in order to use them in practical parts. So, we have

$$\frac{\sqrt{\hat{\alpha}_i^2t_i(\hat{\sigma}^2+\hat{\alpha}_i^2t_i)}-\hat{\alpha}_i^2t_i}{\hat{\alpha}_i^2}<\hat{k}<\frac{\hat{\sigma}^2}{2\hat{\alpha}_i^2}, \tag{54}$$

$$\hat{k} = 0.5\left(\frac{\sqrt{\hat{\alpha}_i^2 t_i \ \ (\hat{\sigma}^2 + \hat{\alpha}_i^2 t_i)} \ - \hat{\alpha}_i^2 t_i}{\hat{\alpha}_i^2} + \frac{\hat{\sigma}^2}{2\hat{\alpha}_i^2}\right), \quad (55)$$

and the corresponding optimal d with the unbiased estimators is given as:

$$\hat{d} = \frac{\hat{\alpha}_i^2 \hat{k}^2 + 2t_i \hat{\alpha}_i^2 \hat{k} - \hat{\sigma}^2 t_i}{\hat{k}(\hat{\sigma}^2 - \hat{\alpha}_i^2 \hat{k})}. \quad (56)$$

3. Simulation Study

A Monte Carlo simulation study is completed for comparing the proposed DBR estimator performance with the other mentioned estimators in this section. Following the methods of Gibbons (1981) and Kibria (2003), the explanatory variables are generated as follows

$$x_{ij} = (1-\rho^2)^{1/2} z_{ij} + \rho z_{i,p+1}, i = 1,2,...,n; j = 1,2,..., p, \quad (57)$$

where z_{ij} are independent pseudo-random numbers and follow a standard normal distribution and the correlations of the explanatory variables are considered here as $\rho = 0.90$ and 0.99. The dependent variable y with n observations are considered as:

$$y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_p x_{ip} + e_i, \ \ i = 1,2,\ldots,n, \quad (58)$$

where e_i are i. i. d. $N(0,\sigma^2)$ and in this simulation we choose $p = 3$ and 7. The β values are considered as $\beta'\beta = 1$, Newhouse and Oman (1971). The biasing parameters of the estimators are considered here as $k = 0.3, 0.6, 0.9$ such that Wichern and Churchill (1978) and Kan et al. (2013) said that the ORR estimator has better results when k is between 0 and 1 and $d = 0.2, 0.5, 0.8$. The number of replications in this simulation is 1,000 times for the given sample sizes $n = 50$ and 100 and σ^2 = 1, 25 and 100. We calculate the mean squares error (MSE) criterion for the estimators in each replicate as follows

$$MSE(\alpha^*) = \frac{1}{1000}\sum_{j=1}^{1000}(\alpha_{ij}^* - \alpha_i)'(\alpha_{ij}^* - \alpha_i), \quad (59)$$

where α_{ij}^* is any given estimator and α_i is the given true parameter. The estimated MSE values of the estimators are stated in Tables 1-8. For each row, the smaller MSE value is bolded.

Tables 1-8 clarify that if σ, ρ and p have an increase in their values, then the estimated MSEs also have an increase in their values, while the factor n has an increase in its value, then the estimated MSEs have a decrease in their values. As known, the OLS estimator has the worst one among all given estimators. In addition, the results clarify that the proposed DBR estimator outperforms better than other mentioned estimators in all cases except the Liu estimator gives near MSE values of the DBR estimator when the biasing parameters k and d are very small (near to zero). Thus, the results we got from the simulation are consistent with the results of the theoretical part.

Table 1 Estimated MSE for OLS, ORR, Liu, TP, MRT and DBR when $p=3$, $\rho=0.9$ and $n=50$

k	d	σ	OLS	ORR	Liu	TP	MRT	DBR
0.3	0.2	1	0.22428	0.21052	**0.19120**	0.21325	0.20800	0.19782
		5	5.60637	5.26417	**4.77823**	5.33169	5.19960	4.94571
		10	22.4254	21.0567	**19.1131**	21.3268	20.7986	19.7828
	0.5	1	0.22428	0.21052	0.20328	0.21735	0.20422	**0.19792**
		5	5.60637	5.26417	5.08074	5.43385	5.10510	**4.94865**
		10	22.4254	21.0567	20.3229	21.7355	20.4206	**19.7948**
	0.8	1	0.22428	0.21052	0.21567	0.22144	0.20055	**0.19803**
		5	5.60637	5.26417	5.39290	5.53707	5.01333	**4.95159**
		10	22.4254	21.0567	21.5715	22.1481	20.0533	**19.8064**
0.6	0.2	1	0.22428	0.19813	0.19120	0.20328	0.19351	**0.17556**
		5	5.60637	4.95348	4.77823	5.08074	4.83724	**4.38658**
		10	22.4254	19.8142	19.1131	20.3229	19.3490	**17.5465**
	0.5	1	0.22428	0.19813	0.20328	0.21094	0.18690	**0.17598**
		5	5.60637	4.95348	5.08074	5.27467	4.67071	**4.39656**
		10	22.4254	19.8142	20.3229	21.0988	18.6829	**17.5864**
	0.8	1	0.22428	0.19813	0.21567	0.21892	0.18060	**0.17629**
		5	5.60637	4.95348	5.39290	5.47239	4.51300	**4.40622**
		10	22.4254	19.8142	21.5715	21.8897	18.0520	**17.6249**
0.9	0.2	1	0.22428	0.18690	0.19120	0.19404	0.18060	**0.15666**
		5	5.60637	4.67071	4.77823	4.85068	4.51300	**3.91062**
		10	22.4254	18.6829	19.1131	19.4029	18.0520	**15.6422**
	0.5	1	0.22428	0.18690	0.20328	0.20506	0.17178	**0.15739**
		5	5.60637	4.67071	5.08074	5.12736	4.29166	**3.92952**
		10	22.4254	18.6829	20.3229	20.5096	17.1666	**15.7177**
	0.8	1	0.22428	0.18690	0.21567	0.21651	0.16369	**0.15813**
		5	5.60637	4.67071	5.39290	5.41212	4.08702	**3.94747**
		10	22.4254	18.6829	21.5715	21.6484	16.3478	**15.7896**

4. Application

The famous data that was originally adopted by Woods et al. (1932) is called the Portland cement and here is used to clarify the proposed DBR estimator performance and other given estimators. This data was analyzed in various studies, for example, Kaciranlar et al. (1999), Li and Yang (2012), Lukman et al. (2019a), and Dawoud and Kibria (2020a, 2020b), among others. The regression model of this data is known as

$$y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon_i. \tag{60}$$

Table 2 Estimated MSE for OLS, ORR, Liu, TP, MRT and DBR when $p = 3$, $\rho = 0.99$ and $n = 50$

k	d	σ	OLS	ORR	Liu	TP	MRT	DBR
0.3	0.2	1	2.04246	1.18209	0.71536	1.33528	1.07698	**0.70591**
		5	51.0603	29.5527	17.8825	33.3833	26.9246	**17.6469**
		10	204.240	118.211	71.5303	133.532	107.698	**70.5878**
	0.5	1	2.04246	1.18209	1.13253	1.58287	0.94395	**0.73279**
		5	51.0603	29.5527	28.3137	39.5707	23.5989	**18.3206**
		10	204.240	118.211	113.255	158.282	94.3961	**73.2826**
	0.8	1	2.04246	1.18209	1.64629	1.85146	0.83433	**0.75694**
		5	51.0603	29.5527	41.1575	46.2878	20.8584	**18.9233**
		10	204.240	118.211	164.629	185.150	83.4340	**75.6934**
0.6	0.2	1	2.04246	0.77164	0.71536	0.97692	0.66591	**0.32203**
		5	51.0603	19.2909	17.8825	24.4216	16.6479	**8.04804**
		10	204.240	77.1638	71.5303	97.6864	66.5921	**32.1921**
	0.5	1	2.04246	0.77164	1.13253	1.33056	0.54432	**0.35710**
		5	51.0603	19.2909	28.3137	33.2643	13.6064	**8.92584**
		10	204.240	77.1638	113.255	133.057	54.4258	**35.7033**
	0.8	1	2.04246	0.77164	1.64629	1.73932	0.45360	**0.38734**
		5	51.0603	19.2909	41.1575	43.4832	11.3378	**9.68289**
		10	204.240	77.1638	164.629	173.932	45.3511	**38.7316**
0.9	0.2	1	2.04246	0.54432	0.71536	0.76671	0.45360	**0.17493**
		5	51.0603	13.6064	17.8825	19.1669	11.3378	**4.36863**
		10	204.240	54.4258	71.5303	76.6678	45.3511	**17.4744**
	0.5	1	2.04246	0.54432	1.13253	1.17274	0.35532	**0.20737**
		5	51.0603	13.6064	28.3137	29.3172	8.88079	**5.18017**
		10	204.240	54.4258	113.255	117.269	35.5233	**20.7204**
	0.8	1	2.04246	0.54432	1.64629	1.66561	0.28644	**0.23509**
		5	51.0603	13.6064	41.1575	41.6393	7.15659	**5.87244**
		10	204.240	54.4258	164.629	166.556	28.6263	**23.4896**

To get more information about this data you can see Woods et al. (1932). Some measures of this data are computed as: the variance inflation factors (VIFs) are $VIF_1 = 38.50$, $VIF_2 = 254.42$, $VIF_3 = 46.87$, $VIF_4 = 282.51$. The eigenvalues of S are $t_1 = 44676.206$, $t_2 = 5965.422$, $t_3 = 809.952$, $t_4 = 105.419$ and $t_5 = 0.001218$, and the condition number (CN) of S is approximately 6056.344. The VIFs, the eigenvalues, and the CN tell us that severe multicollinearity appears in this data.

Table 3 Estimated MSE for OLS, ORR, Liu, TP, MRT and DBR when $p=3$, $\rho=0.9$ and $n=100$

k	d	σ	OLS	ORR	Liu	TP	MRT	DBR
0.3	0.2	1	0.11172	0.10836	**0.10311**	0.10899	0.10762	0.10500
		5	2.79415	2.70826	**2.57649**	2.72538	2.69157	2.62552
		10	11.1765	10.8330	**10.3056**	10.9013	10.7663	10.5018
	0.5	1	0.11172	0.10836	0.10626	0.11004	0.10668	**0.10500**
		5	2.79415	2.70826	2.65702	2.75100	2.66689	**2.62594**
		10	11.1765	10.8330	10.6278	11.0041	10.6675	**10.5034**
	0.8	1	0.11172	0.10836	0.10951	0.11109	0.10573	**0.10510**
		5	2.79415	2.70826	2.73882	2.77683	2.64253	**2.62626**
		10	11.1765	10.8330	10.9552	11.1073	10.5702	**10.5050**
0.6	0.2	1	0.11172	0.10510	0.10311	0.10636	0.10384	**0.09880**
		5	2.79415	2.62657	2.57649	2.65965	2.59497	**2.47054**
		10	11.1765	10.5060	10.3056	10.6383	10.3796	**9.88186**
	0.5	1	0.11172	0.10510	0.10626	0.10836	0.10195	**0.09891**
		5	2.79415	2.62657	2.65702	2.70963	2.54866	**2.47201**
		10	11.1765	10.5060	10.6278	10.8385	10.1944	**9.88774**
	0.8	1	0.11172	0.10510	0.10951	0.11046	0.10017	**0.09901**
		5	2.79415	2.62657	2.73882	2.76013	2.50362	**2.47338**
		10	11.1765	10.5060	10.9552	11.0406	10.0143	**9.89341**
0.9	0.2	1	0.11172	0.10195	0.10311	0.10384	0.10017	**0.09313**
		5	2.79415	2.54866	2.57649	2.59675	2.50362	**2.32785**
		10	11.1765	10.1944	10.3056	10.3870	10.0143	**9.31108**
	0.5	1	0.11172	0.10195	0.10626	0.10678	0.09754	**0.09324**
		5	2.79415	2.54866	2.65702	2.66994	2.43841	**2.33079**
		10	11.1765	10.1944	10.6278	10.6795	9.75355	**9.32295**
	0.8	1	0.11172	0.10195	0.10951	0.10972	0.09513	**0.09345**
		5	2.79415	2.54866	2.73882	2.74407	2.37594	**2.33373**
		10	11.1765	10.1944	10.9552	10.9762	9.50334	**9.33460**

Using Equations (54), (55) and (56) to find the interval and the midpoint value of this interval for $\hat{k}$ and then to get the corresponding value of $\hat{d}$ for the proposed DBR estimator respectively i.e., for $i=1$, $143.910936<\hat{k}<144.142719$, the midpoint value of this interval is $\hat{k}=144.026828$ and the corresponding $\hat{d}=0.499999$, then $MSE(\hat{\alpha}_{DBR})=3892.2163$, for $i=2$, $281.405584<\hat{k}<288.042927$, the midpoint value of this interval is $\hat{k}=284.724256$ and the corresponding $\hat{d}=0.499934$, then $MSE(\hat{\alpha}_{DBR})=3892.9157$, for $i=3$, $11.413290<\hat{k}<11.493704$, the midpoint value of this interval is $\hat{k}=11.453497$ and the corresponding $\hat{d}=0.499994$, then $MSE(\hat{\alpha}_{DBR})=3890.0616$, for $i=4$, $1.236135<\hat{k}<1.243382$, the midpoint value of this interval is

$\hat{k} = 1.239759$ and the corresponding $\hat{d} = 0.499996$, then $MSE(\hat{\alpha}_{DBR}) = 3887.6948$, finally for $i = 5$, $0.000614 < \hat{k} < 0.000768$, the midpoint value of this interval is $\hat{k} = 0.000691$ and the corresponding $\hat{d} = 0.494889$, then $MSE(\hat{\alpha}_{DBR}) = 2170.9595$. Then, the best value is for $MSE(\hat{\alpha}_{DBR}) = 2170.9595$ when $\hat{k} = 0.000691$ and $\hat{d} = 0.494889$ i.e., the best MSE value of the proposed DBR estimator goes for the lowest interval $0.000614 < \hat{k} < 0.000768$ and the midpoint of this interval is $\hat{k} = 0.000691$. Moreover, the estimated parameters and the MSEs of the estimators using the given estimates of their biasing parameters are stated in Table 9. It appears that the proposed DBR estimator outperforms the best.

Table 4 Estimated MSE for OLS, ORR, Liu, TP, MRT and DBR when $p = 3$, $\rho = 0.99$ and $n = 100$

k	d	σ	OLS	ORR	Liu	TP	MRT	DBR
0.3	0.2	1	1.04086	0.78183	**0.55524**	0.83065	0.74203	0.59335
		5	26.0211	19.5465	**13.8818**	20.7655	18.5506	14.8333
		10	104.084	78.1862	**55.5270**	83.0623	74.2024	59.3333
	0.5	1	1.04086	0.78183	0.71925	0.90657	0.68785	**0.59997**
		5	26.0211	19.5465	17.9818	22.6653	17.1963	**15.0002**
		10	104.084	78.1862	71.9275	90.6609	68.7856	**60.0006**
	0.8	1	1.04086	0.78183	0.90499	0.98605	0.63955	**0.60627**
		5	26.0211	19.5465	22.6245	24.6503	15.9880	**15.1554**
		10	104.084	78.1862	90.4982	98.6012	63.9522	**60.6220**
0.6	0.2	1	1.04086	0.61015	0.55524	0.68691	0.55723	**0.37012**
		5	26.0211	15.2532	13.8818	17.1726	13.9305	**9.25165**
		10	104.084	61.0130	55.5270	68.6906	55.7221	**37.0065**
	0.5	1	1.04086	0.61015	0.71925	0.81081	0.49014	**0.38346**
		5	26.0211	15.2532	17.9818	20.2713	12.2521	**9.58597**
		10	104.084	61.0130	71.9275	81.0851	49.0086	**38.3438**
	0.8	1	1.04086	0.61015	0.90499	0.94531	0.43459	**0.39543**
		5	26.0211	15.2532	22.6245	23.6334	10.8643	**9.88533**
		10	104.084	61.0130	90.4982	94.5333	43.4570	**39.5413**
0.9	0.2	1	1.04086	0.49014	0.55524	0.58348	0.43459	**0.24654**
		5	26.0211	12.2521	13.8818	14.5867	10.8643	**6.16234**
		10	104.084	49.0086	55.5270	58.3469	43.4570	**24.6490**
	0.5	1	1.04086	0.49014	0.71925	0.73930	0.36771	**0.26323**
		5	26.0211	12.2521	17.9818	18.4815	9.19201	**6.57877**
		10	104.084	49.0086	71.9275	73.9264	36.7679	**26.3150**
	0.8	1	1.04086	0.49014	0.90499	0.91392	0.31531	**0.27783**
		5	26.0211	12.2521	22.6245	22.8481	7.88308	**6.94344**
		10	104.084	49.0086	90.4982	91.3924	31.5323	**27.7736**

Table 5 Estimated MSE for OLS, ORR, Liu, TP, MRT and DBR when $p=7$, $\rho=0.9$ and $n=50$

k	d	σ	OLS	ORR	Liu	TP	MRT	DBR
0.3	0.2	1	0.78078	0.70276	**0.60753**	0.71788	0.68890	0.63546
		5	19.5182	17.5687	**15.1886**	17.9480	17.2233	15.8858
		10	78.0728	70.2750	**60.7546**	71.7920	68.8934	63.5433
	0.5	1	0.78078	0.70276	0.66927	0.74109	0.66916	**0.63661**
		5	19.5182	17.5687	16.7319	18.5269	16.7291	**15.9158**
		10	78.0728	70.2750	66.9277	74.1075	66.9164	**63.6633**
	0.8	1	0.78078	0.70276	0.73489	0.76471	0.65047	**0.63777**
		5	19.5182	17.5687	18.3715	19.1176	16.2613	**15.9446**
		10	78.0728	70.2750	73.4862	76.4707	65.0455	**63.7785**
0.6	0.2	1	0.78078	0.63850	0.60753	0.66549	0.61593	**0.53046**
		5	19.5182	15.9631	15.1886	16.6374	15.3971	**13.2620**
		10	78.0728	63.8528	60.7546	66.5497	61.5882	**53.0482**
	0.5	1	0.78078	0.63850	0.66927	0.70728	0.58464	**0.53371**
		5	19.5182	15.9631	16.7319	17.6832	14.6159	**13.3435**
		10	78.0728	63.8528	66.9277	70.7330	58.4635	**53.3742**
	0.8	1	0.78078	0.63850	0.73489	0.75085	0.55629	**0.53676**
		5	19.5182	15.9631	18.3715	18.7704	13.9061	**13.4196**
		10	78.0728	63.8528	73.4862	75.0818	55.6243	**53.6787**
0.9	0.2	1	0.78078	0.58464	0.60753	0.62097	0.55629	**0.45150**
		5	19.5182	14.6159	15.1886	15.5239	13.9061	**11.2876**
		10	78.0728	58.4635	60.7546	62.0958	55.6243	**45.1502**
	0.5	1	0.78078	0.58464	0.66927	0.67819	0.51817	**0.45675**
		5	19.5182	14.6159	16.7319	16.9539	12.9546	**11.4178**
		10	78.0728	58.4635	66.9277	67.8157	51.8185	**45.6713**
	0.8	1	0.78078	0.58464	0.73489	0.73857	0.48468	**0.46147**
		5	19.5182	14.6159	18.3715	18.4654	12.1167	**11.5373**
		10	78.0728	58.4635	73.4862	73.8614	48.4671	**46.1488**

Also, to show the prediction performance of the proposed DBR estimator and other existing estimators with their biasing parameter estimators' values stated in Table 9, we calculate the prediction mean squared error (PMSE) and the mean absolute error (MAE) criteria which are defined respectively as $PMSE=\frac{1}{n}\sum_{i=1}^{n}(y_i-x_i'\widetilde{\beta})^2$ and $MAE=\frac{1}{n}\sum_{i=1}^{n}|y_i-x_i'\widetilde{\beta}|$ where x_i' is the i^{th} row vector of the matrix X and $\widetilde{\beta}$ is any estimator of β. So, the PMSE results of the above estimators are $PMSE(\hat{\alpha})=3.6818$, $PMSE(\hat{\alpha}_k)=3.9533$, $PMSE(\hat{\alpha}_d)=3.7957$, $PMSE(\hat{\alpha}_{TP})=3.7952$, $PMSE(\hat{\alpha}_{MRT})=3.7953$ and $PMSE(\hat{\alpha}_{DBR})=3.7950$, which indicate that the OLS and the proposed DBR estimators are better than others by the PMSE criterion. Moreover, the MAE results of the above estimators are $MAE(\hat{\alpha}) = 1.5871$, $MAE(\hat{\alpha}_k) = 1.5427$, $MAE(\hat{\alpha}_d) = 1.5617$, $MAE(\hat{\alpha}_{TP}) = 1.5581$, $MAE(\hat{\alpha}_{MRT}) = 1.5582$ and $MAE(\hat{\alpha}_{DBR}) = 1.5579$, which indicate that the ORR and the proposed DBR estimators are better than others by the MAE criterion. As a result, we can say that the proposed DBR estimator is the best by the MSE criterion and is better than most of other estimators by the PMSE and the MAE criteria because the performance of the estimators depends almost on the selection of the biasing parameters.

Table 6 Estimated MSE for OLS, ORR, Liu, TP, MRT and DBR when $p = 7$, $\rho = 0.99$ and $n = 50$

k	d	σ	OLS	ORR	Liu	TP	MRT	DBR
0.3	0.2	1	7.39074	3.50878	2.14221	4.14540	3.15052	**1.93935**
		5	184.769	87.7200	53.5558	103.634	78.7625	**48.4841**
		10	739.078	350.879	214.223	414.539	315.049	**193.936**
	0.5	1	7.39074	3.50878	3.70303	5.23131	2.71834	**2.04277**
		5	184.769	87.7200	92.5765	130.782	67.9594	**51.0684**
		10	739.078	350.879	370.305	523.134	271.837	**204.273**
	0.8	1	7.39074	3.50878	5.75274	6.47461	2.37741	**2.13328**
		5	184.769	87.7200	143.818	161.863	59.4358	**53.3332**
		10	739.078	350.879	575.274	647.457	237.743	**213.332**
0.6	0.2	1	7.39074	2.18767	2.14221	2.93958	1.87509	**0.88672**
		5	184.769	54.6919	53.5558	73.4891	46.8780	**22.1678**
		10	739.078	218.767	214.223	293.955	187.512	**88.6710**
	0.5	1	7.39074	2.18767	3.70303	4.33807	1.52481	**0.99057**
		5	184.769	54.6919	92.5765	108.451	38.1209	**24.7642**
		10	739.078	218.767	370.305	433.808	152.483	**99.0570**
	0.8	1	7.39074	2.18767	5.75274	6.06144	1.26850	**1.08034**
		5	184.769	54.6919	143.818	151.535	31.7124	**27.0082**
		10	739.078	218.767	575.274	606.143	126.849	**108.032**
0.9	0.2	1	7.39074	1.52481	2.14221	2.29530	1.26850	**0.49245**
		5	184.769	38.1209	53.5558	57.3828	31.7124	**12.3113**
		10	739.078	152.483	214.223	229.531	126.849	**49.2452**
	0.5	1	7.39074	1.52481	3.70303	3.82861	0.99393	**0.58191**
		5	184.769	38.1209	92.5765	95.7143	24.8479	**14.5476**
		10	739.078	152.483	370.305	382.856	99.3916	**58.1906**
	0.8	1	7.39074	1.52481	5.75274	5.81490	0.80209	**0.65898**
		5	184.769	38.1209	143.818	145.371	20.0508	**16.4733**
		10	739.078	152.483	575.274	581.487	80.2033	**65.8933**

Table 7 Estimated MSE for OLS, ORR, Liu, TP, MRT and DBR when $p=7$, $\rho=0.9$ and $n=100$

k	d	σ	OLS	ORR	Liu	TP	MRT	DBR
0.3	0.2	1	0.34272	0.33085	**0.31290**	0.33327	0.32854	0.31951
		5	8.56810	8.27190	**7.82292**	8.33059	8.21467	7.98787
		10	34.2722	33.0873	**31.2916**	33.3225	32.8584	31.9516
	0.5	1	0.34272	0.33085	0.32392	0.33673	0.32518	**0.31962**
		5	8.56810	8.27190	8.09802	8.41921	8.12994	**7.98945**
		10	34.2722	33.0873	32.3920	33.6769	32.5199	**31.9579**
	0.8	1	0.34272	0.33085	0.33516	0.34030	0.32182	**0.31962**
		5	8.56810	8.27190	8.37826	8.50836	8.04678	**7.99102**
		10	34.2722	33.0873	33.5132	34.0334	32.1871	**31.9642**
0.6	0.2	1	0.34272	0.31972	0.31290	0.32424	0.31542	**0.29851**
		5	8.56810	7.99207	7.82292	8.10547	7.88445	**7.46203**
		10	34.2722	31.9683	31.2916	32.4221	31.5379	**29.8480**
	0.5	1	0.34272	0.31972	0.32392	0.33106	0.30912	**0.29872**
		5	8.56810	7.99207	8.09802	8.27736	7.72747	**7.46760**
		10	34.2722	31.9683	32.3920	33.1093	30.9098	**29.8706**
	0.8	1	0.34272	0.31972	0.33516	0.33799	0.30303	**0.29893**
		5	8.56810	7.99207	8.37826	8.45113	7.57543	**7.47316**
		10	34.2722	31.9683	33.5132	33.8044	30.3018	**29.8927**
0.9	0.2	1	0.34272	0.30912	0.31290	0.31563	0.30303	**0.27940**
		5	8.56810	7.72747	7.82292	7.89180	7.57543	**6.98386**
		10	34.2722	30.9098	31.2916	31.5671	30.3018	**27.9355**
	0.5	1	0.34272	0.30912	0.32392	0.32571	0.29421	**0.27982**
		5	8.56810	7.72747	8.09802	8.14180	7.35640	**6.99531**
		10	34.2722	30.9098	32.3920	32.5673	29.4256	**27.9812**
	0.8	1	0.34272	0.30912	0.33516	0.33589	0.28591	**0.28024**
		5	8.56810	7.72747	8.37826	8.39611	7.14745	**7.00633**
		10	34.2722	30.9098	33.5132	33.5845	28.5899	**28.0253**

Table 8 Estimated MSE for OLS, ORR, Liu, TP, MRT and DBR when $p=7$, $\rho=0.99$ and $n=100$

k	d	σ	OLS	ORR	Liu	TP	MRT	DBR
0.3	0.2	1	3.25321	2.38717	**1.67674**	2.54887	2.25834	1.78122
		5	81.3297	59.6791	**41.9197**	63.7208	56.4591	44.5297
		10	325.318	238.716	**167.678**	254.883	225.836	178.118
	0.5	1	3.25321	2.38717	2.20405	2.80213	2.08488	**1.80432**
		5	81.3297	59.6791	55.1021	70.0538	52.1226	**45.1093**
		10	325.318	238.716	220.407	280.214	208.490	**180.437**
	0.8	1	3.25321	2.38717	2.80801	3.06841	1.93168	**1.82574**
		5	81.3297	59.6791	70.2000	76.7111	48.2920	**45.6447**
		10	325.318	238.716	280.799	306.844	193.167	**182.578**
0.6	0.2	1	3.25321	1.83918	1.67674	2.08771	1.67391	**1.09441**
		5	81.3297	45.9796	41.9197	52.1938	41.8476	**27.3594**
		10	325.318	183.918	167.678	208.774	167.390	**109.437**
	0.5	1	3.25321	1.83918	2.20405	2.49270	1.46632	**1.13757**
		5	81.3297	45.9796	55.1021	62.3174	36.6575	**28.4385**
		10	325.318	183.918	220.407	249.269	146.629	**113.753**
	0.8	1	3.25321	1.83918	2.80801	2.93611	1.29622	**1.17600**
		5	81.3297	45.9796	70.2000	73.4038	32.4066	**29.3992**
		10	325.318	183.918	280.799	293.614	129.626	**117.596**
0.9	0.2	1	3.25321	1.46632	1.67674	1.76400	1.29622	**0.72492**
		5	81.3297	36.6575	41.9197	44.1010	32.4066	**18.1217**
		10	325.318	146.629	167.678	176.403	129.626	**72.4871**
	0.5	1	3.25321	1.46632	2.20405	2.26653	1.09315	**0.77658**
		5	81.3297	36.6575	55.1021	56.6639	27.3284	**19.4145**
		10	325.318	146.629	220.407	226.655	109.313	**77.6582**
	0.8	1	3.25321	1.46632	2.80801	2.83615	0.93534	**0.82173**
		5	81.3297	36.6575	70.2000	70.9043	23.3826	**20.5433**
		10	325.318	146.629	280.799	283.616	93.5308	**82.1733**

Table 9 The results of the regression coe□cients and the corresponding MSEs

Coef.	OLS	ORR	Liu	TP	MRT	DBR
α_0	62.4053	8.58715	27.6657	27.6200	27.6229	27.6204
α_1	1.55110	2.10461	1.90080	1.9089	1.9089	1.90891
α_2	0.51016	1.06484	0.86996	0.8687	0.8686	0.86867
α_3	0.10190	0.66808	0.46192	0.4679	0.4679	0.467889
α_4	−0.14406	0.39959	0.20801	0.2073	0.2073	0.207317
MSE	4912.0902	2989.8202	2170.9669	2170.9600	2170.9601	2170.9595
k	−	0.007676	−	0.0015419	0.001534	0.000691
d	−	−	0.442224	0.001536	0.001536	0.494889

5. Concluding Remarks

In this paper, we proposed a new kind of two-parameter regression estimator, namely, the Dawoud Biased Regression (DBR) estimator to tackle the multicollinearity problem. We theoretically compared the proposed DBR estimator with some existing estimators, for examples, the ordinary least squares (OLS), the ordinary ridge regression (ORR), the Liu, the two-parameter (TP) of Ozkale and Kaçiranlar (2007), and the modified ridge type (MRT) estimators and then we found the biasing parameters k and d. A simulation study is done for comparing the proposed DBR estimator performance with other mentioned estimators. The main finding of the performed simulation is that the proposed DBR estimator gives better results than other estimators under some specific conditions. Also, real-life data is used and analyzed for confirming the DBR estimator performance and the mentioned estimators using the mean squares error criterion and then we calculated the prediction mean squared error and the mean absolute error criteria to clarify the prediction performance of the proposed DBR estimator and other estimators. Finally, we can say that the proposed DBR estimator is the best among others in many cases by using different criteria and the performance of the estimators depends almost on the selection of the biasing parameters. So, we encourage authors to propose many different estimators of the biasing parameters for the proposed DBR estimator and discuss their performances for future studies.

References

Akdeniz F, Kaçiranlar S. On the almost unbiased generalized Liu estimator and unbiased estimation of the bias and mse. Comm Stat-Theory Methods. 1995; 24(7): 1789-1797.

Akdeniz F, Roozbeh M. Efficiency of the generalized difference- based weighted mixed almost unbiased two parameter estimator in Partially Linear Model. Comm Stat-Theory Methods. 2017; 46: 12259-12280.

Akdeniz F, Roozbeh M. Generalized di□erence- based weighted mixed almost unbiased ridge estimator in partially linear models. Statist Papers. 2019; 60(5): 1717-1739.

Amini M, Roozbeh M. Optimal partial ridge estimation in restricted semiparametric regression models. J Multivariate Analysis. 2015; 136: 26-40.

Dawoud I, Kibria BMG. A new biased estimator to combat the multicollinearity of the Gaussian linear regression model. Stats. 2020a; 3(4): 526-541.

Dawoud I, Kibria BMG. A new two parameter biased estimator for the unrestricted linear regression model: theory, simulation and application. Int J Clin Biostat Biom. 2020b; 6: 031.

Farebrother RW. Further results on the mean square error of ridge regression. J R Stat Soc B. 1976; 38(3): 248-250.

Gibbons DG. A simulation study of some ridge estimators. J Am Stat Assoc. 1981; 76(373): 131-139.

Hoerl AE, Kennard RW. Ridge regression: biased estimation for nonorthogonal problems. Technometrics. 1970; 12(1): 55-67.

Hoerl AE, Kannard RW, Baldwin KF. Ridge regression: some simulations. Commun Stat. 1975; 4(2): 105-123.

Kaciranlar S, Sakallioglu S, Akdeniz F, Styan GPH, Werner HJ. A new biased estimator in linear regression and a detailed analysis of the widely- analysed dataset on Portland cement. Sankhya: Indian J Stat B. 1999; 61: 443-459.

Kan B, Alpu O, Yazıcı B. Robust ridge and robust Liu estimator for regression based on the LTS estimator. J Appl Stat. 2013; 40(3): 644-655.

Khalaf G, Shukur G. Choosing ridge parameter for regression problems. Commun Stat-Theory Methods. 2005; 34(5): 1177-1182.

Kibria BMG. Performance of some new ridge regression estimators. Commun Stat-Simul Comput.2003; 32(1): 419-435.

Kibria BMG, Banik S. Some ridge regression estimators and their performances. J M Appl Stat Methods. 2016; 15(1): 206-238.

Liu K. A new class of biased estimate in linear regression. Commun Stat Theory Methods. 1993; 22: 393-402.

Li Y, Yang H. Anew Liu-type estimator in linear regression model. Stat Pap. 2012; 53(2): 427-437.

Lukman AF, Ayinde K. Review and classifications of the ridge parameter estimation techniques. Hacet J Math Stat. 2017; 46(5): 953-967.

Lukman AF, Ayinde K, Binuomote S, Clement OA. Modified ridge- type estimator to combat multicollinearity: application to chemical data. J Chemom. 2019a; 33(5): e3125.

Lukman AF, Ayinde K, Sek SK, Adewuyi E. A modified new two- parameter estimator in a linear regression model. Model Simul Eng. 2019b; Article ID 6342702: 10 pages.

Månsson K, Kibria BMG, Shukur G. Performance of some weighted Liu estimators for logit regression model: an application to Swedish accident data. Commun Stat-Theory Methods. 2015; 44(2): 363-375.

Massy W F. Principal components regression in exploratory statistical research. J Am Stat Assoc. 1965; 60(309): 234-256.

Mayer LS, Willke TA. On biased estimation in linear models. Technometrics. 1973; 15(3): 497-508.

Newhouse JP, Oman SD. An evaluation of ridge estimators, a report prepared for United States air force project RAND. 1971.

Ozkale MR, Kaçiranlar S. The restricted and unrestricted two- parameter estimators. Commun Stat-Theory Methods. 2007; 36(15): 2707-2725.

Roozbeh M. Optimal QR-based estimation in partially linear regression models with correlated errors using GCV criterion. Comput Stat Data Anal. 2018; 117: 45–61.

Roozbeh M, Hesamian G, Akbari MG. Ridge estimation in semi-parametric regression models under the stochastic restriction and correlated elliptically contoured errors. J Comput Appl Math. 2020; 378: 112940.

Roozbeh M, Hamzah NA. Uncertain stochastic ridge estimation in partially linear regression models with elliptically distributed errors. J Theor Appl Stat. 2020; 54: 494-523.

Sakallıoglu S, Kaçıranlar S. A new biased estimator based on ridge estimation. Stat Pap. 2008; 49(4): 669-689.

Stein C. Inadmissibility of the usual estimator for mean of multivariate normal distribution. In Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability: Contributions to the Theory of Statistics, 1956; 1: 197-206.

Swindel BF. Good ridge estimators based on prior information. Commun Stat-Theory Methods. 1976; 5(11): 1065-1075.

Trenkler G, Toutenburg H. Mean squared error matrix comparisons between biased estimators- an overview of recent results. Statist Papers. 1990; 31(1): 165-179.

Wichern DW, Churchill GA. A comparison of ridge estimators. Technometrics. 1978; 20(3): 301-311.

Woods H, Steinour HH, Starke HR. E□ect of composition of Portland cement on heat evolved during hardening. Ind Eng Chem. 1932; 24(11): 1207-1214.

Yang H, Chang X. A new two- parameter estimator in linear regression. Commun Stat-Theory Methods. 2010; 39(6): 923-934.