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Efficient Class of Estimators for Population Variance using Auxiliary Information in Presence of Non-response

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Abstract

This paper has great potential to estimate the population variance in presence of non-response. Improved class of estimators is suggested, and their properties are discussed. To prove the theoretical findings, simulation studies are carried out and empirical studies are also performed on real data sets, which show the dominance of the proposed class of estimators over traditional ratio estimator under non-response. Results are interpreted and suitable recommendations have been made to the survey practitioners.

Keywords: Non response, Hansen and Hurwitz, bias, mean square error, simulation study.

1. Introduction

In sample surveys, the use of auxiliary information has shown its significance in improving the precision of estimates of population parameters such as population mean, population variance, population median etc. The estimation of finite population variance attracts the attention of survey practitioners for several practical applications. In recent years, Srivastava and Jhajj (1980), Isaki (1983), Upadhyaya and Singh (1999), Kadilar and Cingi (2006), Khoshnevisan et al. (2007), Singh and Vishwakarma (2008), Khan and Sabbir (2013), Yadav and Kadilar (2013). Shabbir and Gupta (2014), Subramani and Kumarapandiyan (2012, 2015), Saini (2015), Pal and Singh (2017), Saini and Kumar (2017), Yadav and Subramani (2019), Yadav et al. (2019) and Saini and Kumar (2019) among others proposed the ratio, product and regression type estimators for estimating population variance by utilizing the auxiliary information to increase the efficiency of the estimator.

It is profound phenomenon in sample survey; the problem of non-response is inevitable fact, which causes to decrease the quality of data and challenge for survey practitioners. To overcome such difficulties in sample surveys, Hansen and Hurwitz (1946) originated an effective sub sampling method to reduce the non-response. Some works have been done by Sinha and Kumar (2015), Bhat et al. (2018), Shahzad et al. (2017) to reduce the effect of non-response by using Hansen and Hurwitz (1946) technique in the estimation of population variance.

Motivated with the above works, in this paper we have proposed efficient class of estimators for estimating population variance in presence of non-response, when non-responses observed only on study variable and when non-response is observed on study and auxiliary variable both. The properties of the proposed class of estimators have been studies and their efficacies are demonstrated through simulation and empirical studies. Suitable recommendations are put forward for survey practitioners.

2. Notations

Consider a finite population $U=\{1,2,3,...,N\}$ having N units and let y_i and x_i respectively be the values of study variable Y and auxiliary variable X associated with i^{th} unit. Let N_1 and N_2 be the number of units in the population that belong to the response and non- response class respectively. Let a sample of size n has been drawn from the population in which n_1 units responds and $n_2=n-n_1$ units do not respond. A sub-sample of size $r=n_2 f_h^{-1}(f_h \geq 1)$ from non-responding units has been drawn using simple random sampling without replacement (SRSWOR) method. We adopted the following notations for their further use:

$\bar{X}(\bar{Y})$: Population means of the auxiliary (study) variable $x(y)$

$\bar{x}(\bar{y})$: Sample mean of the variable x and y, respectively

$s_x^2(s_y^2)$: Sample variance of the variable $x(y)$

$s_x^{*2}(s_y^{*2})$: Sample variance of the variable $x(y)$ in presence of non-response

C_y, C_x : Population coefficient of variation for the variables shown in subscripts

ρ_{yx} : Population correlation coefficient between the variables y and x

D_i : The i^{th} decile of the auxiliary variable x

M_d : Median of the auxiliary variable x

$Q_i\ (i=1,2,3)$ denote the first, second and third quartile of the auxiliary variable x. Q_r, Q_d and Q_a are the functions of quartiles defined as

$$Q_r=(Q_3-Q_1), Q_d=\frac{(Q_3-Q_1)}{2}, Q_a=\frac{(Q_3+Q_1)}{2} \text{ and } \pi=\left(\frac{1}{n}-\frac{1}{N}\right).$$

To estimate the population variance of y, we consider the s_y^{*2} as in Sinha and Kumar (2015) and variance defined as $V\left(s_y^{*2}\right)=\left[\pi\left(S_y^2\right)^2\left\{\beta_{2(y)}-1\right\}+W_2\frac{f_h-1}{n}\left(S_{y(2)}^2\right)^2\left(\beta_{2(2y)}-1\right)\right]$,

where $S_y^2=\frac{1}{N-1}\sum_{U_N}\left(y_i-\bar{Y}\right)^2$, $S_{y(2)}^2=\frac{1}{N_2-1}\sum_{U_{N_2}}\left(y_i-\bar{Y}_{(2)}\right)^2$, $\bar{Y}_{(2)}=\frac{1}{N_2}\sum_{U_{N_2}} y_i$,

$\bar{X}_{(2)}=\frac{1}{N_2}\sum_{U_{N_2}} x_i$, $\beta_2(y)=\frac{\mu_{40}}{\mu_{20}^2}$, $\beta_{2(2)}(y)=\frac{\mu_{40(2)}}{\mu_{20(2)}^2}$, $\beta_2(x)=\frac{\mu_{04}}{\mu_{02}^2}$, $\beta_{2(2)}(x)=\frac{\mu_{04(2)}}{\mu_{02(2)}^2}$, $W_2=\frac{N_2}{N}$,

$\mu_{rs}=\frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^r\left(x_i-\bar{X}\right)^s$, $\mu_{rs(2)}=\frac{1}{N_2-1}\sum_{U_{N_2}}\left(y_i-\bar{Y}_{(2)}\right)^r\left(x_i-\bar{X}_{(2)}\right)^s$, $\lambda_{22}=\frac{\mu_{22}}{\mu_{20}\mu_{02}}$, and

$\lambda_{22(2)}=\frac{\mu_{22(2)}}{\mu_{20(2)}\mu_{02(2)}}$.

Bias and mean square error of the suggested class of estimators in presence of non-response is solved up to $O(n^{-1})$, under the transformation given as

$$s_y^{*2} = S_y^2(1+e_0); \quad s_x^2 = S_x^2(1+e_1); \quad s_x^{*2} = S_x^2(1+e_2).$$

Such that $E(e_i) = 0$ and $|e_i| \le 1, \forall\, i = 0,1,2$.

$$E(e_0^2) = \left\{\pi\left(\beta_{2(y)}-1\right)+\frac{\left(S_{y(2)}^2\right)^2}{S_y^4}W_2\frac{f_h-1}{n}\left(\beta_{2(2y)}-1\right)\right\},\ E(e_1^2)=\pi\left\{\beta_{2(x)}-1\right\},$$

$$E(e_2^2) = \left\{\pi\left(\beta_{2(x)}-1\right)+\frac{\left(S_{x(2)}^2\right)^2}{S_x^4}W_2\frac{f_h-1}{n}\left(\beta_{2(2x)}-1\right)\right\},\ E(e_0e_1)=\pi\left\{\lambda_{22}-1\right\},$$

$$E(e_0e_2) = \left\{\pi\left(\lambda_{22}-1\right)+\frac{S_{y(2)}^2 S_{x(2)}^2}{S_y^2 S_x^2}W_2\frac{f_h-1}{n}\left(\lambda_{22(2)}-1\right)\right\}.$$

3. Proposed Class of Estimators

Following Khoshnevisan et al. (2007), the class of estimators for estimating population variance S_y^2 in presence of non-response are given as

Case 1: When non-response on study variable Y,

$$\gamma_y^* = s_y^{*2}\left[\frac{a S_x^2 + b}{\alpha\left(a s_x^2+b\right)+(1-\alpha)\left(a S_x^2+b\right)}\right]^g, \tag{1}$$

where α, g are suitable chosen constants and $a(a \ne 0)$, b are either real numbers or function of known parameters of the auxiliary variable X.

By using the above mentioned transformations, the estimator γ_y^* makes the form as follows

$$\gamma_y^* = S_y^2(1+e_0)(1+\alpha\lambda e_1)^{-g} = S_y^2\left[1+e_0-\alpha\lambda g e_1+\frac{g(g+1)}{2}\alpha^2\lambda^2e_1^2-\alpha\lambda g e_0 e_1\right], \tag{2}$$

where $\lambda = \dfrac{aS_x^2}{aS_x^2+b}$. Bias of the proposed estimator is as follow

$$B\left(\gamma_y^*\right) = S_y^2\left[\alpha^2\lambda^2\pi g(g+1)\frac{1}{2}\left\{\beta_{2(x)}-1\right\}-\alpha\lambda g\pi\left(\lambda_{22}-1\right)\right]. \tag{3}$$

Squaring both sides in (2) and ignoring higher order terms

$$\left(\gamma_y^*-S_y^2\right)^2 = S_y^4\left(e_0-\alpha\lambda g e_1\right)^2 = S_y^4\left(e_0^2+\alpha^2\lambda^2g^2e_1^2-2\alpha\lambda g e_0 e_1\right).$$

Taking expectations on both sides putting the values mean square error of the proposed estimator is as follows

$$MSE\left(\gamma_y^*\right) = S_y^4\left[\left\{\pi\left(\beta_{2(y)}-1\right)+\frac{\left(S_{y(2)}^2\right)^2}{S_y^4}W_2\frac{f_h-1}{n}\left(\beta_{2(2y)}-1\right)\right\}+\alpha^2\lambda^2g^2\pi\left(\beta_{2(x)}-1\right)-2\alpha\lambda g\pi\left(\lambda_{22}-1\right)\right]. \tag{4}$$

Partially differentiating (4) with respect to α, the optimum value of α is as follows

$$\alpha_{opt}=\frac{(\lambda_{22}-1)}{\lambda g\left(\beta_{2(x)}-1\right)}.$$

Putting the value of α in (4), we get the minimum mean square error of the proposed estimator is as follows

$$MSE\left(\gamma_y^*\right)_{opt}=S_y^4\left[\pi\left(\beta_{2(y)}-1\right)+\frac{\left(S_{y(2)}^2\right)^2}{S_y^4}W_2\frac{f_h-1}{n}\left(\beta_{2(2y)}-1\right)-\frac{\pi(\lambda_{22}-1)^2}{\left(\beta_{2(x)}-1\right)}\right]. \quad (5)$$

Case 2: Non response on study and auxiliary variable Y and X both

$$\gamma_{yx}^*=s_y^{*2}\left[\frac{a\,S_x^2+b}{\alpha\left(a\,s_x^{*2}+b\right)+(1-\alpha)\left(a\,S_x^2+b\right)}\right]^g, \quad (6)$$

where $\alpha,\ g$ are suitable chosen constants and $a\,(a\neq 0),\ b$ are either real numbers or function of known parameters of the auxiliary variable X. By using the above mentioned transformations, we have

$$\gamma_{yx}^*=S_y^2(1+e_0)(1+\alpha\lambda e_2)^{-g}=S_y^2\left[1+e_0-\alpha\lambda g e_2+\frac{g(g+1)}{2}\alpha^2\lambda^2e_2^2-\alpha\lambda g e_0e_2\right], \quad (7)$$

where $\lambda=\dfrac{aS_x^2}{aS_x^2+b}$. Bias of the proposed estimator is as follow

$$B\left(\gamma_{yx}^*\right)=S_y^2\left[\begin{array}{l} g(g+1)\frac{1}{2}\alpha^2\lambda^2\left\{\pi\left(\beta_{2(x)}-1\right)+\frac{\left(S_{x(2)}^2\right)^2}{S_x^4}W_2\frac{f_h-1}{n}\left(\beta_{2(2x)}-1\right)\right\}\\ -\alpha\lambda g\left\{\pi(\lambda_{22}-1)+\frac{S_{y(2)}^2S_{x(2)}^2}{S_y^2S_x^2}W_2\frac{f_h-1}{n}\left(\lambda_{22(2)}-1\right)\right\}\end{array}\right]. \quad (8)$$

Squaring both sides in (7) and ignoring higher order terms

$$\left(\gamma_{yx}^*-S_y^2\right)^2=S_y^4(e_0-\alpha\lambda g e_2)^2=S_y^4\left(e_0^2+\alpha^2\lambda^2g^2e_2^2-2\alpha\lambda g e_0e_2\right).$$

Taking expectations on both sides putting the values, mean square error of the proposed estimator is as follows

$$MSE\left(\gamma_{yx}^*\right)=S_y^4\left[\begin{array}{l}\left\{\pi\left(\beta_{2(y)}-1\right)+\frac{\left(S_{y(2)}^2\right)^2}{S_y^4}W_2\frac{f_h-1}{n}\left(\beta_{2(2y)}-1\right)\right\}+\alpha^2\lambda^2g^2\left\{\begin{array}{l}\pi\left(\beta_{2(x)}-1\right)+\\ \frac{\left(S_{x(2)}^2\right)^2}{S_x^4}W_2\frac{f_h-1}{n}\left(\beta_{2(2x)}-1\right)\end{array}\right\}\\ -2\alpha\lambda g\left\{\pi(\lambda_{22}-1)+\frac{S_{y(2)}^2S_{x(2)}^2}{S_y^2S_x^2}W_2\frac{f_h-1}{n}\left(\lambda_{22(2)}-1\right)\right\}\end{array}\right]. \quad (9)$$

Partially differentiating (9) with respect to α,

$$\alpha_{opt} = \frac{\left\{\pi\left(\lambda_{22}-1\right)+\frac{S_{y(2)}^{2}S_{x(2)}^{2}}{S_{y}^{4}S_{x}^{4}}W_{2}\frac{f_{h}-1}{n}\left(\lambda_{22(2)}-1\right)\right\}}{\lambda g\left\{\pi\left(\beta_{2(x)}-1\right)+\frac{\left(S_{x(2)}^{2}\right)^{2}}{S_{x}^{4}}W_{2}\frac{f_{h}-1}{n}\left(\beta_{2(2x)}-1\right)\right\}}.$$

Putting the value of α in (9), we get the minimum mean square error of the proposed estimator is as follows

$$MSE\left(\gamma_{yx}^{*}\right)_{opt} = S_{y}^{4}\left[\left\{\pi\left(\beta_{2(y)}-1\right)+\frac{\left(S_{y(2)}^{2}\right)^{2}}{S_{y}^{4}}W_{2}\frac{f_{h}-1}{n}\left(\beta_{2(2y)}-1\right)\right\}-\frac{\left\{\pi\left(\lambda_{22}-1\right)+\frac{S_{y(2)}^{2}S_{x(2)}^{2}}{S_{y}^{2}S_{x}^{2}}W_{2}\frac{f_{h}-1}{n}\left(\lambda_{22(2)}-1\right)\right\}}{\left\{\pi\left(\beta_{2(x)}-1\right)+\frac{\left(S_{x(2)}^{2}\right)^{2}}{S_{x}^{4}}W_{2}\frac{f_{h}-1}{n}\left(\beta_{2(2x)}-1\right)\right\}}\right]. \tag{10}$$

In Tables 1 and 2, member of suggested class of estimators $\Upsilon_i^{(1)}$ and $\Upsilon_i^{(2)}$ are listed as follows

Table 1 Members of the suggested class of estimators $\Upsilon_i^{(1)}$

Values of constants				Estimator
α	a	b	g	
1	1	0	1	$\Upsilon_1^{(1)} = s_y^{*2}\left[S_x^2/s_x^2\right]$ The Usual ratio estimator
1	1	C_x	1	$\Upsilon_2^{(1)} = s_y^{*2}\left[S_x^2+C_x/s_x^2+C_x\right]$ Kadilar and Cingi (2006)
1	1	$\beta_{2(x)}$	1	$\Upsilon_3^{(1)} = s_y^{*2}\left[S_x^2+\beta_{2(x)}/s_x^2+\beta_{2(x)}\right]$ Upadhaya and Singh (1999)
1	1	$\beta_{1(x)}$	1	$\Upsilon_4^{(1)} = s_y^{*2}\left[S_x^2+\beta_{1(x)}/s_x^2+\beta_{1(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	1	ρ	1	$\Upsilon_5^{(1)} = s_y^{*2}\left[S_x^2+\rho/s_x^2+\rho\right]$ Subramani and Kumarapandiyan (2015)
1	1	S_x	1	$\Upsilon_6^{(1)} = s_y^{*2}\left[S_x^2+S_x/s_x^2+S_x\right]$ Subramani and Kumarapandiyan (2015)
1	1	M_d	1	$\Upsilon_7^{(1)} = s_y^{*2}\left[S_x^2+M_d/s_x^2+M_d\right]$ Subramani and Kumarapandiyan (2012)
1	1	Q_1	1	$\Upsilon_8^{(1)} = s_y^{*2}\left[S_x^2+Q_1/s_x^2+Q_1\right]$ Subramani and Kumarapandiyan (2012)
1	1	Q_3	1	$\Upsilon_9^{(1)} = s_y^{*2}\left[S_x^2+Q_3/s_x^2+Q_3\right]$ Subramani and Kumarapandiyan (2012)
1	1	Q_r	1	$\Upsilon_{10}^{(1)} = s_y^{*2}\left[S_x^2+Q_r/s_x^2+Q_r\right]$ Subramani and Kumarapandiyan (2012)

Table 1 (Continued)

Values of constants				Estimator
α	a	b	g	
1	1	Q_d	1	$\Upsilon_{11}^{(1)} = s_y^{*2}\left[S_x^2 + Q_d / s_x^2 + Q_d\right]$ Subramani and Kumarapandiyan (2012)
1	1	Q_a	1	$\Upsilon_{12}^{(1)} = s_y^{*2}\left[S_x^2 + Q_a / s_x^2 + Q_a\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_1	1	$\Upsilon_{13}^{(1)} = s_y^{*2}\left[S_x^2 + D_1 / s_x^2 + D_1\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_2	1	$\Upsilon_{14}^{(1)} = s_y^{*2}\left[S_x^2 + D_2 / s_x^2 + D_2\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_3	1	$\Upsilon_{15}^{(1)} = s_y^{*2}\left[S_x^2 + D_3 / s_x^2 + D_3\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_4	1	$\Upsilon_{16}^{(1)} = s_y^{*2}\left[S_x^2 + D_4 / s_x^2 + D_4\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_5	1	$\Upsilon_{17}^{(1)} = s_y^{*2}\left[S_x^2 + D_5 / s_x^2 + D_5\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_6	1	$\Upsilon_{18}^{(1)} = s_y^{*2}\left[S_x^2 + D_6 / s_x^2 + D_6\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_7	1	$\Upsilon_{19}^{(1)} = s_y^{*2}\left[S_x^2 + D_7 / s_x^2 + D_7\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_8	1	$\Upsilon_{20}^{(1)} = s_y^{*2}\left[S_x^2 + D_8 / s_x^2 + D_8\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_9	1	$\Upsilon_{21}^{(1)} = s_y^{*2}\left[S_x^2 + D_9 / s_x^2 + D_9\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_{10}	1	$\Upsilon_{22}^{(1)} = s_y^{*2}\left[S_x^2 + D_{10} / s_x^2 + D_{10}\right]$ Subramani and Kumarapandiyan (2012)
1	$\beta_{2(x)}$	C_x	1	$\Upsilon_{23}^{(1)} = s_y^{*2}\left[\beta_{2(x)} S_x^2 + C_x / \beta_{2(x)} s_x^2 + C_x\right]$ Kadilar and Cingi (2006)
1	C_x	$\beta_{2(x)}$	1	$\Upsilon_{24}^{(1)} = s_y^{*2}\left[C_x S_x^2 + \beta_{2(x)} / C_x s_x^2 + \beta_{2(x)}\right]$ Kadilar and Cingi (2006)
1	$\beta_{1(x)}$	C_x	1	$\Upsilon_{25}^{(1)} = s_y^{*2}\left[\beta_{1(x)} S_x^2 + C_x / \beta_{1(x)} s_x^2 + C_x\right]$ Subramani and Kumarapandiyan (2015)
1	C_x	$\beta_{1(x)}$	1	$\Upsilon_{26}^{(1)} = s_y^{*2}\left[C_x S_x^2 + \beta_{1(x)} / C_x s_x^2 + \beta_{1(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	ρ	C_x	1	$\Upsilon_{27}^{(1)} = s_y^{*2}\left[\rho S_x^2 + C_x / \rho s_x^2 + C_x\right]$ Subramani and Kumarapandiyan (2015)

Table 1 (Continued)

Values of constants				Estimator
α	a	b	g	
1	C_x	ρ	1	$\Upsilon_{28}^{(1)} = s_y^{*2}\left[C_x S_x^2 + \rho / C_x s_x^2 + \rho\right]$ Subramani and Kumarapandiyan (2015)
1	S_x	C_x	1	$\Upsilon_{29}^{(1)} = s_y^{*2}\left[S_x S_x^2 + C_x / S_x s_x^2 + C_x\right]$ Subramani and Kumarapandiyan (2015)
1	C_x	S_x	1	$\Upsilon_{30}^{(1)} = s_y^{*2}\left[C_x S_x^2 + S_x / C_x s_x^2 + S_x\right]$ Subramani and Kumarapandiyan (2015)
1	M_d	C_x	1	$\Upsilon_{31}^{(1)} = s_y^{*2}\left[M_d S_x^2 + C_x / M_d s_x^2 + C_x\right]$ Subramani and Kumarapandiyan (2015)
1	C_x	M_d	1	$\Upsilon_{32}^{(1)} = s_y^{*2}\left[C_x S_x^2 + M_d / C_x s_x^2 + M_d\right]$ Subramani and Kumarapandiyan (2013)
1	$\beta_{1(x)}$	$\beta_{2(x)}$	1	$\Upsilon_{33}^{(1)} = s_y^{*2}\left[\beta_{1(x)} S_x^2 + \beta_{2(x)} / \beta_{1(x)} s_x^2 + \beta_{2(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	$\beta_{2(x)}$	$\beta_{1(x)}$	1	$\Upsilon_{34}^{(1)} = s_y^{*2}\left[\beta_{2(x)} S_x^2 + \beta_{1(x)} / \beta_{2(x)} s_x^2 + \beta_{1(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	ρ	$\beta_{2(x)}$	1	$\Upsilon_{35}^{(1)} = s_y^{*2}\left[\rho S_x^2 + \beta_{2(x)} / \rho s_x^2 + \beta_{2(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	$\beta_{2(x)}$	ρ	1	$\Upsilon_{36}^{(1)} = s_y^{*2}\left[\beta_{2(x)} S_x^2 + \rho / \beta_{2(x)} s_x^2 + \rho\right]$ Subramani and Kumarapandiyan (2015)
1	S_x	$\beta_{2(x)}$	1	$\Upsilon_{37}^{(1)} = s_y^{*2}\left[S_x S_x^2 + \beta_{2(x)} / S_x s_x^2 + \beta_{2(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	$\beta_{2(x)}$	S_x	1	$\Upsilon_{38}^{(1)} = s_y^{*2}\left[\beta_{2(x)} S_x^2 + S_x / \beta_{2(x)} s_x^2 + S_x\right]$ Subramani and Kumarapandiyan (2015)
1	M_d	$\beta_{2(x)}$	1	$\Upsilon_{39}^{(1)} = s_y^{*2}\left[M_d S_x^2 + \beta_{2(x)} / M_d s_x^2 + \beta_{2(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	$\beta_{2(x)}$	M_d	1	$\Upsilon_{40}^{(1)} = s_y^{*2}\left[\beta_{2(x)} S_x^2 + M_d / \beta_{2(x)} s_x^2 + M_d\right]$ Subramani and Kumarapandiyan (2015)
1	ρ	$\beta_{1(x)}$	1	$\Upsilon_{41}^{(1)} = s_y^{*2}\left[\rho S_x^2 + \beta_{1(x)} / \rho s_x^2 + \beta_{1(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	$\beta_{1(x)}$	ρ	1	$\Upsilon_{42}^{(1)} = s_y^{*2}\left[\beta_{1(x)} S_x^2 + \rho / \beta_{1(x)} s_x^2 + \rho\right]$ Subramani and Kumarapandiyan (2015)
1	S_x	$\beta_{1(x)}$	1	$\Upsilon_{43}^{(1)} = s_y^{*2}\left[S_x S_x^2 + \beta_{1(x)} / S_x s_x^2 + \beta_{1(x)}\right]$ Subramani and Kumarapandiyan (2015)

Table 1 (Continued)

Values of constants				Estimator
α	a	b	g	
1	$\beta_{1(x)}$	S_x	1	$\Upsilon_{44}^{(1)} = s_y^{*2}\left[\beta_{1(x)}S_x^2 + S_x \big/ \beta_{1(x)}s_x^2 + S_x\right]$ Subramani and Kumarapandiyan (2015)
1	M_d	$\beta_{1(x)}$	1	$\Upsilon_{45}^{(1)} = s_y^{*2}\left[M_d S_x^2 + \beta_{1(x)} \big/ M_d s_x^2 + \beta_{1(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	$\beta_{1(x)}$	M_d	1	$\Upsilon_{46}^{(1)} = s_y^{*2}\left[\beta_{1(x)}S_x^2 + M_d \big/ \beta_{1(x)}s_x^2 + M_d\right]$ Subramani and Kumarapandiyan (2015)
1	S_x	ρ	1	$\Upsilon_{47}^{(1)} = s_y^{*2}\left[S_x S_x^2 + \rho \big/ S_x s_x^2 + \rho\right]$ Subramani and Kumarapandiyan (2015)
1	ρ	S_x	1	$\Upsilon_{48}^{(1)} = s_y^{*2}\left[\rho S_x^2 + S_x \big/ \rho s_x^2 + S_x\right]$ Subramani and Kumarapandiyan (2015)
1	M_d	ρ	1	$\Upsilon_{49}^{(1)} = s_y^{*2}\left[M_d S_x^2 + \rho \big/ M_d s_x^2 + \rho\right]$ Subramani and Kumarapandiyan (2015)
1	ρ	M_d	1	$\Upsilon_{50}^{(1)} = s_y^{*2}\left[\rho S_x^2 + M_d \big/ \rho s_x^2 + M_d\right]$ Subramani and Kumarapandiyan (2015)
1	M_d	S_x	1	$\Upsilon_{51}^{(1)} = s_y^{*2}\left[M_d S_x^2 + S_x \big/ M_d s_x^2 + S_x\right]$ Subramani and Kumarapandiyan (2015)
1	S_x	M_d	1	$\Upsilon_{52}^{(1)} = s_y^{*2}\left[S_x S_x^2 + M_d \big/ S_x s_x^2 + M_d\right]$ Subramani and Kumarapandiyan (2015)

Table 2 Members of the suggested class of estimators $\Upsilon_i^{(2)}$

Values of constants				Estimator
α	a	b	g	
1	1	0	1	$\Upsilon_1^{(2)} = s_y^{*2}\left[S_x^2 \big/ s_x^{*2}\right]$ The usual ratio estimator
1	1	C_x	1	$\Upsilon_2^{(2)} = s_y^{*2}\left[S_x^2 + C_x \big/ s_x^{*2} + C_x\right]$ Kadilar and Cingi (2006)
1	1	$\beta_{2(x)}$	1	$\Upsilon_3^{(2)} = s_y^{*2}\left[S_x^2 + \beta_{2(x)} \big/ s_x^{*2} + \beta_{2(x)}\right]$ Upadhaya and Singh (1999)
1	1	$\beta_{1(x)}$	1	$\Upsilon_4^{(2)} = s_y^{*2}\left[S_x^2 + \beta_{1(x)} \big/ s_x^{*2} + \beta_{1(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	1	ρ	1	$\Upsilon_5^{(2)} = s_y^{*2}\left[S_x^2 + \rho \big/ s_x^{*2} + \rho\right]$ Subramani and Kumarapandiyan (2015)
1	1	S_x	1	$\Upsilon_6^{(2)} = s_y^{*2}\left[S_x^2 + S_x \big/ s_x^{*2} + S_x\right]$ Subramani and Kumarapandiyan (2015)

Table 2 (Continued)

Values of constants				Estimator
α	a	b	g	
1	1	M_d	1	$\Upsilon_7^{(2)} = s_y^{*2}\left[S_x^2 + M_d \big/ s_x^{*2} + M_d\right]$ Subramani and Kumarapandiyan (2012)
1	1	Q_1	1	$\Upsilon_8^{(2)} = s_y^{*2}\left[S_x^2 + Q_1 \big/ s_x^{*2} + Q_1\right]$ Subramani and Kumarapandiyan (2012)
1	1	Q_3	1	$\Upsilon_9^{(2)} = s_y^{*2}\left[S_x^2 + Q_3 \big/ s_x^{*2} + Q_3\right]$ Subramani and Kumarapandiyan (2012)
1	1	Q_r	1	$\Upsilon_{10}^{(2)} = s_y^{*2}\left[S_x^2 + Q_r \big/ s_x^{*2} + Q_r\right]$ Subramani and Kumarapandiyan (2012)
1	1	Q_d	1	$\Upsilon_{11}^{(2)} = s_y^{*2}\left[S_x^2 + Q_d \big/ s_x^{*2} + Q_d\right]$ Subramani and Kumarapandiyan (2012)
1	1	Q_a	1	$\Upsilon_{12}^{(2)} = s_y^{*2}\left[S_x^2 + Q_a \big/ s_x^{*2} + Q_a\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_1	1	$\Upsilon_{13}^{(2)} = s_y^{*2}\left[S_x^2 + D_1 \big/ s_x^{*2} + D_1\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_2	1	$\Upsilon_{14}^{(2)} = s_y^{*2}\left[S_x^2 + D_2 \big/ s_x^{*2} + D_2\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_3	1	$\Upsilon_{15}^{(2)} = s_y^{*2}\left[S_x^2 + D_3 \big/ s_x^{*2} + D_3\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_4	1	$\Upsilon_{16}^{(2)} = s_y^{*2}\left[S_x^2 + D_4 \big/ s_x^{*2} + D_4\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_5	1	$\Upsilon_{17}^{(2)} = s_y^{*2}\left[S_x^2 + D_5 \big/ s_x^{*2} + D_5\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_6	1	$\Upsilon_{18}^{(2)} = s_y^{*2}\left[S_x^2 + D_6 \big/ s_x^{*2} + D_6\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_7	1	$\Upsilon_{19}^{(2)} = s_y^{*2}\left[S_x^2 + D_7 \big/ s_x^{*2} + D_7\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_8	1	$\Upsilon_{20}^{(2)} = s_y^{*2}\left[S_x^2 + D_8 \big/ s_x^{*2} + D_8\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_9	1	$\Upsilon_{21}^{(2)} = s_y^{*2}\left[S_x^2 + D_9 \big/ s_x^{*2} + D_9\right]$ Subramani and Kumarapandiyan (2012)
1	1	D_{10}	1	$\Upsilon_{22}^{(2)} = s_y^{*2}\left[S_x^2 + D_{10} \big/ s_x^{*2} + D_{10}\right]$ Subramani and Kumarapandiyan (2012)

Table 2 (Continued)

Values of constants				Estimator
α	a	b	g	
1	$\beta_{2(x)}$	C_x	1	$\Upsilon_{23}^{(2)} = s_y^{*2}\left[\beta_{2(x)}S_x^2 + C_x \middle/ \beta_{2(x)}s_x^{*2} + C_x\right]$ Kadilar and Cingi (2006)
1	C_x	$\beta_{2(x)}$	1	$\Upsilon_{24}^{(2)} = s_y^{*2}\left[C_xS_x^2 + \beta_{2(x)} \middle/ C_xs_x^{*2} + \beta_{2(x)}\right]$ Kadilar and Cingi (2006)
1	$\beta_{1(x)}$	C_x	1	$\Upsilon_{25}^{(2)} = s_y^{*2}\left[\beta_{1(x)}S_x^2 + C_x \middle/ \beta_{1(x)}s_x^{*2} + C_x\right]$ Subramani and Kumarapandiyan (2015)
1	C_x	$\beta_{1(x)}$	1	$\Upsilon_{26}^{(2)} = s_y^{*2}\left[C_xS_x^2 + \beta_{1(x)} \middle/ C_xs_x^{*2} + \beta_{1(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	ρ	C_x	1	$\Upsilon_{27}^{(2)} = s_y^{*2}\left[\rho S_x^2 + C_x \middle/ \rho s_x^{*2} + C_x\right]$ Subramani and Kumarapandiyan (2015)
1	C_x	ρ	1	$\Upsilon_{28}^{(2)} = s_y^{*2}\left[C_xS_x^2 + \rho \middle/ C_xs_x^{*2} + \rho\right]$ Subramani and Kumarapandiyan (2015)
1	S_x	C_x	1	$\Upsilon_{29}^{(2)} = s_y^{*2}\left[S_xS_x^2 + C_x \middle/ S_xs_x^{*2} + C_x\right]$ Subramani and Kumarapandiyan (2015)
1	C_x	S_x	1	$\Upsilon_{30}^{(2)} = s_y^{*2}\left[C_xS_x^2 + S_x \middle/ C_xs_x^{*2} + S_x\right]$ Subramani and Kumarapandiyan (2015)
1	M_d	C_x	1	$\Upsilon_{31}^{(2)} = s_y^{*2}\left[M_dS_x^2 + C_x \middle/ M_ds_x^{*2} + C_x\right]$ Subramani and Kumarapandiyan (2015)
1	C_x	M_d	1	$\Upsilon_{32}^{(2)} = s_y^{*2}\left[C_xS_x^2 + M_d \middle/ C_xs_x^{*2} + M_d\right]$ Subramani and Kumarapandiyan (2013)
1	$\beta_{1(x)}$	$\beta_{2(x)}$	1	$\Upsilon_{33}^{(2)} = s_y^{*2}\left[\beta_{1(x)}S_x^2 + \beta_{2(x)} \middle/ \beta_{1(x)}s_x^{*2} + \beta_{2(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	$\beta_{2(x)}$	$\beta_{1(x)}$	1	$\Upsilon_{34}^{(2)} = s_y^{*2}\left[\beta_{2(x)}S_x^2 + \beta_{1(x)} \middle/ \beta_{2(x)}s_x^{*2} + \beta_{1(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	ρ	$\beta_{2(x)}$	1	$\Upsilon_{35}^{(2)} = s_y^{*2}\left[\rho S_x^2 + \beta_{2(x)} \middle/ \rho s_x^{*2} + \beta_{2(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	$\beta_{2(x)}$	ρ	1	$\Upsilon_{36}^{(2)} = s_y^{*2}\left[\beta_{2(x)}S_x^2 + \rho \middle/ \beta_{2(x)}s_x^{*2} + \rho\right]$ Subramani and Kumarapandiyan (2015)
1	S_x	$\beta_{2(x)}$	1	$\Upsilon_{37}^{(2)} = s_y^{*2}\left[S_xS_x^2 + \beta_{2(x)} \middle/ S_xs_x^{*2} + \beta_{2(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	$\beta_{2(x)}$	S_x	1	$\Upsilon_{38}^{(2)} = s_y^{*2}\left[\beta_{2(x)}S_x^2 + S_x \middle/ \beta_{2(x)}s_x^{*2} + S_x\right]$ Subramani and Kumarapandiyan (2015)
1	M_d	$\beta_{2(x)}$	1	$\Upsilon_{39}^{(2)} = s_y^{*2}\left[M_dS_x^2 + \beta_{2(x)} \middle/ M_ds_x^{*2} + \beta_{2(x)}\right]$ Subramani and Kumarapandiyan (2015)

Table 2 (Continued)

Values of constants				Estimator
α	a	b	g	
1	$\beta_{2(x)}$	M_d	1	$\Upsilon_{40}^{(2)}=s_y^{*2}\left[\beta_{2(x)}S_x^2+M_d\middle/\beta_{2(x)}s_x^{*2}+M_d\right]$ Subramani and Kumarapandiyan (2015)
1	ρ	$\beta_{1(x)}$	1	$\Upsilon_{41}^{(2)}=s_y^{*2}\left[\rho S_x^2+\beta_{1(x)}\middle/\rho s_x^{*2}+\beta_{1(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	$\beta_{1(x)}$	ρ	1	$\Upsilon_{42}^{(2)}=s_y^{*2}\left[\beta_{1(x)}S_x^2+\rho\middle/\beta_{1(x)}s_x^{*2}+\rho\right]$ Subramani and Kumarapandiyan (2015)
1	S_x	$\beta_{1(x)}$	1	$\Upsilon_{43}^{(2)}=s_y^{*2}\left[S_xS_x^2+\beta_{1(x)}\middle/S_xs_x^{*2}+\beta_{1(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	$\beta_{1(x)}$	S_x	1	$\Upsilon_{44}^{(2)}=s_y^{*2}\left[\beta_{1(x)}S_x^2+S_x\middle/\beta_{1(x)}s_x^{*2}+S_x\right]$ Subramani and Kumarapandiyan (2015)
1	M_d	$\beta_{1(x)}$	1	$\Upsilon_{45}^{(2)}=s_y^{*2}\left[M_dS_x^2+\beta_{1(x)}\middle/M_ds_x^{*2}+\beta_{1(x)}\right]$ Subramani and Kumarapandiyan (2015)
1	$\beta_{1(x)}$	M_d	1	$\Upsilon_{46}^{(2)}=s_y^{*2}\left[\beta_{1(x)}S_x^2+M_d\middle/\beta_{1(x)}s_x^{*2}+M_d\right]$ Subramani and Kumarapandiyan (2015)
1	S_x	ρ	1	$\Upsilon_{47}^{(2)}=s_y^{*2}\left[S_xS_x^2+\rho\middle/S_xs_x^{*2}+\rho\right]$ Subramani and Kumarapandiyan (2015)
1	ρ	S_x	1	$\Upsilon_{48}^{(2)}=s_y^{*2}\left[\rho S_x^2+S_x\middle/\rho s_x^{*2}+S_x\right]$ Subramani and Kumarapandiyan (2015)
1	M_d	ρ	1	$\Upsilon_{49}^{(2)}=s_y^{*2}\left[M_dS_x^2+\rho\middle/M_ds_x^{*2}+\rho\right]$ Subramani and Kumarapandiyan (2015)
1	ρ	M_d	1	$\Upsilon_{50}^{(2)}=s_y^{*2}\left[\rho S_x^2+M_d\middle/\rho s_x^{*2}+M_d\right]$ Subramani and Kumarapandiyan (2015)
1	M_d	S_x	1	$\Upsilon_{51}^{(2)}=s_y^{*2}\left[M_dS_x^2+S_x\middle/M_ds_x^{*2}+S_x\right]$ Subramani and Kumarapandiyan (2015)
1	S_x	M_d	1	$\Upsilon_{52}^{(2)}=s_y^{*2}\left[S_xS_x^2+M_d\middle/S_xs_x^{*2}+M_d\right]$ Subramani and Kumarapandiyan (2015)

4. Empirical Studies

In this section, the performances of the proposed class of estimators over competitive estimator are evaluated by considering four real populations as follows:

Population I: Source: Murthy (1967)
Y : Output for 80 factories in a region,
X : Fixed capital,

$N=80, n=24, \bar{Y}=5182.638, \bar{X}=1126.463, S_y^2=3369642.209, S_x^2=715055.8214, \beta_{2y}=2.238,$
$\beta_{2x}=2.830, \beta_{2(2y)}=2.346, \beta_{2(2x)}=2.738, \lambda_{22}=2.293, \lambda_{22(2)}=2.407, Q_1=517.5, Q_2=757.5,$
$Q_3=1693.75, C_x=0.750, Q_a=1105.625, Q_r=1176.25, Q_d=588.125, D_1=369.7, D_2=460.4,$
$D_3=597, D_4=676.8, D_5=757.5, D_6=850.2, D_7=1484.5, D_8=1810, D_9=2500, D_{10}=3485,$
$\rho=0.94, S_{y2}^2=426528.6, S_{x2}^2=158247.6, \beta_{1x}=1.048.$

Population II: Source: Sarjinder Singh (2003)
Y: Real estate farms loans,
X : Non real estate farms loans,
$N=50, n=15, \bar{Y}=555.43, \bar{X}=878.16, S_y^2=342021.45, S_x^2=1176526.36, \beta_{2y}=3.655, \beta_{2x}=4.524$
$\beta_{2(2y)}=1.665, \beta_{2(2x)}=3.742, \lambda_{22}=2.899, \lambda_{22(2)}=1.785, Q_1=63.450, Q_2=452.517, Q_3=1177.151,$
$C_x=1.235, Q_a=620.30, Q_r=1113.7, Q_d=556.85, D_1=19.097, D_2=49.887, D_3=194.614,$
$D_4=387.913, D_5=452.517, D_6=563.188, D_7=936.2075, D_8=1401.95, D_9=2583.331,$
$D_{10}=3928.732, \rho=0.80, S_{y2}^2=244951.8, S_{x2}^2=1209413.308, \beta_{1x}=2.539.$

Population III: Source: Murthy (1967)
Y: Output for 80 factories in a region,
X : Number of workers,
$N=80, n=24, \bar{Y}=5182.638, \bar{X}=285.125, S_y^2=3369642.209, S_x^2=73132.085, \beta_{2y}=2.238,$
$\beta_{2x}=3.536, \beta_{2(2y)}=2.346, \beta_{2(2x)}=1.907, \lambda_{22}=2.294, \lambda_{22(2)}=2.046, Q_1=86.5, Q_2=148,$
$Q_3=445.25, C_x=0.9484, Q_a=265.875, Q_r=358.75, Q_d=179.375, D_1=64.5, D_2=77.6,$
$D_3=95.8, D_4=123.4, D_5=148, D_6=203.2, D_7=358.9, D_8=501.6, D_9=709.5, D_{10}=1095,$
$\rho=0.92, S_{y2}^2=426528.6, S_{x2}^2=749.5, \beta_{1x}=1.607.$

Population IV: Source: Singh and Chaudhary (1986)
Y: Cultivated area (acres),
X : Population,
$N=70, n=21, \bar{Y}=982.714, \bar{X}=1751.714, S_y^2=375327.1, S_x^2=1986916, \beta_{2y}=4.378, \beta_{2x}=6.981$
$\beta_{2(2y)}=2.328, \beta_{2(2x)}=3.211, \lambda_{22}=4.534, \lambda_{22(2)}=2.559, Q_1=801.5, Q_2=1215, Q_3=2250.25,$
$C_x=0.8046, Q_a=1525.875, Q_r=1448.75, Q_d=724.375, D_1=662.6, D_2=756, D_3=850.1,$
$D_4=1086.4, D_5=1215, D_6=1495, D_7=1963, D_8=2588.4, D_9=3207.4, D_{10}=7780, \rho=0.92,$
$S_{y2}^2=70244.725, S_{x2}^2=282218.115, \beta_{1x}=1.101.$

Percent relative efficiency (PRE) of the proposed class of estimators over ratio estimator in empirical and simulation studies are calculated using the following formula given as

$$PRE=\frac{MSE(\Upsilon_1^{(1)})}{MSE(\gamma_y^*)}\times 100, \quad PRE=\frac{MSE(\Upsilon_1^{(1)})}{MSE(\gamma_{yx}^*)}\times 100.$$

Table 3 Percent relative efficiencies of the proposed class of estimator when non-response occurs on study variable Y

Estimator	Population 1	Population 2	Population 3	Population 4
Proposed	146.4760	146.8423	203.0861	176.0811
$\Upsilon_{1}^{(1)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{2}^{(1)}$	100.0001	100.0002	100.0027	100.0001
$\Upsilon_{3}^{(1)}$	100.0005	100.0009	100.0100	100.0007
$\Upsilon_{4}^{(1)}$	100.0003	100.0003	100.0046	100.0001
$\Upsilon_{5}^{(1)}$	100.0001	100.0300	100.0026	100.0001
$\Upsilon_{6}^{(1)}$	100.1185	100.1400	100.7667	100.1498
$\Upsilon_{7}^{(1)}$	100.0495	100.0003	100.4195	100.1291
$\Upsilon_{8}^{(1)}$	100.0069	100.2567	100.2452	100.0852
$\Upsilon_{9}^{(1)}$	100.1287	100.2300	101.2624	100.2392
$\Upsilon_{10}^{(1)}$	100.1218	100.1572	101.0171	100.1540
$\Upsilon_{11}^{(1)}$	100.0609	100.5139	100.5085	100.0770
$\Upsilon_{12}^{(1)}$	100.0678	100.3570	100.7537	100.1622
$\Upsilon_{13}^{(1)}$	100.0021	100.1786	100.1828	100.0704
$\Upsilon_{14}^{(1)}$	100.0055	100.3356	100.2200	100.0804
$\Upsilon_{15}^{(1)}$	100.0213	100.1123	100.2716	100.0904
$\Upsilon_{16}^{(1)}$	100.0424	100.1398	100.3498	100.1155
$\Upsilon_{17}^{(1)}$	100.0495	100.1813	100.4195	100.1291
$\Upsilon_{18}^{(1)}$	100.0616	100.2055	100.5760	100.1589
$\Upsilon_{19}^{(1)}$	100.1024	100.2300	101.0175	100.2086
$\Upsilon_{20}^{(1)}$	100.1533	100.2581	101.4222	100.2751
$\Upsilon_{21}^{(1)}$	100.2822	100.4505	102.0119	100.3409
$\Upsilon_{22}^{(1)}$	100.4288	100.5492	103.1054	100.8266
$\Upsilon_{23}^{(1)}$	100.0000	100.7581	100.0008	100.0000
$\Upsilon_{24}^{(1)}$	100.0004	101.0560	100.0106	100.0009
$\Upsilon_{25}^{(1)}$	100.0001	100.0001	100.0017	100.0001
$\Upsilon_{26}^{(1)}$	100.0002	100.0011	100.0048	100.0001
$\Upsilon_{27}^{(1)}$	100.0002	100.0002	100.0029	100.0001
$\Upsilon_{28}^{(1)}$	100.0001	100.0004	100.0027	100.0001
$\Upsilon_{29}^{(1)}$	100.0000	100.0002	100.0000	100.0000
$\Upsilon_{30}^{(1)}$	100.0960	100.0004	100.8084	100.1862
$\Upsilon_{31}^{(1)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{32}^{(1)}$	100.0401	100.3420	100.4424	100.1605

Table 3 (Continued)

Estimator	Population 1	Population 2	Population 3	Population 4
$\Upsilon_{33}^{(1)}$	100.0002	100.0000	100.0062	100.0007
$\Upsilon_{34}^{(1)}$	100.0001	100.3063	100.0013	100.0000
$\Upsilon_{35}^{(1)}$	100.0006	100.0008	100.0110	100.0010
$\Upsilon_{36}^{(1)}$	100.0000	100.0001	100.0007	100.0000
$\Upsilon_{37}^{(1)}$	100.0000	100.0009	100.0000	100.0000
$\Upsilon_{38}^{(1)}$	100.0262	100.0001	100.2168	100.0215
$\Upsilon_{39}^{(1)}$	100.0000	100.0000	100.0001	100.0000
$\Upsilon_{40}^{(1)}$	100.0109	100.0813	100.1186	100.0185
$\Upsilon_{41}^{(1)}$	100.0003	100.0003	100.0050	100.0002
$\Upsilon_{42}^{(1)}$	100.0000	100.0003	100.0016	100.0001
$\Upsilon_{43}^{(1)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{44}^{(1)}$	100.0467	100.2450	100.4768	100.1360
$\Upsilon_{45}^{(1)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{46}^{(1)}$	100.0195	100.2195	100.2609	100.1173
$\Upsilon_{47}^{(1)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{48}^{(1)}$	100.1481	100.2728	100.8383	100.1926
$\Upsilon_{49}^{(1)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{50}^{(1)}$	100.0619	100.2444	100.4588	100.1660
$\Upsilon_{51}^{(1)}$	100.0003	100.0003	100.0052	100.0001
$\Upsilon_{52}^{(1)}$	100.0000	100.0003	100.0016	100.0001

Table 4 Percent relative efficiencies of the proposed class of estimator when non-response occurs on study variable and auxiliary variables both Y and X

Estimator	Population 1	Population 2	Population 3	Population 4
Proposed	193.8789	150.7173	203.0904	176.2962
$\Upsilon_{1}^{(2)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{2}^{(2)}$	100.0002	100.0002	100.0027	100.0001
$\Upsilon_{3}^{(2)}$	100.0006	100.0009	100.0100	100.0007
$\Upsilon_{4}^{(2)}$	100.0004	100.0003	100.0046	100.0001
$\Upsilon_{5}^{(2)}$	100.0001	100.0003	100.0026	100.0001
$\Upsilon_{6}^{(2)}$	100.1504	100.2553	100.7667	100.1498
$\Upsilon_{7}^{(2)}$	100.0627	100.2287	100.4195	100.1291
$\Upsilon_{8}^{(2)}$	100.0088	100.1563	100.2452	100.0852
$\Upsilon_{9}^{(2)}$	100.1632	100.5110	101.2624	100.2391

Table 4 (Continued)

Estimator	Population 1	Population 2	Population 3	Population 4
$\Upsilon_{10}^{(2)}$	100.1544	100.3550	101.0171	100.1540
$\Upsilon_{11}^{(2)}$	100.0772	100.1776	100.5085	100.0770
$\Upsilon_{12}^{(2)}$	100.0860	100.3337	100.7537	100.1621
$\Upsilon_{13}^{(2)}$	100.0026	100.1116	100.1828	100.0704
$\Upsilon_{14}^{(2)}$	100.0069	100.1390	100.2200	100.0803
$\Upsilon_{15}^{(2)}$	100.0270	100.1802	100.2716	100.0903
$\Upsilon_{16}^{(2)}$	100.0538	100.2043	100.3498	100.1155
$\Upsilon_{17}^{(2)}$	100.0627	100.2287	100.4195	100.1291
$\Upsilon_{18}^{(2)}$	100.0781	100.2567	100.5760	100.1589
$\Upsilon_{19}^{(2)}$	100.1298	100.4480	101.0175	100.2086
$\Upsilon_{20}^{(2)}$	100.1943	100.5461	101.4222	100.2750
$\Upsilon_{21}^{(2)}$	100.3580	100.7539	102.0119	100.3408
$\Upsilon_{22}^{(2)}$	100.5443	101.0502	103.1054	100.8264
$\Upsilon_{23}^{(2)}$	100.0000	100.0001	100.0008	100.0000
$\Upsilon_{24}^{(2)}$	100.0005	100.0011	100.0106	100.0009
$\Upsilon_{25}^{(2)}$	100.0001	100.0002	100.0017	100.0001
$\Upsilon_{26}^{(2)}$	100.0003	100.0004	100.0048	100.0001
$\Upsilon_{27}^{(2)}$	100.0002	100.0002	100.0029	100.0001
$\Upsilon_{28}^{(2)}$	100.0001	100.0004	100.0027	100.0001
$\Upsilon_{29}^{(2)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{30}^{(2)}$	100.1217	100.3400	100.8084	100.1862
$\Upsilon_{31}^{(2)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{32}^{(2)}$	100.0508	100.3046	100.4424	100.1605
$\Upsilon_{33}^{(2)}$	100.0002	100.0008	100.0062	100.0007
$\Upsilon_{34}^{(2)}$	100.0001	100.0001	100.0013	100.0000
$\Upsilon_{35}^{(2)}$	100.0008	100.0009	100.0110	100.0010
$\Upsilon_{36}^{(2)}$	100.0000	100.0001	100.0007	100.0000
$\Upsilon_{37}^{(2)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{38}^{(2)}$	100.0332	100.0902	100.2168	100.0215
$\Upsilon_{39}^{(2)}$	100.0000	100.0000	100.0001	100.0000
$\Upsilon_{40}^{(2)}$	100.0139	100.0808	100.1186	100.0185
$\Upsilon_{41}^{(2)}$	100.0004	100.0003	100.0050	100.0002
$\Upsilon_{42}^{(2)}$	100.0000	100.0003	100.0016	100.0001

Table 4 (Continued)

Estimator	Population 1	Population 2	Population 3	Population 4
$\Upsilon_{43}^{(2)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{44}^{(2)}$	100.0592	100.2436	100.4768	100.1360
$\Upsilon_{45}^{(2)}$	100.0247	100.0000	100.0000	100.0000
$\Upsilon_{46}^{(2)}$	100.0110	100.2182	100.2609	100.1172
$\Upsilon_{47}^{(2)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{48}^{(2)}$	100.1879	100.2713	100.8383	100.1925
$\Upsilon_{49}^{(2)}$	100.0000	100.0000	100.0000	100.0000
$\Upsilon_{50}^{(2)}$	100.0784	100.2430	100.4588	100.1660
$\Upsilon_{51}^{(2)}$	100.0003	100.0003	100.0052	100.0001
$\Upsilon_{52}^{(2)}$	100.0001	100.0003	100.0016	100.0001

5. Simulation Study

In this section, simulation study is carried out to evaluate the performance of the proposed class of estimators with respect to traditional ratio estimator. For this study we have generated a population of size $N = 1000$ from standard normal distribution using MVRNORM package in software R where study and auxiliary variable are correlated with correlation $\rho = 0.7$, draw sample of size $n = 200$ with 35% non-response. The whole simulation process starting from drawing sample from variable Y and auxiliary variable X from Normal population and calculating the estimates was repeated 50,000 times.

Table 5 Percent relative efficiencies of the proposed class of estimators when non-response occurs on study variable Y

Estimator	PRE	Estimator	PRE
Proposed	108.6676	$\Upsilon_{12}^{(1)}$	100.3258
$\Upsilon_{1}^{(1)}$	100.0000	$\Upsilon_{13}^{(1)}$	102.9987
$\Upsilon_{2}^{(1)}$	100.1450	$\Upsilon_{14}^{(1)}$	100.9977
$\Upsilon_{3}^{(1)}$	102.0021	$\Upsilon_{15}^{(1)}$	102.5423
$\Upsilon_{4}^{(1)}$	102.8271	$\Upsilon_{16}^{(1)}$	102.0001
$\Upsilon_{5}^{(1)}$	103.5146	$\Upsilon_{17}^{(1)}$	102.3573
$\Upsilon_{6}^{(1)}$	103.0001	$\Upsilon_{18}^{(1)}$	102.3321
$\Upsilon_{7}^{(1)}$	103.4870	$\Upsilon_{19}^{(1)}$	102.0045
$\Upsilon_{8}^{(1)}$	102.1495	$\Upsilon_{20}^{(1)}$	103.1105
$\Upsilon_{9}^{(1)}$	103.0025	$\Upsilon_{21}^{(1)}$	102.6684
$\Upsilon_{10}^{(1)}$	102.6684	$\Upsilon_{22}^{(1)}$	103.8887
$\Upsilon_{11}^{(1)}$	101.2257	$\Upsilon_{23}^{(1)}$	101.0025

Table 5 (Continued)

Estimator	PRE	Estimator	PRE
$\Upsilon_{24}^{(1)}$	101.3258	$\Upsilon_{39}^{(1)}$	102.3545
$\Upsilon_{25}^{(1)}$	102.6587	$\Upsilon_{40}^{(1)}$	102.1545
$\Upsilon_{26}^{(1)}$	102.0045	$\Upsilon_{41}^{(1)}$	102.0026
$\Upsilon_{27}^{(1)}$	103.3420	$\Upsilon_{42}^{(1)}$	100.3654
$\Upsilon_{28}^{(1)}$	103.0009	$\Upsilon_{43}^{(1)}$	102.0000
$\Upsilon_{29}^{(1)}$	103.1186	$\Upsilon_{44}^{(1)}$	102.2624
$\Upsilon_{30}^{(1)}$	103.6658	$\Upsilon_{45}^{(1)}$	102.6874
$\Upsilon_{31}^{(1)}$	102.7890	$\Upsilon_{46}^{(1)}$	102.1540
$\Upsilon_{32}^{(1)}$	102.6684	$\Upsilon_{47}^{(1)}$	103.0023
$\Upsilon_{33}^{(1)}$	103.8879	$\Upsilon_{48}^{(1)}$	100.8872
$\Upsilon_{34}^{(1)}$	103.5570	$\Upsilon_{49}^{(1)}$	103.6510
$\Upsilon_{35}^{(1)}$	103.0005	$\Upsilon_{50}^{(1)}$	102.2194
$\Upsilon_{36}^{(1)}$	102.0367	$\Upsilon_{51}^{(1)}$	100.5871
$\Upsilon_{37}^{(1)}$	103.6910	$\Upsilon_{52}^{(1)}$	100.2450
$\Upsilon_{38}^{(1)}$	103.5702		

Table 6 Percent relative efficiencies of the proposed class of estimators when non-response occurs on study variable and auxiliary variables both Y and X

Estimator	PRE	Estimator	PRE
Proposed	122.7672	$\Upsilon_{11}^{(2)}$	104.3680
$\Upsilon_{1}^{(2)}$	100.0000	$\Upsilon_{12}^{(2)}$	103.2251
$\Upsilon_{2}^{(2)}$	102.0032	$\Upsilon_{13}^{(2)}$	103.2579
$\Upsilon_{3}^{(2)}$	102.0255	$\Upsilon_{14}^{(2)}$	103.0025
$\Upsilon_{4}^{(2)}$	104.6842	$\Upsilon_{15}^{(2)}$	103.8872
$\Upsilon_{5}^{(2)}$	104.1536	$\Upsilon_{16}^{(2)}$	104.6692
$\Upsilon_{6}^{(2)}$	104.9684	$\Upsilon_{17}^{(2)}$	104.3581
$\Upsilon_{7}^{(2)}$	103.7668	$\Upsilon_{18}^{(2)}$	104.3218
$\Upsilon_{8}^{(2)}$	103.3498	$\Upsilon_{19}^{(2)}$	104.0064
$\Upsilon_{9}^{(2)}$	102.1166	$\Upsilon_{20}^{(2)}$	102.5580
$\Upsilon_{10}^{(2)}$	102.6698	$\Upsilon_{21}^{(2)}$	103.4780

Table 6 (Continued)

Estimator	PRE	Estimator	PRE
$\Upsilon_{22}^{(2)}$	105.6870	$\Upsilon_{43}^{(2)}$	104.6682
$\Upsilon_{23}^{(2)}$	104.3657	$\Upsilon_{44}^{(2)}$	104.6688
$\Upsilon_{24}^{(2)}$	103.2571	$\Upsilon_{45}^{(2)}$	103.0258
$\Upsilon_{25}^{(2)}$	103.2547	$\Upsilon_{46}^{(2)}$	104.0001
$\Upsilon_{26}^{(2)}$	104.2187	$\Upsilon_{47}^{(2)}$	104.6587
$\Upsilon_{27}^{(2)}$	104.6578	$\Upsilon_{48}^{(2)}$	101.2200
$\Upsilon_{28}^{(2)}$	105.3525	$\Upsilon_{49}^{(2)}$	103.5580
$\Upsilon_{29}^{(2)}$	102.6687	$\Upsilon_{50}^{(2)}$	101.2257
$\Upsilon_{30}^{(2)}$	101.3325	$\Upsilon_{51}^{(2)}$	102.6680
$\Upsilon_{31}^{(2)}$	100.8720	$\Upsilon_{52}^{(2)}$	104.3258
$\Upsilon_{32}^{(2)}$	104.6698	$\Upsilon_{43}^{(2)}$	104.6981
$\Upsilon_{33}^{(2)}$	103.2257	$\Upsilon_{44}^{(2)}$	104.8792
$\Upsilon_{34}^{(2)}$	103.8922	$\Upsilon_{45}^{(2)}$	103.3321
$\Upsilon_{35}^{(2)}$	104.3358	$\Upsilon_{46}^{(2)}$	103.5870
$\Upsilon_{36}^{(2)}$	103.2576	$\Upsilon_{47}^{(2)}$	103.2201
$\Upsilon_{37}^{(2)}$	103.2257		

Simulation and empirical studies are performed in Tables 3-6 for proposed class of estimators $\left(\gamma_y^*, \gamma_{yx}^*\right)$ and read out that the values of percent relative efficiencies of the proposed class of estimators with respect to ratio estimator are always greater than 100. Hence from this we may interpret that the proposed class of estimators are more efficient.

6. Conclusions and Recommendations

From the above analysis it may be concluded that the proposed class of estimators are rewarding in terms of percent relative efficiencies on both real and artificial data set. Thus, the proposed class of estimators in this work may be utilize effectively to handle the problem of non-response and recommended to the survey practitioners for their practical uses.

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