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Bootstrap Confidence Intervals for the Parameter of Zero-truncated Poisson-Ishita Distribution

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Abstract

Numerous phenomena interact with count data without zero values, such as the length of a hospital stay and the number of car passengers. Recent research has proposed the zero-truncated Poisson-Ishita distribution (ZTPID) for such data, but its statistical inference, especially interval estimation for the parameter, has not been examined. In this article, the percentile, simple, biased-corrected and accelerated (BCa) bootstrap confidence intervals, as well as the bootstrap-t interval, are examined in terms of coverage probability and average interval length, which are estimated from the Monte Carlo method. The parameter values of ZTPID are varied, resulting in numerous populations with variances ranging from tiny to large values. The results indicate that small sample sizes are inadequate to attain the nominal level of confidence for all settings and bootstrap methods. When a sample size is large enough, all methods do not substantially differ. Overall, it is observed that the bias-corrected and accelerated bootstrap approach outperforms the other methods, even with small sample sizes. Lastly, each of the bootstrap intervals is calculated for two numerical examples, and the results match those of the simulation.

Keywords: Interval estimation, zero-truncated Poisson-Ishita distribution, bootstrap interval

1. Introduction

The Poisson distribution is a discrete distribution that measures the probability of a given number of events happening in specific regions of time or space. (Kissell and Poserina 2017; Mohr et al. 2022). Some random variables might follow a Poisson distribution: the number of orders your firm receives tomorrow, the number of people who apply for a job tomorrow to your human resources division, the number of defects in a finished product, the number of patients arriving in an emergency room between 9.00 and 10.00 pm., etc. (Siegel 2016).

The probability mass function (p.m.f.) of a Poisson distribution is defined as

$$p(x;\theta)=\frac{e^{-\theta}\theta^{x}}{x!},\quad x=0,1,2,...,\ \theta>0, \tag{1}$$

where e is a constant approximately equal to 2.71828 and θ is the parameter of the Poisson distribution. This probability model is usually used in analysis of data containing zero and positive events that have low probabilities of occurrence within some definite time or area range (Sangnawakij 2021). However, probability models are truncated when a range of possible values for the variables is either disregarded or impossible to observe. The models' zero truncation is another truncation event in which one tries to simulate count data without zero. David and Johnson (1952) developed the zero-truncated Poisson (ZTP) distribution, which can be found in several datasets, including the length of hospital stay, which is recorded as a minimum of one day, the number of journal articles published in various disciplines, the number of children ever born to a sample of mothers over 40 years old, and the number of occupants in passenger cars (Hussain 2020). The zero-truncated distribution's p.m.f. can be represented as

$$p(x;\theta) = \frac{p_0(x;\theta)}{1 - p_0(0;\theta)}, \quad x = 1,2,3,..., \tag{2}$$

where $p_0(x;\theta)$ is the p.m.f. of the un-truncated distribution. Several distributions have been introduced as an alternative to zero-truncated Poisson distribution on handling the over-dispersion on data, such as zero-truncated Poisson Lindley (ZTPL) distribution (Ghitany et al. 2008), zero-truncated Poisson-Sujatha (ZTPS) distribution (Shanker and Hagos 2015) and zero-truncated Poisson-Akash (ZTPA) distribution (Shanker 2017b).

Recently, zero-truncated Poisson-Ishita (ZTPI) distribution and its applications were proposed by Shukla et al. (2020). The moment, coefficient of variation, skewness, kurtosis and the index of dispersion of ZTPID had been presented. The method of maximum likelihood and the method of moments had also discussed for estimating its parameter. Furthermore, ZTPI distribution was applied on two real data sets to test its goodness of fit. It was more suitable than ZTP, ZTPL, ZTPS and ZTPA distributions.

In the review literature, there is no research study for estimating the bootstrap confidence intervals for the parameter of ZTPI distribution. Bootstrap confidence intervals provide a way of quantifying the uncertainties in the inferences that can be drawn from a sample of data. The idea is to use a simulation, based on the actual data, to estimate the likely extent of sampling error (Wood 2004). Therefore, the objective of the paper is to study the efficiency of bootstrap confidence intervals for the parameter of ZTPI distribution in four methods, namely, percentile bootstrap, simple bootstrap, bias-corrected and accelerated bootstrap (BCa), and bootstrap-t methods. Because a theoretical comparison is not possible, we conduct a simulation study to compare the performance of these bootstrap confidence intervals, and use these results to suggest a bootstrap confidence interval with coverage probability that attained a nominal confidence level and short average length for practitioners.

The structure of the paper is as follows. In Section 2, theoretical background of the Poisson-Ishita distribution and the ZTPI distribution are explained. We also derive the bootstrap confidence interval methods for the parameter of ZTPI distribution in Section 3. In Section 4, we investigate the performance of all bootstrap confidence intervals using Monte Carlo simulation in various situations. Two numerical examples are illustrated in Section 5. Finally, the discussion and conclusions are presented in the final section.

2. Theoretical Background

Compounding of probability distributions is a sound and innovative technique to obtain new probability distributions to fit data sets not adequately fit by common parametric distributions. Shukla and Shanker (2019) proposed a new compounding distribution by compounding Poisson distribution with Ishita distribution, as there is a need to find more flexible model for analyzing statistical data. The p.m.f. of the Poisson-Ishita distribution is given by

$$p_0(x;\theta)=\frac{\theta^3}{(\theta^3+2)}\frac{x^2+3x+(\theta^3+2\theta^2+\theta+2)}{(\theta+1)^{x+3}},\ x=0,1,2,...,\ \theta>0. \tag{3}$$

Let X be a random variable which follow ZTPI distribution with parameter $\theta,$ it is denoted as $X\sim \text{ZTPI}(\theta).$ Using Equations (2) and (3), the p.m.f. of ZTPI distribution can be obtained as

$$p(x;\theta)=\frac{\theta^3}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}\frac{x^2+3x+(\theta^3+2\theta^2+\theta+2)}{(\theta+1)^x},\ x=1,2,3,...,\ \theta>0. \tag{4}$$

The plots of ZTPI distribution with some specified parameter values θ shown in Figure 1.

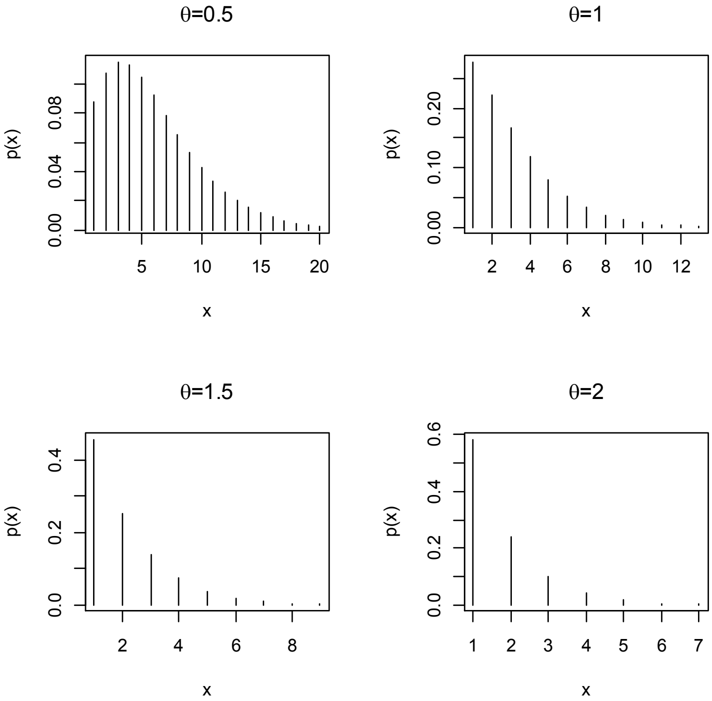

Figure 1 The plots of the mass function of the ZTPI distribution with $\theta=0.5,\ 1,\ 1.5$ and 2

The expected value and variance of X are as follows:

$$E(X)=\frac{\theta^6+3\theta^5+3\theta^4+7\theta^3+18\theta^2+18\theta+6}{\theta(\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2)} \text{ and}$$

$$\text{var}(X)=\frac{(\theta+1)(\theta^{10}+4\theta^9+6\theta^8+27\theta^7+69\theta^6+98\theta^5+136\theta^4+208\theta^3+180\theta^2+72\theta+12)}{\theta^2(\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2)^2}.$$

The point estimator of θ is obtained by maximizing the log-likelihood function $\log L(x_i;\theta)$ or the logarithm of joint p.m.f. of $X_1,X_2,...,X_n.$ Thus, the maximum likelihood (ML) estimator for θ of the ZTPI distribution is derived by the following processes:

$$\frac{\partial}{\partial\theta}\log L(x_i;\theta)=\frac{\partial}{\partial\theta}\left[\begin{array}{l} n\log\left(\dfrac{\theta^3}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}\right)-\sum_{i=1}^{n}x_i\log(\theta+1)\\ +\sum_{i=1}^{n}\log\left[x_i^2+3x_i+\left(\theta^3+2\theta^2+\theta+2\right)\right]\end{array}\right]$$

$$=\frac{3n}{\theta}-\frac{n\left(5\theta^4+8\theta^3+3\theta^2+12\theta+6\right)}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}-\frac{n\bar{x}}{\theta+1}+\sum_{i=1}^{n}\frac{\left(3\theta^2+4\theta+1\right)}{x_i^2+3x_i+\left(\theta^3+2\theta^2+\theta+2\right)}.$$

Solving the equation $\frac{\partial}{\partial\theta}\log L(x_i;\theta)=0$ for $\theta,$ we have the non-linear equation

$$\frac{3n}{\theta}-\frac{n\left(5\theta^4+8\theta^3+3\theta^2+12\theta+6\right)}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}-\frac{n\bar{x}}{\theta+1}+\sum_{i=1}^{n}\frac{\left(3\theta^2+4\theta+1\right)}{x_i^2+3x_i+\left(\theta^3+2\theta^2+\theta+2\right)}=0,$$

where $\bar{x}=\sum_{i=1}^{n}x_i/n$ denotes the sample mean. Since the ML estimator for θ does not provide the closed-form solution, the non-linear equation can be solved by the numerical iteration methods such as Newton-Raphson method, bisection method and Ragula-Falsi method. In this paper, we use maxLik package (Henningsen and Toomet 2011) for ML estimation in the statistical software R.

3. Bootstrap Confidence Interval Methods

In this paper, we focus on the four bootstrap confidence interval methods that are most popular in practice: percentile bootstrap, simple bootstrap, bias-corrected and accelerated bootstrap, and bootstrap-t confidence intervals.

3.1. Percentile bootstrap (PB) method

The percentile bootstrap confidence interval is the interval between the $(\alpha/2)\times 100$ and $(1-(\alpha/2))\times 100$ percentiles of the distribution of θ estimates obtained from resampling or the distribution of $\hat{\theta}^*$, where θ represents a parameter of interest and α is the level of significance (e.g., $\alpha=0.05$ for 95% confidence intervals) (Efron 1982). A percentile bootstrap confidence interval for θ can be obtained as follows:

1) B random bootstrap samples are generated,
2) a parameter estimate $\hat{\theta}^*$ is calculated from each bootstrap sample,
3) all B bootstrap parameter estimates are ordered from the lowest to highest, and
4) the $(1-\alpha)100\%$ percentile bootstrap confidence interval is constructed as follows:

$$CI_{PB}=\left[\hat{\theta}^*_{(r)},\hat{\theta}^*_{(s)}\right], \tag{5}$$

where $\hat{\theta}^*_{(\alpha)}$ denotes the α^{th} percentile of the distribution of $\hat{\theta}^*$ and $0\le r<s\le 100.$ For example, a 95% percentile bootstrap confidence interval with 1000 bootstrap samples is the interval between the 2.5 percentile value and the 97.5 percentile value of the 1000 bootstrap parameter estimates.

3.2. Simple bootstrap (SB) method

The simple bootstrap method is sometimes called the basic bootstrap method and is a method as easy to apply as the percentile bootstrap method. Suppose that the quantity of interest is θ and that the estimator of θ is $\hat{\theta}.$ The simple bootstrap method assumes that the distributions of $\hat{\theta}-\theta$ and

$\hat{\theta}^* - \hat{\theta}$ are approximately the same (Meeker et al., 2017). The $(1-\alpha)100\%$ simple bootstrap confidence interval for θ is

$$CI_{SB} = \left[2\hat{\theta} - \hat{\theta}^*_{(s)}, 2\hat{\theta} - \hat{\theta}^*_{(r)}\right], \tag{6}$$

where the quantiles $\hat{\theta}^*_{(r)}$ and $\hat{\theta}^*_{(s)}$ are the same percentile of empirical distribution of bootstrap estimates $\hat{\theta}^*$ used in (5) for the percentile bootstrap method.

3.3. Bias-corrected and accelerated (BCa) bootstrap method

To overcome the overcoverage issues in percentile bootstrap confidence intervals (Efron and Tibshirani 1993), the BCa bootstrap method corrects for both bias and skewness of the bootstrap parameter estimates by incorporating a bias-correction factor and an acceleration factor (Efron 1987; Efron and Tibshirani 1993). The bias-correction factor $\hat{z}_0$ is estimated as the proportion of the bootstrap estimates less than the original parameter estimate $\hat{\theta}$,

$$\hat{z}_0 = \Phi^{-1}\left(\frac{\#\{\hat{\theta}^* \le \hat{\theta}\}}{B}\right),$$

where Φ^{-1} is the inverse function of a standard normal cumulative distribution function (e.g., $\Phi^{-1}(0.975) = 1.96).$ The acceleration factor $\hat{a}$ is estimated through jackknife resampling (i.e., "leave one out" resampling), which involves generating n replicates of the original sample, where n is the number of observations in the sample. The first jackknife replicate is obtained by leaving out the first case $(i=1)$ of the original sample, the second by leaving out the second case $(i=2),$ and so on, until n samples of size $n-1$ are obtained. For each of the jackknife resamples, $\hat{\theta}_{(-i)}$ is obtained. The average of these estimates is

$$\hat{\theta}_{(\cdot)} = \frac{\sum_{i=1}^{n} \hat{\theta}_{(-i)}}{n}.$$

Then, the acceleration factor $\hat{a}$ is calculated as follow,

$$\hat{a} = \frac{\sum_{i=1}^{n}\left(\hat{\theta}_{(\cdot)} - \hat{\theta}_{(-i)}\right)^3}{6\left\{\sum_{i=1}^{n}\left(\hat{\theta}_{(\cdot)} - \hat{\theta}_{(-i)}\right)^2\right\}^{3/2}}.$$

With the values of $\hat{z}_0$ and $\hat{a},$ the values α_1 and α_2 are calculated,

$$\alpha_1 = \Phi\left\{\hat{z}_0 + \frac{\hat{z}_0 + z_{\alpha/2}}{1-\hat{a}\left(\hat{z}_0 + z_{\alpha/2}\right)}\right\} \text{ and } \alpha_2 = \Phi\left\{\hat{z}_0 + \frac{\hat{z}_0 + z_{1-\alpha/2}}{1-\hat{a}\left(\hat{z}_0 + z_{1-\alpha/2}\right)}\right\},$$

where $z_{\alpha/2}$ is the α quantile of the standard normal distribution (e.g. $z_{0.05/2} = -1.96).$ Then, the $(1-\alpha)100\%$ BCa bootstrap confidence interval for θ is as follows

$$CI_{BCa} = \left[\hat{\theta}^*_{(\alpha_1)}, \hat{\theta}^*_{(\alpha_2)}\right], \tag{7}$$

where $\hat{\theta}^*_{(\alpha)}$ denotes the α^{th} percentile of the distribution of $\hat{\theta}^*$.

3.4. Bootstrap-t (B-t) method

Suppose that the quantity of interest is θ and that from the given data one can compute the estimate $\hat{\theta}$ and $s.e.(\hat{\theta}),$ a corresponding estimate of the standard error of $\hat{\theta}.$ Then, the bootstrap estimates $\hat{\theta}_j^*$ and their corresponding estimated standard errors $s.e.(\hat{\theta}_j^*)$ are computed from each bootstrap sample $j = 1,2,...,B.$ From these, the bootstrap-t (studentized) statistics

$$R_j^* = \frac{\hat{\theta}_j^* - \hat{\theta}}{s.e.(\hat{\theta}_j^*)}, \quad j = 1,2,...,B,$$

are computed. The $(1-\alpha)100\%$ bootstrap-t CI for θ is

$$CI_{B-t} = \left[\hat{\theta} - t_r^* \times s.e.(\hat{\theta}), \hat{\theta} - t_s^* \times s.e.(\hat{\theta})\right], \tag{8}$$

where $r = 1-(\alpha/2)$ and $s = \alpha/2,$ and t_q^* denotes the q quantile of the distribution of $R_j^*.$

4. Simulation Study

In this study, the bootstrap confidence intervals for the parameter of ZTPI distribution are determined. Because a theoretical comparison is not possible, a Monte Carlo simulation study was designed using R version 4.1.3 statistical software (Ihaka and Gentleman 1996) and conducted to compare the performances of four bootstrap confidence intervals for the parameter in a ZTPI distribution. The study was designed to cover cases with different sample sizes, as n = 10, 30, 50, 100 and 500, reflecting small to large samples. To observe the effect of small and large variances, the true parameter (θ) was given by 0.25, 0.5, 1, 2, and 3, and the variance of random variables will decrease as the value of θ increases. The larger the value of the parameter is, the smaller the generated data will be. The $B = 1000$ bootstrap samples of size n are generated from the original sample and repeated the simulation 1000 times. Without loss of generality, the confidence level $(1-\alpha)$ was set at 0.95. The bootstrap confidence intervals were compared in terms of their coverage probabilities and the average lengths of their performances. A bootstrap confidence interval which has a coverage probability greater than or close to the nominal confidence level means that it contains the true value with a given probability. In other words, it can precisely estimate the parameter of interest. The bootstrap confidence interval that satisfies the criterion is the best in comparison.

The results of the study are reported in Table 1. When the sample size is only 10, the coverage probabilities tend to be less than 0.90, except in a few cases where the values of θ are greater or equal to two. The nominal confidence level is difficult to reach in circumstances where $\theta \geq 1$ and $n \leq 30.$ Generally, as sample size increases, the coverage probability tends to increase and approach 0.95. The average length also obviously increases when the value of θ increases; this is because of the relationship between the variance and θ value. Unsurprisingly, as sample size increases, the average length falls. It can be as small as approximately 0.028 when θ is at 0.25 and the sample size is 10; the largest average length, 8.062, occurs when $\theta = 3$ and $n = 10$ in the case of bootstrap-t method.

When four types of confidence intervals are compared, they can differ when the variance of the distribution is small, i.e., $\mathrm{var}(X) = 1.21, 0.53$ for $\theta = 2,3,$ respectively, and n is very small, i.e., $n \leq 30;$ the percentile bootstrap and BCa bootstrap approaches outperform the others in terms of coverage probability. Given the same sample sizes, scenarios with larger variances, i.e.,

$\text{var}(X) = 60.6, 17.8$ for $\theta = 0.25, 0.50$, respectively, lead to the conclusion that all intervals perform approximately the same.

Table 1. Coverage probability and average length of the 95% bootstrap confidence intervals for θ in the zero-truncated Poisson-Ishita distribution

n	θ	Coverage probability				Average length			
		PB	SB	BCa	B-t	PB	SB	BCa	B-t
10	0.25	0.868	0.864	0.883	0.882	0.213	0.210	0.204	0.186
	0.5	0.878	0.881	0.899	0.882	0.464	0.459	0.447	0.410
	1	0.894	0.875	0.902	0.832	1.217	1.069	1.101	0.846
	2	0.928	0.864	0.935	0.803	4.394	3.427	4.261	3.615
	3	0.923	0.824	0.947	0.793	5.832	5.961	5.842	8.062
30	0.25	0.934	0.941	0.935	0.932	0.116	0.118	0.116	0.114
	0.5	0.938	0.939	0.938	0.935	0.256	0.256	0.248	0.245
	1	0.918	0.907	0.935	0.913	0.565	0.554	0.537	0.525
	2	0.925	0.910	0.928	0.891	2.101	1.925	1.775	1.437
	3	0.921	0.862	0.928	0.917	6.478	4.452	5.602	4.068
50	0.25	0.942	0.929	0.943	0.935	0.090	0.090	0.088	0.090
	0.5	0.948	0.934	0.945	0.947	0.199	0.218	0.204	0.192
	1	0.921	0.924	0.937	0.928	0.453	0.471	0.435	0.458
	2	0.930	0.926	0.933	0.910	1.347	1.553	1.240	1.217
	3	0.932	0.906	0.928	0.928	4.102	3.377	3.252	2.675
100	0.25	0.951	0.932	0.946	0.945	0.064	0.062	0.063	0.063
	0.5	0.938	0.944	0.918	0.925	0.137	0.136	0.137	0.136
	1	0.932	0.938	0.942	0.938	0.297	0.294	0.290	0.289
	2	0.941	0.940	0.939	0.935	0.819	0.827	0.791	0.754
	3	0.936	0.934	0.942	0.947	1.975	2.004	1.890	1.712
500	0.25	0.962	0.948	0.948	0.949	0.028	0.028	0.028	0.028
	0.5	0.939	0.948	0.952	0.940	0.061	0.061	0.061	0.061
	1	0.948	0.954	0.949	0.941	0.130	0.131	0.130	0.130
	2	0.950	0.942	0.954	0.931	0.341	0.344	0.336	0.334
	3	0.942	0.962	0.952	0.952	0.749	0.761	0.756	0.734

5. Numerical Examples

We used two real-world examples to demonstrate the application of the bootstrap confidence intervals for the parameter of the ZTPI distribution established in the preceding section.

5.1 Flower heads example

The first dataset, shown in Table 2, is the number of flower heads as per the number of fly eggs reported by Finney and Varley (1955); the total sample size is 88. For the chi-square goodness-of-fit test (Turhan 2020), the chi-square statistic was 3.7681 and p-value was 0.7080. It was found that these data fitted well to the ZTPI distribution with the parameter $\hat{\theta} = 1.01406$. The 95% bootstrap confidence intervals for the parameter of the ZTPI distribution were calculated and reported in Table 3. Similar to simulation results when $\theta = 1$ and $n = 100$, the width of all confidence intervals is around 0.29, but the largest interval is from the percentile bootstrap. Also, from the simulation results, it is expected that the coverage probability is about the same, at approximately 0.94.

Table 2. The number of flower heads as per the number of fly eggs

Number of fly eggs	1	2	3	4	5	6	≥ 7
Observed frequency	22	18	18	11	9	6	4
Expected frequency	24.9287	19.7204	14.6526	10.2922	6.9078	4.4711	7.0272

Table 3. The 95% bootstrap confidence intervals and corresponding widths using all intervals for the parameter in the flower heads example

Methods	Confidence intervals	Widths
PB	(0.8919, 1.1654)	0.2735
SB	(0.8603, 1.1320)	0.2718
BCa	(0.8938, 1.1632)	0.2694
B-t	(0.8824, 1.1521)	0.2697

5.2 European red mites example

Garman (1923) reported the number of European red mites on apple leaves in the second dataset given in Table 4; the total sample size is 80. For the chi-square goodness-of-fit test (Turhan 2020), the chi-square statistic was 2.5111 and the p-value was 0.8672. It was found that these data fitted well to the ZTPI distribution with parameter $\hat{\theta}$ of 1.4924. The 95% bootstrap confidence intervals for the parameter of the ZTPI distribution were calculated and reported in Table 5. The estimated parameter $\hat{\theta}$ is between 1 $\theta = 1$ and 2 $\theta = 2.$ The results correspond with the simulation results with $n = 100$ because- the average length in BCa and B-t methods is shorter than in PB and SB methods. According to the simulation results, the coverage probability is expected to be 0.94.

Table 4. The number of European red mites on apple leaves

Number of European red mites	1	2	3	4	5	6	≥ 7
Observed frequency	38	17	10	9	3	2	1
Expected frequency	36.2338	20.2491	11.1797	6.0178	3.1522	1.6100	1.5574

Table 5. The 95% bootstrap confidence intervals and corresponding widths using all intervals for the parameter in the European red mites example

Methods	Confidence intervals	Widths
PB	(1.2717, 1.7985)	0.5268
SB	(1.1631, 1.7171)	0.5540
BCa	(1.2548, 1.7772)	0.5224
B-t	(1.2548, 1.7676)	0.5128

6. Conclusions and Discussion

The bootstrap confidence intervals of the parameter of the zero-truncated Poisson-Ishita distribution are investigated in this study. At $n = 10,$ all coverage probabilities are substantially lower than 0.95. A sample size of 30 is still insufficient to achieve the nominal confidence level for all $\theta's$ and bootstrap intervals. When the sample size is large enough, i.e., greater than or equal to 50, the coverage probabilities from four intervals, as well as the average length, are not markedly different.

According to our findings, the bias-corrected and accelerated bootstrap approach performs best even with small sample sizes as long as the variance of ZTPID is not too large. Future research could focus on the other approaches to compare with the bootstrap methods.

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References

Beran R. Prepivoting to reduce level error of confidence sets. Biometrika. 1987; 74(3): 457-468.

David F, Johnson N. The truncated Poisson. Biometrics. 1952; 8(4): 275-285.

Donna LM, Wilson WJ, Freund RJ. Statistical methods. London: Academic Press; 2022.

Efron B. (1982). The Jackknife, the bootstrap, and other resampling plans, in CBMS-NSF Regional Conference Series in Applied Mathematics, Monograph 38 (Philadelphia, PA: SIAM).

Efron B. Better bootstrap confidence intervals. J Am Stat Assoc. 1987; 82(397), 171-185.

Efron B, Tibshirani RJ. An Introduction to the Bootstrap. New York: Chapman and Hall; 1993.

Finney DJ, Varley GC. An example of the truncated Poisson distribution, Biometrics. 1955; 11(3), 387-394.

Garman P. (1923) The European red mites in Connecticut apple orchards, Connecticut Agri Exper Station Bull 252: 103-125.

Ghitany ME, Al-Mutairi DK and Nadarajah S. Zero-truncated Poisson-Lindley distribution and its application. Math Comput Simul. 2008; 79(3): 279-287.

Henningsen A, Toomet O. maxLik: A package for maximum likelihood estimation in R. Comput Stat. 2011; 26(3): 443-458.

Hussain T. A zero truncated discrete distribution: Theory and applications to count data. Pak J Stat Oper Res. 2020; 16(1): 167-190.

Ihaka R., Gentleman R. (1996). R: a language for data analysis and graphics. J Comput Graph Stat. 5(3): 299-314.

Kissell R, Poserina J. Optimal sports math, statistics, and fantasy. London: Academic Press; 2017.

Letson D, McCullough BD. Better confidence intervals: the double bootstrap with no pivot. Am J Agric Econ. 1998; 80(3), 552-559.

Lindley DV. Fiducial distributions and Bayes' theorem. J R Stat Soc Ser B. 1958; 20(1): 102-107.

McCullough BD, Vinod HD. (1998). Implementing the double bootstrap. Comput Econ. 1998; 12(1): 79-95.

Meeker WQ, Hahn GJ, Escobar LA. Statistical intervals: A guide for practitioners and researchers. New York: John Wiley and Sons; 2017.

Nankervis JC. Computational algorithms for double bootstrap confidence intervals. Comput Stat Data Anal. 2005; 49(2): 461-475.

Sangnawakij P. Confidence interval for the parameter of the zero-truncated Poisson distribution. J Appl Sci. 2021; 20(2): 13-22.

Shanker R. The discrete Poisson-Akash distribution. Int J Prob Stat. 2017a; 6(1): 1-10.

Shanker R. Zero-truncated Poisson-Akash distribution and its applications. Am J Math Stat. 2017b; 7(6): 227-236.

Shanker R. Akash distribution and its applications. Int J Prob Stat. 2015; 4(3): 65-75.

Shanker R, Hagos F. Zero-truncated Poisson-Sujatha distribution with applications. J Ethiopian Stat Assoc. 2015; 24: 55-63.

Shanker R, Shukla KK. Ishita distribution and its application to model lifetime data. Biomet Biostat Int J. 2017; 5(2): 1-9.

Shukla KK, Shanker R. A discrete Poisson-Ishita distribution and its applications. Int J stat Econ. 2017; 20(2): 109-122.

Shukla KK, Shanker R, Tiwari MK. Zero-truncated Poisson-Ishita distribution and its application. J Sci Res. 2020; 64(2): 287-294.

Siegel AF. Practical business statistics. London: Academic Press; 2016.

Simonoff JS. Analyzing categorical data. New York: Springer; 2003.

Turhan NS. Karl Pearson's chi-square tests. Educ Res Rev. 2020; 15(9): 575-580.

Wood M. Statistical inference using bootstrap confidence intervals. Significance. 2004; 1(4): 180-182.