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Trivariate Copulas on the MEWMA Control Chart

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Abstract

Copulas are used to model multivariate distribution with continuous margins and dependence observations. This paper proposes four types of copulas on the multivariate exponentially weighted moving average (MEWMA) control chart for trivariate case. Observations are generated by an exponential distribution based on the Monte Carlo simulation for the Clayton, Frank, Gumbel and normal copulas and the mean shifts are 1.05, 1.25, 1.5, 2, 2.5 and 3. The performance of the control charts describes in terms of the average run length (ARL). Levels of the dependence of random variables are measured by Kendall's tau (τ) as 0.2, 0.5 and 0.8 for small, moderate and large dependencies. The results reveal that the Clayton copula performs the ARL_1 values less than the others for almost mean shifts.

Keywords: Average run length (ARL), Monte Carlo simulation, Kendall's tau.

1. Introduction

Control charts are statistical and visual tools that are designed to detect shifts in a process under the assumption of observations. A univariate control chart is planned to monitor the quality of a single process characteristic. However, modern processes often monitor more than one of the quality characteristics and are referred to as multivariate statistical process control (MSPC) procedures. MSPC is the most rapidly developing section of statistical process control and raises interest in the simultaneous examination of several related quality characteristics (Fuchs and Kenett 1998, Lowry and Montgomery 1995). Most multivariate detection procedures are based on a multi-normality assumption and independence, but many processes found a non-normality and correlation.

The copula approach is a popular tool for modeling non-linearity, asymmetry and tail dependence in several fields. It can be used in the study of the dependence or association among random variables. Copula modeling is based on a representation of Sklar's theorem (Sklar 1973). It can estimate the joint distribution of non-linear outcomes and explain the dependence structure among variables through the joint distribution by eliminating the effect of univariate marginals.

Currently, many researchers have published about copulas on the control charts. Fatahi et al. (2012) studied copula-based bivariate zero inflated Poisson distribution to monitor rare events.

Hryniewicz (2012) presented the robustness of Shewhart control charts in the case of dependent data for auto-correlated and normal data. Dokouhaki and Noorossana (2013) described a copula Markov CUSUM chart for monitoring of two auto-correlated data sets. Verdier (2013) introduced a new approach for non-normal multivariate cases by Cauchy, Student's t and normal distributions. Kuvattana et al. (2016) proposed bivariate copulas on the exponentially weighted moving average control chart. Sukparungsee et al. (2017) studied multivariate copulas on the MCUSUM control chart for trivariate case. Tiengket et al. (2020) constructed bivariate copulas on the Hotelling's $\mathbf{T}^2$ control chart for five types of copulas.

In industrial processes, a continuous distribution is widely used for monitoring the time in successful occurrences of events, namely exponential distribution. This paper proposes four types of copulas on the multivariate exponentially weighted moving average (MEWMA) control chart for trivariate case when observations are generated by an exponential distribution. The dependence among random variables is specified by Kendall's tau (τ) and the mean shifts are 1.05, 1.25, 1.5, 2, 2.5 and 3.

2. The MEWMA Control Chart

The multivariate exponentially weighted moving average (MEWMA) control chart has been proposed by Lowry et al. (1992). Let $W_1, W_2, \ldots$ be observations from a *d*-variate Gaussian distribution $\mathrm{N}(\boldsymbol{\mu}, \Sigma)$ for $i = 1, 2, \ldots,$ which define recursively,

$$\mathbf{Z}_i = \mathbf{\Lambda}\mathbf{W}_i + (1 - \mathbf{\Lambda})\mathbf{Z}_{i-1}, \tag{1}$$

where $\mathbf{Z}_0$ denotes the vector of variable values from the historical data, and $\mathbf{\Lambda}$ is a diagonal matrix with entries $\lambda_1, \ldots, \lambda_d$. The plotted quantity is written by

$$\mathbf{T}_i^2 = \mathbf{Z}_i' \textstyle\sum_i^{-1} \mathbf{Z}_i, \tag{2}$$

where $\sum_i = \dfrac{\lambda}{2-\lambda}\left\{1-(1-\lambda)^{2i}\right\}\sum$ and $\lambda_1 = \ldots = \lambda_d = \lambda \in (0,1)$ as assumed in the paper. The control chart signals a shift in the mean vector when $\mathbf{T}_i^2 > h,$ where h is the control limit chosen to achieve a desired in-control. If $\lambda = 1$ in (1), the MEWMA control chart statistic reduces to $\mathbf{T}_i^2 = \mathbf{Z}_i' \sum_i^{-1} \mathbf{Z}_i,$ the statistic used in the Hotelling's $\mathbf{T}^2$ control chart (Runger et al. 1999, Montgomery 2009, Kuvattana et al. 2016).

3. Copulas Modeling

Copulas explain a general form of multivariate distribution with marginal distributions. The multivariate copula function is used for capturing the dependence between two or more random variables. Suppose that a random vector $X_1, \ldots, X_d$ has a joint distribution function $H(x_1, \ldots, x_d)$ with continuous marginal distribution function $F_i(x_i) = u_i,$ where U_i has uniform distribution [0,1] for $i = 1, \ldots, d;$ then, there exists a unique *d*-dimensional copula $C.$ (see Sklar 1959, Trivedi and Zimmer 2005, Genest and McKay 1986, Joe 2015, Sukparungsee et al. 2017). Theory of copulas describes as follows:

3.1. The Fréchet-Hoeffding bounds

The Fréchet-Hoeffding bounds for the d-variate joint cumulative distribution function (cdf) $H(x_1,...,x_d)$ with univariate marginal cdfs $F_1,...,F_d$ are $\mathcal{F}$ $(F_1,...,F_d)$. The Fréchet-Hoeffding lower and upper bounds are expressed below:

$$\max\left[\sum_{i=1}^{d} F_i - d + 1, 0\right] \le H(x_1,...,x_d) \le \min\left[F_1,...,F_d\right] \tag{3}$$

(see Joe 1997).

3.2. Sklar's theorem

Sklar's theorem is the most important result regarding copulas and use in essentially all applications of copulas.

Theorem 1. (Sklar's theorem for d-variate) *Let $F \in \mathcal{F}(F_1,...,F_d)$ with i^{th} univariate margin F_i. The copula associated with H is a distribution function $C:[0,1]^d \to [0,1]$ with $U(0,1)$ that satisfies*

$$H(x_1,...,x_d) = C\left(F_1(x_1),...,F_d(x_d)\right);\ x_1,...,x_d \in \mathbf{R}^d. \tag{4}$$

(a) If F is a continuous d-variate distribution function with univariate margins $F_1,...,F_d$ and quantile functions $F_1^{-1},...,F_d^{-1}$, then

$$C(u_1,...,u_d) = H\left(F_1^{-1}(u_1),...,F_d^{-1}(u_d)\right);\ u_1,...,u_d \in [0,1]^d \tag{5}$$

is unique.

(b) If F is a d-variate distribution function of discrete random variables, then the copula is unique only on the set

$$Range(F_1) \times ... \times Range(F_d), \tag{6}$$

where $F_i^{-1}(u_i)$ is the quantile function of F.

3.3. Families of copulas

This section considers two families of copulas, the elliptical and Archimedean copulas. The paper focuses on the normal copula and three types of Archimedean copulas, i.e., the Clayton, Frank and Gumbel copulas, because these are the well-known copulas. The densities of copulas are illustrated in Figures 1-3.

3.3.1. Normal copula

The Normal copula is an elliptical copula. For the d-variate, multivariate normal distribution with zero means, unit variances and correlation matrix Σ, the trivariate normal copula is written by

$$C(u_1,u_2,u_3;\Sigma) = \Phi\left(\Phi^{-1}(u_1),\Phi^{-1}(u_2),\Phi^{-1}(u_3);\Sigma\right); \tag{7}$$

$-1 \le \theta \le 1$, $\Sigma = \begin{bmatrix} 1 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & 1 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & 1 \end{bmatrix}$ and $\theta = (\mu_{x_1},\mu_{x_2},\mu_{x_3},\sigma_{x_1},\sigma_{x_2},\sigma_{x_3},\rho)$, where $\Phi(\cdot;\Sigma)$ is the trivariate normal cdf, Φ is the univariate normal cdf, and Φ^{-1} is the univariate normal inverse cdf or quantile function (Joe 2015, Ganguli and Reddy 2013, Sukparungsee et al. 2017).

3.3.2. Archimedean copulas

Let ϕ denotes a continuous strictly decreasing function from $\mathbf{I}$ to $[0,\infty]$ such that $\phi(0)=\infty$ and $\phi(1)=0,$ and let ϕ^{-1} denote the inverse of $\phi.$ The d-dimensional function from $\mathbf{I}^d$ to $\mathbf{I}$ is given by

$$C(u_1,...,u_d)=\phi^{[-1]}\left(\phi(u_1)+...+\phi(u_d)\right), \tag{8}$$

where $u_i=F_i(x_i)$ is the marginal cdf of X_i and $\phi^{[-1]}$ is the pseudo-inverse of an Archimedean generator ϕ and it is completely monotonic on $[0,\infty],$ i.e. $\phi^{[-1]}=\phi^{-1}$ (see Trivedi and Zimmer 2005, Nelsen 2006). The Archimedean copulas in this paper are generated as follows:

3.3.2.1. Clayton copula

Let $\phi(t)=(t^{-\theta}-1)/\theta$ for $\theta>0,$ which generates a subfamily of the bivariate Clayton copula. Then, $C(u_1,u_2)=\left[\max(u_1^{-\theta}+u_2^{-\theta}-1,0)\right]^{-1/\theta}.$ The trivariate Clayton copula can be generalized from the bivariate copula by

$$C(u_1,u_2,u_3)=(u_1^{-\theta}+u_2^{-\theta}+u_3^{-\theta}-2)^{-1/\theta}\,;\theta\ge 0. \tag{9}$$

3.3.2.2. Frank copula

Let $\phi(t)=-\ln\left(\dfrac{e^{-\theta t}-1}{e^{-\theta}-1}\right)$ for $\theta>0,$ which generates a subfamily of the bivariate Frank copula. Then, $C(u_1,u_2)=-\dfrac{1}{\theta}\ln\left[1+\dfrac{(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)}{e^{-\theta}-1}\right].$ The trivariate Frank copula can be generalized from the bivariate copula by

$$C(u_1,u_2,u_3)=-\frac{1}{\theta}\ln\left[1+\frac{(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)(e^{-\theta u_3}-1)}{(e^{-\theta}-1)^2}\right];\quad \theta\ge 0. \tag{10}$$

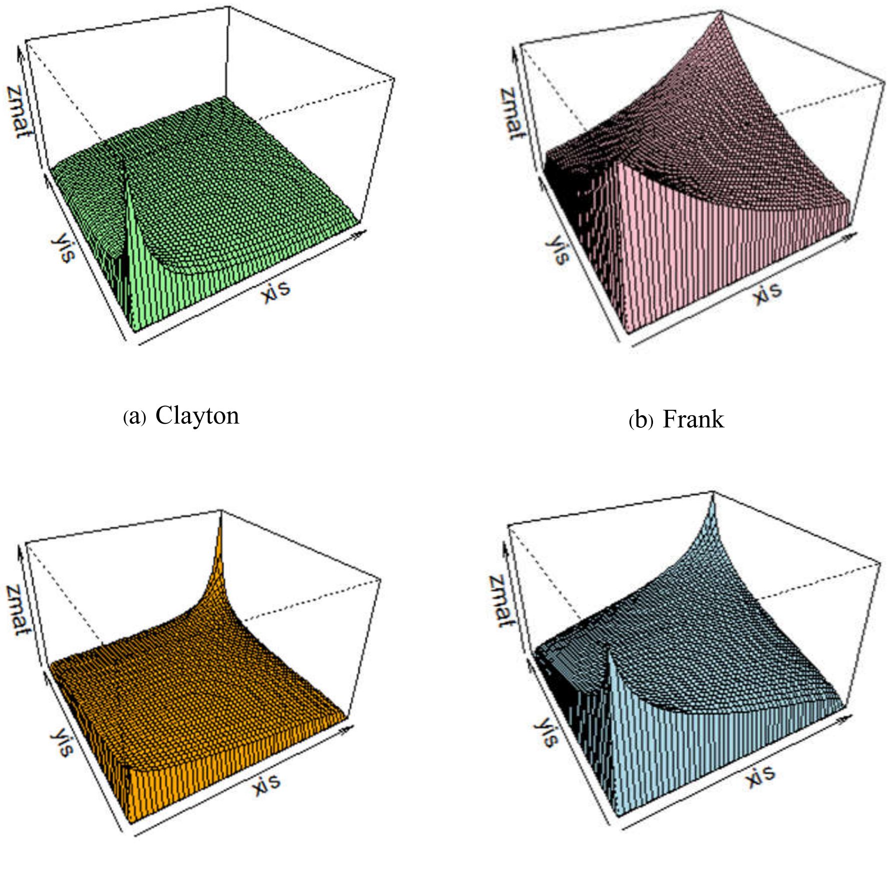
Let $\phi(t)=[-\ln(t)]^{\theta}$ for $\theta\in[1,\infty),$ which generates a subfamily of the bivariate Gumbel copula. Then, $C(u_1,u_2)=\exp\left[-\left\{(-\ln u_1)^{\theta}+(-\ln u_2)^{\theta}\right\}^{1/\theta}\right].$ The trivariate Gumbel copula can be generalized from the bivariate copula by

$$C(u_1,u_2,u_3)=\exp\left[-\left\{(-\ln u_1)^{\theta_2}+(-\ln u_2)^{\theta_2}\right\}^{\theta_1/\theta_2}+(-\ln u_3)^{\theta_1}\right]^{1/\theta_1}\,;\theta\ge 0 \tag{11}$$

(see Orcel et al. 2020, Latif and Mustafa 2020, Wong et al. 2010).

4. Measuring Dependence and Average Run Length

Spearman's rho and Kendall's tau are the well-known measures of multivariate relationship. Kendall's tau is a non-parametric measure for association among random variables and can be applied to copulas. In the bivariate case, we assume that X_1 and X_2 denote continuous random variables whose copula is $C,$ Kendall's tau is given by $\tau_c=4\iint_{\mathbf{I}^2}C(u_1,u_2)\,dC(u_1,u_2)-1,$ where τ_c is Kendall's tau of copula C and the unit square $\mathbf{I}^2$ is the product of $\mathbf{I}\times\mathbf{I}$ where $\mathbf{I}=[0,1].$ The expected value is the function $C(u_1,u_2)$ of uniform (0,1) random variables U_1 and $U_2,$ whose joint distribution function is C, i.e., $\tau_c = 4E\left[C(U_i, U_2)\right] - 1$ (Nelsen 2006). Considering two random variables of Archimedean copula C generated by ϕ; then, $\tau_{Arch} = 4\int_0^1 \frac{\phi(t)}{\phi'(t)}\,dt + 1$, where τ_{Arch} is Kendall's tau of C. In this paper, τ_c of the trivariate and bivariate cases is similarly estimated (Genest and McKay 1986).

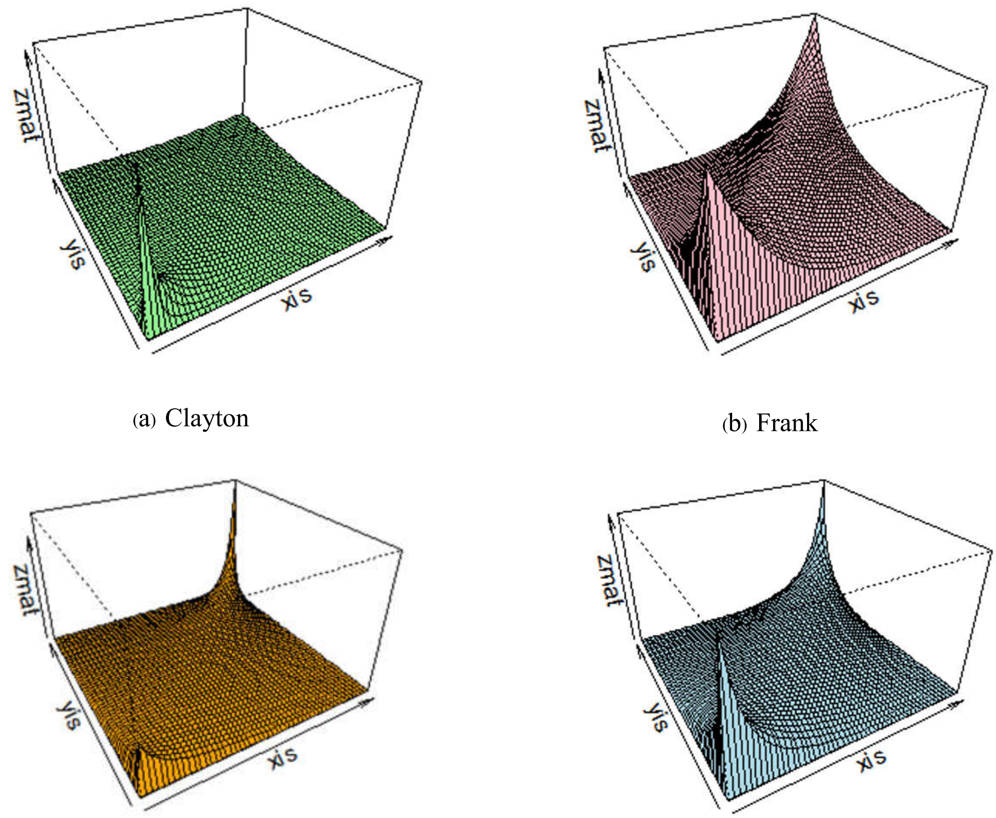


(a) Clayton (b) Frank

(c) Gumbel (d) Normal

Figure 1 Plots of the corresponding parameter under 3-dimensional copulas $(\tau = 0.2)$

The average run length (ARL) is typically used to measure the performance of control charts. ARL is split into ARL_0 for in-control and ARL_1 for out-of-control. In-control process, ARL_0 should be large, while out-of-control process, ARL_1 should be small.



(a) Clayton (b) Frank

(c) Gumbel (d) Normal

Figure 2 Plots of the corresponding parameter under 3-dimensional copulas ($\tau = 0.5$)

Table 1 Kendall's tau and four different copula functions

Copula	Type	Kendall's tau (τ)	Parameter space of θ
Clayton	Asymmetric	$\theta / (\theta + 2)$	$[-1, \infty) \setminus \{0\}$
Frank	Asymmetric	$1 + 4\left[\frac{1}{\theta}\int_0^{\theta} \frac{t}{e^t - 1} dt - 1\right] / \theta$	$(-\infty, \infty) \setminus \{0\}$
Gumbel	Asymmetric	$(\theta - 1) / \theta$	$[1, \infty)$
Normal	symmetric	$\arcsin(\theta) / (\pi / 2)$	$[-1, 1]$

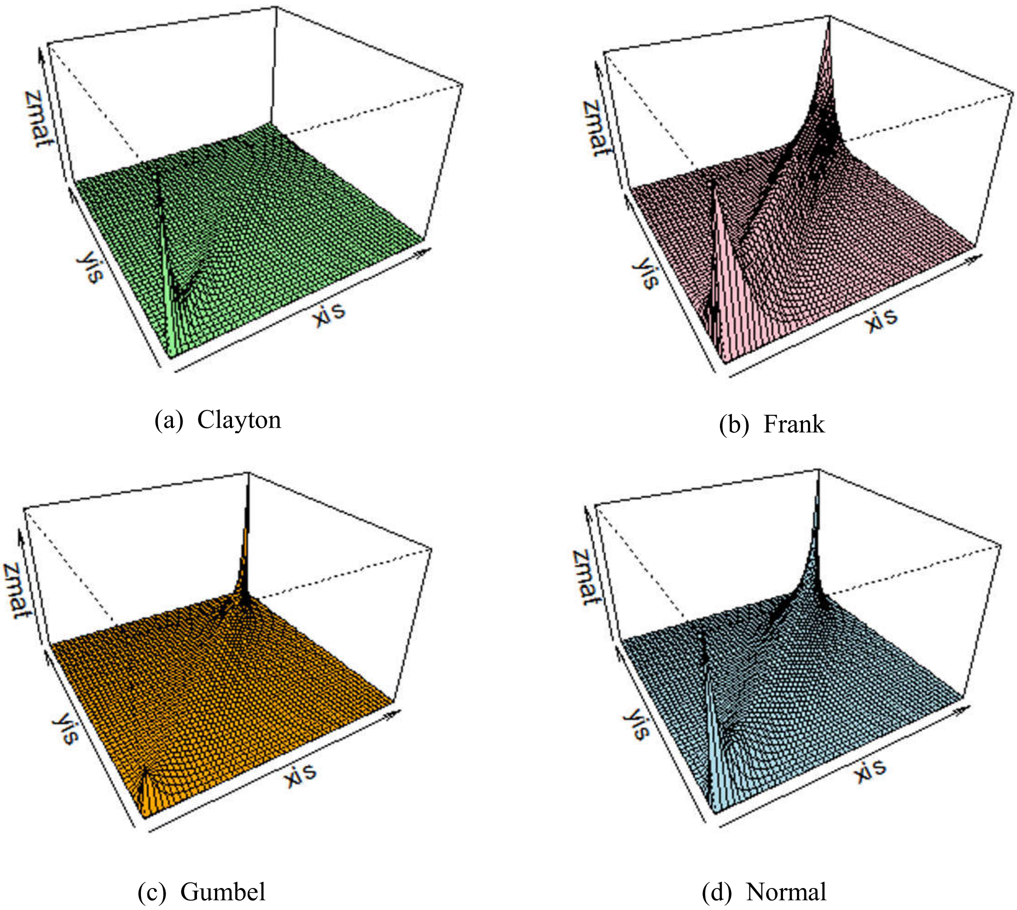


Figure 3 Plots of the corresponding parameter under 3-dimensional copulas $(\tau = 0.8)$

5. Results and Discussion

This research used R program to simulate observations, control chart and copulas by Monte Carlo simulation technique with 50,000 simulations and sample sizes of 1,000. (see Yan 2007, Hofert et al. 2012). Let α and μ denote the parameters of an exponential distribution and the mean of process, respectively. For the quantity measurement, empirical model defined δ as the shift size reported in $\delta = \mu - \mu_0$. For the in-control process, observations were from parameters $\alpha = 1$, $\delta = 0$ and $\mu = 1$. The mean (μ) shifts of out-of-control processes were 1.05, 1.25, 1.5, 2, 2.5 and 3.

The performance of the MEWMA control chart with $\lambda = 0.1$ was evaluated by simulation technique. All copula functions were set by θ to be the parameters of Kendall's tau in the case of positive dependence. Kendall's tau values are 0.2, 0.5 and 0.8 for small, moderate and large dependencies, respectively.

Tables 2-4 showed the results for empirical observations. The different values of exponential parameters were (μ_1, μ_2, μ_3) for the variables (X_1, X_2, X_3). The MEWMA control chart was chosen by setting the desired ARL_0 = 370 for in-control processes, and $\mu_1 = \mu_2 = \mu_3 = 1$ was fixed for each copula. The results were illustrated in Table 2 which showed the mean shifts of $\tau = 0.2$ and the ARL_1 values of the Clayton copula were less than the other copulas for all shifts.

Table 3 showed the case of moderate dependence that was (1) in the case of all parameters shifts, the ARL_1 values of the Clayton copula were less than the other copulas for small shifts $(1.05 \le \mu_i \le 2)$ and the ARL_1 values of the normal copula were less than the other copulas for large shifts $(2.5 \le \mu_i \le 3)$, and (2) in the case of two parameters shifts, the ARL_1 values of the Clayton copula were less than the other copulas for small shifts $(1.05 \le \mu_i \le 1.5)$ and the ARL_1 values of the normal copula were less than the other copulas for large shifts $(2 \le \mu_i \le 2.5)$.

For large dependence in Table 4 showed that (1) in the case of all parameters shifts, the ARL_1 values of the Clayton copula were less than the other copulas for small shifts $(1.05 \le \mu_i \le 1.5)$ and the ARL_1 values of the normal copula were less than the other copulas for large shifts $(2 \le \mu_i \le 3)$, and (2) in the case of two parameters shifts, the ARL_1 values of the Gumbel copula were less than the other copulas for almost all shifts. Finally, the performance comparison of Kendall's tau was shown in Table 5.

Table 2 ARL of the MEWMA control chart for $\tau = 0.2$

Mean shifts			ARL			
μ_1	μ_2	μ_3	Clayton	Frank	Gumbel	Normal
1	1	1	369.960	370.023	369.910	369.898
1.05	1.05	1.05	**279.697**	280.963	291.751	282.670
1.25	1.25	1.25	**103.737**	106.080	123.101	110.093
1.5	1.5	1.5	**40.564**	42.498	51.952	44.980
2	2	2	**10.846**	11.264	13.317	11.740
2.5	2.5	2.5	**4.263**	4.385	4.980	4.494
3	3	3	**2.047**	2.088	2.313	2.150
1	1.05	1.05	**305.351**	307.866	315.478	308.176
1	1.25	1.25	**142.619**	146.951	164.207	150.500
1	1.5	1.5	**66.019**	67.852	80.088	70.644
1	2	2	**20.030**	20.487	23.912	21.264
1	2.5	2.5	**8.314**	8.612	9.621	8.848
1	3	3	**4.256**	4.300	4.776	4.428
1.05	1	1.05	309.581	**308.056**	317.455	308.341
1.25	1	1.25	**144.788**	146.860	164.245	149.418
1.5	1	1.5	**65.303**	67.768	79.747	70.710
2	1	2	**20.016**	20.617	23.726	21.316
2.5	1	2.5	**8.398**	8.547	9.667	8.740
3	1	3	**4.231**	4.325	4.686	4.366
1.05	1.05	1	306.453	**305.916**	316.567	309.526
1.25	1.25	1	**144.707**	146.655	163.661	150.549
1.5	1.5	1	**65.396**	68.173	79.922	71.053
2	2	1	**20.106**	20.698	23.926	21.395
2.5	2.5	1	**8.454**	8.575	9.499	8.828
3	3	1	**4.276**	4.304	4.731	4.364

Note: bold number is minimum ARL_1

Table 3 ARL of the MEWMA control chart for $\tau = 0.5$

Mean shifts			ARL			
μ_1	μ_2	μ_3	Clayton	Frank	Gumbel	Normal
1	1	1	370.134	370.066	370.018	369.904
1.05	1.05	1.05	**285.216**	287.826	301.343	290.994
1.25	1.25	1.25	**108.527**	114.974	140.117	123.149
1.5	1.5	1.5	**45.978**	50.082	62.085	54.089
2	2	2	**13.973**	14.994	16.002	14.551
2.5	2.5	2.5	5.920	6.226	5.930	**5.622**
3	3	3	3.090	3.159	2.745	**2.726**
1	1.05	1.05	**309.609**	310.308	320.415	313.618
1	1.25	1.25	**148.064**	152.182	172.172	158.909
1	1.5	1.5	**69.929**	73.205	82.481	76.728
1	2	2	22.862	23.254	22.372	**21.803**
1	2.5	2.5	9.617	9.524	8.510	**8.476**
1	3	3	4.918	4.808	**4.051**	4.174
1.05	1	1.05	**308.817**	313.213	321.498	313.355
1.25	1	1.25	**147.383**	152.309	173.409	158.011
1.5	1	1.5	**69.814**	72.880	82.690	76.586
2	1	2	22.823	23.269	22.290	**21.851**
2.5	1	2.5	9.531	9.534	8.493	**8.476**
3	1	3	4.909	4.735	**4.016**	4.224
1.05	1.05	1	**307.960**	313.901	319.746	312.560
1.25	1.25	1	**147.269**	152.124	172.348	156.370
1.5	1.5	1	**70.327**	73.361	81.957	75.464
2	2	1	22.652	23.093	22.327	**21.715**
2.5	2.5	1	9.499	9.584	8.415	**8.419**
3	3	1	4.917	4.781	**4.039**	4.166

Note: bold number is minimum ARL_1

6. Conclusions

Copula models have become more and more popular, especially in recent years. Measuring dependence of more than one variable can be shown in terms of copulas by Sklar's theorem. The authors simulate observations for four types of copulas on the MEWMA control chart based on trivariate copulas. The results reveal that the Clayton and the Gumbel copulas perform the ARL_1 values less than the others for small and large dependencies, respectively. Further research may be extended to other copulas such as Farlie-Gumbel-Morgenstern, Plackett, and Ali-Mikhail-Haq. Moreover, we design to compare with various control charts for the advanced development based on the proposed copulas modeling, and also extend our trivariate copulas with real data application.

Table 4 ARL of the MEWMA control chart for $\tau = 0.8$

Mean shifts			ARL			
μ_1	μ_2	μ_3	Clayton	Frank	Gumbel	Normal
1	1	1	369.860	370.130	370.010	370.054
1.05	1.05	1.05	**296.336**	301.590	305.480	304.027
1.25	1.25	1.25	**129.621**	137.993	153.416	146.314
1.5	1.5	1.5	**61.340**	69.736	73.379	69.273
2	2	2	23.788	27.651	21.676	**20.105**
2.5	2.5	2.5	11.819	13.226	8.559	**8.047**
3	3	3	6.884	7.440	4.125	**3.901**
1	1.05	1.05	**316.529**	319.383	321.317	319.507
1	1.25	1.25	159.352	163.231	157.127	**150.631**
1	1.5	1.5	79.654	81.950	**60.262**	60.780
1	2	2	27.170	25.274	**12.717**	13.287
1	2.5	2.5	11.114	9.613	**4.431**	4.727
1	3	3	5.382	4.652	**1.914**	2.045
1.05	1	1.05	**316.634**	319.858	321.715	320.339
1.25	1	1.25	159.168	163.343	157.315	**150.283**
1.5	1	1.5	78.969	81.642	**59.997**	61.118
2	1	2	27.420	25.424	**12.684**	13.267
2.5	1	2.5	11.037	9.559	**4.473**	4.746
3	1	3	5.431	4.663	**1.903**	2.061
1.05	1.05	1	**317.182**	318.797	321.614	319.081
1.25	1.25	1	160.801	162.747	156.781	**151.073**
1.5	1.5	1	79.373	82.035	**59.787**	61.776
2	2	1	27.376	25.319	**12.783**	13.306
2.5	2.5	1	11.057	9.520	**4.450**	4.740
3	3	1	5.369	4.667	**1.903**	2.010

Note: bold number is minimum ARL_1

Table 5 The performance comparison of Kendall's tau

Parameters	Shifts	Kendall's tau (τ)		
		$\tau = 0.2$	$\tau = 0.5$	$\tau = 0.8$
Two parameters	Small shift	Frank	Clayton	Clayton
	Moderate shifts	Clayton	Clayton	Gumbel, Normal
	Large shifts	Clayton	Normal	Gumbel
Three parameters	Small shift	Clayton	Clayton, Normal	Clayton
	Moderate shifts	Clayton	Clayton, Normal	Clayton, Normal
	Large shifts	Clayton	Normal	Normal

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