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Estimating of α-series Process Parameters Using Particle Swarm Optimization Algorithm

Shaimaa Waleed Mahmood*
Department of Statistics and Informatics, College of Computer science & Mathematics, University of Mosul, Mosul, Iraq.
*Corresponding author; e-mail: shaimaa.waleed@uomosul.edu.iq

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Abstract

The α -series stochastic process has special importance in many life areas. In this paper, the particle swarm optimization (PSO) and the least squares (LS) methods are used to estimate parameters of the α -series process. The real data results show that the PSO algorithm is better in estimation compared to the LS method, in term of mean absolute error (MAE).

Keywords: α -series process, Laplace's test, particle swarm, nonparametric estimation.

1. Introduction

The α-series process is a stochastic process related to the geometric process, and it is also monotone process which has special importance in the many of life areas. In addition, it has many features, including the possible treatment of many problems in different areas and ease of procedures.

The model of the α -series stochastic process is an important tool for studying deteriorating systems that can be repaired in maintenance problems, as a main tool in data analysis with a monotone trend, and it has many uses as the optimal inspection repair replacement policy (Chen et al. 2010).

Swarm intelligence is a branch of artificial intelligence that is widely used for solving optimization problems, and it took its inspiration from biology (Premalatha and Natarajan 2009). Particle swarm optimization is a simple model of social learning whose nascent behavior gained popularity as a technique to solve complex optimization problems in a reliable and simple manner.

In this paper, a least squares and particle swarm optimization algorithm are used to estimate the α -series process parameters with application on a dataset. The results are compared between the two using methods.

2. α-Series Process

The α-series process is a monotone stochastic process. Its idea belongs to Braun et al. (2005), who studied their properties. It has many features including the executions and the possibility of its application in reliability and stochastic scheduling, which explains the many uses for the model this process. It is called the α -power process in which the constant process is raised to power.

Consider the stochastic process $\{N(t), t \geq 0\}$ that is a counting process. x_j is the time interval between the two events times j and $j-1$. The counting process with random nonnegative variables is the α-series process for the parameter α, if there a real value α such that (Aydoğdu and Kara 2012)

$$y_j = j^{\alpha} x_j, \quad j = 1,2,\ldots \tag{1}$$

where y_j is a renewal process (RP), which are independent and identically distributed (i.i.d.) random variables. The cumulative distribution function can be expresses as (Braun et al. 2008)

$$F_j(x) = F\left(j^{\alpha} x_j\right), \quad j = 1,2,\ldots \tag{2}$$

By taking the derivative of (2) with respect to x, you can get the probability density function for this process (Lam 1988).

$$\frac{\partial F_j(x)}{\partial x} = f_j(x) = j^{\alpha} f(j^{\alpha} x), \quad -\infty < \alpha < \infty. \tag{3}$$

The α -series process is stochastically decreasing when $\alpha > 0$ and stochastically increasing when $\alpha < 0$. When $\alpha = 0$, the α -series process reduces to a renewal process, and all the random variables are identically distributed. If F is an exponentiation distribution function and $\alpha = 1$, then the α -series process is a linear birth process (Aydogdu et al. 2010, Braun et al. 2008, and Tang and Liu 2006).

3. Parameters of α-Series Process

Assume that $\{x_j, j = 1,2,\ldots.\}$ are events times for the α -series process. Then

$$x_j = \frac{y_j}{j^{\alpha}}, \tag{4}$$

where $y_j = j^{\alpha} x_j$, which is a sequence of random variables (i.i.d.), then

$$E\left(x_j\right) = \frac{E\left(y_j\right)}{j^{\alpha}},$$

such that $E\left(y_j\right) = \mu$, then

$$E\left(x_j\right) = \frac{\mu}{j^{\alpha}}, \tag{5}$$

while the variance x_j can be expressed as

$$Var\left(x_j\right) = \frac{Var\left(y_j\right)}{(j^{\alpha})^2},$$

such that $Var\left(y_j\right) = \sigma^2$, then

$$Var\left(x_j\right) = \frac{\sigma^2}{j^{2\alpha}}. \tag{6}$$

The main parameters for the α -series process are α, μ and σ^2 because they specify the mean and variance of x_j (Aydoğdu and Kara 2012).

4. Testing α-Series Process

Testing the α -series process is a good and necessary subject for studying processes. When implementing this process on real data, we have many problems. The most important problem is the appropriateness of the study data for this process. The procedure for testing can be performed in two steps.

4.1. Testing for monotone trend of the data

To test the α-series process, we should first test if there is a monotone trend of data. If the test proves that there is no monotone trend of data, the renewal process will be best for use. There are many techniques used to test for the monotone trend of a stochastic process, including Laplace's test, technical graph and Mann test. Moreover, many techniques have been discussed, e.g., (Cox and Lewis 1966) and (Ascher and Feingold 1984). In this paper, Laplace's test is used to test the monotone trend in the data. Laplace's test is a nonparametric test, and it is commonly used to test the monotone trend in a dataset according to these hypotheses. The null and alternative hypotheses are as follows:

H_0 : Data do not have monotone trend

H_1 : Data have a monotone trend.

The statistical test for this hypothesis test will be the following (Xavier et al. 2013):

$$U = \frac{\frac{\sum_{i=1}^{n-1} t_{i-1}}{n-1} - \frac{t_n}{2}}{t_n \sqrt{\left(\frac{1}{12(n-1)}\right)}}. \tag{7}$$

U is Laplace's test; $U \sim N(0,1)$. Considering the p-value in $P(|Z| \geq U)$, then if the p-value is bigger than 0.05, then we accept H_0 and refuse H_1.

4.2. Testing data for the α-series process

The second step after knowing the trend is to test whether data are accepted for the α -series process. To solve this problem, assume (Aydoğdu and Kara 2012)

$$y_j = j^{\alpha} x_j, \qquad j = 1,2,\ldots.$$

By taking the logarithm, we obtain

$$\ln y_j = \alpha \ln j + \ln x_j. \tag{8}$$

The y_j are i.i.d. random variables, and the simple linear regression model can be used with

$$\ln y_j = \gamma + e_j, j = 1,2,\ldots \tag{9}$$

$$E(\ln y_j) = \gamma. \tag{10}$$

The e_j are the sequence of i.i.d. random variables with mean 0 and variance σ_e^2. From (8), we can obtain the model.

$$\ln x_j = \ln y_j - \alpha \ln j. \tag{11}$$

Using (9) and (11), we get

$$\ln x_j = (\gamma + e_j) - \alpha \ln j\ . \tag{12}$$

Repeating (12), we get the model

$$\ln x_j = \gamma - \alpha \ln j + e_j, j = 1,2,\ldots \tag{13}$$

The plot of $\ln x_j$ against $\ln j,$ can be used to note whether there is a linear relationship between them or not. If the plot shows that there is a linear relationship between them, we conclude that the data x_j come from an α-series process.

5. Parameter Estimation of α-series Process

As we know, the α -series process has three important parameters, α, μ and σ^2. When applying this process on real data, we need to estimate these parameters. There are many methods suggested to estimate these parameters, including nonparametric, parametric and other methods (Aydogdu et al. 2010).

In this paper, we use the nonparametric least squares method, which is known method in estimation, and particle swarm optimization algorithm, which is an intelligent technique method.

5.1. Least squares method

The least squares method is one of the important nonparametric methods in estimation. The time rate for events in the α-series process is good with this method. This method depends on decreasing the sum of squares error to get the best to estimator parameters in this process. Assuming that the stochastic process $(x_j, j = 1,2,\ldots)$ represents the event times of the α -series process, the sum of squares error with the logarithm is

$$SSE = \sum_{j=1}^{n} \left[\ln y_j - E\left(\ln y_j \right) \right]^2 . \tag{14}$$

Using (8), (10) and (14), we obtain

$$SEE = \sum_{j=1}^{n} \left[\ln x_j + \alpha \ln j - \gamma \right]^2 . \tag{15}$$

To find the minimum of the sum of squares error, take the first partial derivative of (15) with respect to the parameters α and $\gamma,$ and set it equal zero shown in Equations (16) and (17):

$$\frac{\partial SEE}{\partial \alpha} = 2\sum_{j=1}^{n} \left[\ln x_j + \alpha \ln j - \gamma \right] \ln j = 0 \tag{16}$$

$$\frac{\partial SEE}{\partial \gamma} = -2\sum_{j=1}^{n} \left[\ln x_j + \alpha \ln j - \gamma \right] = 0 . \tag{17}$$

The procedure of the counting process and simplification of the least squares estimator for parameters α and γ can be expressed in Equations (18) and (19) (Braun et al. 2005)

$$\hat{\alpha} = \frac{\sum_{j=1}^{n} \ln x_j \sum_{j=1}^{n} \ln j - n\sum_{j=1}^{n} \ln x_j \ln j}{n\sum_{j=1}^{n} \left(\ln j \right)^2 - \left(\sum_{j=1}^{n} \ln j \right)^2}, \tag{18}$$

$$\hat{\gamma} = \frac{\sum_{j=1}^{n} \ln j \sum_{j=1}^{n} \ln x_j \ln j - \sum_{j=1}^{n} \left(\ln j \right)^2 \sum_{j=1}^{n} \ln x_j}{\left(\sum_{j=1}^{n} \ln j \right)^2 - n\sum_{j=1}^{n} \left(\ln j \right)^2}. \tag{19}$$

The estimator parameters μ and σ^2 can be obtained from the following equations (Aydoğdu and Kara 2012)

$$\hat{\mu} = \frac{s_n}{\sum_{j=1}^{n} j^{-\hat{\alpha}}} = \frac{\sum_{j=1}^{n} x_j}{\sum_{j=1}^{n} j^{-\hat{\alpha}}}, \tag{20}$$

$$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{j=1}^{n}\left(\hat{y}_j - \overline{\hat{y}}\right)^2, \tag{21}$$

$$\hat{y}_j = j^{\hat{\alpha}} x_j, \quad \overline{\hat{y}} = n^{-1}\sum_{j=1}^{n} \hat{y}_i.$$

5.2. Particle swarm optimization algorithm

Particle swarm optimization (PSO) is one of the intelligent techniques for solving optimization issues and this algorithm. This algorithm was considered by Kennedy and Eberhart (1995). Consider an unconstrained minimization issue (Bai 2010):

$$\text{Minimize } f(h) \ , \ H^l \le H \le H^u,$$

where as H^l and H^u indicate the lower and upper bounds on H. The steps of the PSO algorithm can be implemented as follows.

1. The volume of the swarm number of particles is N. To decrease the total number of function evaluations needed to discover a solution, we must assume a smaller volume of the swarm. In this situation, it can take a long time to discover the perfect solution. Ordinarily a volume of 20 to 30 particles is assumed for the swarm as a compromise.

2. Generate the initial population of H in the extent $\left[H^l, H^u\right]$ randomly assigned like $H_1, H_2, \ldots, H_N$. After that, the position of j and its velocity in iteration i are indicated $H_j(i)$ and $Q_j(i)$. Thus, the particles generated initially are indicated by $H_1(0), H_2(0), \ldots, H_N(0)$. The vectors $H_j(0),\ j = 1,2,\ldots,N$ are called particles. Then, evaluate the objective function values corresponding to the particles as $f\left[H_1(0)\right], f\left[H_2(0)\right], \ldots, f\left[H_N(0)\right]$.

3. Find the velocity of particles. All particles will be moving to the optimal point with velocity. Initially, all particle velocities are supposed to be 0. Set the iteration number $i = 1$.

4. In the i^{th} iteration, we discover the two important parameters used by particle j:

a. the best position for the particle,

b. the speed of particle j in the i^{th} iteration as in

$$Q_j(i) = Q_j(i-1) + c_1 r_1\left[P_{best} - H_j(i-1)\right] + c_2 r_2\left[G_{best} - H_j(i-1)\right], j = 1,2,...,N \ , \tag{22}$$

where

$Q_j(i)$ is the particle velocity in i^{th} iteration;

c_1, c_2 are the acceleration coefficients, ordinarily take value 2;

r_1, r_2 are the random values in the extent 0 to 1;

P_{best} is the best position to the particle swarm;

G_{best} is the best position to the particle including the entire swarm; and

c. Find the position of the j^{th} particle in i^{th} iteration.

$$H_j(i) = H_j(i-1) + Q_j(i) \ , j = 1,2,\ldots,N. \tag{23}$$

Then, evaluate the objective function values corresponding to the particle as $f\left[H_1(i)\right], f\left[H_2(i)\right], \ldots, f\left[H_N(i)\right]$.

5. Test the convergence of the current solution. If the position of all particles converges to itself as a set of values, then the method is assumed to have converged. If the convergence criterion is not satisfied, step 4 is repeated updating the iteration number to be $i = i + 1,$ and by computing the new values of P_{best} and G_{best}. The iterative process is continued until all particles converge to the same optimum solution.

6. Goodness of Fit Test

There are many measures used to test goodness of fit. In this paper, mean absolute error (MAE) is used. To compare a particle swarm optimization with the least squares, it is defined as

$$MAE = \frac{1}{n}\sum_{j=1}^{n}\left|x_j - \hat{x}_j\right| . \tag{24}$$

7. Application

In this section, two datasets with different sample sizes are used, the first dataset is operating times (hours) for machine building (Al-Saffawy and Al-Jammal 2006). The second dataset is the operating times between two stops (Ramadan 2017). For PSO, the swarm number was 30, and the number of iterations was 500. The first step in the statistical analysis for the α-series stochastic process on the two datasets is to test whether the process has a monotone trend, Laplace's test is used to ensure existence of the monotone trend for this data according to the hypotheses.

H_0 : Data do not have a monotone trend.

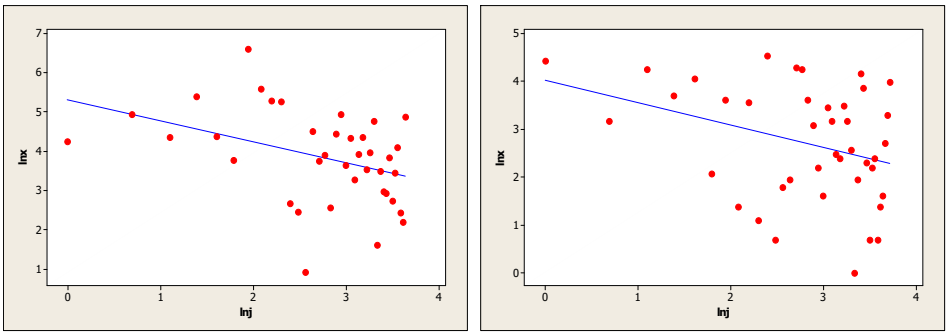
H_1 : Data have a monotone trend.

Table 1 shows the value of Laplace's test U and P_U for the two datasets.

Table 1 Laplace's test for two datasets

Data	U	P_U
Data 1	3.4095	0.0006
Data 2	2.1286	0.034

From Table 1, we note that the P_U values for the two datasets are less than 0.05. We reject the null hypothesis and accept the alternative hypothesis that the data have a monotone trend. The second step for analysis after knowing the monotone general trend of the data is testing whether the data come from an α-series process through plotting the logarithm for data with logarithm for time as shown in Figure 1.



a) Data 1 b) Data 2

Figure 1 Testing two datasets for α-series process

We can see that there are linear relationships between the logarithm for data and logarithm for time, and therefore we consider the two datasets as coming from α-series processes. We using nonparametric (LS) and intelligent technique (PSO) methods in estimating parameters of these processes, with α, μ and σ^2 for both sets as shown in Table 2.

Table 2 LS and PSO estimators to the α-series process parameters

Data	$\hat{\alpha}_{LS}$	$\hat{\mu}_{LS}$	$\hat{\sigma}^2_{LS}$	$\hat{\alpha}_{PSO}$	$\hat{\mu}_{PSO}$	$\hat{\sigma}^2_{PSO}$
Data 1	0.538	333.826	146843.144	0.2706	178.7363	47550.8898
Data 2	0.469	89.5584	7976.91098	0.4417	84.0082	6734.8639

To compare the PSO algorithm with least squares, MAE is used as a comparison criterion as shown in Table 3.

Table 3 MAE values of estimation methods for α-series process parameters

Data	LS	PSO
Data 1	70.2357	67.8431
Data 2	17.8925	16.0025

From Table 3, we note that the MAE values for the two datasets in PSO are less than that of LS. Therefore, the PSO algorithm is the best as compared with the LS method in estimation of parameters.

8. Conclusions

From the logarithm plot for the two datasets with the logarithm for time, the α value which is greater than zero for all the estimation methods used in this study. This shows that the data belong to the α-series process decreasing with the time. Through MAE values, we conclude that the PSO algorithm is the best as compared with the LS method in estimation of parameters.

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