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Bayesian Analysis of Inverse Rayleigh Distribution under Non-Informative Prior for Different Loss Functions

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Abstract

In this paper, we tend to acquire Bayes estimators for the unknown parameter of an inverse Rayleigh distribution (IRD). Bayes estimators are obtained beneath symmetric squared error loss function (SELF) and asymmetric loss functions by employing a non-informative prior. The performance of the estimators is assessed on the idea of their relative risk under the different loss functions. We also obtained the risk functions and risk efficiencies associated with the different Bayes estimators under the different loss functions and compared the performance of these estimators through simulation study. Finally, a numerical study is provided from which we concluded that minimum expected loss function is better than SELF, De-Groot loss function and precautionary loss function.

Keywords: Inverse Rayleigh distribution (IRD), Bayes estimator, squared error loss function, precautionary loss function, De-Groot loss function, minimum expected loss function, risk function, risk efficiency.

1. Introduction

The inverse Rayleigh distribution (IRD) plays a vital role in reliability and life testing study. It is also characterized as the failure time distribution. However, one parameter inverse Rayleigh distribution (IRD) with probability density function (pdf) and the reliability function is respectively given by

$$f(x,\lambda)=\frac{2e^{-\frac{1}{\lambda x^2}}}{\lambda x^3};x>0,\lambda>0. \tag{1}$$

$$R(t)=1-F(t)=1-e^{-\frac{1}{\lambda t^2}};t>0. \tag{2}$$

Voda (1972) mentioned that, the distribution of lifetimes of many sorts of experimental units is often approximated by the inverse Rayleigh distribution. The necessary feature of this distribution is that its variance and upper order moments don’t exist. Several authors have studied the estimation of inverse Rayleigh distribution. Some estimators and prediction results are developed by Abdel-Monem (2003). Soliman et al. (2010) studied the estimation and prediction of inverse Rayleigh distribution based on lower record values and Bayes estimator have been developed under squared error and zero-one loss functions. Dey (2012) discussed the Bayesian estimation of the parameter and reliability function of the inverse Rayleigh distribution by using squared error, linex loss function. Sindhua et al. (2013) studied the Bayes estimation of the parameters of the inverse Rayleigh distribution for left censored data under different loss functions (symmetric and asymmetric). Prakash (2013) discussed the Bayes estimation in the inverse Rayleigh model under two different loss functions (squared error, linex). Fan (2015) discussed Bayes estimation for the inverse Rayleigh model under different loss functions like squared error loss, linex loss function and entropy loss functions. Rasheed et al. (2015) discussed the comparison of the classical estimators with the Bayes estimator of one parameter inverse Rayleigh distribution under the generalized squared error loss function. Abdullah and Aref (2016) discussed the Bayesian approach for estimating the scale parameter of inverse Rayleigh distribution under different loss functions. Fatima and Ahmed (2017) studied the estimation and prediction of inverse Rayleigh distribution depending on lower record values. Sharma et al. (2019) obtained entropy of the inverse Rayleigh distribution and its order statistics.

The layout of the paper is as follows: In Section 2, we introduced the prior distribution and loss functions employed in this paper. In Section 3, Bayes estimators of the parameter of inverse Rayleigh distribution under different loss functions are obtained. In Section 4, we obtained the risk functions of all the estimators. In Section 5, the risk efficiencies of different estimators under different loss functions are obtained. In Section 6, a numerical example is provided using simulation and finally conclusion of the results is drawn in Section 7.

2. Prior and Loss Function

The use of the symmetrical loss function is considered to be inappropriate for estimating the mean failure time or reliability function as it has been recognized that overestimation is more serious than underestimation (Basu and Ebrahimi 1991). Varian (1975) has suggested that linex loss function should be used in case the overestimation and underestimation are not equally serious. But, again the linex loss function is suitable for the estimation of the location parameter only and is not so appropriate in case we want to estimate the others parameter like shape and scale parameter. So, we use precautionary loss function, De-Groot loss function and minimum expected loss function along with the squared error loss function for estimating the scale parameter of the IRD. In this paper we have used both the symmetrical and asymmetrical loss functions for better comprehension of the Bayesian analysis.

In Bayesian analysis there is a belief or idea that in most of the practical situations there is subjective prior information is available about the probable values of the parameter to be estimated. But, if there may arise a situation when the prior information about the parameter is not available, then we move towards the non-informative prior. Since, we don't have prior information regarding the probable values of the scale parameter of IRD. That is why we have decided to use Jeffrey's prior as a non-informative prior.

In this paper we use the following non-informative prior viz: Jeffrey's prior

$$g(\lambda) \propto \frac{1}{\lambda}, \lambda > 0. \tag{3}$$

Taking under consideration the Jeffrey's prior, we make use of four different loss functions for the considered model:

(i) Squared error loss function (SELF) which is symmetrical in the nature,
(ii) Precautionary loss function that is asymmetric loss function,
(iii) De-Groot loss function,

(iv) Minimum expected loss function,

(v) The squared error loss function is given by

$$L\left(\hat{\lambda},\lambda\right)=\left(\hat{\lambda},\lambda\right),^{2} \tag{4}$$

which is symmetric and $\hat{\lambda}$ is an estimate of λ. The squared error loss function is used in estimators like linear regression, calculation of unbiased statistics, and many areas of machine learning.

(vi) The precautionary loss function is given by

$$L\left(\hat{\lambda},\lambda\right)=\frac{\left(\hat{\lambda},\lambda\right)^{2}}{\hat{\lambda}}, \tag{5}$$

which is asymmetric, λ and $\hat{\lambda}$ represent the true and estimated values of the parameter. This loss function is used when the under estimation is more serious consequences.

(vii) The third loss function is the De-Groot loss function and it is given by

$$L\left(\hat{\lambda},\lambda\right)=\frac{\left(\hat{\lambda},\lambda\right)^{2}}{\hat{\lambda}^{2}}. \tag{6}$$

DeGroot (1970) discussed this asymmetric loss function defined for the positive values of the parameter.

(viii) The minimum expected loss function is given by

$$L\left(\hat{\lambda},\lambda\right)=\frac{\left(\hat{\lambda},\lambda\right)^{2}}{\lambda^{2}}. \tag{7}$$

3. Bayesian Estimation

Let $\underline{x}=\left(x_1,x_2,...,x_n\right)$ be a random sample of size n having the probability density function as

$$f\left(x,\lambda\right)=\frac{2e^{-\frac{1}{\lambda x^{2}}}}{\lambda x^{3}};x>0,\lambda>0.$$

The likelihood function of the random sample $\underline{x}=\left(x_1,x_2,...,x_n\right)$ is given by

$$L\left(\frac{\underline{x}}{\lambda}\right)=f\left(\underline{x},\lambda\right)=\frac{2^{n}}{\lambda^{n}}\prod_{i=1}^{n}\frac{1}{x_i^{3}}e^{-\frac{\sum_{i=1}^{n}\frac{1}{x_i^{2}}}{\lambda}}. \tag{8}$$

The maximum likelihood estimator of λ is

$$\hat{\lambda}_{mle}=\frac{\sum_{i=1}^{n}\frac{1}{x_i^{2}}}{n}. \tag{9}$$

The prior distribution is Jeffrey's prior which is given by

$$g(\lambda)\propto\frac{1}{\lambda},\lambda>0. \tag{10}$$

The posterior density function is given by

$$P\left(\frac{\lambda}{\underline{x}}\right)=\frac{L\left(\frac{\underline{x}}{\lambda}\right)g(\lambda)}{\int_0^\infty L\left(\frac{\underline{x}}{\lambda}\right)g(\lambda)d\lambda}. \tag{11}$$

On solving (11) by using (8) and (10), we get

$$P\left(\frac{\lambda}{\underline{x}}\right)=\frac{\left(\sum_{i=1}^{n}\frac{1}{x_i^2}\right)^n e^{-\frac{\sum_{i=1}^{n}\frac{1}{x_i^2}}{\lambda}}}{\lambda^{n+1}\Gamma n};\lambda\geq 0. \tag{12}$$

3.1. Bayes estimator of λ by using the squared error loss function

Under the squared error loss function $L\left(\hat{\lambda},\lambda\right)=\left(\hat{\lambda}-\lambda\right)^2$, the Bayes estimator is obtained by minimizing $E\left\{L\left(\hat{\lambda},\lambda\right)\right\}$, where

$$E\left\{L\left(\hat{\lambda},\lambda\right)\right\}=\int_0^\infty\left(\hat{\lambda}-\lambda\right)^2 P\left(\frac{\lambda}{\underline{x}}\right)d\lambda=\int_0^\infty\left(\hat{\lambda}-\lambda\right)^2\frac{\left(\sum_{i=1}^{n}\frac{1}{x_i^2}\right)^n e^{-\frac{\sum_{i=1}^{n}\frac{1}{x_i^2}}{\lambda}}}{\lambda^{n+1}\Gamma n}d\lambda,$$

and hence the Bayes estimator under SELF is

$$\hat{\lambda}_{SB}=\frac{\left(\sum_{i=1}^{n}\frac{1}{x_i^2}\right)}{n-1}. \tag{13}$$

3.2. Bayes estimator of λ by using the precautionary loss function

Under the precautionary loss function $L\left(\hat{\lambda},\lambda\right)=\frac{\left(\hat{\lambda}-\lambda\right)^2}{\hat{\lambda}}$, the Bayes estimator is obtained by minimizing $E\left\{L\left(\hat{\lambda},\lambda\right)\right\}$, where

$$E\left\{L\left(\hat{\lambda},\lambda\right)\right\}=\int_0^\infty\frac{\left(\hat{\lambda}-\lambda\right)^2}{\hat{\lambda}}P\left(\frac{\lambda}{\underline{x}}\right)d\lambda.$$

The Bayes estimator under the precautionary loss function is

$$\hat{\lambda}_{PB}=\frac{\left(\sum_{i=1}^{n}\frac{1}{x_i^2}\right)}{\sqrt{(n-1)(n-2)}}. \tag{14}$$

3.3. Bayes estimator of λ by using the De-Groot loss function

Under the De-Groot loss function $L\left(\hat{\lambda},\lambda\right)=\frac{\left(\hat{\lambda}-\lambda\right)^2}{\hat{\lambda}^2}$, the Bayes estimator is obtained by minimizing $E\left\{L\left(\hat{\lambda},\lambda\right)\right\}$, where

$$E\left\{L\left(\hat{\lambda},\lambda\right)\right\}=\int_0^\infty \frac{\left(\hat{\lambda}-\lambda\right)^2}{\hat{\lambda}^2}P\left(\frac{\lambda}{\underline{x}}\right)d\lambda.$$

The Bayes estimator under the De-Groot loss function is

$$\hat{\lambda}_{DG}=\frac{\left(\sum_{i=1}^{n}\frac{1}{x_i^2}\right)}{n-2}. \tag{15}$$

3.4. Bayes Estimator of λ by using the minimum expected loss function

Under the minimum expected loss function $L\left(\hat{\lambda},\lambda\right)=\frac{\left(\hat{\lambda}-\lambda\right)^2}{\lambda^2}$, the Bayes estimator is obtained by minimizing $E\left\{L\left(\hat{\lambda},\lambda\right)\right\}$, where

$$E\left\{L\left(\hat{\lambda},\lambda\right)\right\}=\int_0^\infty \frac{\left(\hat{\lambda}-\lambda\right)^2}{\lambda^2}P\left(\frac{\lambda}{\underline{x}}\right)d\lambda.$$

The Bayes estimator under the minimum expected loss function is

$$\hat{\lambda}_{ME}=\frac{\left(\sum_{i=1}^{n}\frac{1}{x_i^2}\right)}{n+1}. \tag{16}$$

4. Risk Function

In decision theory, a good decision function is that one which is having a small value of the risk function. The risk function is defined as

$$R\left(\hat{\lambda},\lambda\right)=E\left\{L\left(\hat{\lambda},\lambda\right)\right\}=\int_0^\infty L\left(\hat{\lambda},\lambda\right)f\left(x,\lambda\right)dx.$$

The likelihood function of the random sample $\underline{x}=\left(x_1,x_2,...,x_n\right)$ is given by

$$L\left(\underline{x},\lambda\right)=\frac{2^n}{\lambda^n}\prod_{i=1}^{n}\frac{1}{x_i^3}e^{-\frac{\sum_{i=1}^{n}\frac{1}{x_i^2}}{\lambda}}.$$

Let $S=\sum_{i=1}^{n}\frac{1}{x_i^2}$ and since $x_i,\left(i=1,2,...,3\right)$ follows inverse Rayleigh distribution, so $\frac{1}{x_i}$ will follow Rayleigh distribution, that means $\frac{1}{x_i^2}$ will follow the exponential distribution. Also using the fact that the sum of the independent exponentially distributed random variables forms a gamma distributed variable which gives the probability density function of S as

$$h(s)=\frac{s^{n-1}e^{-\frac{s}{\lambda}}}{\lambda^n\Gamma n};s>0,\lambda>0. \tag{17}$$

4.1. The risk function of the estimator $\hat{\lambda}_{SB}$ under SELF

The risk function of the estimator $\hat{\lambda}_{SB}$ under SELF is

$$R_S\left(\hat{\lambda}_{SB},\lambda\right)=E\left\{L\left(\hat{\lambda}_{SB},\lambda\right)\right\}=\int_0^\infty\left(\hat{\lambda}_{SB}-\lambda\right)^2 h(s)\,ds=\lambda^2\left\{1-\frac{2n}{(n-1)}+\frac{n(n+1)}{(n-1)^2}\right\}. \quad (18)$$

4.2. The risk function of $\hat{\lambda}_{PB}$ under the precautionary loss function

The risk function of $\hat{\lambda}_{PB}$ under the precautionary loss function is

$$R_P\left(\hat{\lambda}_{PB},\lambda\right)=E\left\{L\left(\hat{\lambda}_{PB},\lambda\right)\right\}=\int_0^\infty\frac{\left(\hat{\lambda}_{PB}-\lambda\right)^2}{\hat{\lambda}_{PB}}h(s)\,ds=\lambda\left\{\frac{n}{\sqrt{(n-1)(n-2)}}-2+\frac{\sqrt{(n-1)(n-2)}}{(n-1)}\right\}. \quad (19)$$

4.3. The risk function of $\hat{\lambda}_{DG}$ under the De-Groot loss function

The risk function of $\hat{\lambda}_{DG}$ under the De-Groot loss function is

$$R_D\left(\hat{\lambda}_{DG},\lambda\right)=E\left\{L\left(\hat{\lambda}_{DG},\lambda\right)\right\}=\int_0^\infty\frac{\left(\hat{\lambda}_{DG}-\lambda\right)^2}{\hat{\lambda}^2_{DG}}h(s)\,ds=\left\{1-\frac{(n-2)}{(n-1)}\right\}. \quad (20)$$

4.4. The risk function of $\hat{\lambda}_{ME}$ under the minimum expected loss function

The risk function of $\hat{\lambda}_{ME}$ under the minimum expected loss function is

$$R_E\left(\hat{\lambda}_{ME},\lambda\right)=E\left\{L\left(\hat{\lambda}_{ME},\lambda\right)\right\}==\int_0^\infty\frac{\left(\hat{\lambda}_{ME}-\lambda\right)^2}{\lambda^2}h(s)\,ds=\left\{1-\frac{n}{(n+1)}\right\}. \quad (21)$$

5. Risk Efficiencies

The risk efficiencies of the $\hat{\lambda}_{SB}$, $\hat{\lambda}_{PB}$, $\hat{\lambda}_{DG}$ and $\hat{\lambda}_{ME}$ with respect to each other under different loss functions are summarized below:

5.1. The risk efficiencies of $\hat{\lambda}_{SB}$, $\hat{\lambda}_{PB}$, $\hat{\lambda}_{DG}$ and $\hat{\lambda}_{ME}$ with respect to each other under SELF

In this section, we are going to find the risk efficiencies of $\hat{\lambda}_{SB}$, $\hat{\lambda}_{PB}$, $\hat{\lambda}_{DG}$ and $\hat{\lambda}_{ME}$ relative to SELF. For that our risk functions are given as

$$R_S\left(\hat{\lambda}_{SB},\lambda\right)=\lambda^2\left\{1-\frac{2n}{(n-1)}+\frac{n(n+1)}{(n-1)^2}\right\}=E\left\{L\left(\hat{\lambda}_{PB},\lambda\right)\right\}=\int_0^\infty\left(\hat{\lambda}_{PB}-\lambda\right)^2 h(s)\,ds \quad (22)$$

$$R_S\left(\hat{\lambda}_{PB},\lambda\right)=\lambda^2\left\{1-\frac{2n}{\sqrt{(n-1)(n-2)}}+\frac{n(n+1)}{(n-1)(n-2)}\right\}. \quad (23)$$

Now,

$$R_S\left(\hat{\lambda}_{DG},\lambda\right)=E\left\{L\left(\hat{\lambda}_{DG},\lambda\right)\right\}=\int_0^\infty\left(\hat{\lambda}_{DG}-\lambda\right)^2 h(s)\,ds=\lambda^2\left\{1-\frac{2n}{(n-2)}+\frac{n(n+1)}{(n-2)^2}\right\}, \tag{24}$$

$$R_S\left(\hat{\lambda}_{ME},\lambda\right)=E\left\{L\left(\hat{\lambda}_{ME},\lambda\right)\right\}=\int_0^\infty\left(\hat{\lambda}_{ME}-\lambda\right)^2 h(s)\,ds=\lambda^2\left\{1-\frac{2n}{(n+1)}+\frac{n(n+1)}{(n+1)^2}\right\}. \tag{25}$$

The risk efficiencies of $\hat{\lambda}_{SB}$, $\hat{\lambda}_{PB}$, $\hat{\lambda}_{DG}$ and $\hat{\lambda}_{ME}$ with respect to each other under SELF are given below:

$RE_S\left(\hat{\lambda}_{PB},\hat{\lambda}_{SB}\right)=\dfrac{R_S\left(\hat{\lambda}_{PB},\lambda\right)}{R_S\left(\hat{\lambda}_{SB},\lambda\right)}$ which gives

$$RE_S\left(\hat{\lambda}_{PB},\hat{\lambda}_{SB}\right)=\frac{\lambda^2\left\{1-\frac{2n}{\sqrt{(n-1)(n-2)}}+\frac{n(n+1)}{(n-1)(n-2)}\right\}}{\lambda^2\left\{1-\frac{2n}{(n-2)}+\frac{n(n+1)}{(n-2)^2}\right\}}. \tag{26}$$

$RE_S\left(\hat{\lambda}_{PB},\hat{\lambda}_{DG}\right)=\dfrac{R_S\left(\hat{\lambda}_{PB},\lambda\right)}{R_S\left(\hat{\lambda}_{DG},\lambda\right)}$ which gives

$$RE_S\left(\hat{\lambda}_{PB},\hat{\lambda}_{DG}\right)=\frac{\lambda^2\left\{1-\frac{2n}{\sqrt{(n-1)(n-2)}}+\frac{n(n+1)}{(n-1)(n-2)}\right\}}{\lambda^2\left\{1-\frac{2n}{(n-2)}+\frac{n(n+1)}{(n-2)^2}\right\}}. \tag{27}$$

$RE_S\left(\hat{\lambda}_{ME},\hat{\lambda}_{PB}\right)=\dfrac{R_S\left(\hat{\lambda}_{ME},\lambda\right)}{R_S\left(\hat{\lambda}_{PB},\lambda\right)}$ which gives

$$RE_S\left(\hat{\lambda}_{ME},\hat{\lambda}_{PB}\right)=\frac{\lambda^2\left\{1-\frac{2n}{(n+1)}+\frac{n(n+1)}{(n+1)^2}\right\}}{\lambda^2\left\{1-\frac{2n}{(n-2)}+\frac{n(n+1)}{(n-2)^2}\right\}}. \tag{28}$$

$RE_S\left(\hat{\lambda}_{ME},\hat{\lambda}_{DG}\right)=\dfrac{R_S\left(\hat{\lambda}_{ME},\lambda\right)}{R_S\left(\hat{\lambda}_{DG},\lambda\right)}$ which gives

$$RE_S\left(\hat{\lambda}_{ME},\hat{\lambda}_{DG}\right)=\frac{\lambda^2\left\{1-\frac{2n}{(n+1)}+\frac{n(n+1)}{(n+1)^2}\right\}}{\lambda^2\left\{1-\frac{2n}{(n-2)}+\frac{n(n+1)}{(n-2)^2}\right\}}. \tag{29}$$

$$RE_S\left(\hat{\lambda}_{SB},\hat{\lambda}_{DG}\right)=\frac{R_S\left(\hat{\lambda}_{SB},\lambda\right)}{R_S\left(\hat{\lambda}_{DG},\lambda\right)} \text{ which gives}$$

$$RE_S\left(\hat{\lambda}_{SB},\hat{\lambda}_{DG}\right)=\frac{\lambda^2\left\{1-\frac{2n}{(n-1)}+\frac{n(n+1)}{(n-1)^2}\right\}}{\lambda^2\left\{1-\frac{2n}{(n-2)}+\frac{n(n+1)}{(n-2)^2}\right\}}. \tag{30}$$

$$RE_S\left(\hat{\lambda}_{ME},\hat{\lambda}_{SB}\right)=\frac{R_S\left(\hat{\lambda}_{ME},\lambda\right)}{R_S\left(\hat{\lambda}_{SB},\lambda\right)} \text{ which gives}$$

$$RE_S\left(\hat{\lambda}_{ME},\hat{\lambda}_{SB}\right)=\frac{\lambda^2\left\{1-\frac{2n}{(n+1)}+\frac{n(n+1)}{(n+1)^2}\right\}}{\lambda^2\left\{1-\frac{2n}{(n-1)}+\frac{n(n+1)}{(n-1)^2}\right\}}. \tag{31}$$

5.2. The risk efficiencies of $\hat{\lambda}_{SB}$, $\hat{\lambda}_{PB}$, $\hat{\lambda}_{DG}$ and $\hat{\lambda}_{ME}$ with respect to each other under the precautionary loss function

In this section, we obtained the risk efficiencies of $\hat{\lambda}_{SB}$, $\hat{\lambda}_{PB}$, $\hat{\lambda}_{DG}$ and $\hat{\lambda}_{ME}$ relative to the precautionary loss function. For that our risk functions are given as

$$R_P\left(\hat{\lambda}_{PB},\lambda\right)=\lambda\left\{\frac{n}{\sqrt{(n-1)(n-2)}}-2+\frac{\sqrt{(n-1)(n-2)}}{(n-1)}\right\}. \tag{32}$$

Now

$$R_P\left(\hat{\lambda}_{SB},\lambda\right)=E\left\{L\left(\hat{\lambda}_{SB},\lambda\right)\right\}=\int_0^\infty\frac{\left(\hat{\lambda}_{SB}-\lambda\right)^2}{\hat{\lambda}_{SB}}h(s)\,ds=\left\{\frac{\lambda}{(n-1)}\right\}. \tag{33}$$

$$R_P\left(\hat{\lambda}_{DG},\lambda\right)=E\left\{L\left(\hat{\lambda}_{DG}-\lambda\right)\right\}=\int_0^\infty\frac{\left(\hat{\lambda}_{DG}-\lambda\right)^2}{\hat{\lambda}_{DG}}h(s)\,ds=\lambda\left\{\frac{n}{(n-2)}-2+\frac{(n-2)}{(n-1)}\right\}. \tag{34}$$

$$R_P\left(\hat{\lambda}_{ME},\lambda\right)=E\left\{L\left(\hat{\lambda}_{ME}-\lambda\right)\right\}=\int_0^\infty\frac{\left(\hat{\lambda}_{ME}-\lambda\right)^2}{\hat{\lambda}_{ME}}h(s)\,ds=\lambda\left\{\frac{n}{(n+1)}-2+\frac{(n+1)}{(n-1)}\right\}. \tag{35}$$

The risk efficiencies of $\hat{\lambda}_{SB}$, $\hat{\lambda}_{PB}$, $\hat{\lambda}_{DG}$ and $\hat{\lambda}_{ME}$ with respect to each other under the precautionary loss function are as under

$$RE_P\left(\hat{\lambda}_{SB},\hat{\lambda}_{PB}\right)=\frac{R_P\left(\hat{\lambda}_{SB},\lambda\right)}{R_P\left(\hat{\lambda}_{PB},\lambda\right)} \text{ which gives}$$

$$RE_P\left(\hat{\lambda}_{SB},\hat{\lambda}_{PB}\right)=\frac{\left\{\frac{\lambda}{(n-1)}\right\}}{\lambda\left\{\frac{n}{\sqrt{(n-1)(n-2)}}-2+\frac{\sqrt{(n-1)(n-2)}}{(n-1)}\right\}}. \tag{36}$$

$RE_P\left(\hat{\lambda}_{SB},\hat{\lambda}_{DG}\right)=\frac{R_P\left(\hat{\lambda}_{SB},\lambda\right)}{R_P\left(\hat{\lambda}_{DG},\lambda\right)}$ which gives

$$RE_P\left(\hat{\lambda}_{SB},\hat{\lambda}_{DG}\right)=\frac{\left\{\frac{\lambda}{(n-1)}\right\}}{\lambda\left\{\frac{n}{(n-2)}-2+\frac{(n-2)}{(n-1)}\right\}}. \tag{37}$$

$RE_P\left(\hat{\lambda}_{DG},\hat{\lambda}_{ME}\right)=\frac{R_P\left(\hat{\lambda}_{DG},\lambda\right)}{R_P\left(\hat{\lambda}_{ME},\lambda\right)}$ which gives

$$RE_P\left(\hat{\lambda}_{DG},\hat{\lambda}_{ME}\right)=\frac{\lambda\left\{\frac{n}{(n-2)}-2+\frac{(n-2)}{(n-1)}\right\}}{\lambda\left\{\frac{n}{(n+1)}-2+\frac{(n+1)}{(n-1)}\right\}}. \tag{38}$$

$RE_P\left(\hat{\lambda}_{ME},\hat{\lambda}_{PB}\right)=\frac{R_P\left(\hat{\lambda}_{ME},\lambda\right)}{R_P\left(\hat{\lambda}_{PB},\lambda\right)}$ which gives

$$RE_P\left(\hat{\lambda}_{ME},\hat{\lambda}_{PB}\right)=\frac{\lambda\left\{\frac{n}{(n+1)}-2+\frac{(n+1)}{(n-1)}\right\}}{\lambda\left\{\frac{n}{\sqrt{(n-1)(n-2)}}-2+\frac{\sqrt{(n-1)(n-2)}}{(n-1)}\right\}}. \tag{39}$$

5.3. The risk efficiencies of $\hat{\lambda}_{SB}$, $\hat{\lambda}_{PB}$, $\hat{\lambda}_{DG}$ and $\hat{\lambda}_{ME}$ with respect to each other under the De-Groot loss function

In this section, we are going to find the risk efficiencies of $\hat{\lambda}_{SB}$, $\hat{\lambda}_{PB}$, $\hat{\lambda}_{DG}$ and $\hat{\lambda}_{ME}$ relative to the De-Groot loss function. For that our risk functions are given as

$$R_D\left(\hat{\lambda}_{DG},\lambda\right)=\left\{\frac{1}{(n-1)}\right\}. \tag{40}$$

$$R_D\left(\hat{\lambda}_{SB},\lambda\right)=E\left\{L\left(\hat{\lambda}_{SB}-\lambda\right)\right\}=\int_0^\infty\frac{\left(\hat{\lambda}_{SB}-\lambda\right)^2}{\hat{\lambda}_{SB}^2}h(s)\,ds=\left\{\frac{1}{(n-2)}\right\}. \tag{41}$$

$$R_D\left(\hat{\lambda}_{PB},\lambda\right)=E\left\{L\left(\hat{\lambda}_{PB}-\lambda\right)\right\}=\int_0^\infty\frac{\left(\hat{\lambda}_{PB}-\lambda\right)^2}{\hat{\lambda}_{PB}^2}h(s)\,ds=\left\{2-2\frac{\sqrt{(n-1)(n-2)}}{n}\right\}. \tag{42}$$

$$R_D\left(\hat{\lambda}_{ME},\lambda\right)=E\left\{L\left(\hat{\lambda}_{ME}-\lambda\right)\right\}=\int_0^\infty \frac{\left(\hat{\lambda}_{ME}-\lambda\right)^2}{\hat{\lambda}_{ME}^2}h(s)\,ds=\left\{1+\frac{(n+1)^2}{(n-1)(n-2)}-2\frac{(n+1)}{(n-1)}\right\}. \quad (43)$$

Now, the risk efficiencies of $\hat{\lambda}_{SB}$, $\hat{\lambda}_{PB}$, $\hat{\lambda}_{DG}$ and $\hat{\lambda}_{ME}$ under the De-Groot loss function are as under

$RE_D\left(\hat{\lambda}_{SB},\hat{\lambda}_{DG}\right)=\dfrac{R_D\left(\hat{\lambda}_{SB},\lambda\right)}{R_D\left(\hat{\lambda}_{DG},\lambda\right)}$ which gives

$$RE_D\left(\hat{\lambda}_{SB},\hat{\lambda}_{DG}\right)=\frac{\left\{\frac{1}{(n-1)}\right\}}{\left\{\frac{1}{(n-2)}\right\}}. \quad (44)$$

$RE_D\left(\hat{\lambda}_{SB},\hat{\lambda}_{PB}\right)=\dfrac{R_D\left(\hat{\lambda}_{SB},\lambda\right)}{R_D\left(\hat{\lambda}_{PB},\lambda\right)}$ which gives

$$RE_D\left(\hat{\lambda}_{SB},\hat{\lambda}_{PB}\right)=\frac{\left\{\frac{1}{(n-2)}\right\}}{\left\{2-2\frac{\sqrt{(n-1)(n-2)}}{n}\right\}}. \quad (45)$$

$RE_D\left(\hat{\lambda}_{SB},\hat{\lambda}_{ME}\right)=\dfrac{R_D\left(\hat{\lambda}_{SB},\lambda\right)}{R_D\left(\hat{\lambda}_{ME},\lambda\right)}$ which gives

$$RE_D\left(\hat{\lambda}_{SB},\hat{\lambda}_{ME}\right)=\frac{\left\{\frac{1}{(n-2)}\right\}}{\left\{1+\frac{(n+1)^2}{(n-1)(n-2)}-2\frac{(n+1)}{(n-1)}\right\}}. \quad (46)$$

$RE_D\left(\hat{\lambda}_{PB},\hat{\lambda}_{DG}\right)=\dfrac{R_D\left(\hat{\lambda}_{PB},\lambda\right)}{R_D\left(\hat{\lambda}_{DG},\lambda\right)}$ which gives

$$RE_D\left(\hat{\lambda}_{PB},\hat{\lambda}_{DG}\right)=\frac{\left\{2-2\frac{\sqrt{(n-1)(n-2)}}{n}\right\}}{\left\{\frac{1}{(n-1)}\right\}}. \quad (47)$$

$RE_D\left(\hat{\lambda}_{PB},\hat{\lambda}_{ME}\right)=\dfrac{R_D\left(\hat{\lambda}_{PB},\lambda\right)}{R_D\left(\hat{\lambda}_{ME},\lambda\right)}$ which gives

$$RE_D\left(\hat{\lambda}_{PB},\hat{\lambda}_{ME}\right)=\frac{\left\{2-2\frac{\sqrt{(n-1)(n-2)}}{n}\right\}}{\left\{1+\frac{(n+1)^2}{(n-1)(n-2)}-2\frac{(n+1)}{(n-1)}\right\}}. \tag{48}$$

$RE_D\left(\hat{\lambda}_{ME},\hat{\lambda}_{DG}\right)=\frac{R_D\left(\hat{\lambda}_{ME},\lambda\right)}{R_D\left(\hat{\lambda}_{DG},\lambda\right)}$ which gives

$$RE_D\left(\hat{\lambda}_{ME},\hat{\lambda}_{DG}\right)=\frac{\left\{1+\frac{(n+1)^2}{(n-1)(n-2)}-2\frac{(n+1)}{(n-1)}\right\}}{\left\{\frac{1}{(n-1)}\right\}}. \tag{49}$$

5.4. The risk efficiencies of $\hat{\lambda}_{SB}$, $\hat{\lambda}_{PB}$, $\hat{\lambda}_{DG}$ and $\hat{\lambda}_{ME}$ with respect to each other under the minimum expected loss function

Under this section, we obtained the risk efficiencies of different Bayes estimators. For that firstly we obtain the risk functions of different Bayes estimator under the minimum expected loss function which are as under

$$R_E\left(\hat{\lambda}_{ME},\lambda\right)=\left\{1-\frac{n}{n+1}\right\}. \tag{50}$$

$$R_E\left(\hat{\lambda}_{SB},\lambda\right)=E\left\{L\left(\hat{\lambda}_{SB}-\lambda\right)\right\}=\int_0^\infty\frac{\left(\hat{\lambda}_{SB}-\lambda\right)^2}{\lambda^2}h(s)\,ds=\left\{1-\frac{2n}{(n-1)}+\frac{n(n+1)}{(n-1)^2}\right\}. \tag{51}$$

$$R_E\left(\hat{\lambda}_{PB},\lambda\right)=E\left\{L\left(\hat{\lambda}_{PB}-\lambda\right)\right\}=\int_0^\infty\frac{\left(\hat{\lambda}_{PB}-\lambda\right)^2}{\lambda^2}h(s)\,ds=\left\{1-\frac{2n}{\sqrt{(n-1)(n-2)}}+\frac{n(n+1)}{(n-1)(n-2)}\right\}. \tag{52}$$

$$R_E\left(\hat{\lambda}_{DG},\lambda\right)=E\left\{L\left(\hat{\lambda}_{DG}-\lambda\right)\right\}=\int_0^\infty\frac{\left(\hat{\lambda}_{DG}-\lambda\right)^2}{\lambda^2}h(s)\,ds=\left\{1-\frac{2n}{(n-2)}+\frac{n(n+1)}{(n-2)^2}\right\}. \tag{53}$$

Now, the risk efficiencies of $\hat{\lambda}_{SB}$, $\hat{\lambda}_{PB}$, $\hat{\lambda}_{DG}$ and $\hat{\lambda}_{ME}$ with respect to each other under the minimum expected loss function which are as follows

$$RE_E\left(\hat{\lambda}_{SB},\hat{\lambda}_{PB}\right)=\frac{R_E\left(\hat{\lambda}_{SB},\lambda\right)}{R_E\left(\hat{\lambda}_{PB},\lambda\right)}=\frac{\left\{1-\frac{2n}{(n-1)}+\frac{n(n+1)}{(n-1)^2}\right\}}{\left\{1-\frac{2n}{\sqrt{(n-1)(n-2)}}+\frac{n(n+1)}{(n-1)(n-2)}\right\}}. \tag{54}$$

$$RE_E\left(\hat{\lambda}_{SB},\hat{\lambda}_{DG}\right)=\frac{R_E\left(\hat{\lambda}_{SB},\lambda\right)}{R_E\left(\hat{\lambda}_{DG},\lambda\right)}=\frac{\left\{1-\frac{2n}{\sqrt{(n-1)(n-2)}}+\frac{n(n+1)}{(n-1)(n-2)}\right\}}{\left\{1-\frac{2n}{(n-2)}+\frac{n(n+1)}{(n-2)^2}\right\}}. \tag{55}$$

$$RE_E\left(\hat{\lambda}_{SB},\hat{\lambda}_{ME}\right)=\frac{R_E\left(\hat{\lambda}_{SB},\lambda\right)}{R_E\left(\hat{\lambda}_{ME},\lambda\right)}=\frac{\left\{1-\frac{2n}{(n-1)}+\frac{n(n+1)}{(n-1)^2}\right\}}{\left\{1-\frac{n}{n+1}\right\}}. \tag{56}$$

$$RE_E\left(\hat{\lambda}_{PB},\hat{\lambda}_{DG}\right)=\frac{R_E\left(\hat{\lambda}_{PB},\lambda\right)}{R_E\left(\hat{\lambda}_{DG},\lambda\right)}=\frac{\left\{1-\frac{2n}{\sqrt{(n-1)(n-2)}}+\frac{n(n+1)}{(n-1)(n-2)}\right\}}{\left\{1-\frac{2n}{(n-2)}+\frac{n(n+1)}{(n-2)^2}\right\}}. \tag{57}$$

$$RE_E\left(\hat{\lambda}_{PB},\hat{\lambda}_{ME}\right)=\frac{R_E\left(\hat{\lambda}_{PB},\lambda\right)}{R_E\left(\hat{\lambda}_{ME},\lambda\right)}=\frac{\left\{1-\frac{2n}{\sqrt{(n-1)(n-2)}}+\frac{n(n+1)}{(n-1)(n-2)}\right\}}{\left\{1-\frac{n}{n+1}\right\}}. \tag{58}$$

$$RE_E\left(\hat{\lambda}_{DG},\hat{\lambda}_{ME}\right)=\frac{R_E\left(\hat{\lambda}_{DG},\lambda\right)}{R_E\left(\hat{\lambda}_{ME},\lambda\right)}=\frac{\left\{1-\frac{2n}{(n-2)}+\frac{n(n+1)}{(n-2)^2}\right\}}{\left\{1-\frac{n}{n+1}\right\}}. \tag{59}$$

6. Simulation Study and Results

In order to check the statistical performance of Bayes estimators, a simulation study is conducted. The random samples are generated from (1) with value of the parameter $\lambda = 1$ for various samples of sizes (n = 20, 40, 60, 80, 100 120, 140, 160). We tend to use MATLAB package to get these samples. The results of simulation study are supported by 500 repetitions. Here, Bayes estimators, risk functions and risk efficiencies of Bayes estimators are computed under completely different loss functions. The estimators for the parameter and the risk functions and risk efficiencies are averaged over the total number of repetitions. The results of the simulation study are summarized through the tables from 1 to 8. Graphs are plotted by taking risk along the y-axis and sample size along the x-axis under the different loss functions to see the behavior of risk function of Bayes estimators and to find an admissible estimator under SELF, precautionary loss function, De-Groot loss function and minimum expected loss function.

Tables 1-4 shows that the Bayes estimator and the risk associated with the minimum expected loss function is minimum as compared to the precautionary, De-Groot and SELF and risk goes on decreasing as the sample size increases.

Table 1 Bayes estimators of λ and the risk functions of Bayes estimator with $\lambda=1$

n	$\hat{\lambda}_{mle}$	$\hat{\lambda}_{SB}$	$\hat{\lambda}_{PB}$	$\hat{\lambda}_{DG}$	$\hat{\lambda}_{ME}$	$R_S(\hat{\lambda}_{SB},\lambda)$	$R_S(\hat{\lambda}_{PB},\lambda)$	$R_S(\hat{\lambda}_{DG},\lambda)$	$R_S(\hat{\lambda}_{ME},\lambda)$
20	0.4374	0.4605	0.4731	0.4861	0.4166	0.0005	0.0012	0.0023	0.0004
40	0.4085	0.4190	0.4244	0.4300	0.3985	0.0001	0.0002	0.0004	0.00009
60	0.4215	0.4286	0.4323	0.4360	0.4145	0.00005	0.00011	0.0002	0.00004
80	0.4960	0.5022	0.5054	0.5087	0.4898	0.00003	0.00009	0.0001	0.00003
100	0.5195	0.5247	0.5274	0.5301	0.5143	0.00002	0.00006	0.0001	0.00002
120	0.4303	0.4339	0.4358	0.4376	0.4268	0.00001	0.00002	0.00005	0.00001
140	0.4518	0.4550	0.4567	0.4583	0.4486	0.00001	0.00002	0.00004	0.00001
160	0.5529	0.5563	0.5581	0.5599	0.5494	0.00001	0.00002	0.00004	0.00001

Table 2 The risk functions of Bayes estimator with $\lambda=1$

n	$R_P(\hat{\lambda}_{SB},\lambda)$	$R_P(\hat{\lambda}_{PB},\lambda)$	$R_P(\hat{\lambda}_{DG},\lambda)$	$R_P(\hat{\lambda}_{ME},\lambda)$
20	0.0011	0.0026	0.0048	0.0010
40	0.0002	0.0005	0.0010	0.0002
60	0.0001	0.0002	0.0004	0.0001
80	0.00007	0.0001	0.0003	0.00007
100	0.00005	0.0001	0.0002	0.00005
120	0.00003	0.00006	0.0001	0.00002
140	0.00002	0.00005	0.00009	0.00002
160	0.00002	0.00004	0.00008	0.00002

Table 3 The risk functions of Bayes estimator with $\lambda=1$

n	$R_D(\hat{\lambda}_{SB},\lambda)$	$R_D(\hat{\lambda}_{PB},\lambda)$	$R_E(\hat{\lambda}_{DG},\lambda)$	$R_E(\hat{\lambda}_{ME},\lambda)$
20	0.0024	0.0056	0.0100	0.0024
40	0.0006	0.0014	0.0025	0.0006
60	0.0002	0.0006	0.0011	0.0002
80	0.00015	0.00035	0.00062	0.00015
100	0.00010	0.00022	0.00039	0.00010
120	0.00006	0.00015	0.00027	0.00006
140	0.00005	0.00011	0.00020	0.00005
160	0.00003	0.00008	0.00015	0.00003

Table 4 The risk functions of Bayes estimator with $\lambda=1$

n	$R_E(\hat{\lambda}_{SB},\lambda)$	$R_E(\hat{\lambda}_{PB},\lambda)$	$R_E(\hat{\lambda}_{DG},\lambda)$	$R_E(\hat{\lambda}_{ME},\lambda)$
20	0.0024	0.0056	0.0100	0.0024
40	0.0006	0.0014	0.0025	0.0006
60	0.0002	0.0006	0.0011	0.0002
80	0.00015	0.00035	0.00062	0.00015
100	0.00010	0.00022	0.00039	0.00010
120	0.00006	0.00015	0.00027	0.00006
140	0.00005	0.00011	0.00020	0.00005
160	0.00003	0.00008	0.00015	0.00003

Table 5 Risk efficiencies of Bayes estimators under squared error loss function with $\lambda = 1$

n	$RE_S\left(\hat{\lambda}_{ME},\hat{\lambda}_{DG}\right)$	$RE_S\left(\hat{\lambda}_{PB},\hat{\lambda}_{ME}\right)$	$RE_S\left(\hat{\lambda}_{ME},\hat{\lambda}_{DG}\right)$	$RE_S\left(\hat{\lambda}_{SB},\hat{\lambda}_{DG}\right)$	$RE_S\left(\hat{\lambda}_{ME},\hat{\lambda}_{SB}\right)$
20	0.5377	0.3415	0.1836	0.2243	0.8185
40	0.5504	0.3901	0.2147	0.2373	0.9048
60	0.5545	0.4075	0.2260	0.2415	0.9355
80	0.5565	0.4165	0.2318	0.2437	0.9512
100	0.5577	0.4219	0.2353	0.2449	0.9607
120	0.5585	0.4256	0.2377	0.2458	0.9672
140	0.5591	0.4283	0.2394	0.2464	0.9718
160	0.5595	0.4302	0.2407	0.2468	0.9753

Table 5 shows that the risk efficiencies of Bayes estimator under squared error loss function are increasing as the sample size increases.

Table 6 Risk efficiencies of Bayes estimators under precautionary loss function with $\lambda = 1$

n	$RE_P\left(\hat{\lambda}_{SB},\hat{\lambda}_{DG}\right)$	$RE_P\left(\hat{\lambda}_{DG},\hat{\lambda}_{ME}\right)$	$RE_P\left(\hat{\lambda}_{ME},\hat{\lambda}_{PB}\right)$	$RE_P\left(\hat{\lambda}_{SB},\hat{\lambda}_{PB}\right)$
20	0.2368	0.2142	0.3878	0.4287
40	0.2435	0.2317	0.4155	0.4368
60	0.2457	0.2377	0.4250	0.4394
80	0.2468	0.2407	0.4298	0.4406
100	0.2474	0.2425	0.4327	0.4414
120	0.2478	0.2438	0.4346	0.4419
140	0.2482	0.2446	0.4360	0.4423
160	0.2484	0.2453	0.4370	0.4425

Table 6 shows that in some cases the risk efficiencies under De-Groot loss function increases with the increase of the sample size whereas in some cases the risk efficiencies decreases.

Table 7 Risk efficiencies of Bayes estimators under De-Groot loss function with $\lambda = 1$

n	$RE_E\left(\hat{\lambda}_{SB},\hat{\lambda}_{ME}\right)$	$RE_E\left(\hat{\lambda}_{PB},\hat{\lambda}_{DG}\right)$	$RE_E\left(\hat{\lambda}_{PB},\hat{\lambda}_{ME}\right)$	$RE_E\left(\hat{\lambda}_{ME},\hat{\lambda}_{DG}\right)$	$RE_E\left(\hat{\lambda}_{SB},\hat{\lambda}_{PB}\right)$	$RE_E\left(\hat{\lambda}_{SB},\hat{\lambda}_{DG}\right)$
20	1.2216	0.5377	2.9275	0.1836	0.4172	0.2243
40	1.1051	0.5504	2.5631	0.2147	0.4311	0.2373
60	1.0689	0.5545	2.4535	0.2260	0.4356	0.2415
80	1.0512	0.5565	2.4007	0.2318	0.4378	0.2437
100	1.0408	0.5577	2.3697	0.2353	0.4392	0.2449
120	1.0338	0.5585	2.3492	0.2377	0.4400	0.2458
140	1.0289	0.5591	2.3347	0.2394	0.4407	0.2464
160	1.0253	0.5595	2.3240	0.2407	0.4411	0.2468

Table 7 shows that in some cases the risk efficiencies under De-Groot loss function increases with the increase of the sample size whereas in some cases the risk efficiencies decreases.

Table 8 Risk efficiencies of Bayes estimators under minimum expected loss function with $\lambda = 1$

n	$RE_E\left(\hat{\lambda}_{SB}, \hat{\lambda}_{ME}\right)$	$RE_E\left(\hat{\lambda}_{PB}, \hat{\lambda}_{DG}\right)$	$RE_E\left(\hat{\lambda}_{PB}, \hat{\lambda}_{ME}\right)$	$RE_E\left(\hat{\lambda}_{ME}, \hat{\lambda}_{DG}\right)$	$RE_E\left(\hat{\lambda}_{SB}, \hat{\lambda}_{PB}\right)$	$RE_E\left(\hat{\lambda}_{SB}, \hat{\lambda}_{DG}\right)$
20	1.2216	0.5377	2.9275	0.1836	0.4172	0.2243
40	1.1051	0.5504	2.5631	0.2147	0.4311	0.2373
60	1.0689	0.5545	2.4535	0.2260	0.4356	0.2415
80	1.0512	0.5565	2.4007	0.2318	0.4378	0.2437
100	1.0408	0.5577	2.3697	0.2353	0.4392	0.2449
120	1.0338	0.5585	2.3492	0.2377	0.4400	0.2458
140	1.0289	0.5591	2.3347	0.2394	0.4407	0.2464
160	1.0253	0.5595	2.3240	0.2407	0.4411	0.2468

Table 8 shows that in some cases the risk efficiencies under minimum expected loss function increases with the increase of the sample size whereas in some cases the risk efficiencies decreases.

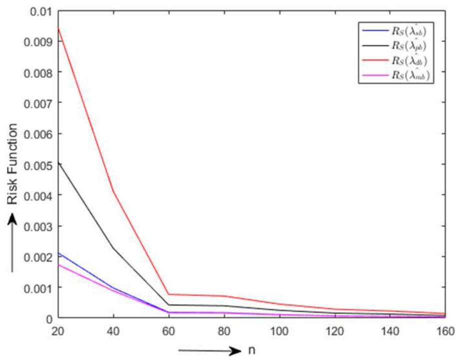

Figure 1 Behavior of risk function of Bayes estimator under squared error loss function for $\lambda = 1$

From Figure 1, we can see that the risk associated with the minimum expected loss function is minimum under the SELF as compared to the other loss functions and at the same time we can also observe that as the sample size increases risk goes on decreasing.

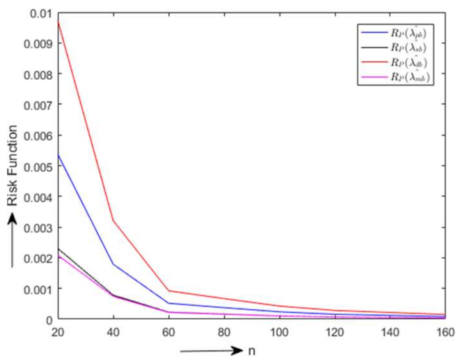

Figure 2 Behavior of risk function of Bayes estimator under precautionary loss function for $\lambda = 1$

From Figure 2, we can see that the risk associated with the minimum expected loss function is minimum under the precautionary loss function as compared to the other loss functions and at the same time we can also observe that as the sample size increases risk goes on decreasing.

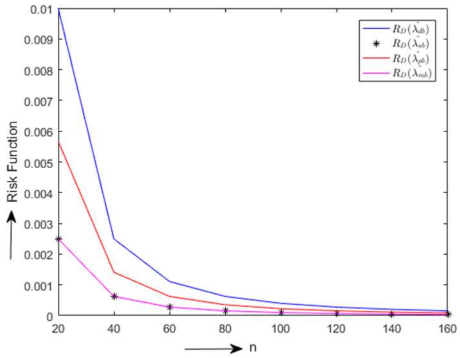

Figure 3 Behavior of risk function of Bayes estimator under De-Groot loss function for $\lambda = 1$

From Figure 3, we can see that the risk associated with the minimum expected loss function is minimum under the De-Groot loss function as compared to the other loss functions and at the same time we can also observe that as the sample size increases risk goes on decreasing.

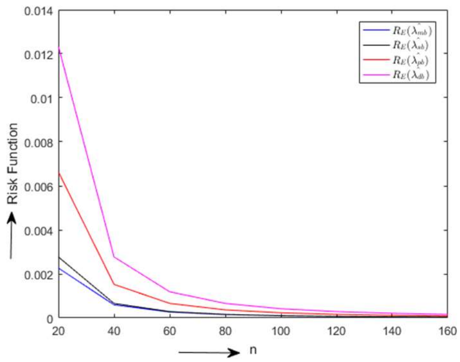

Figure 4 Behavior of risk function of Bayes estimator under minimum expected loss function for $\lambda = 1$

From Figure 4, we can see that the risk associated with the minimum expected loss function is minimum under the minimum expected loss function as compared to the other loss functions and at the same time we can also observe that as the sample size increases risk goes on decreasing.

7. Conclusions

In this paper we obtained the Bayes estimators of parameter of inverse Rayleigh distribution using four different loss functions: SELF, precautionary loss function, De-Groot loss function and minimum expected loss function. From the simulation study, we observed that the Bayes estimators under the minimum expected loss function shows better performance as there is less risk involved during estimation as compare to other loss functions under non-informative prior. So, it is suggested to use Bayesian approach under the minimum expected loss function for estimating the scale parameter of inverse Rayleigh distribution under non-informative prior.

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