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A New Nonparametric Tukey CUSUM-MA Control Chart for Detecting Mean Shifts

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Abstract

This research aims to create a new non-parametric control chart, called Tukey cumulative sum-moving average control chart (MCM-TCC) used for detecting parameter changes in asymmetrical process mean. The proposed control chart efficiency was compared with that of the cumulative sum (CUSUM), moving average (MA), mixed cumulative sum-moving average (MCM), mixed moving average-cumulative sum (MMC), mixed cumulative sum-Tukey's (CUSUM-TCC) and mixed moving average-Tukey's (MA-TCC) control charts at different levels of parameter changes by using average run length (ARL) and median run length (MRL), via Monte Carlo simulation (MC). The results of the study found that the MCM-TCC chart was efficiency more than other control charts, when the small parameter changes, and if the moderate-to-large parameter changes the MA-TCC had more efficiency, for the case of exponential distribution. In the case of the gamma distribution, the MMC control chart had more efficiency to detect the small-to-moderate parameter changes, if the large parameter changes the MA-TCC had more efficiency. For the application of the MCM-TCC chart to two datasets. It was found that the proposed control chart was almost as fast as the CUSUM-TCC chart, when the observations had an exponential distribution.

Keywords: Tukey cumulative sum-moving average control chart, non-parametric control chart, average run length, median run length, Monte Carlo simulation.

1. Introduction

Statistical process control (SPC) is an important tool for monitoring and improving production processes because it can help to achieve quality standards and streamline the production process. Control charts, which comprise the most popular method for applying SPC, can be divided into two types: variable control charts used to measure product quality by monitoring the mean and variation of a process ($\bar{X}$ chart, S chart, etc.) and attribute control charts used for monitoring the number of defective units in various samples (p chart, c chart, etc.).

The first control chart presented by Shewhart (1931), can be used to detect large parameter changes in the process mean. Whereas, the cumulative sum (CUSUM) control chart (Page 1954) can be employed to detect small changes in the process mean, as can the moving average (MA) control chart (Khoo 2004). Many researchers have designed CUSUM and MA control charts for different situations; see Abbas et al. (2018), Alves et al. (2019), Hussain et al. (2020), Abid et al. (2020), Saengsura et al. (2022), Taboran et al. (2019) and Sukparungsee et al. (2020). They are both very effective control charts for detecting small changes in the process mean under the assumption that the process observations are normality distributed.

In reality, the distribution of the process observations is sometimes unknown, and thus nonparametric control chart can be used to solve this problem. The Arcsine control chart was formulated by Ryan (2000) for detecting shifts is a good control chart for detecting process mean. Later, Tukey's control chart (TCC) presented by Alemi (2004) can be used on a single observation for detecting changes in the process mean. In 2012, Sukparungsee improved Tukey's control chart (TCC) for the both normal and non-normal distributed observations. Many researchers have combined Tukey's control chart with other control charts to provide better performance than either on their own. For example, the Tukey-Cumulative Sum (TCC-CUSUM) control chart (Khaliq and Riaz 2016), exponentially weighted moving average-Tukey's (EWMA-TCC) control chart (Khaliq et al. 2016), and mixed Tukey EWMA-CUSUM (MEC-TCC) control chart (Riaz et al. 2017). A combination of the mixed EWMA-CUSUM and mixed cumulative sum-Tukey's (CUSUM-TCC) control chart was presented by Thitisoowaranon et al. (2019) for detecting process dispersion using a range when the observations follow either symmetric and asymmetric distributions. A mixed double exponentially weighted moving average-Tukey's (MDEWMA-TCC) control chart was presented by Phantu and Sukparungsee (2020) for monitoring process change parameters when observations are either symmetrically or asymmetrically distributed. A new nonparametric Tukey MA-EWMA (MME-TCC) control chart was presented by Taboran et al. (2020) for detecting changes in the process mean with observations from either symmetric or asymmetric distributions. Finally, a mixed Tukey-double moving average (TCC-DMA) control chart was presented by Sukparungsee et al. (2021) for monitoring changes in the process mean by using either symmetrically or asymmetrically distributed process observations.

Herein, a new non-parametric Tukey CUSUM-MA control chart to detect the changes in the process mean where the observations are asymmetrically distributed is proposed. Moreover, the efficacy of the MCM-TCC control chart is compared with CUSUM, MA, MCM, MMC, CUSUM-TCC and MA-TCC control charts, by using the criteria to measure the efficacy were average run length (ARL) and median run length (MRL) via Monte Carlo simulation (MC) and applied to two sets of real data with were have exponential and gamma distributions.

2. The Design of the Control Chart

2.1. The cumulative sum control chart (CUSUM)

The CUSUM control chart is a very effective control chart for detecting small changes and this control chart has two statistical values as follows

$$\begin{aligned} C_i^+ &= \max(0, C_{i-1}^+ + X_i - \mu_0 - k) \\ C_i^- &= \min(0, C_{i-1}^- + X_i - \mu_0 + k), \end{aligned} \qquad (1)$$

where C_i^+ and C_i^- are the statistical values of the CUSUM control chart with a value of zero, X_i is the observation at time, μ_0 is the mean of process and k is the reference value. The control limits of CUSUM control chart are shown in (2).

$$\begin{aligned} UCL &= K_1 \\ LCL &= -K_1, \end{aligned} \qquad (2)$$

where K_1 is the coefficient of the control limits for the CUSUM control chart.

2.2. The moving average control chart (MA)

The MA control chart is best suited for detecting small changes. In this chart, w is the width at time and the statistics of MA control chart defined by Montgomery (2009) as follows

$$MA_i = \begin{cases} \dfrac{X_i + X_{i-1} + X_{i-2} + \ldots}{i} & , i < w \\ \dfrac{X_i + X_{i-1} + \ldots + X_{i-w+1}}{w} & , i \geq w, \end{cases} \tag{3}$$

where X_i and w are the observations and width at time i, respectively. The MA control chart has the control limits as follows:

$$UCL/LCL = \begin{cases} \mu_0 \pm \dfrac{K_2\sigma_0}{\sqrt{i}} & , i < w \\ \mu_0 \pm \dfrac{K_2\sigma_0}{\sqrt{w}} & , i \geq w, \end{cases} \tag{4}$$

where μ_0 is the mean of the process, σ_0 is the standard deviation of the process, and K_2 is the coefficient of the control limits for the MA control chart.

2.3. The mixed cumulative sum-moving average control chart (MCM)

The MCM control chart is a mix of the CUSUM and MA control charts, the CUSUM statistics in (1) are used as inputs for the MA control chart, and this control chart has statistical values as follows

$$MCM_i^+ = \begin{cases} \dfrac{C_i^+ + C_{i-1}^+ + C_{i-2}^+ + \ldots}{i} & , i < w \\ \dfrac{C_i^+ + C_{i-1}^+ + \ldots + C_{i-w+1}^+}{w} & , i \geq w, \end{cases} \tag{5}$$

$$MCM_i^- = \begin{cases} \dfrac{C_i^- + C_{i-1}^- + C_{i-2}^- + \ldots}{i} & , i < w \\ \dfrac{C_i^- + C_{i-1}^- + \ldots + C_{i-w+1}^-}{w} & , i \geq w, \end{cases} \tag{6}$$

where C_i^+ and C_i^- are the CUSUM statistics, and w is the width of the MA control chart. The control limits of MCM control chart are shown in (7).

$$UCL/LCL = \begin{cases} \mu_0 \pm \dfrac{K_3\sigma_0}{\sqrt{i}} & , i < w \\ \mu_0 \pm \dfrac{K_3\sigma_0}{\sqrt{w}} & , i \geq w, \end{cases} \tag{7}$$

where μ_0 is the mean of the process, σ_0 is the standard deviation of the process, and K_3 is the coefficient of the control limits for the MCM control chart.

2.4. The mixed moving average-cumulative sum control chart (MMC)

The MMC control chart is a mix of the MA and CUSUM control charts, the MA statistics in (3) are used as inputs for the CUSUM control chart, and this control chart has statistical values as follows:

$$\begin{aligned} MMC_i^+ &= \max(0, MMC_{i-1}^+ + MA_i - \mu_0 - k) \\ MMC_i^- &= \min(0, MMC_{i-1}^- + MA_i - \mu_0 + k), \end{aligned} \tag{8}$$

where MMC_i^+ and MMC_i^- are the statistical values of the MMC control chart with a value of zero, MA_i is the observation at time i, μ_0 is the mean of the process and k is the reference value. The MMC control chart has the control limits, for $i < w$ are shown in (9) as follows

$$\begin{aligned} UCL &= K_4 \frac{\sigma_0}{\sqrt{i}} \\ LCL &= -K_4 \frac{\sigma_0}{\sqrt{i}}, \end{aligned} \tag{9}$$

and for $i \geq w,$ the MMC control chart has the control limits are shown in (10) as follows:

$$\begin{aligned} UCL &= K_4 \frac{\sigma_0}{\sqrt{w}} \\ LCL &= -K_4 \frac{\sigma_0}{\sqrt{w}}, \end{aligned} \tag{10}$$

where K_4 is the coefficient of the control limits for the MMC control chart, and σ_0 is the standard deviation of the process.

2.5. The Tukey's control chart (TCC)

The TCC control chart is a non-parametric control chart, when the distribution of the process is unknown or the subsample is 1 $(n = 1)$. The TCC control chart has the control limits are shown in (11) as follows

$$\begin{aligned} UCL &= Q_3 + K(IQR) \\ LCL &= Q_1 - K(IQR), \end{aligned} \tag{11}$$

where Q_1 is the first quartile, Q_3 is the third quartile, K is the coefficient of the control limits for the TCC control chart, and IQR is the quartile range $(Q_3 - Q_1)$.

2.6. The mixed cumulative sum-Tukey's control chart (CUSUM-TCC)

The CUSUM-TCC control chart is a non-parametric control chart that combines the CUSUM and TCC control charts. The statistics belong to the CUSUM control chart and the control limit belongs to the TCC control chart as follows

$$\begin{aligned} C_i^+ &= \max(0, C_{i-1}^+ + X_i - Q_3 - k) \\ C_i^- &= \min(0, C_{i-1}^- + X_i - Q_1 + k), \end{aligned} \tag{12}$$

where C_i^+ and C_i^- are the statistical values of the CUSUM control chart with a value of zero, X_i is the observation at time i, Q_1 is the first quartile, Q_3 is the third quartile, and k is the reference value. The CUSUM-TCC control chart has the control limits are shown in (13) as follows

$$\begin{aligned} UCL &= K_5(IQR) \\ LCL &= -K_5(IQR), \end{aligned} \tag{13}$$

where K_5 is the coefficient of the control limits for the CUSUM-TCC control chart and IQR is the quartile range $(Q_3 - Q_1)$.

2.7. The mixed moving average-Tukey's control chart (MA-TCC)

The MA-TCC control chart is a non-parametric control chart that combines the MA and TCC control charts. The statistics belong to the MA control chart and the control limit belongs to the TCC control chart. The MA-TCC control chart has control limits, for $i < w$ are shown in (14) as follows

$$\begin{aligned} UCL &= Q_3 + \frac{K_6(IQR)}{\sqrt{i}} \\ LCL &= Q_1 - \frac{K_6(IQR)}{\sqrt{i}}, \end{aligned} \tag{14}$$

and for $i \geq w,$ the MA-TCC control chart has the control limits are shown in (15) as follows

$$\begin{aligned} UCL &= Q_3 + \frac{K_6(IQR)}{\sqrt{w}} \\ LCL &= Q_1 - \frac{K_6(IQR)}{\sqrt{w}}, \end{aligned} \tag{15}$$

where Q_1 is the first quartile, Q_3 is the third quartile, K_6 is the coefficient of the control limits for the MA-TCC control chart, and IQR is the quartile range $(Q_3 - Q_1)$.

2.8. The Tukey cumulative sum-moving average control chart (MCM-TCC)

The MCM-TCC control chart is a non-parametric control chart. It is designed from combination of the MCM and TCC control charts. The statistics belong to the MCM control chart and the control limit belongs to the TCC control chart. The MCM-TCC control chart has the control limits, for $i < w$ are shown in (16) as follows

$$\begin{aligned} UCL &= Q_3 + \frac{K_7(IQR)}{\sqrt{i}} \\ LCL &= Q_1 - \frac{K_7(IQR)}{\sqrt{i}}, \end{aligned} \tag{16}$$

and for $i \geq w,$ the MCM-TCC control chart has the control limits are shown in (17) as follows

$$\begin{aligned} UCL &= Q_3 + \frac{K_7(IQR)}{\sqrt{w}} \\ LCL &= Q_1 - \frac{K_7(IQR)}{\sqrt{w}}, \end{aligned} \tag{17}$$

where Q_1 is the first quartile, Q_3 is the third quartile, K_7 is the coefficient of the control limits for the MCM-TCC control chart, and IQR is the quartile range $(Q_3 - Q_1)$.

3. Performance Comparisons

The most widely used criterion used to compare the performances of control charts is the average run length (ARL). It is the average number of observations that must be monitored until the first out-of-control process is detected. Two aspects of the ARL must be ascertained: ARL_0 (when the process is in-control) and ARL_1 (when the process is out-of-control). In addition, the median of the run length (MRL) (Gan 1993) can also be used. Estimating the ARL and MRL in this research was achieved as follows

$$ARL = \frac{\sum_{i=1}^{R} RL_i}{R}, \tag{18}$$

$$MRL = Median(RL_i), \tag{19}$$

where RL_i is the number of observation data that must be monitored until the first out-of-control process is detected, i is the number of data simulation, and R is the number of experiment repetition. In this research, we set the other values as follows:

(i) The number of sample size of each experiment repetition $(n = 5{,}000)$.

(ii) The number of the experiment repetition $(R = 200{,}000)$.

(iii) The in-control average run length is 370.

4. Simulation Study Results

We compared the performance of the MCM-TCC control chart with those of the CUSUM, MA, MCM, MMC, CUSUM-TCC, and MA-TCC control charts with observations that were asymmetrically distributed (exponential(1) and gamma(4,1)) by detecting a change in the process mean where $\delta \in [0,4]$. The criteria used to evaluate the efficacy of the control charts were ARL and MRL, the lowest values of which identify the most efficacious control chart in each set of circumstances.

For exponential distributed observations with $\lambda = 1$ (Table 1 and Figure 1), the MCM-TCC control chart provided $K_7 = 11.239$ and lower ARL_1 and MRL values than the other for parameter change levels of 0.05, 0.10, and 0.25, while, MA-TCC control chart achieved $K_6 = 4.784$ and lower ARL_1 and MRL values than the other for parameter change levels of 0.50, 0.75, 1.00, 1.50, 2.00, 3.00 and 4.00.

For gamma distributed observations with parameters $\alpha = 4$ and $\beta = 1$ (Table 2 and Figure 2), the MMC control chart attained $K_4 = 6.331$ and lower ARL_1 and MRL values than the other for parameter change levels of 0.05, 0.10, 0.25, 0.50 and 0.75, while, MA-TCC control chart provided $K_6 = 4.014$ and lower ARL_1 and MRL values than the other for parameter change levels of 1.00, 1.50, 2.00, 3.00 and 4.00.

5. Application with Real Data

Here, the control charts are applied to processes comprising two real datasets.

5.1. Cancer survival times

The first dataset comprises 58 observations from the survival time of patients suffering from head and neck cancer disease and treated using radiotherapy (Efran 1988). After testing the data by using statistical methods, the results show they follow an exponential distribution. When applied to this dataset, the CUSUM-TCC, MCM-TCC, MA-TCC, CUSUM and MCM control charts could detect a change in the process mean at the 46th, 47th, 49th, 53rd, 55th, and 56th observation, respectively (Figure 3).

Table 1 ARL and MRL performance of MCM-TCC versus CUSUM, MA, MCM, MMC, CUSUM-TCC and MA-TCC control charts for exponential(1) distribution

Shift	Measure	CUSUM	MA	MCM	MMC	CUSUM-TCC	MA-TCC	MCM-TCC
		K_1=6.128	K_2=3.339	K_3=8.645	K_4=8.241	K_5=6.951	K_6=4.784	K_7=11.239
0	ARL	370.04	370.00	370.01	370.01	370.02	370.05	370.03
	MRL	257.00	257.00	259.00	254.00	249.00	254.00	259.00
0.05	ARL	247.11	252.28	246.79	241.11	253.35	222.73	**186.98**
	MRL	172.00	174.00	173.00	165.00	177.00	144.00	**131.00**
0.10	ARL	173.53	180.08	172.47	165.37	177.13	157.26	**134.39**
	MRL	122.00	125.00	121.00	113.00	124.00	101.00	**94.00**
0.25	ARL	74.01	79.42	73.89	67.58	75.24	66.80	**61.30**
	MRL	52.00	55.00	53.00	46.00	53.00	**42.00**	44.00
0.50	ARL	29.38	31.50	30.24	26.05	29.70	**25.04**	26.54
	MRL	21.00	22.00	22.00	18.00	22.00	**15.00**	20.00
0.75	ARL	16.49	17.04	17.62	14.47	16.63	**12.92**	15.82
	MRL	12.00	12.00	14.00	10.00	12.00	**7.00**	12.00
1.00	ARL	11.09	10.99	12.27	9.78	11.17	**7.99**	11.11
	MRL	9.00	8.00	10.00	7.00	9.00	**4.00**	9.00
1.50	ARL	6.58	6.10	7.74	5.93	6.62	**4.09**	6.97
	MRL	5.00	4.00	6.00	4.00	5.00	**1.00**	6.00
2.00	ARL	4.68	4.15	5.76	4.33	4.71	**2.59**	5.11
	MRL	4.00	3.00	5.00	3.00	4.00	**1.00**	5.00
3.00	ARL	3.03	2.58	3.92	2.92	3.05	**1.40**	3.38
	MRL	2.00	1.00	3.00	2.00	2.00	**0.00**	3.00
4.00	ARL	2.32	1.97	3.03	2.30	2.33	**0.93**	2.56
	MRL	2.00	1.00	3.00	2.00	2.00	**0.00**	2.00

The italic and bold number are minimal of ARL and MRL.

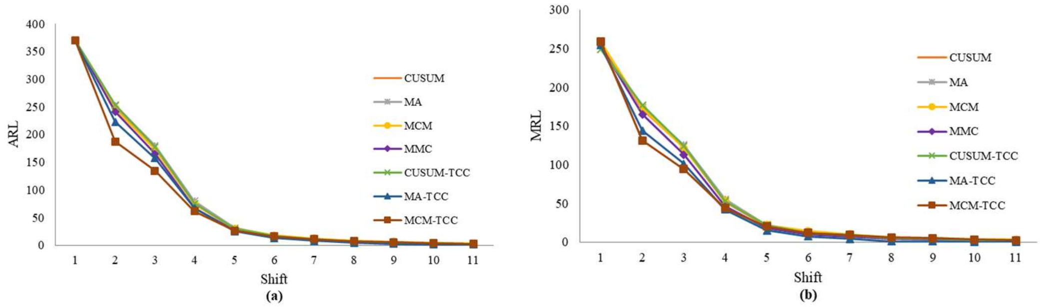

Figure 1 ARL(a) and MRL(b) curves of MCM-TCC versus CUSUM, MA, MCM, MMC, CUSUM-TCC and MA-TCC control charts for exponential(1) distribution

5.2. Stock market data

The second dataset comprises 36 historical data observations from the S&P 500 index from 2015-2018 (Finance 2018). After testing the data by using statistical methods, it was found that they followed a gamma distribution. The application of the control chart to this set of data showed that MA, MMC, and CUSUM control charts, could detect a change in the process mean at the 1st, 4th, and 7th observations, respectively (Figure 4). Meanwhile, the MCM, CUSUM-TCC and MCM-TCC control charts all detected the change at the 8th observation.

Table 2 ARL and MRL performance of MCM-TCC versus CUSUM, MA, MCM, MMC, CUSUM-TCC and MA-TCC control charts for gamma(4,1) distribution

Shift	Measure	CUSUM	MA	MCM	MMC	CUSUM-TCC	MA-TCC	MCM-TCC
		K_1=15.028	K_2=1.511	K_3=5.095	K_4=6.331	K_5=3.186	K_6=4.014	K_7=5.063
0	ARL	370.00	370.21	370.00	370.02	370.01	370.00	370.01
	MRL	258.00	256.00	260.00	255.00	260.00	254.00	260.00
0.05	ARL	248.89	284.76	248.90	**243.34**	328.99	277.63	331.04
	MRL	175.00	197.00	175.00	**168.00**	232.00	174.00	234.00
0.10	ARL	170.51	219.93	170.36	**163.60**	217.10	212.04	217.17
	MRL	121.00	152.00	122.00	**114.00**	154.00	132.00	154.00
0.25	ARL	68.09	108.69	68.40	**62.06**	79.67	100.61	80.04
	MRL	51.00	76.00	51.00	**45.00**	60.00	61.00	60.00
0.50	ARL	25.72	41.37	26.45	**22.26**	28.47	35.38	29.28
	MRL	21.00	29.00	22.00	**18.00**	24.00	21.00	25.00
0.75	ARL	14.60	19.46	15.54	**12.21**	16.01	15.07	16.99
	MRL	13.00	14.00	14.00	10.00	14.00	**8.00**	15.00
1.00	ARL	9.95	10.72	11.06	8.25	10.88	**7.51**	11.99
	MRL	9.00	8.00	10.00	7.00	10.00	**3.00**	11.00
1.50	ARL	6.58	4.51	7.19	4.98	6.45	**2.45**	7.75
	MRL	5.00	3.00	7.00	4.00	6.00	**1.00**	7.00
2.00	ARL	4.68	2.52	5.48	3.64	4.47	**0.98**	5.86
	MRL	4.00	2.00	5.00	3.00	4.00	**0.00**	6.00
3.00	ARL	3.03	1.32	3.74	2.44	2.65	**0.21**	3.83
	MRL	2.00	1.00	4.00	2.00	3.00	**0.00**	4.00
4.00	ARL	2.32	1.05	2.74	1.86	1.81	**0.05**	2.54
	MRL	2.00	1.00	3.00	2.00	2.00	**0.00**	2.00

The italic and bold number are minimal of ARL and MRL.

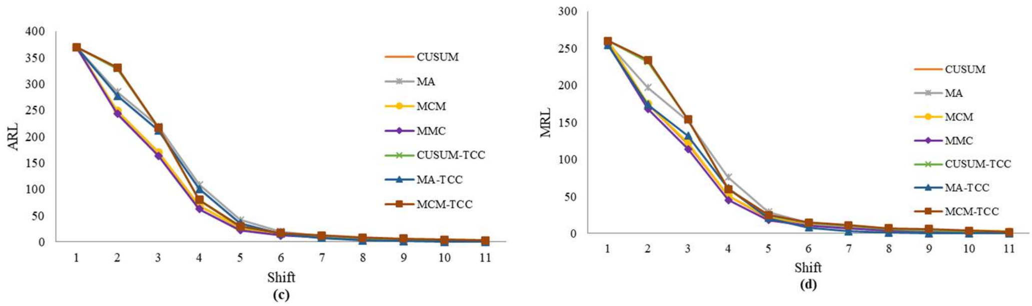

Figure 2 ARL(c) and MRL(d) curves of CM-TCC versus CUSUM, MA, MCM, MMC, CUSUM-TCC and MA-TCC control charts for gamma(4,1) distribution

6. Conclusions

The MCM-TCC control chart was proposed and its performance was compared with the CUSUM, MA, MCM, MMC, CUSUM-TCC and MA-TCC control charts with observations following an exponential(1) and gamma(4,1) distributions. The MCM-TCC control chart was more efficacious than the others when the shift in the parameter was small and the observations followed an exponential distribution. However, for a moderate-to-large shift in the parameter, the MA-TCC control chart was better than the proposed control chart. When the observations followed a gamma distribution, the MMC control chart was better at detecting small-to-moderate shift and the MA-TCC control chart was better at detecting large shift in the process parameter than the proposed control chart. When applying the methods to real dataset, the proposed control chart was almost as quick as the CUSUM-TCC control chart at detecting a parameter shift when the observations followed an exponential distribution, which is consistent with Taboran et al. (2021). Therefore, the MCM-TCC control chart is a good alternative to the CUSUM-TCC control chart for this scenario. In further research, process observations following a symmetrical distribution (e.g., normal or Laplace) or comparisons with different skewness levels and sample sizes could be explored.

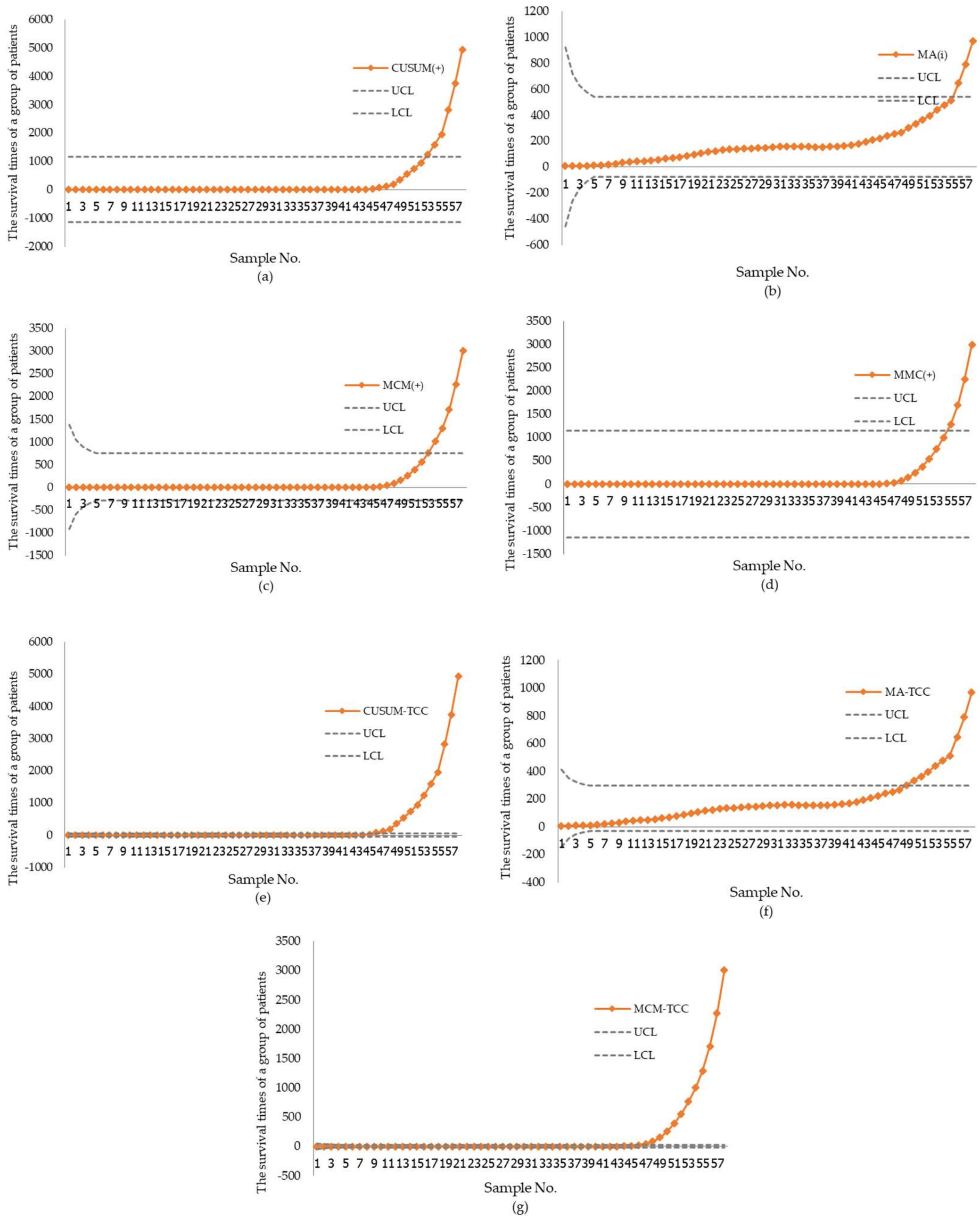

Figure 3 Applying the first set of data to the control charts: (a) CUSUM chart, (b) MA chart, (c) MCM chart, (d) MMC chart, (e) CUSUM-TCC chart, (f) MA-TCC chart and (g) MCM-TCC chart

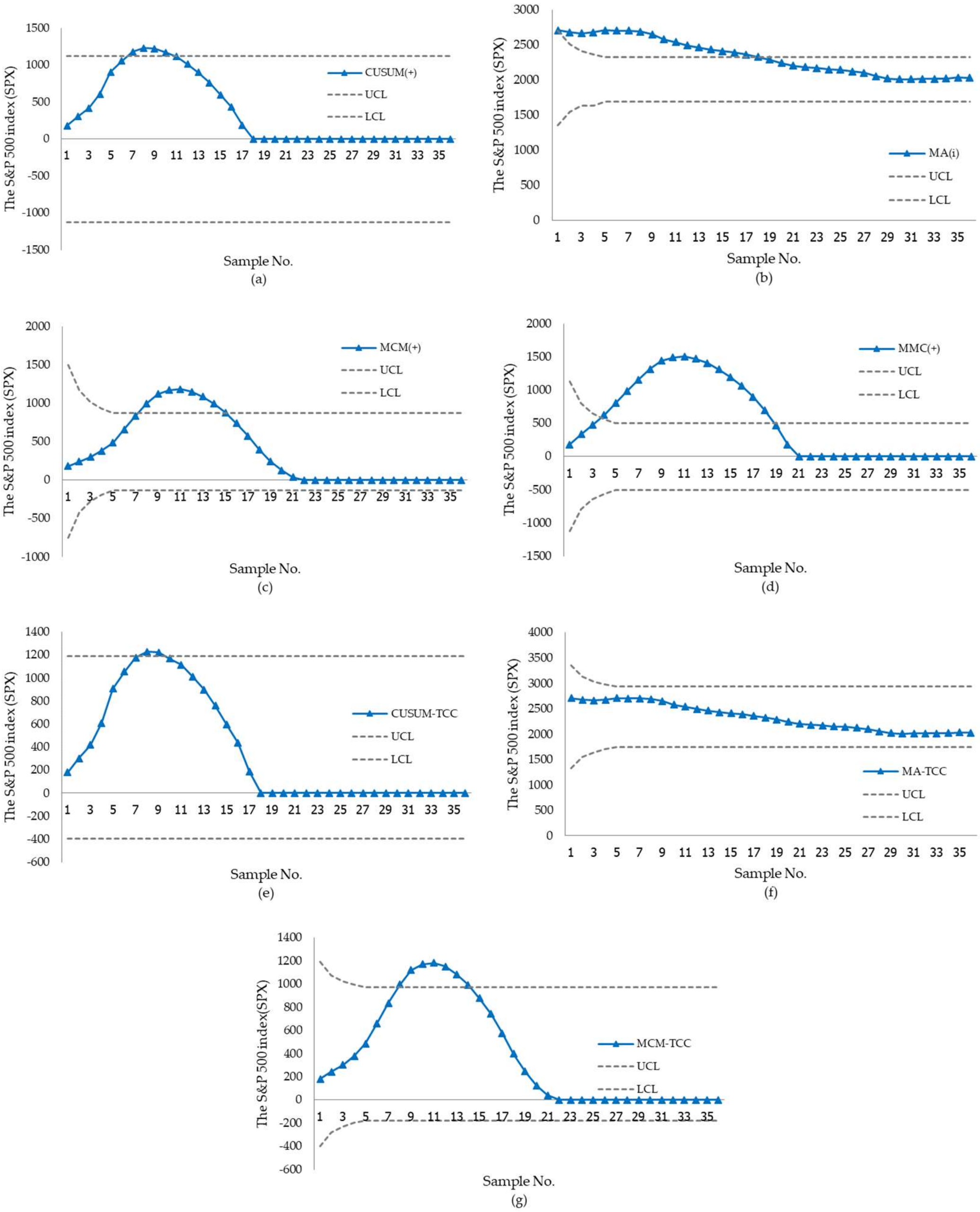

Figure 4 Applying the second set of data to the control charts: (a) CUSUM chart, (b) MA chart, (c) MCM chart, (d) MMC chart, (e) CUSUM-TCC chart, (f) MA-TCC chart and (g) MCM-TCC chart

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