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Partial Elimination of Cause of Death under Dynamic Set Up and Its Applications

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Abstract

As the overall pattern of mortality rates is changing, like in a dynamic life table, then it is quite obvious that these changes are due to its different constituent causes of death. Taking this assertion, in the present paper, we propose that the probabilities of death from different causes could also be shown from the dynamic approach of mortality change. The evaluation of the impact of changing incidence of death from various causes, particularly from non-communicable diseases (NCDs), can better be shown through this approach. Since, cause eliminated life tables are another method of assessing life tables and corresponding life expectancy in which a particular cause of death has been hypothetically eliminated and as the total elimination is usually not possible, therefore an attempt is made to formulate the technique of partial elimination of a particular cause of death under dynamic consideration. An application of this technique has been illustrated with the help of the data taken from Japan and the USA for the cardiovascular disease and neoplasm for the year 2013 based on 2003. A comparative analysis has also made between the potential gains of dynamic and usual life expectancies obtained after partial elimination of the cause of death and shows that the gains in dynamic life expectancy after eliminating partially the cause cardiovascular diseases or neoplasm are lower than the corresponding values under usual consideration for both the countries.

Keywords: Dynamic, partial elimination, gain in life expectancy.

1. Introduction

Life tables analyzed according to cause of death can be divided into two forms: the multiple decrement life table and the associated single decrement life tables, which are often called ‘cause eliminated’ life table. (Chiang 1978, Namboodari and Suchindran 1987, Tseng 1983, Preston et al., 2001). The former provides the probabilities of dying from a certain cause of death in the presence of other causes and the later answers the questions concerning what would be the life expectancy if certain causes were eliminated (Carey 1989). The cause-eliminated life table can provide a more precise and comprehensive evaluation of the cause of death on the expected lifetime. However, the complete elimination of some causes of death is usually not possible from practical point of view (Choudhury and Rajbongshi 2006, Gulati et al. 2015). In such cases, one might want to know the

effect on mortality due to the change in a particular cause of death and also the effect of its removal by 1% or say, 5% at each age.

The period life table traces the mortality experience of a cohort throughout their entire life under the assumption that the age specific death rates are constant over time (Shryock and Seigel 1976). This assumption of a constant mortality rates of a period life table is extended by Denton and Spencer (2011), where the possibility of further changes in probabilities of death are allowed and thus named as dynamic life table (Denton and Spencer 2011, Sharma et al. 2017). However, our assertion is that if the overall mortality rates change in a dynamic life table, then there is a likelihood that its constituent causes of death rate will also change as concerted efforts are being made through the World Health Organization and health programs of individual countries to reduce mortality due to specific diseases (Chiang 1978). Owing to the changing mortality scenario, one might be keenly interested to know what would happen to the expectation of life if certain percentage of the deaths due to a specific cause were eliminated at each age under the assumption that the mortality rates due to that specific cause were changing over a period of time. In other words, one may be interested in estimating the effect of the causes of death on dynamic expectation of life and thereby measuring the potential gain in dynamic life expectancy when certain causes of death is partially eliminated.

Thus, this paper is devoted to the development of partial elimination of a particular cause of death under dynamic set up, which further requires in defining the force of mortality under dynamic consideration and then followed by its application. To demonstrate the application of the development there is a considerable need for reliable data on deaths specific for age, sex and cause of death. As a consequence, the application of the partial elimination technique under dynamic consideration has been demonstrated with the help of the data taken from some developed countries, namely Japan and the United States of America. Accordingly, the first objective of the present study is to formulate the mechanism for partial elimination of a particular cause of death under dynamic consideration. And then illustrate the above with an application to the data taken from Japan and the United States of America by partially eliminating the two causes of death, namely, cardiovascular disease and neoplasm at 1% and 5% level for both males and females for the year 2013 based over the previous year 2003.

2. Force of Mortality under Dynamic Set Up

The theoretical statements on mortality are often expressed most simply in terms of the age specific death rates in a narrow age interval dx (Keyfitz and Caswell 1977), usually known as force of mortality or hazard rate and is denoted as $\mu(x)$. We denote the dynamic analogue of $\mu(x)$ as $\mu(x, y)$, which is given by

$$\mu(x, y) = L_t \frac{l(x, y) - l(x, y + \Delta y)}{l(x, y) \Delta y} \quad (1)$$

i.e., $\mu(x, y)$ denotes the proportionate decrement of the l_{xy} population of the dynamic life table between ages y when $\Delta y \rightarrow 0$. Note that l_{xy} and $l(x, y)$ carry the same meaning, where the former represents the population of initial age group $[x, x + n]$ that survives to age group $[y, y + n]$ (Denton and Spencer 2011) and the later represents the same in a continuous case.

3. Dynamic Framework for Partial Elimination of a Particular Cause of Death

Keyfitz and Caswell (1977) has shown that if there is 100δ percentage change in the death rate due to cause α (say), then the effect of this small change δ on the expectation of life can be obtained

by

$$\frac{\Delta e_0^0}{e_0^0 \delta} = \frac{\int_0^w [\log l_\alpha(x)] l(x) dx}{e_0^0} \quad (2)$$

where $l_\alpha(x)$ is the survival function for those alive at age x and are subjected to the risk of dying from only cause α , $e_0^0 = \int_0^w l(x) dx$, is the expectation of life at birth and the integration is for the entire age range from age 0 to w , the highest age to which any one lives. Now, if H_α is defined as

$$H_\alpha = \frac{-\int_0^w [\log l_\alpha(x)] l(x) dx}{\int_0^w l(x) dx} \quad (3)$$

which is the weighted average of logarithms $(\log l_\alpha(x))$, the weights being the $l(x)$ column of the life table (Keyfitz and Caswell 1977). Then, using (2) and (3), we have

$$\frac{\Delta e_0^0}{e_0^0 \delta} = -H_\alpha \Rightarrow \frac{\Delta e_0^0}{e_0^0} = -H_\alpha \delta. \quad (4)$$

A computational formula for H_α has also been derived by Namboodari and Suchindran (1987) from the expression given in (4), which is

$$H_\alpha = 1 - \frac{1}{e_0^0} \sum_x \frac{l_{x,\alpha} l_x}{_n M_{x,\alpha} + _n M_{x,+}} \left\{ 1 - \exp \left[- \left({}_n M_{x,\alpha} + {}_n M_{x,+} \right) \right] \right\}, \quad (5)$$

where

e_0^0 is the life expectancy at birth, obtained from ordinary life table based on all causes combined,

${}_n M_{x,+}$ is the observed age specific death rate for all causes combined,

${}_n M_{x,\alpha}$ is the observed age specific death rate for the cause α ,

l_x is the survival function for those alive at age x and are subjected to risk of dying from all causes combined. These are based on the values ${}_n q_x$, i.e., the probability of death at age x of the ordinary life table,

$l_{x,\alpha}$ is the survival function for those alive at age x and are subjected to the deaths from the cause α . These are based on the values ${}_n q_{x,\alpha}$, i.e., the cause specific probabilities of death at age x due to cause α .

Since under independence of the different causes, force of mortality due to all causes can be expressed as the sum of the force of mortality due to cause α and due to all other causes combined, which in turn implies that the survival probability $l_x = l_\alpha(x)l_{(-\alpha)}(x)$, with $l_0 = 1$ (Keyfitz and Caswell 1977, Namboodari and Suchindran 1987). Here also, $l_{x,\alpha}$ and $l_\alpha(x)$ carry the same meaning. Therefore, the above formula (5) may be derived using the concept that

$${}_n L_x = \frac{{}_n d_x}{{}_n M_x} = \frac{l_x \left[1 - \exp \left(- \int_0^x \mu(t) dt \right) \right]}{{}_n M_x}$$

and on the assumption that force of mortality in a given interval can be approximated by the corresponding age specific death rates.

Now, extending to dynamic consideration, we first need to construct the dynamic life table by using the method as proposed by Denton and Spencer (2011). Here, two period life tables which are t years apart are taken and then the annual rate of change of probabilities of death for any age group $[x, x+n)$ is calculated by

$${}_n r_x = \left(\frac{{}_n q_x}{{}_n \bar{q}_x} \right)^{1/t} - 1, \quad (6)$$

where ${}_n q_x$ is the probability of death for the age group $[x, x+n)$ in the reference period life table and ${}_n \bar{q}_x$ is the corresponding probability in the earlier period life table (Denton and Spencer 2011).

Then, the dynamic probabilities of death from all causes can be estimated by

$${}_n q_{xy} = {}_n q_y (1 + {}_n r_y)^{y-x}, \quad (7)$$

where ${}_n q_y$ is the probability of death in the age group $[y, y+n)$ in the reference period, ${}_n r_y$ is the annual rate of change of that probability and $y-x$ is the number of years between the subsequent age and the initial age group (Denton and Spencer 2011).

Now, using the similar process for obtaining the probabilities of death for cause α as mentioned by Chiang (1978, 1984) and Preston et al. (2001) and assuming that the probabilities of death due to cause α are changing at constant rate, we can obtain the dynamic probabilities of death due to the specified cause α . Since, the observed cause of death ratio and the life table cause of death ratio are assumed to be equal (Namboodari and Suchindran 1987), we have the dynamic cause specific probabilities of death, ${}_n q_{xy,\alpha}$, as

$${}_n q_{xy,\alpha} = 1 - (1 - {}_n q_{xy})^{\left(\frac{{}_n d_{xy,\alpha}}{{}_n d_{xy,+}} \right)}, \quad (8)$$

where we define ${}_n d_{xy,\alpha}$ and ${}_n d_{xy,+}$ as the number of deaths of the dynamic life table population who lived up to age x but died in the age interval y to $y+n$ due to cause α and due to all causes combined, respectively.

Following the procedures as given by Keyfitz and Caswell (1977), we have the new dynamic expectation of life for a small percentage change δ in $\mu(x, y)$, which is given by:

$$\begin{aligned} e_{00}^{0*} &= \int_0^w l^*(x, y) dy = \int_0^w l_\alpha^{1+\delta}(x, y) l_{(-\alpha)}(x, y) dy \\ &= \int_0^w l_\alpha^\delta(x, y) l_\alpha(x, y) l_{(-\alpha)}(x, y) dy = \int_0^w l_\alpha^\delta(x, y) l(x, y) dy. \end{aligned} \quad (9)$$

Therefore,

$$\frac{de_{00}^{0*}}{d\delta} = \int_0^w [\log l_\alpha(x, y)] l_\alpha^\delta(x, y) l(x, y) dy \quad (10)$$

gives the effect of δ on the dynamic expectation of life,

$$\Rightarrow \frac{de_{00}^0}{d\delta} = \int_0^w [\log l_\alpha(x, y)] l(x, y) dy, \text{ at } \delta = 0. \quad (11)$$

Further, we denote the dynamic analogue of H_α by H_α^D , which is given by

$$H_{\alpha}^D = \frac{-\int_0^w [\log l_{\alpha}(x, y)] l(x, y) dy}{\int_0^w l(x, y) dy}. \quad (12)$$

Then for small and finite δ , we have from (12)

$$\frac{\Delta e_{00}^0}{e_{00}^0} \approx -H_{\alpha}^D \delta. \quad (13)$$

So, one can also obtain a computational formula for H_{α}^D from (13) in the similar way as given by Namboodari and Suchindran (1987), which is

$$H_{\alpha}^D = 1 - \frac{1}{e_{00}^0} \sum_y \frac{l_{xy, \alpha} l_{xy}}{{}_n M_{xy, \alpha} + {}_n M_{xy, +}} \left\{ 1 - \exp \left[- ({}_n M_{xy, \alpha} + {}_n M_{xy, +}) \right] \right\}, \quad (14)$$

where

the summation is over the entire age range of y ,

e_{00}^0 is the dynamic life expectancy at birth, obtained from dynamic life table based on all causes combined,

l_{xy} is the population of initial age x (l_{xx}) that survived to age $[y, y+n]$ and are subjected to risk of dying from all causes combined. These are based on the dynamic probabilities of death (${}_n q_{xy}$) of the dynamic life table,

$l_{xy, \alpha}$ is the population of initial age x (i.e. l_{xx} cohort of the dynamic life table) that survived to age $[y, y+n]$ and are subjected to only deaths from the cause α . These are based on the values ${}_n q_{xy, \alpha}$. Note that here the values of ${}_n M_{xy, +}$, ${}_n M_{xy, \alpha}$ are approximated by ${}_n m_{xy, +}$, ${}_n m_{xy, \alpha}$ which are again obtained from the respective values of ${}_n q_{xy}$ and ${}_n q_{xy, \alpha}$.

Similarly, one may define H_{α}^D by replacing the dynamic life expectancy at birth by dynamic life expectancy at any age y and restricting the summation from age y to w . After H_{α}^D is obtained, the amount of gain in dynamic life expectancy at any age can be derived by $\delta H_{\alpha}^D e_{xx}^0$, where δ is the percentage level of elimination of a cause of death, viz, 1%, 5% etc.

4. Application

The causes of death under consideration show characteristics values of H_{α}^D (or, H_{α}) and these values are worth studying for what they tell us about the effect on expectation of life of eradication of a small part of each cause (Keyfitz and Caswell 1977). For illustrating the given method, we have taken the data for the countries Japan and the United States of America, as a representation of most developed Asian and North American countries, for the year 2003 and 2013 from human cause of death database. The corresponding life tables for the countries have been accessed from human mortality database. The partial elimination technique under both dynamic and traditional consideration were exemplify for the causes CVD and neoplasm, as these were more prevalent among the citizens of both the countries. The prerequisite values in the expression (5) were obtained from the ordinary life table for 2013 and the corresponding values for cause CVD and neoplasm were based on the values taken from human cause of death database. Similarly, in expression (14), the values corresponding to dynamic life tables from all causes combined are computed separately and the values of ${}_n M_{xy, \alpha}$ are estimated from the dynamic probabilities of death due to cause CVD and

neoplasm. However, the values of $l_{xy,\alpha}$, which gives the number of survivals from death due to only cause CVD (or only cause neoplasm), are estimated separately from the dynamic probabilities of death when only CVD (or neoplasm) were operating.

4.1. Results and discussion

The values of H_α^D for two causes of death, namely, CVD and neoplasm are presented in Table 1 for Japan and in Table 2 for USA. For the sake of space, the corresponding values of H_α in usual consideration have been presented in Appendix (Tables 5 and 6).

Table 1 Values of H_α^D , giving effect on dynamic life expectancy (e_{xx}^0) by a small fractional decrease in cause CVD and neoplasm at some selected ages, Japan, 2013, males and females

Age	Males			Females		
	e_{xx}^0	H_α^D (CVD)	H_α^D (Neoplasm)	e_{xx}^0	H_α^D (CVD)	H_α^D (Neoplasm)
0	88.6	0.0137	0.0358	93.0	0.0150	0.0312
10	78.9	0.0188	0.0436	83.2	0.0196	0.0377
20	69.0	0.0221	0.0505	73.3	0.0224	0.0429
30	59.3	0.0293	0.0622	63.4	0.0272	0.0509
40	49.5	0.0366	0.0760	53.5	0.0339	0.0617
50	39.7	0.0476	0.0965	43.8	0.0458	0.0793
60	30.1	0.0661	0.1306	34.2	0.0639	0.1051
70	21.1	0.1183	0.2057	24.8	0.0967	0.1463
80	12.2	0.2033	0.3334	15.3	0.1539	0.2136
90	5.9	0.5563	0.6883	7.1	0.3906	0.4331

From Table 1, it is seen that the dynamic life expectancy at birth when all causes were effective was 88.6 years in case of males. The value of H_α^D for CVD at birth is 0.0137 and for neoplasm it is 0.0358. Thus, we see that for the males of Japan, a drop of one percent in CVD as well as in neoplasm deaths uniformly at all ages would result in an increase of 0.0137 and 0.0358 percent in the dynamic expectation of life at birth. Correspondingly, in case of females, a uniform reduction in the deaths due to CVD and neoplasm at all ages by one percent would result in a 0.0150 and 0.0312 percent increase in the dynamic life expectancy at birth, which is 93.0 years. Again, if we consider the age 60 (say) and if the deaths due to CVD were reduced uniformly by 1 percent at all ages from age 60 onwards then the percentage increase in the dynamic life expectancy at the given age would be 0.0661 for males and 0.0639 for females. For the same age, if the deaths due to neoplasm were reduced by 1 percent then the corresponding values of H_α^D is 0.1306 and 0.1051 for both males and females respectively. A similar interpretation can be made for H_α^D values in the remaining ages for both males and females of Japan. It is observed that these values of H_α^D , that is the percentage increase in the dynamic life expectancy were more in males than in females for both the cause of death under consideration in almost all ages, specifically the higher ages from age 30. This indicates that the males of Japan were more exposed to the risk of non-communicable diseases like CVD and neoplasm as compared to their female counterparts.

Further, we observed from Table 2 that the dynamic life expectancy at birth in 2013 for males of the USA was 88.4 years and that for females was 91.3 years, when all causes were effective and if

the corresponding rates of change of the probabilities of death were maintained over a period of 10 years (that is based on 2003). Consequently, if one percent in deaths due to CVD were dropped consistently at all ages, then there would be an increase of $H_\alpha^D = 0.0163$ (0.0092) percent in the dynamic life expectancy at birth for males (females) in the USA. Analogously, the percentage increase in the dynamic expectation of life at birth, after uniformly reducing deaths from neoplasm at all ages, is 0.0244 (0.0245) percent in males (females). The effects of reduction on dynamic life expectancy in the remaining ages are also seen after eradication of a small fraction of each cause of death. Taking, for example, at the age 40 it has been observed that the H_α^D values for CVD and neoplasm were respectively 0.0623 (0.0357) and 0.0764 (0.0619) for males (females). Thus, it indicates that if CVD and neoplasm deaths were reduced uniformly by one percent at age 40 and above, then the resultant increase in the dynamic life expectancy at age 40 would have been 0.0623 and 0.0764 percent in males and 0.0357 and 0.0619 percent for females.

From the values of H_α^D and H_α , one can obtain the subsequent gain in life expectancy (by using $\delta H_\alpha^D e_{xx}^0$) and thus a comparison of gain in life expectancy after eliminating CVD (neoplasm) under the dynamic and usual approach has been represented vide Tables 3 and 4 respectively for Japan and the USA for both males and females.

Table 2 Values of H_α^D , giving effect on dynamic life expectancy (e_{xx}^0) by a small fractional decrease in cause CVD and neoplasm at some selected ages, United States, 2013, males and females

Age	e_{xx}^0	Males		Females	
		H_α^D (CVD)	H_α^D (Neoplasm)	e_{xx}^0	H_α^D (CVD)
0	88.4	0.0163	0.0244	91.3	0.0092
10	79.1	0.0263	0.0354	81.9	0.0169
20	69.3	0.0310	0.0413	72.0	0.0194
30	60.0	0.0449	0.0569	62.3	0.0257
40	50.7	0.0623	0.0764	52.8	0.0357
50	41.3	0.0804	0.0979	43.4	0.0507
60	33.5	0.1397	0.1617	34.8	0.0881
70	25.1	0.1968	0.2250	25.6	0.1208
80	16.8	0.2756	0.3099	17.1	0.1933
90	9.1	0.4337	0.4612	9.5	0.3515

Table 3 depicts that in Japan, the gain in dynamic life expectancy at birth is 0.0121 years for males and 0.0140 years for females, if the changing mortality rates due to CVD could be reduced by one percent at all ages. The corresponding values under usual consideration are 0.0148 years for males and 0.0150 years for females. This gives the differential among the usual and dynamic consideration, where the gains in life expectancy under usual approach were found to be more than those obtained under dynamic approach. A similar pattern could also be observed when the deaths from CVD are reduced by 5 percent at all ages for both males and females of Japan. Considering the cause neoplasm, it is observed from Table 3 that if neoplasm is eliminated throughout the ages by one percent then the potential gain expected at birth under dynamic consideration is 0.0317 (0.0290) years for males (females) of Japan. On the contrary, if similar reduction is made in the cause of death due to neoplasm then the gain in life expectancy at birth would have been 0.0320 years for males and 0.0288 years for females.

Table 3 Gain in dynamic life expectancies with alternate gain in life expectancies by various degrees of elimination of CVD and Neoplasm, Japan, 2013, males and females

Age	Gain in dynamic life expectancy				Gain in life expectancy			
	1% elimination		5 % elimination		1% elimination		5 % elimination	
	Males	Females	Males	Females	Males	Females	Males	Females
CVD								
0	0.0121	0.0140	0.0606	0.0698	0.0148	0.0150	0.0739	0.0748
10	0.0148	0.0163	0.0741	0.0817	0.0173	0.0172	0.0864	0.0860
20	0.0153	0.0164	0.0763	0.0822	0.0180	0.0175	0.0902	0.0877
30	0.0174	0.0173	0.0869	0.0863	0.0204	0.0186	0.1019	0.0930
40	0.0181	0.0181	0.0906	0.0907	0.0224	0.0200	0.1121	0.1000
50	0.0189	0.0201	0.0944	0.1004	0.0258	0.0226	0.1290	0.1129
60	0.0199	0.0219	0.0995	0.1094	0.0317	0.0265	0.1587	0.1325
70	0.0250	0.0240	0.1250	0.1198	0.0392	0.0310	0.1960	0.1551
80	0.0247	0.0235	0.1235	0.1177	0.0418	0.0353	0.2092	0.1766
90	0.0327	0.0278	0.1635	0.1390	0.0361	0.0366	0.1803	0.1830
Neoplasm								
0	0.0317	0.0290	0.1586	0.1451	0.0320	0.0288	0.1600	0.1441
10	0.0344	0.0314	0.1720	0.1569	0.0345	0.0311	0.1725	0.1553
20	0.0348	0.0315	0.1741	0.1573	0.0352	0.0314	0.1762	0.1569
30	0.0369	0.0322	0.1845	0.1612	0.0375	0.0324	0.1877	0.1620
40	0.0376	0.0330	0.1879	0.1652	0.0395	0.0337	0.1976	0.1685
50	0.0383	0.0348	0.1916	0.1738	0.0428	0.0359	0.2140	0.1795
60	0.0393	0.0360	0.1965	0.1798	0.0482	0.0387	0.2409	0.1937
70	0.0435	0.0363	0.2174	0.1813	0.0531	0.0408	0.2657	0.2038
80	0.0405	0.0327	0.2026	0.1633	0.0499	0.0406	0.2495	0.2031
90	0.0405	0.0308	0.2023	0.1541	0.0377	0.0371	0.1885	0.1855

From Table 4, it is observed that if 1% reduction in mortality rate due to CVD could be made at all ages for males of USA, then the resulting gain in life expectancy at birth under dynamic consideration is 0.0144 years and that under usual consideration is 0.0254 years, indicating that the gain is more in usual consideration. Similarly, from the same table, it has been observed that the gain in dynamic life expectancy at birth for females is 0.0084 years and the corresponding gain in usual life expectancy at birth is 0.0225 years. Again, if CVD were reduced by 5 percent then the gain in dynamic life expectancy at birth would be 0.7196 years for males and 0.4206 years for females. However, the corresponding gain at birth is more in case of usual consideration with 1.2680 years for males and 1.1235 years for females. Similarly, by eliminating deaths from neoplasm by one percent would increase the dynamic life expectancy at birth by 0.0216 (0.0224) years for males (females). However, the respective gains under usual consideration would be 0.0264 years for males and 0.0267 years for females.

Further, from the Tables 3 and 4, it is seen that the gain in life expectancy (both dynamic and usual) after elimination of CVD deaths by 1 percent or 5 percent is generally more at higher age groups in males as well as in females. It is clear from the above tables that the gains after eliminating CVD or neoplasm by one percent were smaller in dynamic consideration by a fraction of years as compared to the usual consideration in both the countries and for almost all the ages. Likewise, the differences among the gains after eliminating neoplasm by 5 percent are also inevitable in the countries.

Table 4 Gain in dynamic life expectancies with alternate gain in life expectancies by various degrees of elimination of CVD and Neoplasm, USA, 2013, males and females

Age	Gain in dynamic life expectancy				Gain in life expectancy			
	1% elimination		5% elimination		1% elimination		5% elimination	
	Males	Females	Males	Females	Males	Females	Males	Females
CVD								
0	0.0144	0.0084	0.0720	0.0421	0.0254	0.0225	0.1268	0.1124
10	0.0208	0.0138	0.1040	0.0692	0.0309	0.0274	0.1547	0.1369
20	0.0214	0.0140	0.1072	0.0699	0.0323	0.0278	0.1613	0.1391
30	0.0269	0.0160	0.1347	0.0802	0.0372	0.0296	0.1859	0.1479
40	0.0316	0.0188	0.1581	0.0941	0.0410	0.0319	0.2048	0.1595
50	0.0332	0.0220	0.1661	0.1100	0.0456	0.0358	0.2280	0.1792
60	0.0467	0.0306	0.2337	0.1531	0.0531	0.0419	0.2656	0.2096
70	0.0494	0.0310	0.2472	0.1549	0.0574	0.0471	0.2872	0.2355
80	0.0463	0.0331	0.2315	0.1657	0.0538	0.0490	0.2688	0.2452
90	0.0395	0.0333	0.1975	0.1663	0.0388	0.0405	0.1939	0.2027
Neoplasm								
0	0.0216	0.0224	0.1080	0.1118	0.0264	0.0267	0.1322	0.1337
10	0.0280	0.0278	0.1401	0.1388	0.0320	0.0316	0.1601	0.1582
20	0.0286	0.0279	0.1432	0.1395	0.0333	0.0321	0.1666	0.1603
30	0.0341	0.0299	0.1705	0.1496	0.0382	0.0338	0.1910	0.1689
40	0.0388	0.0327	0.1939	0.1633	0.0420	0.0361	0.2099	0.1803
50	0.0405	0.0357	0.2024	0.1785	0.0467	0.0398	0.2336	0.1992
60	0.0541	0.0440	0.2705	0.2200	0.0544	0.0454	0.2720	0.2270
70	0.0565	0.0435	0.2826	0.2177	0.0584	0.0491	0.2918	0.2454
80	0.0521	0.0435	0.2603	0.2175	0.0535	0.0484	0.2673	0.2421
90	0.0420	0.0391	0.2101	0.1955	0.038	0.0388	0.1900	0.1941

It is noteworthy to mention that the lesser value of gain in dynamic life expectancy after eliminating CVD or neoplasm is attributable to the fact that the dynamic probabilities of death when only CVD or neoplasm is present are lower than the corresponding value under usual consideration in almost all ages (except a few). Moreover, the striking gain in life expectancy (dynamic or usual) is a reflection of more deaths from that cause and vice versa. Thus, this reflects that if the rate of change of the probabilities of death due to CVD or neoplasm were maintained over a period of 10 years and if the resultant dynamic probabilities of death were smaller than the usual probabilities then it is quite obvious that the subsequent gains in life expectancy will be lesser under dynamic consideration.

It is worth mentioning that the gain in life expectancy at various degree of partial elimination could invariably be obtained from the values given at 1 percent level. For example, the gain in dynamic life expectancy after elimination of neoplasm by 5 percent at the age 40 (say) for females of the USA is 1.6325 years (tables 3 and 4), which is 5 times the gain in dynamic life expectancy after elimination of neoplasm by 1 percent (i.e., $5 \times 0.0327 = 0.16325$ years). This is also true in case of gain obtained under usual procedure. However, one should not use to interpret the same for 100 percent elimination of the cause under consideration. As H_{α}^D (or, H_{α}) is applicable only to small uniform percentage changes at all ages, it is not strictly proper to multiply it by the life expectancy to find the result of completely eliminating the given cause (Keyfitz and Caswell 1977).

5. Conclusion

The importance of the various causes of death is often measured by the gain in life expectancy when a specified cause of death is eliminated (Tsai et al. 1978), partially; as such it becomes more imperative to measure the impact of eradicating causes of death when their corresponding mortality rates were assumed to be changing over the years. Moreover, it is evident that when the probabilities of death decrease, the life expectancy will increase and so the influence of the two major causes of death, viz., CVD and neoplasm, on the overall mortality pattern of Japan and the USA has been ascertained by the potential gains in the life expectancy after partial elimination of that cause. The findings of this analysis reveals that the gain in dynamic life expectancy after partial elimination of the cause CVD and neoplasm are lesser than the corresponding values under usual consideration. Although the gain in dynamic consideration were low, but the expectation of life at all ages due to all cause, CVD only, neoplasm only were more than the usual one. This ascertains the situation that rates of cause specific probabilities of death were gradually declining, thus giving the higher values of life expectancy under dynamic consideration from all cause, CVD only and neoplasm only, which resulted in low gain as compared to those obtained under usual one.

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