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On Designing of a New EWMA-DMA Control Chart for Detecting Mean Shifts and Its Application

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Abstract

The control chart based on the statistical principle is used as a tool to detect and control the production process to meet the required quality. This research aimed to present the exponentially weighted moving average-double moving average chart (EWMA-DMA chart) for detecting the change of mean of the process with normal, Laplace, exponential, and gamma distribution, and compare the efficiency of the change detection of the EWMA-DMA control chart and EWMA, MA, DMA, and DMA-EWMA control chart at different levels of the parameter change. The criteria used for measuring the efficiency included the average run length for out of control process (ARL_1) and median run length (MRL) using the Monte Carlo simulation (MC). Research results showed that the results of the processes under all distribution conditions were consistent. The proposed control chart had the best detection efficiency when the change level was $-0.75 \le \delta \le 0.75$. However, DMA had better detection efficiency at other change levels. The adaptation results of the proposed control chart to two sets of actual data corresponded to the research results.

Keywords: Monte Carlo simulation (MC), double moving average control chart, median run length (MRL), EWMA-DMA control chart, parametric control chart.

1. Introduction

The production of quality and reliable products for the consumers and good services are the key factors affecting the success of the business and industry areas. Quality products mean the products have a good and appropriate quality to usage and satisfy the consumers. Determining the quality of each product begins with the survey or research to study the customer demand to set the product standard or specification limits to control the production to meet such specification limits. It is obvious that the consumer is the person who determines the product quality, not the manufacturer. Thus, quality control is the work process management applying the techniques and methods to make the products and services satisfy the consumers or service users.

Quality control is important in manufacturing. Raw materials used in the production process are often sourced from several suppliers and thus vary in quality, which directly influences the quality of the final product. There are other factors that can affect the product quality such as the quantity and capability of employees. For this reason, the methods of product quality control have been developed systematically to control the quality of each product since the time before World War II. The methods of quality control are various so the manufacturer should choose the appropriate method for the product quality to control the quality of the product successfully.

Statistical process control or SPC is the application of the statistical method to control the production process to maintain the product quality to fulfill the consumer demand and enhance the production process. When SPC is used to resolve the problem with quality, quality control, and quality development, the sample data collected from the production process is needed to analyze and interpret to determine the actual state of production. Before the data collection, the objective should be determined to know the reason for data collection to select the simple and accurate statistical methods or tools for data analysis. The statistical methods or tools for controlling the production process includes six main tools, which are a check sheet, histogram, pareto diagram, fishbone diagram or cause and effect diagram, scatter diagram, and control chart (Grant and Leavenworth 1996).

The Shewhart control chart is one of the popular SPC to detect and control the production control to prevent and resolve the problems with quality and avoid the quality problem in the mass production. The control chart that was invented by Dr. Walter Andrew Shewhart was firstly used in 1924. Three main functions of the control chart are to determine the production process, such as setting the average of product specification, to achieve the production objective, and to upgrade the production (Montgomery 2005). In 1931, Shewhart proposed the Shewhart control chart which the assumption is the production has the normal distribution and is independent. Shewhart control chart can detect the change of average of the production process efficiently when there is a great change. Anyhow, the Shewhart control chart does not focus on the past data. Later, the control charts that focus on the past data were developed, such as the cumulative sum control chart (CUSUM chart) which was proposed by Page (1954). Afterward, Robert (1959) introduced the exponentially weighted moving average control chart (EWMA), which can well detect the change in the small process (Montgomery 2009). Then, Khoo (2004) developed the moving average control chart (MA), which calculates the moving average with the period of moving average (w) that detects the small change very well, and applies to the continuous and discontinuous distribution. Then, Khoo and Wong (2008) developed the double moving average control chart (DMA), which is the chart that determines the moving average from the statistic value of the MA control chart, and compared the efficiency with MA, EWMA, and CUSUM control chart. Findings indicated that the DMA control chart was efficient to detect the change of average of the small process.

Further, some research examined and developed EWMA and DMA control chart under different conditions and circumstances, such as Crowder (1989), Areepong (2013), Noor-ul-Amin et al. (2018), Giner-Bosch et al. (2019), Noor-ul-Amin et al. (2020), Supharakonsakun (2021) and Raweesawat and Sukparungsee (2022), etc. Moreover, some researchers introduced the mixed control charts, such as Muhammad Aslam et al. (2017) proposed a double moving average-EWMA control chart for exponentially distributed quality by comparing it with the DMA control chart under the exponential distribution and using the ARL as the criteria to measure the efficiency. It was found that the proposed control chart had better efficiency to detect the change in the process than the DMA control chart. Next, the EWMA-MA control chart was introduced by Sukparungsee et al. (2020), which is the combination of EWMA and MA control chart, and compared the introduced control chart to Shewhart, MA, EWMA, MA-EWMA, and EWMA-MA control chart by using average run length

(ARL), the standard deviation of run length (SDRL), and median run length (MRL) as the criteria to compare the efficiency. Research results revealed that the proposed control chart was more efficient to detect the change in the process with the positive skewness than other control charts at all average levels.

Besides, other control charts were introduced, such as Tukey-CUSUM (Khaliq and Riaz 2014), Tukey-EWMA (Khaliq et al. 2016), Tukey EWMA-CUSUM (Riaz et al. 2017), Tukey MA-EWMA (Taboran et al. 2020), Tukey-DMA (Sukparungsee et al. 2021), etc. Presently, the researchers put more importance on the development of a control chart for the faster detection of the change and apply it to many incidents with the fewest limitations, and to be practical in various areas, such as finance (Sadeghi et al. 2017), economy (Wang et al. 2018), medicine (Suman et al. 2018), industry (Aslam et al. 2019), etc.

The previous research combined the strengths of the DMA and EWMA control chart. However, the authors would like to propose the combination of EWMA and DMA control chart in the different methods by introducing the EWMA-DMA control chart to detect the change of mean of the process with the normal, Laplace, exponential, and gamma distribution, and to compare the efficiency of change detection of EWMA-DMA and EWMA, MA, DMA, and DMA-EWMA control chart at different levels of parameter change. The criteria for measuring the efficiency were the average run length for out of control process (ARL_1) and median run length (MRL) using Monte Carlo simulation (MC). Further, it was applied to the actual data, which had the data of the diameter of a workpiece from an industrial factory, and data of the S&P 500 index (SPX) during 2015-2018.

2. Research Methodology

The objective of this research was to propose the EWMA-DMA control chart and to compare the efficiency of change detection of the parameter of the proposed control chart to the MA, EWMA, DMA, and DMA-EWMA control chart to consider which control chart had the lowest efficiency of ARL_1 and MRL so it was the most efficient control chart. This section will explain the methodology as the following details.

2.1. Parameter setting

(i) When the process is out of control, the parameter is set at $\mu_1 = \mu_0 + \delta\sigma$ and the size of average change (δ) was ± 4, ± 3, ± 2, ± 1.5, ± 1, ± 0.75, ± 0.50, ± 0.25, ± 0.10, and ± 0.05.

(ii) Set the weighted parameter (λ) of EWMA, DMA-EWMA, and EWMA-DMA control chart was 0.25.

(iii) Set the width of the moving average of MA and DMA was 2, 5, and 10.

(iv) The average run length of the in-control process (ARL_0) was 370.

2.2. Control limit determination of each chart

2.2.1 Exponentially weighted moving average control chart (EWMA chart)

An EWMA control chart for monitoring the mean of a process is based on the statistic (Robert 1959).

$$Z_i = \lambda X_i + (1-\lambda) Z_{i-1}, \ i = 1,2,..., \tag{1}$$

where λ is the weighing parameter of the data in the past having the values from 0 to 1, and X_i is the observation of the process at time *i*. At the very first time point $Z_0 = \mu_0$ (the steady and initial value), where X_i $(i = 1,2,...)$ are the independent and normally distributed observations, then the mean and variance of Z_i are

$$E(Z_i) = \mu_0 \tag{2}$$

and

$$Var(Z_i) = \sigma^2 \left(\frac{\lambda}{2-\lambda} \left(1-(1-\lambda)^{2i}\right) \right), \; i = 1,2,\ldots . \tag{3}$$

From Equation (3), when $i \to \infty$, the asymptotic variance is

$$Var(Z) = \sigma^2 \left(\frac{\lambda}{2-\lambda} \right). \tag{4}$$

Therefore, the control limits of the EWMA control chart are as the following

$$UCL / LCL = \mu_0 \pm H_1 \sqrt{\sigma^2 \left(\frac{\lambda}{2-\lambda} \right)} , \tag{5}$$

where H_1 is a coefficient of the control limit of the EWMA control chart, μ_0 is the mean of the process and the variance is σ^2.

2.2.2 Moving average control chart (MA chart)

In the moving average control chart, the width (w) and the statistics of the MA control chart (Khoo 2004) at *i* are calculated from the moving average at each w. There are two cases as follows:

$$MA_i = \begin{cases} \dfrac{X_i + X_{i-1} + X_{i-2} + \ldots}{i} & ; \; i < w \\ \dfrac{X_i + X_{i-1} + \ldots + X_{i-w+1}}{w} & ; \; i \geq w, \end{cases} \tag{6}$$

where w is the width of the MA control chart, and the mean and variance of statistics MA_i are

$$E(MA_i) = \mu_0 \tag{7}$$

and

$$Var(MA_i) = \begin{cases} \dfrac{\sigma^2}{i}, & i < w \\ \dfrac{\sigma^2}{w}, & i \geq w. \end{cases} \tag{8}$$

Therefore, the control limits of the MA control chart are as the following,

$$UCL / LCL = \begin{cases} \mu_0 \pm \dfrac{H_2 \sigma}{\sqrt{i}} & ; \; i < w \\ \mu_0 \pm \dfrac{H_2 \sigma}{\sqrt{w}} & ; \; i \geq w, \end{cases} \tag{9}$$

where H_2 is a coefficient of the control limit of an MA control chart, μ_0 is the mean and σ is the standard deviation of the process when it is under control.

2.2.3 Double moving average control chart (DMA chart)

DMA chart is appropriate for detecting the process with small and moderate changes (Khoo and Wong 2008). DMA chart finds the moving average from MA statistic value at w period. DMA statistic value can be determined as follows:

$$DMA_i = \begin{cases} \dfrac{MA_i + MA_{i-1} + MA_{i-2} + \ldots}{i} & ; i \le w \\ \dfrac{MA_i + MA_{i-1} + \ldots + MA_{i-w+1}}{w} & ; w < i < 2w-1 \\ \dfrac{MA_i + MA_{i-1} + \ldots + MA_{i-w+1}}{w} & ; i \ge 2w-1, \end{cases} \tag{10}$$

where MA_i is a statistic of MA chart, then the mean and variance of DMA_i are

$$E(DMA_i) = \mu_0, \tag{11}$$

$$Var(DMA_i) = \begin{cases} \dfrac{\sigma^2}{i^2}\displaystyle\sum_{j=1}^{i}\frac{1}{j} & ; \; i \le w \\ \dfrac{\sigma^2}{w^2}\left[\displaystyle\sum_{j=i\text{-}w+1}^{w\text{-}1}\frac{1}{j} + (j-w+1)\left(\frac{1}{w}\right)\right] & ; \; w < i < 2w-1\,. \\ \dfrac{\sigma^2}{w^2} & ; \; i \ge 2w-1 \end{cases} \tag{12}$$

Therefore, the control limit of DMA chart is as the following

$$UCL/LCL = \begin{cases} \mu_0 \pm H_3\dfrac{\sigma}{\sqrt{i}}\sqrt{\displaystyle\sum_{j=1}^{i}\frac{i}{j}} & ; i \le w \\ \mu_0 \pm H_3\dfrac{\sigma}{\sqrt{w}}\sqrt{\displaystyle\sum_{j=i-w+1}^{w-1}\frac{i}{j} + \left(j-w+1\right)\left(\frac{1}{w}\right)} & ; \; w < i < 2w-1 \\ \mu_0 \pm H_3\dfrac{\sigma}{\sqrt{w}} & ; i \ge 2w-1, \end{cases} \tag{13}$$

where H_3 is a coefficient of the control limit of an DMA control chart, μ_0 is the mean and σ is the standard deviation of the process when it is under control.

2.2.4 Double moving average-exponentially weighted moving average control chart (DMA-EWMA chart)

The DMA-EWMA chart combines DMA with EWMA control chart using the statistical value of EWMA control chart to represent X_i in MA control chart, and find the moving average from MA statistic value, at *w* period, results in DMA_i statistic value. Thus, the statistics of DMA-EWMA is

$$DMA_i = \begin{cases} \dfrac{MA_i + MA_{i-1} + MA_{i-2} + \ldots}{i} & , \; i \le w \\ \dfrac{MA_i + MA_{i-1} + \ldots + MA_{i-w+1}}{w} & , \; \mathrm{w}<i < 2w-1 \\ \dfrac{MA_i + MA_{i-1} + \ldots + MA_{i-w+1}}{w} & , \; \mathrm{i} \ge 2w-1, \end{cases} \tag{14}$$

where

$$MA_i = \begin{cases} \dfrac{Z_i + Z_{i-1} + Z_{i-2} + \dots}{i} & , \; i < w \\ \dfrac{Z_i + Z_{i-1} + \dots + Z_{i-w+1}}{w} & , \; i \ge w. \end{cases}$$

Thus, the asymptotical control limit of the DMA-EWMA chart is as follows:

$$UCL / LCL = \begin{cases} \mu_Z \pm \dfrac{H_4 \sigma_z}{i} \sqrt{\sum_{j=1}^{i} \dfrac{1}{j} \left(\dfrac{\lambda}{2-\lambda} \right)} & , \; i \le w \\ \mu_Z \pm \dfrac{H_4 \sigma_z}{w} \sqrt{\sum_{j=i-w+1}^{w-1} \dfrac{1}{j} + (j-w+1) \left(\dfrac{1}{w} \right) \left(\dfrac{\lambda}{2-\lambda} \right)} & , \; w < i < 2w-1 \\ \mu_Z \pm \dfrac{H_4 \sigma_z}{w} \sqrt{\dfrac{\lambda}{2-\lambda}} & , \; i \ge 2w-1, \end{cases} \tag{15}$$

where H_4 is the coefficient of the control limits for the DMA-EWMA chart.

2.2.5 Exponentially weighted moving average- double moving average control chart (EWMADMA chart)

The EWMA-DMA chart combines EWMA with DMA control chart using the statistic value of DMA control chart to represent X_i in EWMA control chart. Thus, the statistics of EWMA-DMA chart is

$$Z_i = \lambda DMA_i + (1-\lambda) Z_{i-1}, \; i = 1, 2, \dots, \tag{16}$$

where λ is the weighing parameter of the data in the past having the values from 0 to 1, Z_0 is the starting value and is set to be equal to the target mean μ_0, then the *UCL* and *LCL* of the EWMA-DMA chart are the expected values for the data, which will be the same value as the DMA chart. Variance will be applied between the EWMA and DMA charts, and the control limits are as follows:

$$UCL / LCL = \mu_{DMA} \pm \frac{H_5 \sigma_{DMA}}{w} \sqrt{\left(\frac{\lambda}{2-\lambda} \right)}, \tag{17}$$

where H_5 is a coefficient of the control limits of the EWMA-DMA chart.

2.3. Steps to determine ARL and MRL of each chart

(i) Simulating data using Monte Carlo simulation

Monte Carlo simulation (MC) was used in this research to find ARL because it is a simple method to understand and can be used to compare the accuracy to other estimation methods. Moreover, Median run length (MRL) and Standard deviation of run length (SDRL) were determined as the criteria for measuring the efficiency of the control chart. Since the distribution applied to this research was the positive skewness distribution, the measuring with MRL would be more appropriate and accurate than considering ARL only (Gan 1994). MC method was from the program to simulate the finding of ARL and the ARL_0 was 370 as follows:

$$ARL = \frac{1}{1 - P(LCL \le Y \le UCL)}, \tag{18}$$

or

$$ARL = \frac{\sum_{t=1}^{M} RL_t}{M}, \tag{19}$$

and

$$MRL = \text{ Median}\left(RL\right), \tag{20}$$

when Y is the statistic of the control charts, RL_t represented the examined sample before the process was out of the limit for the first time. The data simulation at round t, M represented the number of repetition, which M = 200,000. Data simulation with MC was applied when the data had a symmetrical distribution, which were normal(0,1) and Laplace(0,1), and asymmetrical distribution with the right-skewness, which were exponential(1) and gamma(4,1). The details of each distribution were shown in Table 1.

(ii) Calculating the statistic value of EWMA, MA, DMA, DMA-EWMA, and EWMA-DMA control chart as shown in (1), (6), (10), (14), and (16).

(iii) Calculating the control limits as 2.2

(iv) Repeating step (i)-(iii) of all distributions and charts to determine the ARL_1, MRL and SDRL, and consider which control chart had the lowest efficiency of ARL_1 and MRL which meant such control chart had the highest efficiency.

Table 1 Properties of the probability distributions

Distribution	pdf	Parameters
normal	$f(x) = \frac{e^{-(x-\mu_0)^2/2\sigma_0^2}}{\sqrt{2\pi\sigma_0^2}}$	$\mu_0 = 0,\ \sigma_0 = 1$ $\mu_0 \in R,$ (location) $\sigma_0^2 > 0,$ (scale)
Laplace	$f(x) = \frac{e^{\left(\frac{-\lvert x-\mu_0\rvert}{b_0}\right)}}{2b_0}$	$\mu_0 = 0,\ b_0 = 1$ $\mu_0,$ (location) $b_0 > 0,$ (scale)
exponential	$f(x) = \frac{1}{\theta_0} e^{-x/\theta_0}$	$\theta_0 = 1$ $\theta_0 > 0,$ (scale)
gamma	$f(x) = \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} x^{\alpha_0 - 1} e^{-\beta_0 x}$	$\alpha_0 = 4,\ \beta_0 = 1$ $\alpha_0 > 0,$ (shape) $\beta_0 > 0,$ (scale)

3. Research results

The objective of this research was to compare the sensitiveness of detection of mean change of the proposed chart and the MA, EWMA, DMA, and DMA-EWMA control chart under four distributions, which were normal(0,1), Laplace(0,1), exponential(1), and gamma(4,1) distribution.

Table 2 ARL_1 and MRL performance of the proposed chart under normal(0,1) distribution by varying w and given ARL_0 =370 and $\lambda = 0.25$

Shift size (δ)	ARL			MRL		
	w=2	w=5	w=10	w=2	w=5	w=10
	$H_5 = 5.040$	$H_5 = 9.997$	$H_5 = 15.441$	$H_5 = 5.040$	$H_5 = 9.997$	$H_5 = 15.441$
−4.00	0.44	0.17	0.05	0.00	0.00	0.00
−3.00	0.99	0.60	0.29	1.00	1.00	0.00
−2.00	2.05	1.63	1.08	2.00	1.00	1.00
−1.50	3.48	2.86	2.15	3.00	2.00	1.00
−1.00	7.93	6.24	5.04	6.00	4.00	3.00
−0.75	14.82	11.39	8.95	11.00	5.00	4.00
−0.50	35.88	27.67	20.67	26.00	14.00	12.00
−0.25	125.92	104.87	81.40	88.00	57.00	43.00
−0.10	288.05	272.19	245.06	199.00	135.00	127.00
−0.05	346.33	340.65	328.73	238.00	161.00	170.00
0.00	370.69	370.72	369.79	255.00	234.00	190.00
0.05	345.86	339.64	329.32	238.00	161.00	170.00
0.10	287.21	269.78	245.41	198.00	134.00	127.00
0.25	125.26	104.49	80.83	87.00	57.00	43.00
0.50	35.71	27.49	20.63	25.00	14.00	12.00
0.75	14.76	11.33	8.88	11.00	5.00	4.00
1.00	7.89	6.21	5.01	6.00	4.00	2.00
1.50	3.48	2.85	2.14	3.00	2.00	1.00
2.00	2.05	1.63	1.07	2.00	1.00	1.00
3.00	0.99	0.60	0.29	1.00	1.00	0.00
4.00	0.44	0.17	0.05	0.00	0.00	0.00

Table 3 ARL_1 and MRL performance of the proposed chart under exponential (1) distribution by varying w and given ARL_0 =370 and $\lambda = 0.25$

Shift size (δ)	ARL			MRL		
	w=2	w=5	w=10	w=2	w=5	w=10
	$H_5 = 5.947$	$H_5 = 11.75$	$H_5 = 19.271$	$H_5 = 5.947$	$H_5 = 11.75$	$H_5 = 19.271$
0.00	370.41	370.01	369.10	258.00	119.00	109.00
0.05	248.82	233.24	206.57	174.00	79.00	21.00
0.10	175.54	155.38	125.53	123.00	55.00	11.00
0.25	76.95	61.22	40.53	54.00	25.00	7.00
0.50	31.44	22.95	13.26	23.00	11.00	5.00
0.75	17.93	12.77	7.24	14.00	7.00	4.00
1.00	12.13	8.71	5.03	9.00	6.00	4.00
1.50	7.22	5.42	3.26	6.00	4.00	1.00
2.00	5.14	4.04	2.49	4.00	3.00	1.00
3.00	3.30	2.76	1.74	3.00	2.00	1.00
4.00	2.46	2.11	1.37	2.00	1.00	1.00

Table 4 ARL_1 and MRL performance of the proposed chart under normal(0,1) distribution by varying w and given $w=5$, $\lambda = 0.25$ and $\text{ARL}_0 = 370$

δ	ARL					MRL				
	EWMA	MA	DMA	DMA-EWMA	EWMA-DMA	EWMA	MA	DMA	DMA-EWMA	EWMA-DMA
	H_1=2.927	H_2=2.884	H_3=5.235	H_4=25.850	H_5=9.997	H_1=2.927	H_2=2.884	H_3=5.235	H_4=25.850	H_5=9.997
−4.00	1.08	1.00	0.00	0.00	0.17	1.00	1.00	0.00	0.00	0.00
−3.00	1.38	1.10	**0.03**	1.00	0.60	1.00	1.00	**0.00**	1.00	1.00
−2.00	2.51	1.99	**0.45**	2.91	1.63	2.00	2.00	**0.00**	3.00	1.00
−1.50	4.19	3.76	**1.44**	5.00	2.86	4.00	3.00	**0.00**	5.00	2.00
−1.00	9.09	10.17	**5.06**	9.86	6.24	7.00	7.00	**0.00**	9.00	4.00
−0.75	16.72	20.61	11.42	15.85	**11.39**	13.00	15.00	8.00	13.00	**5.00**
−0.50	39.93	51.62	32.66	33.69	**27.67**	29.00	36.00	18.00	25.00	**14.00**
−0.25	136.26	162.27	126.01	113.30	**104.87**	96.00	113.00	67.00	81.00	**57.00**
−0.10	307.28	311.20	289.66	276.44	**272.19**	217.00	216.00	173.00	193.00	**135.00**
−0.05	351.09	353.55	345.01	341.82	**340.65**	260.00	246.00	216.00	238.00	**161.00**
0.00	370	370.36	370.01	370.64	370.72	285.00	258.00	243.00	259.00	234.00
0.05	344.66	354.09	343.88	340.32	**339.64**	263.00	246.00	215.00	237.00	**161.00**
0.10	302.15	311.23	287.02	275.46	**269.78**	226.00	216.00	171.00	193.00	**134.00**
0.25	138.84	162.75	125.49	112.90	**104.49**	98.00	113.00	67.00	81.00	**57.00**
0.50	40.76	51.64	32.54	33.61	**27.49**	29.00	36.00	18.00	25.00	**14.00**
0.75	16.7	20.58	11.36	15.79	**11.33**	12.00	15.00	8.00	13.00	**5.00**
1.00	9.11	10.13	**5.01**	9.82	6.21	7.00	7.00	**0.00**	9.00	4.00
1.50	4.18	3.76	**1.43**	5.00	2.85	4.00	3.00	**0.00**	5.00	2.00
2.00	2.54	1.99	**0.45**	2.91	1.63	2.00	2.00	**0.00**	3.00	1.00
3.00	1.39	1.10	**0.03**	1.00	0.60	1.00	1.00	**0.00**	1.00	1.00
4.00	1.08	1.00	0.00	0.00	0.17	1.00	1.00	0.00	0.00	0.00

The bold is minimal of ARL_1 and MRL of control chart.

The average change level was at a range of $\delta \in [-4, 4]$. The criteria for evaluating the efficiency of the control chart based on ARL_1 and MRL. The control chart with the lowest ARL_1 and MRL would be the most efficient control chart.

3.1. Proposed control chart (EWMA-DMA)

From Tables 2 and 3, when the observation had the normal$(0,1)$ distribution, $\text{ARL}_0 = 370$, and the width of the MA of the control chart was 2, 5, and 10. It was found that when w increased, ARL_1 and MRL decreased. On the other hand, if w decreased, ARL_1 and MRL increased. Moreover, if the observation had the exponential(1) distribution, the results were consistent with the observation with the normal(0,1) distribution, as shown in Table 3.

3.2. Performance comparisons

From Tables 4-7 and Figures 1-2, when the observation had normal(0,1) distribution that H_5= 9.997, the Laplace(0,1) distribution that H_5= 7.411, exponential(1) distribution that H_5= 11.750, and gamma(4,1) distribution that H_5= 7.415, and ARL_0 =370 and $\lambda = 0.25$, ARL_1 and MRL had the consistent results. EWMA-DMA control chart was the most efficient chart to detect the change at level of $-0.75 \le \delta \le 0.75$. However, if the parameter had $\delta < -0.75$ and $\delta > 0.75$, DMA control chart that H_5= 5.235 and the observation had the normal(0,1) distribution and H_5= 4.010, Laplace(0,1) distribution and H_5= 5.950, exponential(1) distribution and H_5= 2.731, gamma(4,1) distribution would have the better efficiency to detect the change of process than EWMA-DMA,

DMA-EWMA, MA, and EWMA control chart.

4. Practical Applications

In this section, the researcher applied the proposed control chart to two sets of the sample, which were the 40 samples that had the diameter of a workpiece from an industrial factory (Buntam 2013) and 36 sets of data of the S&P 500 index (SPX) including historical data from 2015-2018 (Finance 2018). Then, the data were tested with statistical methods (Baghban et al. 2013). It was found that the data had the normal and gamma distribution, respectively.

Table 5 ARL_1 and MRL performance of the proposed chart under Laplace (0,1) distribution by varying w and given w =5, λ = 0.25 and ARL_0 =370

δ	ARL					MRL				
	EWMA	MA	DMA	DMA-EWMA	EWMA-DMA	EWMA	MA	DMA	DMA-EWMA	EWMA-DMA
	H_1=3.396	H_2=3.112	H_3=4.010	H_4=19.200	H_5=7.411	H_1=3.396	H_2=3.112	H_3=4.010	H_4=19.200	H_5=7.411
−4.00	1.79	1.20	**0.08**	1.61	0.85	2.00	1.00	**0.00**	2.00	1.00
−3.00	2.93	1.98	**0.32**	2.91	1.49	9.00	2.00	**0.00**	3.00	1.00
−2.00	6.88	5.37	**1.73**	6.21	3.29	17.00	4.00	**0.00**	6.00	2.00
−1.50	14.01	12.59	**5.58**	9.60	5.93	53.00	9.00	**1.00**	9.00	4.00
−1.00	38.94	38.69	22.35	20.03	**15.39**	84.00	27.00	12.00	16.00	**11.00**
−0.75	74.36	74.40	48.62	36.30	**30.82**	111.00	52.00	26.00	27.00	**21.00**
−0.50	159.05	147.66	111.84	80.30	**74.06**	132.00	102.00	63.00	58.00	**49.00**
−0.25	287.64	277.65	247.47	205.67	**200.71**	176.00	191.00	143.00	145.00	**131.00**
−0.10	357.64	352.11	341.63	329.15	**328.39**	228.00	243.00	214.00	229.00	**198.00**
−0.05	365.47	365.43	360.82	359.22	**358.50**	251.00	253.00	234.00	250.00	**208.00**
0.00	370.42	370.36	369.56	370.78	370.04	259.00	256.00	256.00	259.00	241.00
0.05	362.05	364.71	361.31	358.19	**358.15**	252.00	253.00	233.00	250.00	**209.00**
0.10	349.88	351.64	342.47	328.74	**327.91**	229.00	244.00	213.00	229.00	**198.00**
0.25	283.41	276.56	246.67	206.27	**201.32**	178.00	192.00	142.00	145.00	**131.00**
0.50	150.03	147.88	111.19	80.12	**73.96**	134.00	102.00	62.00	57.00	**49.00**
0.75	72.00	74.46	48.55	36.40	**30.94**	109.00	52.00	26.00	27.00	**21.00**
1.00	38.23	38.79	22.39	20.06	**15.37**	87.00	27.00	12.00	16.00	**11.00**
1.50	13.95	12.52	**5.56**	9.62	5.94	49.00	9.00	**1.00**	9.00	4.00
2.00	6.69	5.35	**1.72**	6.21	3.28	16.00	4.00	**0.00**	6.00	2.00
3.00	2.88	1.98	**0.32**	2.91	1.49	8.00	2.00	**0.00**	3.00	1.00
4.00	1.77	1.20	**0.08**	1.61	0.85	4.00	1.00	**0.00**	2.00	1.00

The bold is minimal of ARL_1 and MRL of control chart.

Table 6 ARL_1 and MRL performance of the proposed chart under exponential (1) distribution by varying w and given w = 5, λ = 0.25 and ARL_0 =370

δ	ARL					MRL				
	EWMA	MA	DMA	DMA-EWMA	EWMA-DMA	EWMA	MA	DMA	DMA-EWMA	EWMA-DMA
	H_1=3.747	H_2=3.339	H_3=5.950	H_4=35.000	H_5=11.750	H_1=3.747	H_2=3.339	H_3=5.950	H_4=35.000	H_5=11.750
0.00	370.00	370.82	370.74	370.36	370.01	278.00	257.00	241.00	290.00	119.00
0.05	269.58	253.40	243.81	365.63	**233.24**	191.00	175.00	156.00	171.00	**79.00**
0.10	193.54	180.02	169.96	329.61	**155.38**	139.50	124.00	108.00	167.00	**55.00**
0.25	85.31	78.99	71.05	150.67	**61.22**	60.00	54.00	44.00	107.00	**25.00**
0.50	33.55	31.40	26.36	51.69	**22.95**	25.00	22.00	16.00	38.00	**11.00**
0.75	19.49	17.01	13.80	25.72	**12.77**	15.00	12.00	8.00	20.00	**7.00**
1.00	13.15	10.95	**8.69**	13.24	8.71	10.00	8.00	**5.00**	9.00	6.00
1.50	8.12	6.09	**4.38**	4.95	5.42	6.00	4.00	**2.00**	3.00	4.00
2.00	5.70	4.15	**2.44**	2.78	4.04	5.00	3.00	**1.00**	2.00	3.00
3.00	3.82	2.59	**0.90**	1.26	2.76	3.00	2.00	**0.00**	1.00	2.00
4.00	3.01	1.97	**0.46**	1.01	2.11	2.00	1.00	**0.00**	1.00	1.00

The bold is minimal of ARL_1 and MRL of control chart.

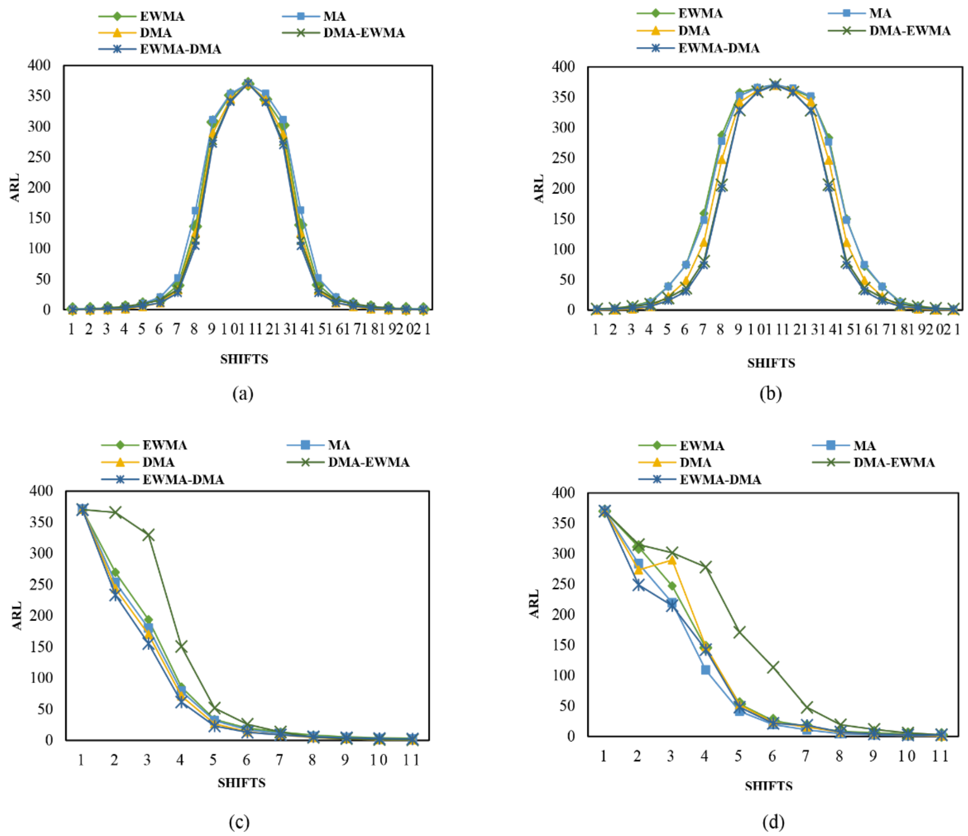

Figure 1 ARL performance of the proposed chart vs. MA, EWMA, DMA, and DMA-EWMA charts: (a) normal(0,1); (b) Laplace(0,1); (c) exponential(1) and (d) gamma(4,1) distribution

Table 7 ARL_1 and MRL performance of the proposed chart under gamma(4,1) distribution by varying w and given $w=5$, $\lambda=0.25$ and $ARL_0=370$

δ	ARL					MRL				
	EWMA	MA	DMA	DMA-EWMA	EWMA-DMA	EWMA	MA	DMA	DMA-EWMA	EWMA-DMA
	H_1=3.549	H_2=3.023	H_3=2.731	H_4=1.295	H_5=7.415	H_1=3.549	H_2=3.023	H_3=2.731	H_4=1.295	H_5=7.415
0.00	370.04	370.52	370.04	369.98	370.18	193.00	258.00	199.00	181.00	178.00
0.05	311.13	283.92	273.40	315.33	**248.86**	182.50	197.00	172.00	170.00	**117.00**
0.10	247.13	219.56	289.68	301.78	**214.90**	165.00	152.00	147.00	152.00	**71.00**
0.25	145.89	108.91	147.56	278.33	**142.43**	71.00	76.00	96.00	102.00	**54.00**
0.50	53.20	41.48	52.14	170.98	**48.07**	42.00	29.00	49.00	69.00	**27.00**
0.75	25.79	19.52	23.00	113.50	**21.45**	29.00	14.00	27.00	35.00	**13.00**
1.00	16.17	10.75	**9.52**	47.82	18.49	18.00	8.00	**7.00**	21.00	9.00
1.50	8.50	4.53	**3.96**	19.23	7.37	9.00	3.00	**2.00**	12.00	5.00
2.00	5.87	2.53	**2.01**	11.70	4.18	5.00	2.00	**2.00**	5.00	2.00
3.00	3.49	1.32	**1.29**	5.72	2.62	2.00	1.00	**0.00**	1.00	1.00
4.00	2.46	1.06	**0.93**	2.72	2.43	1.00	1.00	**0.00**	1.00	1.00

The bold is minimal of ARL_1 and MRL of control chart.

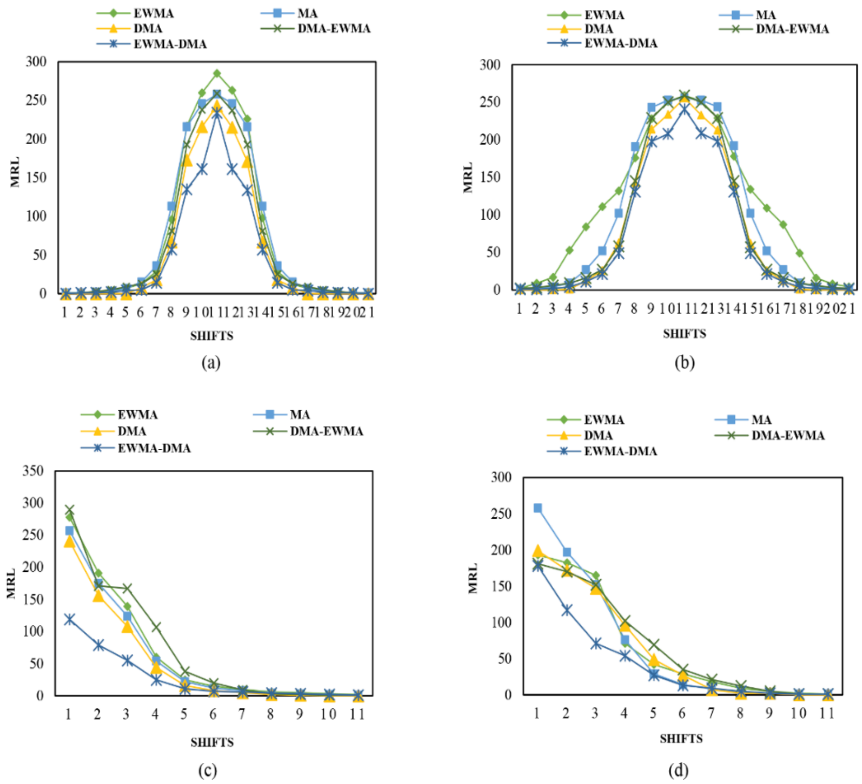

Figure 2 MRL performance of the proposed chart vs. MA, EWMA, DMA, and DMA-EWMA charts: (a) normal(0,1); (b) Laplace(0,1); (c) exponential(1) and (d) gamma(4,1) distribution

The first set was 40 sets of data: 1.94, 1.98, 1.98, 1.98, 1.98, 1.95, 1.96, 1.97, 1.94, 1.96, 1.93, 1.97, 1.98, 1.94, 1.94, 1.90, 1.94, 1.98, 1.93, 1.93, 1.94, 1.92, 1.94, 1.92, 1.91, 1.92, 1.91, 1.94, 1.90, 1.91, 2.00, 1.94, 2.01, 1.95, 1.94, 1.96, 1.95, 2.00, 1.96, 1.97 and the second set was 36 sets of data: 2705.27, 2648.05, 2640.87, 2713.83, 2823.81, 2673.61, 2647.58, 2575.26, 2519.36, 2471.65, 2470.30, 2423.41, 2411.80, 2384.20, 2362.72, 2363.64, 2278.87, 2238.83, 2198.81, 2126.15, 2168.27, 2170.95, 2173.60, 2098.86, 2096.96, 2065.30, 2059.74, 1932.23, 1940.24, 2043.94, 2080.41, 2079.36, 1920.03, 1972.18, 2103.84, and 2063.11.

Such data were used to create MA, EWMA, DMA, DMA-EWMW, and EWMA-DMA control chart to detect the mean change of the process using the statistic value and control limit of each control chart as shown in Equations (5), (9), (13), (15), and (17). The research results were shown in Figures 3 and 4 as follows.

The application results of the control chart and the first set of the data that had the normal distribution, it detected the change of process from the first chart. As for the MA, DMA and DMA-EWMA control chart, they detected the change of process in the 5^{th} time while the EWMA control chart could do so in the 30th time as shown in Figure 3. The application of the control chart to the second set of the data that had the gamma distribution illustrated that the proposed control chart detected the change of process at the 2^{nd} time whereas the DMA and DMA-EWMA could do so at the 3rd and 4th time, and MA and EWMA control chart could do so at the 5^{th} time as shown in Figure 4.

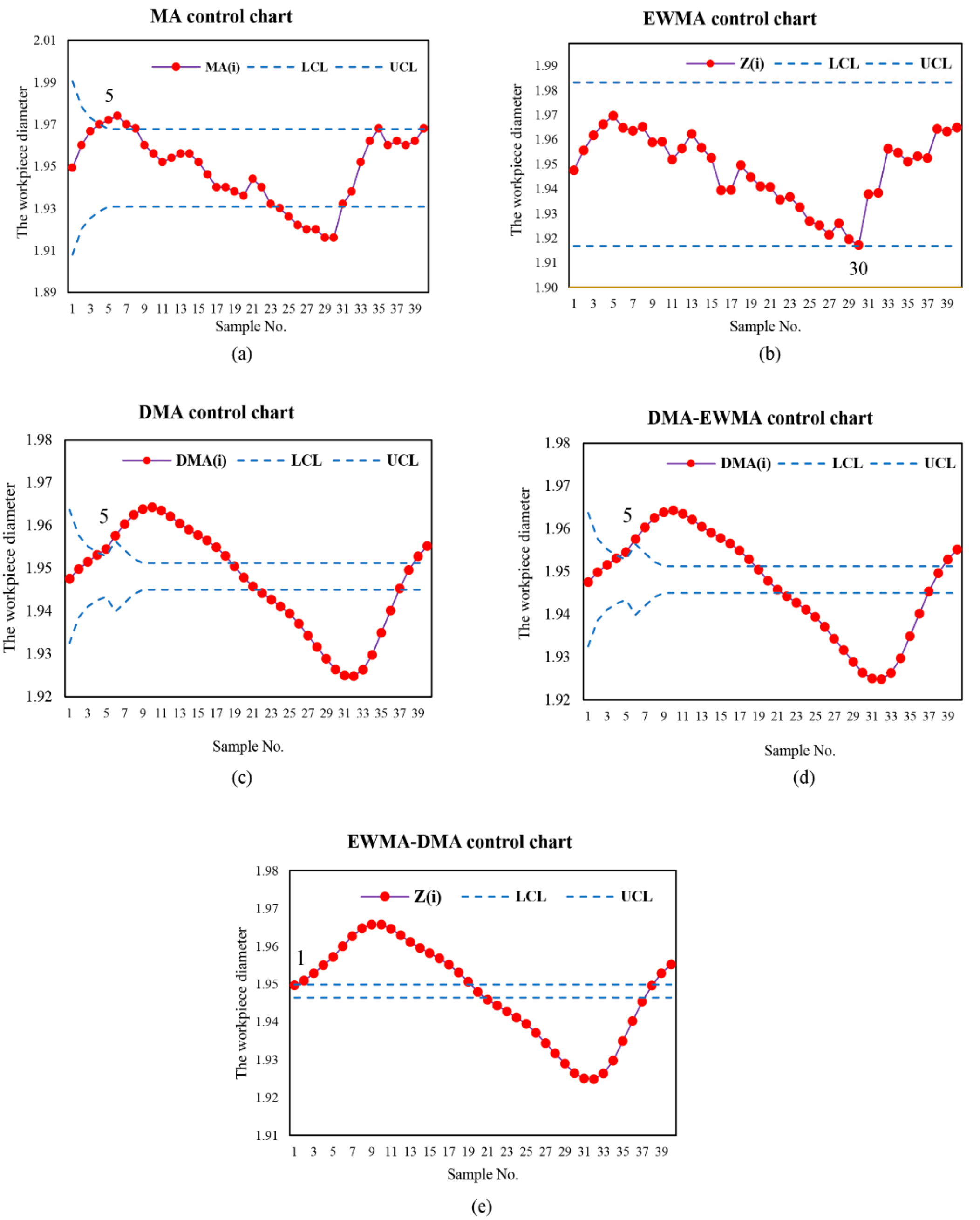

Figure 3 Graphical displays of the illustrative for the diameter of a workpiece from an industrial factory of control charts: (a) MA control chart; (b) EWMA control chart; (c) DMA control chart; (d) DMA-EWMA control chart and (e) EWMA-DMA control chart

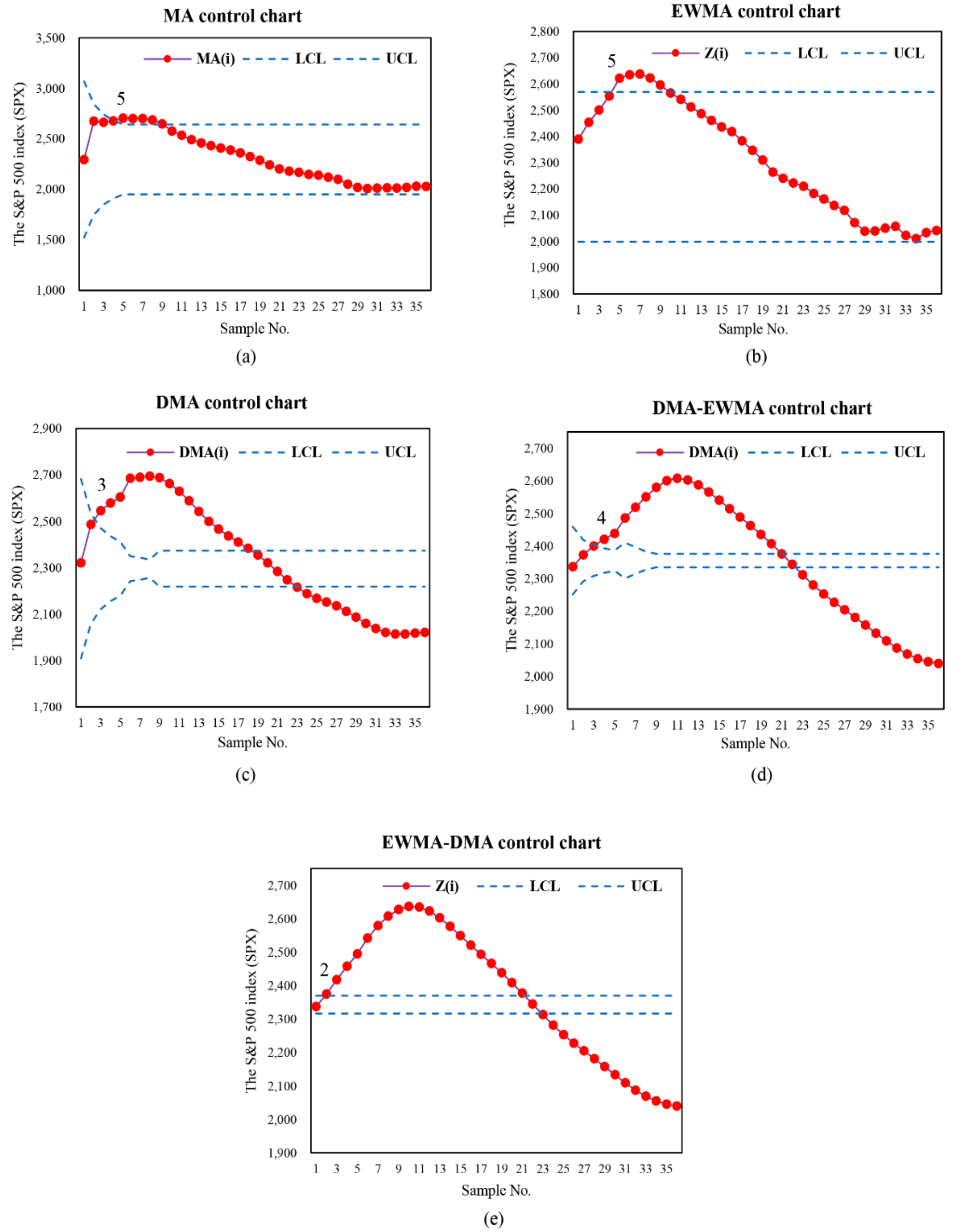

Figure 4 Graphical displays of the illustrative for the S&P 500 index (SPX) during 2015 - 2018 of control charts: (a) MA control chart; (b) EWMA control chart; (c) DMA control chart; (d) DMA-EWMA control chart and (e) EWMA-DMA control chart

5. Conclusions and recommendations

In this study, the efficiency of the proposed control chart was compared with other existing control charts (EWMA, MA, DMA, and DMA-EWMA) by using ARL_1 and MRL as the evaluation criteria. It was found that consistent results are obtained for normal(0,1), Laplace(0,1), exponential(1), and gamma(1) distributions. In addition, the EWMA-DMA chart was found most efficient for detecting the change at a level of $-0.75 \le \delta \le 0.75$. However, if it was the parameter that $\delta < -0.75$ and $\delta > 0.75$, the DMA control chart had a higher efficiency to detect the change in the process than the EWMA-DMA, DMA-EWMA, MA, and EWMA control chart. In addition, ARL_1 and MRL depended on the determination of parameter *w*; if *w* increased, ARL_1 and MRL decreased. In contrast, if *w* decreased, ARL_1 and MRL increased. These conformed to the previous research (Sukparungsee et al. 2020). Therefore, $w = 5$ was viewed as the optimal value one because it was moderate. The application results of the proposed control chart to the actual data, the data of diameter of the workpiece of an industrial factory, and data of the S&P 500 index (SPX) including historical data from 2015-2018, revealed that the proposed control chart had the faster detection of the change in the process than other control charts, which was consistent with the research results.

Besides, the researcher compared the proposed control chart to the Tukey-DEWMA (Supchottharee et al. 2016), Tukey-CUSUM (Khaliq and Riaz 2014) and MA-EWMA control chart (Sukparungsee et al. 2020) when the process had the normal distribution and was under the control, ARL_0=370, and the change of the average was $\delta \in [-4, 4]$ and ARL_1 was used as the criteria to evaluate the efficiency. It was found that the proposed chart had a better efficiency to detect the change than other control charts, except for the parameter change at $\pm$ 1.50 and $\pm$ 2.00 which the Tukey-DEWMA was more efficient than the proposed control chart.

Findings indicated that the EWMA-DMA control chart was an alternative to the parametric control charts. Further research might extend the scope of study to examine the data with other distributions, such as beta and Weibull distribution, or adjust the parameter and the sample size to see whether it affects the efficiency of the control chart or not.

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