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Estimation and Testing Procedures for the Reliability Functions of One Parameter Generalized Exponential Distribution(GED)

Sumit Koul*[a] and Ajit Chaturvedi [b]
[a] Department of Mathematics and Scientific Computing, NIT Hamirpur, Hamirpur, Himachal Pradesh, India
[b] Department of Statistics, University of Delhi, Delhi, India
* Corresponding author; Email: sumit.koul8@gmail.com

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Abstract

This paper has considered the estimation and testing of two reliability function e.g. $R(t) = P(X > t)$ and $P = P(X > Y)$ for one parameter generalized exponential distribution(GED). Uniformly minimum variance unbiased estimators (UMVUES) and maximum likelihood estimators (MLES)techniques are used to estimate the two reliability function under Type II and Type I censoring scheme in point estimation. Asymptotic confidence interval for the parameter θ, based on maximum likelihood estimators (MLES),with dispersion matrix is constructed. A hypothesis testing procedure has been obtained for two parametric functions. Lastly, the simulation study of two reliability procedures has been done and for the illustrative purposes real life data analysis is made.

Keywords: Generalized exponential distribution, point-estimation, interval-estimation, censoring methods, testing of hypothesis

1. Introduction

In some life time situations the exponential distribution is not so good because of its memoryless property. In the course of this process to study the life span of a product, the output will inevitably change if there will be certain damages within its life span. As exponential distribution is not able to withstand with these situations,a very significant distribution is introduced by Gupta and Kundu (1999) known as generalized exponential distribution (GED(θ)). It overcomes the problems associated with one parameter exponential distribution without memory and GED also has many applications in engineering fields like mechanical reliability, survival analysis. GED has gained attention among many distributions available in literature. In place of Weibull or Gamma distributions, Gupta and Kundu (1999) has introduced GED as an alternative distribution in many problems. As is analyzed that the suggested GED posses various significant characteristics and in certain circumstances, GED works better as compared to Weibull or Gamma distribution. Analysis of left censored data from the generalized exponential distribution was considered by Mitra and Kundu (2008). Sarhan (2007) has described the competing risk models of GED using censored and uncensored data. Kundu and Gupta (2011) has modeled the reliability of the stress-strength model depending upon the GED. Kundu and Pradhan (2009) has studied the performance of the GED parameter using hybrid censoring scheme. Raqab and Ahsanullah (2001) has studied the GED parameters namely location and shape with order statistics to analyses the performance. Kaushik et al. (2017) has considered the GED to the analyses the performance of parameters by using the progressive Type-I Censoring Scheme possessing Random Removals and reference cited. We assume that, a complete sample is available where the failure times of all n items are recorded. There are several situations, in which this is neither possible nor desirable. The life testing experiment are destructive in nature and we cannot reuse again. When we have failure censored samples, we put n products on trial and terminate the trial when a preallocated count of products have failed say, r. When we have time censored samples, putting n products on trial and terminate the trial at a preallocated time say, t_o. Many researchers has discussed the modeling of R(t) as well as P under complete and censoring observations for the point estimation in recent years. One can review, Pugh (1963), Bartholomew (1957, 1963), Johnson (1975), Kelley et al. (1976), Tong (1974, 1975), Awad and Gharraf (1986), Chao (1982), Tyagi and Bhattacharya (1989a,b), Chaturvedi and Rani (1997, 1998), , Chaturvedi et al. (2002), Chaturvedi and Tomer (2002), Chaturvedi and Tomer (2002), Chaturvedi et al. (2019) and others.

The objective of the study is many-fold. Baklizi (2008) has worked on strength reliability to obtain the likelihood as well as bayesian statistic on lower record values using one parameter generalised exponential distribution and we have considered this distribution in our formulation. we have incorporated point estimation procedure under two type of censoring viz., Type II and Type I. For Type II censoring. authors have also derived UMVUEs and MLEs by new technique for calculating both $R(t)$ and Stress-Strength reliability. we have obtain confidence interval for Type II censoring for θ, asymptotic confidence interval for P. In Bartholomew (1963) sampling technique, we have proposed new technique for UMVUEs and MLEs for both $R(t)$ and Stress-Strength reliability. we have proposed hypothesis testing procedures. Section 2 includes the definitions and notations used in the formation. In Section 3, we have considered the point estimation, derived the UMVUE and MLE for power, $R(t)$ and P under Type-II censoring, Exact confidence interval for θ and P are obtained. Type I Censoring Scheme for Point Estimators under Bartholomew Scheme is considered, we deduced the UMVUE and MLE for $R(t)$ and P in Section 4. In Section 5, we have proposed hypothesis testing. In Section 6, simulation results are drawn, validity of hypothesis testing procedures and practical and real time values is considered, in Section 7 summary of results is presented and conclusion has been done in Section 8.

2. Preliminaries, Notations and Definitions

Suppose X is random variable which follows one parameter generalized distribution having the pdf and cdf as refereed in Baklizi (2008) can be defined

$$f(x;\theta) = \theta e^{-x}\left(1-e^{-x}\right)^{\theta-1}, x>0, \theta>0 \tag{1}$$

and

$$F(x;\theta) = \left(1-e^{-x}\right)^{\theta}.$$

The reliability $R(t)$ of an item, is the probability of failure free operation untill time t, reliability function for a particular task at instant t is given by

$$\begin{aligned} R(t) &= p(x>t) \\ &= 1-F(t) \\ &= 1-\left(1-e^{-t}\right)^{\theta}. \end{aligned} \tag{2}$$

From (1) and (2), the hazard rate is given by

$$\begin{aligned} h(t) &= \frac{f(t)}{R(t)} \\ &= \frac{\theta e^{-x}\left(1-e^{-x}\right)^{\theta-1}}{1-\left(1-e^{-x}\right)^{\theta}}. \end{aligned}$$

In this paper, other reliability function is used which is described as the lifetime of an individual under random strength variable X subjected to the random stress variable Y is called stress-strength model. Whenever stress exceeds its strength then the failure of an item occurs, mathematically it is expressed as $P = P(X > Y)$.In many practical situations like in engineering sciences, electrical and electronic systems. Suppose X and Y follows GED(θ_1) and GED(θ_2). X and Y are independent,

$$\begin{aligned} P &= P(X > Y) \\ &= \int_{y=0}^{\infty}\int_{x=y}^{\infty} f(x)f(y)\,dx\,dy \\ &= \frac{\theta_2}{\theta_1+\theta_2}. \end{aligned}$$

3. Type II Censoring Scheme for Point Estimators

Consider n product under trial and after the first r product are recorded the trial is terminated. On denoting $0 < X_{(1)} \leq X_{(2)} \leq ..... \leq X_{(r)}$, $0 < r < n$, is taken as the first r product during lifetime. Henceforth, (n-r) product retain until $X_{(r)}$. Before proving the main theorem of this section, we first state Lemma.

Lemma 1 *Let $S_r = \sum_{i=1}^{r} y_i + (n-r)y_r$. The S_r is complete and sufficient for the distribution which is delineate in Eqn. (1). The pdf of S_r is*

$$g(S_r,\theta) = \frac{S_r^{r-1}\,(\theta^r)\,exp\,(-\theta S_r)}{\Gamma(r)}, S_r > 0. \tag{3}$$

Proof: Rewrite Eqn. (1) as,

$$f(x;\theta) = \theta\left(\frac{e^{-x}}{1-e^{-x}}\right)e^{-\theta\left(-ln\left(1-e^{-x}\right)\right)}. \tag{4}$$

From (4), the joint pdf of $0 < X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ is

$$\begin{aligned} f^*\left(x_{(1)},x_{(2)},\ldots,x_{(n)};\theta\right) &= n!\prod_{i=1}^{n}\theta\left(\frac{e^{-x_i}}{1-e^{-x_i}}\right)e^{-\theta\left(-ln\left(1-e^{-x_i}\right)\right)} \\ f^*\left(x_{(1)},x_{(2)},\ldots,x_{(n)};\theta\right) &= n!\theta^n\prod_{i=1}^{n}\left(\frac{e^{-x_i}}{1-e^{-x_i}}\right)e^{-\theta\sum_{i=1}^{n}\left(-ln\left(1-e^{-x_i}\right)\right)}. \end{aligned}$$

Taking the transformation $y_i = -ln\left(1-e^{-x_i}\right)$. The Jacobian transformation is given by $|J| = \frac{1-e^{-x_i}}{e^{-x_i}}$. Then the joint pdf of $Y_{(1)}, Y_{(2)}, \ldots, Y_{(n)}$ is

$$\begin{aligned} f^{**}\left(y_{(1)},y_{(2)},\ldots,y_{(n)};\theta\right) &= n!\theta^n\prod_{i=1}^{n}|J|\left(\frac{e^{-x_i}}{1-e^{-x_i}}\right)e^{-\theta y_i} \\ &= n!\theta^n e^{\left(-\theta\sum_{i=1}^{n}y_i\right)}. \end{aligned} \tag{5}$$

By integrating the $y_{(r+1)}, y_{(r+2)}, \ldots, y_{(n)}$ form Eqn. (5) for the region $y_{(r)} \leq y_{(r+1)} \leq \ldots \leq y_{(n)}$, the joint pdf of $0 < y_{(1)} \leq y_{(2)} \leq \ldots \leq y_{(r)}$ is of the form Sinha (1968),

$$f^{***}\left(y_{(1)},y_{(2)},\ldots,y_{(r)};\theta\right) = \frac{n!}{(n-r)!}\theta^r\left(e^{-\theta S_r}\right).$$

From Fisher-Neyman fractorization theorem Rohtagi (1976) and Eqn. (4) the value of S_r is sufficient for the θ. Considering the transformation, $U = -ln\left(1-e^{-x_i}\right)$, the pdf of U follows Exp($\frac{1}{\alpha}$).

Suppose the modification is taken as $Z_i = (n-i+1)\left(U_{(i)} - U_{(i-1)}\right), i = 1, 2, \ldots, r,$. Since $\sum_{i=1}^{r} Z_i = S_r$, $Z_{i's}$ are independent and identically distributed random variable each having Exp(1). Eqn. (4) follows the additive property of Gamma(S_r, θ) Johnson (1970). Although the distribution of S_r belongs to the exponential family of distributions and is complete Rohtagi and Saleh (2012).

3.1. UMVUE

Theorem 1 *For $q \in (-\infty, \infty)$, the UMVUE of θ^q is*

$$\hat{\theta}^q_{II} = \begin{cases} \frac{\Gamma(r)}{\Gamma(r-q)} S_r^{-q}, & r - q > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Proof: Using (3),

$$\begin{aligned} E\left(S_r^{-q}\right) &= \frac{1}{\theta \Gamma(r)} \int_0^\infty S_r^{r-q-1} e^{(-\theta S_r)} ds_r \\ &= \frac{\theta^q \Gamma(r-q)}{\Gamma(r)}, \quad r > q \end{aligned}$$

Lehmann-Scheffe theorem gives Rohtagi and Saleh (2012) and which follows Eqn. (6).

Theorem 2 *The UMVUE of R(t) during instant t is*

$$\hat{R}_{II}(t) = \begin{cases} 1 - [\left(1 + \frac{ln\left(1-e^{-t}\right)}{S_r}\right)^{r-1}], & -ln\left(1-e^{-t}\right) < S_r \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Proof: By the definition of reliability,

$$\begin{aligned} R(t) &= 1 - F(t) \\ &= 1 - (1 - e^{-t}) \\ &= 1 - e(\theta ln(1 - e^{-t})) \\ &= 1 - \sum_{i=0}^{\infty} \frac{(ln(1-e^{-t}))^i}{i!} \theta^i_{II}. \end{aligned}$$

Using the result of Theorem 1,

$$\begin{aligned} \hat{R}_{II}(t) &= 1 - \sum_{i=0}^{r-1} \frac{(ln(1-e^{-t}))^i}{i!} \hat{\theta}^i_{II} \\ &= 1 - \sum_{i=0}^{r-1} \frac{(ln(1-e^{-t}))^i}{i!} \frac{\Gamma(r)}{\Gamma(r-i)} S_r^{-i} \\ &= 1 - \sum_{i=0}^{r-1} \binom{r-1}{i} [\frac{ln(1-e^{-t})}{S_r}]^i, \end{aligned}$$

and hence the theorem is derived.

Corollary 1 *The UMVUE of $f\,(x;\theta)$ at a particular point x is*

$$\hat{f}_{II}(x;\theta) = \begin{cases} (r-1)\,[1 + \frac{ln\left(1-e^{-x}\right)}{S_r}]^{r-2} \frac{e^{-x}}{S_r(1-e^{-x})}, & -ln\left(1-e^{-t}\right) < S_r \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

Proof: As $F(x,S_r) = f(x,\theta)\, g(S_r,\theta)$ being a continuous function of (X,S_r) on rectangle $[t,\infty)$ x $[0,\infty)$, using the Fubini's theorem Bilodeau et al. (2010) the conditions are satisfied for the change of order of integration. Suppose the expectation of the integral are considered $\int_t^\infty \hat{f}_{II}(x;\theta)\,dx$ w.r.t. S_r then,

$$\begin{aligned} \hat{R}_{II}(t) &= \int_t^\infty \hat{f}_{II}(x;\theta)\,dx \\ \text{or} \quad \frac{-d\hat{R}_{II}(t)}{dt} &= \hat{f}_{II}(t;\theta) \end{aligned} \tag{9}$$

applying Theorem 2 and Eqn. (9) and theorem follows.

Let $X \sim f_1(x;\theta_1)$ and $Y \sim f_2(y;\theta_2)$ be two I.D. random variables following the distributions of classes respectively, where

$$\begin{aligned} f_{1II}(x;\theta_1) &= \frac{\theta_1 e^{-x}}{1-e^{-x}} e^{-\theta_1\left(-ln\left(1-e^{-x}\right)\right)},;x>0,\theta_1>0 \\ \text{and} \quad f_{2II}(y;\theta_2) &= \frac{\theta_2 e^{-y}}{1-e^{-y}} e^{-\theta_2\left(-ln\left(1-e^{-y}\right)\right)},;y>0,\theta_2>0. \end{aligned}$$

Suppose n is the product of X and m is the product of Y which are kept on trial and r,s are the stopping limits for X and Y, respectively.

$$\begin{aligned} S_r &= \sum_{i=1}^{r} y_{1(i)} + (n-r)y_{1(r)} \\ T_s &= \sum_{j=1}^{s} y_{2(j)} + (m-s)y_{2(s)}. \end{aligned}$$

By making the transformation,

$$\begin{aligned} Y_{1(i)} &= -\ln\left(1-e^{-x_i}\right);i=1,2,...,r \\ Y_{2(j)} &= -\ln\left(1-e^{-y_j}\right);j=1,2,...,s, \end{aligned}$$

which determines the UMVUE of P in the following theorem.

Theorem 3 *The UMVUE of P is given by*

$$\hat{P}_{II} = \begin{cases} (s-1)\displaystyle\int_0^{(\ln(1-e^{-T_s}))^{-1}\ln(1-e^{-S_r})} [1-\left(1-\frac{zT_s}{S_r}\right)^{r-1}](1-z)^{s-2}\,dz; & -\ln(1-e^{-S_r}) > -\ln(1-e^{-T_s}) \\ (s-1)\displaystyle\int_0^1 [1-\left(1-\frac{zT_s}{S_r}\right)^{r-1}](1-z)^{s-2}\,dz; & -\ln(1-e^{-S_r}) < -\ln(1-e^{-T_s}). \end{cases}$$

Proof: From Corollary 1,

$$\begin{aligned} \hat{f}_{1II}(x;\theta_1) &= (r-1)\,[1+\frac{\ln(1-e^x)}{S_r}]^{r-2}\frac{e^x}{S_r(1-e^x)};-\ln(1-e^x)<S_r \\ \text{and} \quad \hat{f}_{2II}(y;\theta_2) &= (s-1)\,[1+\frac{\ln(1-e^y)}{T_s}]^{s-2}\frac{e^y}{T_s(1-e^y)};-\ln(1-e^y)<T_s \end{aligned}$$

using the similar argument adopted in Corollary 1,

$$\begin{aligned}\hat{P}_{II} &= \int_{y=0}^{\infty}\int_{x=y}^{\infty}\hat{f}_{1II}(x;\theta_1)\hat{f}_{2II}(y;\theta_2)\,dx\,dy\\ &= \int_{y=0}^{\infty}\hat{R}_{1II}(y;\theta_1)\left\{-\frac{d}{dy}\hat{R}_{2II}(y;\theta_2)\right\}dy\\ &= (s-1)\int_{M}^{\infty}[1-[\left(1+\frac{ln\,(1-e^{-y})}{S_r}\right)^{r-1}]][1+\frac{ln\,(1-e^{y})}{T_s}]^{s-2}\frac{e^{y}}{T_s\,(1-e^{y})}dy, \quad (10)\end{aligned}$$

where, M = $max\left(-ln\left(1-e^{-S_r}\right),-ln\left(1-e^{-T_s}\right)\right)$.

Using Theorem 2 and Eqn. (10) and putting $z=\frac{ln(1-e^{y})}{T_s}$ and hence Theorem 3 is derived.

Corollary 2 *When X and Y belongs to the same families of distributions*

$$\hat{P}_{II}=\begin{cases}(s-1)\int_0^{\frac{S_r}{T_s}}[1-\left(1-\frac{zT_s}{S_r}\right)^{r-1}]\,(1-z)^{s-2}\,dz; & S_r<T_s\\ (s-1)\int_0^{1}[1-\left(1-\frac{zT_s}{S_r}\right)^{r-1}]\,(1-z)^{s-2}\,dz; & S_r>T_s.\end{cases}$$

3.2. MLE

From Eqn. (6) , the MLE of θ under Type II censoring is

$$\widetilde{\theta} = \frac{r}{S_r}. \quad (11)$$

Now, MLE is derived from the following theorem for power θ.

Theorem 4 *The MLE of $\widetilde{\theta}_q$, $q\epsilon\,(-\infty,+\infty)$ is given by*

$$\widetilde{\theta}^q_{II}=\left(\frac{r}{S_r}\right)^q.$$

Proof: Using the Eqn. (11), invariant property of MLEs and the theorem follows.

Obtaining the MLE of $R(t)$ is as follows.

Theorem 5 *The MLE of $\widetilde{R}_{II}(t)$ is delineate as,*

$$\widetilde{R}_{II}(t)=1-\left(1-e^{-t}\right)^{\frac{r}{S_r}}.$$

Corollary 3

$$\widetilde{f}_{II}(x;\theta)=\frac{re^{-x}}{S_r(1-e^{-x})}e^{-\frac{r}{S_r}(-ln(1-e^{-x}))}.$$

Proof: Using the fact that

$$\frac{-d}{dt}\widetilde{R}_{II}(t)=\widetilde{f}_{II}(x;\theta)$$

and hence the theorem is derived.

MLE is derived from the following theorem for P.

Theorem 6 *The MLE of P is given by*

$$\widetilde{P}_{II}=\frac{s}{T_s}\int_0^{\infty}(1-e^{\frac{r}{S_r}ln(y)})z^{\frac{s}{T_s}-1}dz.$$

Proof: Using the similar argument as in Eqn. (10) and then theorem follows.

Theorem 7 *The MLE of the pdf $f(x;\theta)$ at particular point x is delineate as*

$$\widetilde{f}(x;\theta)=\frac{r}{S_r}e^{-x}(1-e^{e^{-x}}).$$

Proof: Eqn. (9) has been used to solve the following theorem.

Corollary 4 *When X and Y belong to same distribution, then the MLE of P*

$$\widetilde{P}_{II}=\frac{\frac{s}{T_s}}{\frac{r}{S_r}+\frac{s}{T_s}}.$$

3.3. Under Type II censoring scheme: Exact confidence interval for MLE and UMVUE

In this section, we construct the problem of two-sided confidence interval for MLE of θ, using the pivotal quantity $2\theta S_r$. By the definition chi-square which is mathematically written as

$$P\left(\chi^2>\chi^2_\alpha\right)=\int_{\chi^2_\alpha}^{\infty}p\left(\chi^2\right)d\chi^2=\alpha, \tag{12}$$

where $p\left(\chi^2\right)$ is the probability distribution function of χ^2-distribution with 2r degree of freedom.

We know that, $2\theta S_r\sim\chi^2_{(2r)}\left(.\right)$

$$\begin{aligned}
P\left[\chi^2_{(2r)}\left(\frac{\alpha}{2}\right)\le\chi^2_{(2r)}\le\chi^2_{(2r)}\left(1-\frac{\alpha}{2}\right)\right] &= 1-\alpha\\
P\left[\chi^2_{(2r)}\left(\frac{\alpha}{2}\right)\le 2\theta S_r\le\chi^2_{(2r)}\left(1-\frac{\alpha}{2}\right)\right] &= 1-\alpha\\
P\left[\frac{2\theta r}{\chi^2_{(2r)}\left(1-\frac{\alpha}{2}\right)}\le\frac{r}{S_r}\le\frac{2\theta r}{\chi^2_{(2r)}\left(\frac{\alpha}{2}\right)}\right] &= 1-\alpha.
\end{aligned}$$

Using Eqn. (12) one can find $\chi^2_{(2r)}\left(1-\frac{\alpha}{2}\right)$ and $\chi^2_{(2r)}\left(\frac{\alpha}{2}\right)$ with 2r df. Therefore, $100(1-\alpha)\%$ confidence interval for MLE of θ is

$$\left[\frac{2\theta r}{\chi^2_{(2r)}\left(1-\frac{\alpha}{2}\right)},\frac{2\theta r}{\chi^2_{(2r)}\left(\frac{\alpha}{2}\right)}\right].$$

Also we construct the confidence interval for MLE of reliability function, $R(t)$. From Eqn. (2), we know that, $R(t;\theta)$ is an increasing function θ and $\widetilde{R}_{II}(t)=1-exp\left(\widetilde{\theta}_{MLE}ln\left(1-e^{-t}\right)\right)$. Hence, $100(1-\alpha)\%$ confidence interval for MLE of $R(t)$ is delineate as

$$\left[\ (1-(exp(\tfrac{2r\theta}{\chi^2_{2r}(\frac{\alpha}{2})}ln(1-e^{-t})))),(1-(exp(\tfrac{2r\theta}{\chi^2_{2r}(1-\frac{\alpha}{2})}ln(1-e^{-t}))))\ \right].$$

In order to construct the problem of two-sided confidence interval for UMVUE of θ, the unbiased estimator of θ for UMVUE is given as $\hat{\theta}=\frac{r-1}{S_r}$. Proceeding in similar way as above, $100(1-\alpha)\%$ confidence interval for UMVUE of θ is of the form

$$\left[\ \tfrac{2(r-1)\theta}{\chi^2_{2r}(1-\frac{\alpha}{2})},\tfrac{2(r-1)\theta}{\chi^2_{2r}(\frac{\alpha}{2})}\ \right].$$

According to Theorem 2, we have

$$\begin{aligned}
\hat{R}_{UMVUE}(t) &= 1-\left[\left(1+\frac{ln\left(1-e^{-t}\right)}{S_r}\right)^{r-1}\right]\\
&= 1-\left[\left(1+\frac{\theta}{r-1}ln\left(1-e^{-t}\right)\right)^{r-1}\right].
\end{aligned}$$

Thus, $100(1-\alpha)\%$ confidence interval for UMVUE of $R(t)$ is delineate as

$$\left[\ (1-\left(1+\frac{2\theta}{\chi^2_{2r}(\frac{\alpha}{2})}ln\,(1-e^{-t})\right)^{r-1}),(1-\left(1+\frac{2\theta}{\chi^2_{2r}(1-\frac{\alpha}{2})}ln\,(1-e^{-t})\right)^{r-1})\ \right].$$

Now we will next derive confidence interval for UMVUE and MLE for P.

Since $\widetilde{\theta}_1=\frac{r}{S_r}$ and $\widetilde{\theta}_2=\frac{s}{T_s}$, and also $P=\frac{\theta_2}{\theta_1+\theta_2}\Rightarrow\widetilde{P}_{MLE}=\frac{1}{\frac{\widetilde{\theta}_1}{\widetilde{\theta}_2}+1}$. By the independent of two random quantities we have, $\frac{\widetilde{\theta}_1\frac{S_r}{r}}{\widetilde{\theta}_2\frac{T_s}{s}}\sim F_{2r,2s}$ a scaled F distribution. It follows that $\widetilde{P}_{MLE}=\frac{1}{\frac{\widetilde{\theta}_1}{\widetilde{\theta}_2}F_{2r,2s}+1}$, by simple transformation techniques we obtained confidence interval for MLE of P as

$$\begin{aligned}
P\left[F_{2r,2s}\left(1-\frac{\alpha}{2}\right)\le F_{2r,2s}\le F_{2r,2s}\left(\frac{\alpha}{2}\right)\right] &= 1-\alpha\\
P\left[F_{2r,2s}\left(1-\frac{\alpha}{2}\right)\le\frac{\theta_1\frac{S_r}{r}}{\theta_2\frac{T_s}{s}}\le F_{2r,2s}\left(\frac{\alpha}{2}\right)\right] &= 1-\alpha\\
P\left[\left(\frac{rT_sF_{2r,2s}\left(\frac{\alpha}{2}\right)}{sS_r}+1\right)^{-1}\le\frac{\theta_2}{\theta_1+\theta_2}\le\left(\frac{rT_sF_{2r,2s}\left(1-\frac{\alpha}{2}\right)}{sS_r}+1\right)^{-1}\right] &= 1-\alpha.
\end{aligned}$$

Therefore, $100(1-\alpha)\%$ confidence interval for MLE of P is delineate as

$$\left[\left(\frac{rT_sF_{2r,2s}\left(\frac{\alpha}{2}\right)}{sS_r}+1\right)^{-1},\left(\frac{rT_sF_{2r,2s}\left(1-\frac{\alpha}{2}\right)}{sS_r}+1\right)^{-1}\right].$$

Now we considered the problem of confidence interval for UMVUE of P, we know that

$$(s-1)\int_0^1\left[1-\left(1-\frac{zT_s}{S_r}\right)^{r-1}\right](1-z)^{s-2}\,dz;\ S_r>T_s.$$

Also,

$$\frac{\widetilde{\theta}_1\frac{S_r}{r}}{\widetilde{\theta}_2\frac{T_s}{s}}\sim F_{2r,2s}$$

$$P\left[F_{2r,2s}\left(1-\frac{\alpha}{2}\right)\le\frac{\theta_1sS_r}{\theta_2rT_s}\le F_{2r,2s}\left(\frac{\alpha}{2}\right)\right]=1-\alpha$$

or

$$P\left[\frac{\theta_2r}{\theta_1s}F_{2r,2s}\left(1-\frac{\alpha}{2}\right)z\le\frac{S_r}{T_s}z\le\frac{\theta_2r}{\theta_1s}F_{2r,2s}\left(\frac{\alpha}{2}\right)z\right]=1-\alpha$$

or

$$P\left[\begin{array}{r}(1-z)^{s-2}\left[1-\left(1-\frac{\theta_2r}{\theta_1s}F_{2r,2s}\left(1-\frac{\alpha}{2}\right)z\right)^{r-1}\right]\le(1-z)^{s-2}\left(1-\frac{S_r}{T_s}z\right)\le\\(1-z)^{s-2}\left[1-\left(1-\frac{\theta_2r}{\theta_1s}F_{2r,2s}\left(\frac{\alpha}{2}\right)z\right)^{r-1}\right]\end{array}\right]=1-\alpha.$$

Thus, $100(1-\alpha)\%$ confidence interval for UMVUE of P is delineate as

$$\left[\begin{array}{c}(s-1)\int_0^1(1-z)^{s-2}\,[1-\left(1-\frac{\theta_2r}{\theta_1s}F_{2s,2r}(1-\frac{\alpha}{2})z\right)^{r-1}],\\(s-1)\int_0^1(1-z)^{s-2}\,[1-\left(1-\frac{\theta_2r}{\theta_1s}F_{2s,2r}(\frac{\alpha}{2})z\right)^{r-1}]\end{array}\right].$$

We can get the result when $S_r<T_s$ in similar manner.

4. Type I Censoring Scheme for Point Estimators

Let us denote $0 < X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ be the failure time of n product on trial from equation (1). The trial starts instantly at $X_{(0)} = 0$ and an organized scheme works till $X_{(1)} = x_{(1)}$, when the earliest failure occurs. The failed product is changed with a fresh product and the system runs till the succeeding failure happens instantly at $X_{(2)} = x_{(2)}$ and so on. The trial is terminated instantly at particular t_0. According to Bartholomew (1963), the count of failures that occurred in the period 0 to t_o only were considered and neglected the duration when failures happened, it was also denoted that the count of failures $N(t_0)$, under such plan, during the interval 0 to t_o has Pois(λ). Before proving the main theorem of this section, we first state lemma.

Lemma 2 *If $N(t_0)$ be the count of failures during the period 0 to t_o, then*

$$P[N(t_0) = r|t_0] = \left(-n\theta \ln\left(1 - e^{-t_0}\right)\right)^r \frac{exp(-n\theta \ln(1 - e^{-t_0}))}{r!}. \quad (13)$$

Proof: Let us make the transformation $W_1 = Y_{(1)}, W_2 = Y_{(2)} - Y_{(1)}, ..., W_n = Y_{(n)} - Y_{(n-1)}$. The pdf of W_1 is $h(w_1) = n\theta \ln(-n\theta w_1)$. Although $w_2, w_3, ..., w_n$ are independent and identically distributed as w_1, using the monotonic property of $-\ln(1 - e^{-x})$,

$$\begin{aligned} P[N(t_0) = r|t_0] &= P\left(X_{(r)} \leq t_0\right) - P\left(X_{(r+1)} \leq t_0\right) \\ &= P[Y_r \leq -ln\left(1 - e^{-t_0}\right)] - P[Y_{r+1} \leq -\ln\left(1 - e^{-t_0}\right)] \\ &= P[n\theta \sum_{i=1}^{r+1} W_i \geq -n\theta \ln\left(1 - e^{-t_0}\right)] \\ &\quad -P[n\theta \sum_{i=1}^{r} W_i \geq -n\theta \ln\left(1 - e^{-t_0}\right)]. \end{aligned} \quad (14)$$

According to the additive property of Expo(θ), Johnson and Kotz (1970), $U = n\theta \sum_{i=1}^{r} W_i$ follows Gamma(u) with pdf

$$g(u) = \frac{1}{\Gamma r} u^{r-1} e^{-u}; u > 0. \quad (15)$$

By taking the result of Patel et. al. (1976) and (14), authors attain from (15) as

$$\begin{aligned} P[N(t_0) = r|t_0] &= \frac{1}{\Gamma(r+1)} \int_{-n\theta \ln(1-e^{-t_0})}^{\infty} e^{-r} u^r du - \frac{1}{\Gamma(r)} \int_{-n\theta \ln(1-e^{-t_0})}^{\infty} e^{-r} u^{r-1} du \\ &= exp\left(-n\theta \ln\left(1 - e^{-t_0}\right)\right) \left[\sum_{j=0}^{r} \frac{[-n\theta \ln(1 - e^{-t_0})]^j}{j!} - \sum_{j=0}^{r-1} \frac{(-n\theta \ln(1 - e^{-t_0}))^j}{j!}\right] \end{aligned}$$

and hence the lemma is obtained.

4.1. UMVUE

Theorem 8 *For $q \in (0, \infty)$, the UMVUE of θ^q is*

$$\hat{\theta}_I^q = \begin{cases} \frac{r!}{(r-q)!} [-n\theta \ln(1 - e^{-t_0})]^{-q}, & r - q > 0 \\ 0 & \text{otherwise.} \end{cases} \quad (16)$$

Proof: By using Neyman-Factorization theorem Rohtagi (1976) and Lemma 2 it signifies that r is sufficient for evaluating θ. Also r is complete as the distribution of r pertains to exponential family Rohtagi and Saleh (2012). Using the result as described below

$$E(r(r-1)...(r-q+1)) = \left(-n\theta \ln\left(1 - e^{-t_0}\right)\right)^q,$$

the theorem follows.

Theorem 9 *The UMVUE of R(t) at instant t is*

$$\hat{R}_I(t) = \begin{cases} \left(1 - \left(1 - \frac{\ln\left(1-e^{-t}\right)}{n\ln(1-e^{-t_0})}\right)^r\right), & \ln\left(1-e^{-t}\right) > n\ln\left(1-e^{-t_0}\right) \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

Proof: By the definition of reliability, we have,

$$\begin{aligned} R(t) &= 1 - \left(1-e^{-t}\right)^{\theta} \\ &= 1 - exp\left(\theta \ln\left(1-e^{-t}\right)\right) \\ &= 1 - \sum_{j=0}^{\infty} \frac{\theta^j \left(\ln\left(1-e^{-t}\right)\right)^j}{j!}. \end{aligned}$$

Now,
$$\hat{R}_I(t) = 1 - \sum_{j=0}^{\infty} \frac{\hat{\theta}_I^j \left(\ln\left(1-e^{-t}\right)\right)^j}{j!}.$$

Using Eqn. (16) we get,

$$\hat{R}_I(t) = 1 - \sum_{j=0}^{r} (-1)^j \binom{r}{j} \left(\frac{\ln\left(1-e^{-t}\right)}{n\ln\left(1-e^{-t_0}\right)}\right)^j.$$

and the theorem is derived.

Corollary 5 *The UMVUE of the sampled pdf at a particular point*

$$\hat{f}_I(x;\theta) = \begin{cases} \frac{-re^{-x}}{n(1-e^{-x})\ln(1-e^{-x})}\left(1 - \frac{\ln\left(1-e^{-t}\right)}{n\ln(1-e^{-t_0})}\right)^{r-1}, & \ln\left(1-e^{-x}\right) > n\ln\left(1-e^{-t_0}\right) \\ 0 & \text{otherwise.} \end{cases}$$

Proof: Theorem follows after adopting the similar fact as used in the proof of Corollary 1.

Now we discuss X and Y as two classes of distribution $\hat{f}_I(x;\theta_1)$ and $\hat{f}_I(y;\theta_2)$. Suppose n product put on trial for X and m for Y. Termination time t_o for X and t_{oo} for Y, before these termination time r, s are the counts of failures respectively, where,

$$\hat{f}_I(x;\theta_1) = \begin{cases} \frac{-re^{-x}}{n(1-e^{-x})\ln(1-e^{-t_0})}\left(1 - \frac{\ln\left(1-e^{-x}\right)}{n\ln(1-e^{-t_0})}\right)^{r-1}, & \ln\left(1-e^{-x}\right) > n\ln\left(1-e^{-t_0}\right) \\ 0 & \text{otherwise} \end{cases}$$

and

$$\hat{f}_I(y;\theta_2) = \begin{cases} \frac{-se^{-y}}{m(1-e^{-y})\ln(1-e^{-t_{00}})}\left(1 - \frac{\ln\left(1-e^{-y}\right)}{m\ln(1-e^{-t_{00}})}\right)^{s-1}, & \ln\left(1-e^{-y}\right) > m\ln\left(1-e^{-t_{00}}\right) \\ 0 & \text{otherwise.} \end{cases}$$

The following theorem is based on UMVUE of P under Type I censoring scheme.

Theorem 10 *The UMVUE of P is given by*

$$\hat{P}_I = \begin{cases} s\int_0^{n\ln\left(1-e^{-t_0}\right)\left(m\ln\left(1-e^{-t_{00}}\right)\right)^{-1}} (1-z)^{s-1}\left(1 - \left(1 - \frac{m\ln\left(1-e^{-t_{00}}\right)z}{n\ln\left(1-e^{-t_0}\right)}\right)^r\right) dz; \\ \qquad n\ln\left(1-e^{-t_0}\right) > m\ln\left(1-e^{-t_{00}}\right) \\ s\int_0^1 (1-z)^{s-1}\left(1 - \left(1 - \frac{m\ln\left(1-e^{-t_{00}}\right)z}{n\ln\left(1-e^{-t_0}\right)}\right)^r\right) dz; \\ \qquad n\ln\left(1-e^{-t_0}\right) < m\ln\left(1-e^{-t_{00}}\right) \end{cases}$$

Proof: Using the similar arguments which are adopted in Theorem 3, we have

$$\begin{aligned}
\hat{P}_I &= \int_{y=0}^{\infty}\int_{x=y}^{\infty} \hat{f}_{1I}(x;\theta_1)\hat{f}_{2I}(y;\theta_2)\,dx\,dy \\
&= \int_{y=0}^{\infty} \hat{R}_{1I}(y;\theta_1)\left\{-\frac{d}{dy}\hat{R}_{2I}(y;\theta_2)\right\}dy \\
&= (s-1)\int_{max(n\ln(1-e^{-t_0}),m\ln(1-e^{-t_{00}}))}^{\infty}\left(1-\left(1-\frac{\ln\left(1-e^{-y}\right)}{n\ln\left(1-e^{-t_0}\right)}\right)^{r}\right) \\
&\quad \frac{-se^{-y}}{m\left(1-e^{-y}\right)\ln\left(1-e^{-t_{00}}\right)}\left(1-\frac{\ln\left(1-e^{-y}\right)}{m\ln\left(1-e^{-t_{00}}\right)}\right)^{s-1}dy
\end{aligned}$$

Putting $z=\frac{\ln\left(1-e^{-y}\right)}{m\ln\left(1-e^{-t_{00}}\right)}$ we get the two cases and theorem follows.

Corollary 6 *When t_0 = t_{00}, X and Y belong to same distribution is delineate as*

$$\hat{P}_I=\begin{cases} s\displaystyle\int_0^{\frac{n}{m}}(1-z)^{s-1}\left(1-\left(1-\frac{mz}{n}\right)^{r}\right)dz; & m>n \\ s\displaystyle\int_0^{1}(1-z)^{s-1}\left(1-\left(1-\frac{mz}{n}\right)^{r}\right)dz; & m<n \end{cases}$$

Proof: When m is greater than n and m is less than n, using Theorem 10 and both the assertion follows.

4.2. MLE

Under the sampling scheme of Bartholomew (1963), it follows from Eqn. (12) that

$$\widetilde{\theta}_I=\left(\frac{-r}{n\ln\left(1-e^{-t_0}\right)}\right). \tag{18}$$

The following theorem of MLE of power is as under

Theorem 11 For q$\in(-\infty,\infty)$, the MLE of θ^q is

$$\widetilde{\theta}_I^q=\left(\frac{-r}{n\ln\left(1-e^{-t_0}\right)}\right).$$

Proof: To obtain the power estimate we differentiate w.r.t. θ and equate it to zero after taking logarithm of Lemma 2.

In preceding theorem the MLE of $R\left(t\right)$ is proved.

Theorem 12 *The MLE of $R(t)$ is given by*

$$\widetilde{R}_I(t)=\left(1-\left(1-e^{-t}\right)^{\left(\frac{-r}{n\ln\left(1-e^{-t_0}\right)}\right)}\right).$$

Proof: Using Eqn. (18), one-to-one and invariant property of MLE and hence the theorem is derived.

Corollary 7 *At a particular point x, the MLE of $f\left(x;\theta\right)$ is given as*

$$\widetilde{f}_I\left(x;\theta\right)=\left\{\frac{re^{-x}}{n\ln(1-e^{-t_0})}\left(1-e^{-x}\right)^{\left(\frac{r}{[-n\ln\left(1-e^{-t_0}\right)]}\right)-1}\right..$$

The MLE of P is proved in the following theorem.

Theorem 13 *The MLE of P is given by*

$$\widetilde{P}_I = \frac{s}{m\ln\left(1-e^{-t_{00}}\right)} \int_0^\infty \left(1 - z^{\left(\frac{r}{[-n\ln\left(1-e^{-t_0}\right)]}\right)}\right) z^{\left(\frac{r}{[-n\ln\left(1-e^{-t_0}\right)]}\right)-1}.$$

Proof: Here,

$$\begin{aligned}
\widetilde{P}_I &= \int_{y=0}^\infty \int_{x=y}^\infty \widetilde{f}_{1I}(x;\theta_1)\widetilde{f}_{2I}(y;\theta_2)\,dx\,dy \\
&= \int_{y=0}^\infty \widetilde{R}_{1I}(y;\theta_1)\left\{-\frac{d}{dy}\widetilde{R}_{2I}(y;\theta_2)\right\} dy \\
&= \frac{s}{m\ln(1-e^{-t_{00}})}\int_{y=0}^\infty \left(1-\left(1-e^{-y}\right)^{\overline{\frac{r}{[-n\ln\left(1-e^{-t_0}\right)]}}}\right) e^{-y}\left(1-e^{-y}\right)^{\overline{\frac{s}{[-m\ln\left(1-e^{-t_{00}}\right)]}}-1} dy
\end{aligned}$$

Putting $1-e^{-y}=z$, the theorem follows.

Corollary 8 *When X and Y have same family of distribution, we have*

$$\widetilde{P}_I = \left\{ \frac{\overline{\frac{s}{mln\left(1-e^{-t_{00}}\right)}}}{\overline{\frac{r}{nln\left(1-e^{-t_0}\right)}}+\overline{\frac{s}{mln\left(1-e^{-t_{00}}\right)}}} \right. .$$

5. Hypotheses Testing

In the following section, we consider an essential hypothesis in life testing experiments, $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. Based on Eqn. (6) the likelihood function observing θ, Sinha (1968) is given by

$$L(\theta|\mathbf{x}) = \frac{n!}{(n-r)!}\theta^r exp\left(-\theta S_r\right).\ \forall \mathbf{x} = \left(x_{(1)}, x_{(2)}, \ldots.., x_{(r)}\right)$$

Under H_0,

$$\sup_{\Theta_0} L(\theta|\mathbf{x}) = \frac{n!}{(n-r)!}\theta_0^r exp\left(-\theta_0 S_r\right)$$

and when $\theta = \frac{r}{S_r}$ then,

$$\sup_{\Theta} L(\theta|\mathbf{x}) = \frac{n!}{(n-r)!}\left(\frac{r}{S_r}\right)^r exp\left(-r\right).$$

The likelihood ratio is delineate as

$$\lambda(\mathbf{x}) = \frac{\sup_{\Theta_0} L(\theta|\mathbf{x})}{\sup_{\Theta} L(\theta|\mathbf{x})} = \left(\frac{\theta_0 S_r}{r}\right)^r exp\left(-\theta_0 S_r + r\right). \quad (19)$$

It is seemed from term Ist and 2nd on the right hand side of Eqn. (19) are monotonically increasing and decreasing in S_r respectively. $\chi^2_{2r}(.)$, is indicated as the test statistic chi-square having degrees of freedom $2r$ and furthermore $2\theta_0 S_r \sim \chi^2_{2r}$, the critical region is delineate as

$$\{0 < S_r < k_0\} \bigcup \left\{k_0^{'} < S_r < \infty\right\},$$

where $k_0=\frac{1}{2\theta_0}\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)$ and $k_0^{'}=\frac{1}{2\theta_0}\chi^2_{2r}\left(\frac{\alpha}{2}\right)$ are obtained such that

$$P\left[\chi^2_{2r}<2\theta_0k_0 \text{ or } 2\theta_0k_0^{'}<\chi^2_{2r}\right]=\alpha.$$

Likewise, it can be shown that under Type I censoring Bartholomew (1963) sampling scheme, the uniform most powerful critical region for testing $H_0:\theta=\theta_0$ vs $H_1:\theta\neq\theta_0$ is given

$$(r<k_1 \text{ or } r>\acute{k_1}), r\sim Poisson\left(-n\theta\ln\left(1-e^{-t_0}\right)\right).$$

Further, let we test the null hypothesis $H_0:\theta\leq\theta_0$ against $H_1:\theta>\theta_0$.
For $\theta_1<\theta_2$, using Eqn. (6) we have,

$$\lambda(\mathbf{x})=\left(\frac{\theta_2}{\theta_1}\right)^r exp\left(-\left(\theta_2-\theta_1\right)S_r\right). \tag{20}$$

It has been observed from Eqn. (20) that the family of sampled pdf has monotonic likelihood in (S_r). Hence, the uniformly most powerful critical region for testing H_0 against H_1 is given by Lehmann (1959)

$$\lambda\left(x_{(1)},x_{(2)},\ldots,x_{(r)}\right)=\begin{cases}1, & S_r\leq\acute{k_0}^{'}\\ 0 & \text{otherwise.}\end{cases}$$

where $\acute{k_0}^{'}=\left(\frac{1}{2\theta_0}\chi^2_{(2r)}\left(1-\acute{\alpha}\right)\right)$ is achieved such that $P\left[\chi^2_{(2r)}<\theta_0 2\acute{k_0}^{'}\right]=\acute{\alpha}$.

Similarly, it is shown by using Eqn. (13), the uniformly most powerful critical region for testing $H_0:\theta\leq\theta_0$ against $H_1:\theta>\theta_0$ under Type I censoring scheme as,

$$\lambda(r)=\begin{cases}1, & r\leq\acute{k_1}^{'}\\ 0 & \text{otherwise}\end{cases}$$

where $\acute{k_1}^{'}$ is obtained such that $P\left[r<\acute{k_1}^{'}\right]=\beta$.

In order to test the null hypothesis $P_0:\theta=\theta_0$ against $P_1:\theta\neq\theta_0$ under Type II censoring. It shows that $H_0:\theta_1=\delta\theta_2$ against $H_1:\theta_1\neq\delta\theta_2$. Under H_0, $\hat{\theta}_{1II}=\frac{\delta(r+s)}{\delta S_r+T_s}$ and $\hat{\theta}_{2II}=\frac{r+s}{\delta S_r+T_s}$. For generic constant η, the likelihood of sampled observation $\mathbf{x}=\left(x_{(1)},x_{(2)},\ldots,x_{(r)}\right)$ and $\mathbf{y}=\left(y_{(1)},y_{(2)},\ldots,y_{(r)}\right)$ is

$$L\left(\theta_1,\theta_2|\mathbf{xy}\right)=K\theta_1^r\theta_2^r exp\left(-\left(\theta_1S_r+\theta_2T_s\right)\right).$$

Under H_0,

$$\sup_{\Theta_0}L\left(\theta_1,\theta_2|\mathbf{xy}\right)=\frac{Kexp\left(-\left(r+s\right)\right)}{\left(S_r+\frac{T_s}{\delta}\right)^{r+s}} \tag{21}$$

Also for whole parametric space Θ=$\{(\theta_1,\theta_2)/\theta_1,\theta_2>0\}$,

$$\sup_{\Theta}L\left(\theta_1,\theta_2|\mathbf{xy}\right)=\frac{Kexp\left(-\left(r+s\right)\right)}{S_r^rT_s^s}. \tag{22}$$

Using Eqns. (21) and (22), the likelihood ratio criterion is

$$\lambda^*\left(\theta_1,\theta_2|\mathbf{xy}\right)=K\left(\frac{S_r}{\delta T_s}\right)^r\frac{1}{\left(\delta\frac{S_r}{T_s}+1\right)^{r+s}}.$$

Furthermore,

$$\frac{S_r}{T_s} \sim \frac{r\theta_2}{s\theta_1} F_{2r,2s}\left(.\right),$$

where $F_{a,b}\left(.\right)$, is the statistic for F having degree of freedom $\left(a, b\right)$.

The critical region is delineate as $\left(\left\{\frac{S_r}{T_r} < k_2\right\} \bigcup \left\{\frac{S_r}{T_r} > k_2^{'}\right\}\right)$, where k_2 and $k_2^{'}$ are achieve so that

$$P\left(\frac{\delta s S_r}{rT_s} < F_{2r,2s} \bigcup \frac{\delta s S_r}{rT_s} > F_{2r,2s}\right) = \acute{\alpha},$$

where, $k_2^{''} = \frac{r}{\delta s} F_{2r,2s}\left(1 - \frac{\acute{\alpha}}{2}\right)$ and $k_2^{'''} = \frac{r}{\delta s} F_{2r,2s}\left(\frac{\acute{\alpha}}{2}\right)$.

Now, considering the case when, $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$. Based on Eqn. (6) the likelihood function observing θ is delineate as

$$L\left(\theta|\mathbf{x}\right) = \frac{n!}{(n-r)!}\theta^r exp\left(-\theta S_r\right).\ \forall \mathbf{x} = \left(x_{(1)}, x_{(2)}, ..., x_{(r)}\right)$$

Under H_0,

$$L\left(\theta_0|\mathbf{x}\right) = \frac{n!}{(n-r)!}\theta_0^r exp\left(-\theta_0 S_r\right)$$

and under H_1, then,

$$L\left(\theta_1|\mathbf{x}\right) = \frac{n!}{(n-r)!}\theta_1^r exp\left(-\theta_1 S_r\right).$$

Suppose m be any positive number. Let C be the set of points where

$$\frac{L\left(\theta_0|\mathbf{x}\right)}{L\left(\theta_1|\mathbf{x}\right)}$$

which provide,

$$S_r \leq \frac{log(m)}{(\theta_1 - \theta_0)} + \frac{rlog\left(\frac{\theta_1}{\theta_0}\right)}{(\theta_1 - \theta_0)}.$$

When $\theta_1 > \theta_0$,

$$S_r \geq \frac{1}{(\theta_1 - \theta_0)}\left(log(m) + rlog\left(\frac{\theta_1}{\theta_0}\right)\right) = a, \tag{23}$$

and when $\theta_1 < \theta_0$,

$$S_r \leq \frac{1}{(\theta_1 - \theta_0)}\left(log(m) + rlog\left(\frac{\theta_1}{\theta_0}\right)\right) = b. \tag{24}$$

Considering case when $\theta_1 > \theta_0$, form (23), the set C = $\{(x_1, x_2, ..., x_n) : S_r \geq a\}$ is a best critical region (BCR)for testing H_0 against H_1. The constant a can be determined so that the size of BCR is α. Using the fact that $2\theta S_r \sim \chi^2_{2r}$,

$\therefore$

$$\{S_r \geq c\} = \left\{\chi^2_{2r} \geq 2\theta c\right\}. \tag{25}$$

Determine the size of the test α and power of test $(1-\beta)$. Let F be the CDF of χ^2_{2r} then using (25)

$$\alpha = P\{X\epsilon c|H_0\} = 1 - F(2\theta_0 a), \tag{26}$$
$$\text{also,} \quad 1-\beta = P\{X\epsilon c|H_1\} = 1 - F(2\theta_1 a). \tag{27}$$

Knowing α, $2\theta_0 a$ is determined and hence a form (26). This value of a then determine power of the test from (27). Now, case when $\theta_1 < \theta_0$, form (24), the set C = $\{(x_1, x_2, ..., x_n) : S_r \leq b\}$ is a best critical region (BCR)for testing H_0 against H_1. The constant b can be determined so that the size of BCR is α. Using the fact that $2\theta S_r \sim \chi^2_{2r}$,
$\therefore$

$$\{S_r \leq c\} = \left\{\chi^2_{2r} \leq 2\theta c\right\}.$$

Determine the size of the test α and power of test $(1-\beta)$. Let F be the CDF of χ^2_{2r} then using (25)

$$\alpha = P\{X\epsilon c|H_0\} = 1 - F(2\theta_0 b), \tag{28}$$
$$\text{also,} \quad 1-\beta = P\{X\epsilon c|H_1\} = 1 - F(2\theta_1 b). \tag{29}$$

Knowing α, $2\theta_0 b$ is determined and hence b form (28). This value of b then determine power of the test from (29). Using χ^2_n variate table.

6. Simulation Result

In this simulation section, Figures 1, 2, 3, demonstrate the shape of the proposed density function, cdf and its hazard function for different parameter values. Authors have preformed simulation to investigate and make a comparative analysis for UMVUE & MLE of power, reliability $R(t)$ and stress-strength reliability $P(X > Y)$ estimates under Type II & I censoring schemes. Firstly, considering Type II censoring scheme, taking $n = 50$, $\theta = 1$, using inverse transformation method and Eqn. (2) we generate 10000 random samples to obtain an average estimate, average bias and mean square error(MSEs) for power, reliability $R(t)$ and stress-strength reliability $P(X > Y)$ is construct on the UMVUE and MLE under types II and I censoring schemes. Taking $q = 0.5$, $r = 35$, power estimate for UMVUE and MLE under Type II censoring scheme, true value $= 1.0000$, UMVUE $(0.9988, -0.0012, 0.0073)$ and MLE $(1.0097, 0.0097, 0.0075)$. Note that in parentheses first term is average estimate, second term is average bias and third term is MSE. Again, for different values of $r = 10, 20, 35$, $t = 1, 1.5, 2, 2.5$, we obtain the reliability estimate for UMVUE and MLE under Type II censoring scheme and its result is shown in Table 1.

Now, by generating the 10000 random samples, form each population of X and Y with $(n = m = 50)$ sizes, by considering the inverse transformation technique and Eqn. (2) for $(\theta_1, \theta_2) = (1, 1), (1, 1.5), (1, 2), (1.5, 2), (2, 1.5)$. We compute the stress-strength reliability for UMVUE and MLE under Type II censoring scheme for different values of $r = s = 10, 20, 35$, see Table 2. In the similar manner, we have preformed the simulation of power, reliability and stress- strength reliability estimates for UMVUE & MLE under Type I censoring scheme. Power estimate under Type I censoring scheme, we compute as the true value $= 1.0000$, UMVUE $(1.0048, 0.0048, 0.0038)$ and MLE $(1.0003, 0.0003, 0.0038)$ for $n = 50$, $\theta = 1$, $to = 0.84$, $q = 0.5$. Again, computing the reliability function R(t) for UMVUE & MLE under Type I censoring scheme, we considered the parameter as $n = 50$, $\theta = 1.5$, $t = 1, 0.5, 2, 2.5$, $to = 0.65, 0.80, 1$, where to is the termination time which is fixed. If to is the instant termination and also fixed then we get the r value. Also replace the failure by operating a new value, see Table 3.

In order to investigate the strength reliability for UMVUE & MLE under Type I censoring scheme, we introduce two cases as $(n > m)$ i.e., $n = 50, m = 40$, $(n < m)$ i.e., $n = 40, m = 50$. The 10000 random sample are generated from Eqn. (2) & using inverse transformation method with $\theta_1 = \theta_2 = (1, 1), (1, 1.5), (1, 2), (1.5, 2), (2, 1.5)$. The value of r (before time to the number of failures in X) is obtained by fixing the termination time to and replacing the failure by operating a new one. Similarly the values of s is evaluated by fixing the termination time at too in Y. Taking $to = too = 0.65, 0.80, 1$, we find the average estimate, average bias and MSE for case I when $(n > m)$ and case II $(n < m)$ as shown in Tables 4 and 5.

Table 1 Performance of the R(t) estimates under Type II when $\theta = 1$

	r	10		20		35	
t	$R_{II}(t)$	$\hat{R}_{II}(t)$	$\widetilde{R}_{II}(t)$	$\hat{R}_{II}(t)$	$\widetilde{R}_{II}(t)$	$\hat{R}_{II}(t)$	$\widetilde{R}_{II}(t)$
1	0.3679	0.3671 -8e-04 **0.0095**	0.3895 0.0216 **0.0101**	0.3679 0.0000 **0.0045**	0.3791 0.0112 **0.0047**	0.3675 -4e-04 **0.0025**	0.3739 0.006 **0.0025**
1.5	0.2231	0.2229 -2e-04 **0.0044**	0.2409 0.0178 **0.0052**	0.2238 6e-04 **0.0021**	0.2326 0.0094 **0.0023**	0.2227 -4e-04 **0.0011**	0.2277 0.0045 **0.0012**
2	0.1353	0.1345 -8e-04 **0.0018**	0.147 0.0117 **0.0022**	0.1351 -2e-04 **0.0012**	0.1411 0.0058 **0.0015**	0.1351 -3e-04 **0.0010**	0.1385 0.0031 **0.0009**
2.5	0.0821	0.0828 7e-04 **0.0015**	0.091 0.0089 **0.0019**	0.0819 -2e-04 **0.0011**	0.0858 0.0037 **0.0014**	0.0821 0.000 **0.0007**	0.0843 0.0022 **0.0005**

Note: Second column represents the true value while third, fifth, seventh column corresponds the results for UMVUE and forth, sixth, eight column corresponds the results for MLE. In each cell, the average length, average bias are provided and the corresponding MSE (in bold) respectively.

Table 2 Performance of the $P(X > Y)$ estimates under Type II

r=s		10		20		35	
$[\theta_1, \theta_2]$	P_{II}	$\hat{P}_{II}$	$\widetilde{P}_{II}$	$\hat{P}_{II}$	$\widetilde{P}_{II}$	$\hat{P}_{II}$	$\widetilde{P}_{II}$
[1, 1]	0.5	0.4987 -0.0013 **0.0129**	0.4996 -4e-04 **0.0121**	0.4997 -3e-04 **0.0064**	0.4997 -3e-04 **0.0061**	0.5003 3e-04 **0.0036**	0.5003 3e-04 **0.0035**
[1,1.5]	0.6	0.6487 0.0487 **0.0081**	0.6542 0.0542 **0.0107**	0.6173 0.0173 **0.0049**	0.6146 0.0146 **0.0046**	0.5818 -0.0182 **0.0032**	0.5807 -0.0193 **0.0031**
[1,2]	0.6667	0.7000 0.0334 **0.0032**	0.7294 0.0627 **0.0091**	0.6791 0.0125 **0.0036**	0.6754 0.0088 **0.0035**	0.6310 -0.0357 **0.0036**	0.6292 -0.0374 **0.0037**
[1.5,2]	0.5714	0.5920 0.0205 **0.0059**	0.5889 0.0174 **0.0056**	0.5683 -0.0031 **0.0032**	0.5666 -0.0048 **0.0031**	0.5506 -0.0208 **0.0024**	0.5499 -0.0215 **0.0024**
[2,1.5]	0.4286	0.4070 -0.0215 **0.0063**	0.4117 -0.0169 **0.0056**	0.4325 0.0040 **0.0032**	0.4342 0.0057 **0.0030**	0.4494 0.0208 **0.0025**	0.4501 0.0215 **0.0025**

Note: Second column represents the true value while third, fifth, seventh column corresponds the results for UMVUE and forth, sixth, eight column corresponds the results for MLE. In each cell, the average length, average bias are provided and the corresponding MSE (in bold) respectively.

Table 3 Performance of the R(t) estimates under Type I when $\theta = 1.5$

to		0.65		0.80		1	
t	$R_I(t)$	$\hat{R}_I(t)$	$\widetilde{R}_I(t)$	$\hat{R}_I(t)$	$\widetilde{R}_I(t)$	$\hat{R}_I(t)$	$\widetilde{R}_I(t)$
1	0.4974	0.1857 -0.3117 **0.0983**	0.1847 -0.3128 **0.0990**	0.2709 -0.2265 **0.0529**	0.2692 -0.2283 **0.0537**	0.3971 -0.1003 **0.0119**	0.3940 -0.1034 **0.0125**
1.5	0.3153	0.1072 -0.2081 **0.0437**	0.1068 -0.2084 **0.0439**	0.1586 -0.1566 **0.0251**	0.1580 -0.1572 **0.0253**	0.2424 -0.0728 **0.0062**	0.2413 -0.0740 **0.0063**
2	0.1960	0.0629 -0.1331 **0.0179**	0.0628 -0.1332 **0.0179**	0.0950 -0.1010 **0.0104**	0.0948 -0.1012 **0.0105**	0.1472 -0.0488 **0.0027**	0.1468 -0.0492 **0.0028**
2.5	0.1206	0.0376 -0.0829 **0.0069**	0.0376 -0.0830 **0.0069**	0.0570 -0.0636 **0.0041**	0.0570 -0.0636 **0.0041**	0.0897 -0.0308 **0.0011**	0.0896 -0.0310 **0.0011**

Note: Second column represents the true value while third, fifth, seventh column corresponds the results for UMVUE and forth, sixth, eight column corresponds the results for MLE. In each cell, the average length, average bias are provided and the corresponding MSE (in bold) respectively.

Table 4 Performance of the $P(X > Y)$ estimates under Type I when $(n > m)$

$to = too$		0.65		0.80		1	
$[\theta_1, \theta_2]$	P_I	$\hat{P}_I$	$\widetilde{P}_I$	$\hat{P}_I$	$\widetilde{P}_I$	$\hat{P}_I$	$\widetilde{P}_I$
[1,1]	0.5	0.4982 -0.0018 **0.0040**	0.4994 -6e-04 **0.0040**	0.4995 -5e-04 **0.0031**	0.5005 5e-04 **0.0031**	0.5000 0.0000 **0.0024**	0.5009 9e-04 **0.0024**
[1,1.5]	0.6	0.6114 0.0114 **0.0048**	0.6128 0.0128 **0.0048**	0.5993 -7e-04 **0.0036**	0.6004 4e-04 **0.0035**	0.5862 -0.0138 **0.0029**	0.5872 -0.0128 **0.0029**
[1,2]	0.6667	0.7002 0.0336 **0.0062**	0.7016 0.0349 **0.0063**	0.6750 0.0083 **0.0041**	0.6762 0.0095 **0.0041**	0.6513 -0.0154 **0.0033**	0.6522 -0.0144 **0.0032**
[1.5,2]	0.5714	0.5978 0.0263 **0.0086**	0.5999 0.0285 **0.0087**	0.5835 0.0120 **0.0056**	0.5851 0.0137 **0.0056**	0.5689 -0.0025 **0.0039**	0.5702 -0.0012 **0.0039**
[2,1.5]	0.4286	0.4017 -0.0269 **0.0082**	0.4039 -0.0247 **0.0081**	0.4156 -0.0129 **0.0054**	0.4173 -0.0112 **0.0054**	0.4307 0.0021 **0.0037**	0.4320 0.0034 **0.0037**

Note: Second column represents the true value while third, fifth, seventh column corresponds the results for UMVUE and forth, sixth, eight column corresponds the results for MLE. In each cell, the average length, average bias are provided and the corresponding MSE (in bold) respectively.

Table 5 Performance of the $P(X > Y)$ estimates under Type I when $(n < m)$

$to = too$		0.65		0.80		1	
$[\theta_1, \theta_2]$	P_I	$\hat{P}_I$	$\widetilde{P}_I$	$\hat{P}_I$	$\widetilde{P}_I$	$\hat{P}_I$	$\widetilde{P}_I$
[1,1]	0.5	0.5010 0.0010 **0.0039**	0.4998 -2e-04 **0.0039**	0.5006 6e-04 **0.0031**	0.4996 -4e-04 **0.0031**	0.5009 9e-04 **0.0024**	0.5001 1e-04 **0.0024**
[1,1.5]	0.6	0.6114 0.0114 **0.0045**	0.6101 0.0101 **0.0045**	0.5982 -0.0018 **0.0035**	0.5971 -0.0029 **0.0035**	0.5863 -0.0137 **0.0028**	0.5854 -0.0146 **0.0028**
[1,2]	0.6667	0.7028 0.0361 **0.0059**	0.7014 0.0347 **0.0058**	0.6762 0.0095 **0.0038**	0.6750 0.0084 **0.0038**	0.6510 -0.0157 **0.0030**	0.6500 -0.0166 **0.0031**
[1.5,2]	0.5714	0.5978 0.0264 **0.0082**	0.5956 0.0242 **0.0081**	0.5824 0.0109 **0.0054**	0.5807 0.0093 **0.0054**	0.5699 -0.0015 **0.0036**	0.5686 -0.0028 **0.0037**
[2,1.5]	0.4286	0.4022 -0.0264 **0.0086**	0.4000 -0.0286 **0.0087**	0.4167 -0.0119 **0.0057**	0.4150 -0.0136 **0.0057**	0.4318 0.0032 **0.0038**	0.4304 0.0019 **0.0038**

Note: Second column represents the true value while third, fifth, seventh column corresponds the results for UMVUE and forth, sixth, eight column corresponds the results for MLE. In each cell, the average length, average bias are provided and the corresponding MSE (in bold) respectively.

6.1. Validation of hypothesis testing

In this section we build out the validation of the theory which is derived in Section 5. Suppose first we want to test the hypothesis under Type II censoring scheme as $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$, sample are generated from Eqn. (1) taking $n = 40$, $r = 30$, $\theta_1 = 1.5$ as follows

Table 6 Generated Sample 1

0.0478, 0.0529, 0.1260, 0.1642, 0.1709, 0.1806, 0.3640, 0.3785, 0.3931, 0.4144, 0.4864, 0.5448, 0.6121, 0.6481, 0.6716, 0.6974, 0.7229, 0.8168, 0.9850, 1.0325, 1.0645, 1.1104, 1.1837, 1.1948, 1.2401, 1.2920, 1.4009, 1.4258, 1.5010, 1.5619, 1.6153, 1.9643, 2.0013, 2.1749, 2.2443, 2.3357, 2.6386, 2.7677, 2.7728, 5.3331.

Using Table 6, it has been seen that after finding $k_0 = 20.24$ and $\acute{k_0} = 41.65$ from the tabulated value of chi-square with significance level at 5%, we accept H_0 as $S_{35} = 38.5338$ lie between k_0 and $\acute{k_0}$. Further from Table 6, considered the hypothesis test as $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$, we accept the null hypothesis H_0 as $S_{35} = 38.5338$ lie between $\acute{\acute{k_0}}$ and $\acute{k_0}$ whereas the value of $\acute{\acute{k_0}} = 21.595$ using tabulated value of chi-square with significance level at 5%. Now, considered the hypothesis test as $P_0 : \theta = \theta_0$ against $P_1 : \theta \neq \theta_0$ under Type II censoring scheme, sample are generated from equation (1) taking $m = 50$, $s = 35$, $\theta_2 = 2$ as follows

Table 7 Generated Sample 2

0.0979, 0.3288, 0.4216, 0.4366, 0.4587, 0.6391, 0.6541, 0.6673, 0.7028, 0.7318, 0.9147, 0.9235,
0.9796, 1.0132, 1.0385, 1.1037, 1.3259, 1.3926, 1.4049, 1.4109, 1.4627, 1.5054, 1.5434, 1.6129,
1.7311, 1.7328, 1.8159, 1.8532, 1.9057, 1.9151, 1.9665, 1.9877, 2.0265, 2.0302, 2.1835, 2.2211,
2.2614, 2.2678, 2.3347, 2.5671, 2.6678, 2.6773, 3.0340, 3.1324,3.3520, 3.4555, 3.6034, 3.6691,
4.3513, 4.8568.

From Tables 6 and 7, the ratio $S_{30}/T_{35} = 0.5028$ is lie between $\acute{k}_2 = 0.2314$ and $\acute{\acute{k}}_2 = 0.6183$, we accept the null hypothesis H_0 using F-table at 5% level of significance as $T_s = 76.6353$.

6.2. Real data

In this section, authors has analyzed two real data sets. Firstly, considering the data set 1 which is used by Nichols and Padgett (2006) formally after that many authors introduced this data set in their study. The data consisting of 100 observations on breaking stress of carbon fibers (in Gba) as follows

Table 8 Real Data Set I

3.70, 2.74, 2.73, 2.50, 3.60, 3.11, 3.27, 2.87, 1.47, 3.11, 4.42, 2.41, 3.19, 3.22, 1.69, 3.28, 3.09, 1.87,
3.15, 4.90, 3.75, 2.43, 2.95, 2.97, 3.39, 2.96, 2.53, 2.67, 2.93, 3.22, 3.39, 2.81, 4.20, 3.33, 2.55, 3.31,
3.31, 2.85, 2.56, 3.56, 3.15, 2.35, 2.55, 2.59, 2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36,
0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19, 1.57, 0.81, 5.56, 1.73, 1.59, 2.00, 1.22, 1.12, 1.71, 2.17, 1.17,
5.08, 2.48, 1.18, 3.51, 2.17, 1.69, 1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.70, 2.03, 1.80,
1.57, 1.08, 2.03, 1.61, 2.12, 1.89, 2.88, 2.82, 2.05, 3.65.

Table 9 Real Data Set II

0.01, 0.01, 0.02, 0.02, 0.02, 0.03, 0.03, 0.04, 0.05, 0.06, 0.07, 0.07, 0.08, 0.09, 0.09, 0.10, 0.10, 0.11,
0.11, 0.12, 0.13, 0.18, 0.19, 0.20, 0.23, 0.24, 0.24, 0.29, 0.34, 0.35, 0.36, 0.38, 0.40, 0.42, 0.43, 0.52,
0.54, 0.56, 0.60, 0.60, 0.63, 0.65, 0.67, 0.68, 0.72, 0.72, 0.72, 0.73, 0.79, 0.79, 0.80, 0.80, 0.83, 0.85,
0.90, 0.92, 0.95, 0.99, 1.00, 1.01, 1.02, 1.03, 1.05, 1.10, 1.10, 1.11, 1.15, 1.18, 1.20, 1.29, 1.31, 1.33,
1.34, 1.40, 1.43, 1.45, 1.50, 1.51, 1.52, 1.53, 1.54, 1.54, 1.55, 1.58, 1.60, 1.63, 1.64, 1.80, 1.80, 1.81,
2.02, 2.05, 2.14, 2.17, 2.33, 3.03, 3.03, 3.34, 4.20, 4.69, 7.89.

Ramos and Louzada (2016) has considered this data set 1 for the EIW distribution. In this context, we have considered data set 1 to check whether the one parameter generalized exponential distribution fits or not by utilizing the Kolmogorov-Smirnov (K-S)test which gives us, the MLE of unknown parameter $(\widetilde{\theta})$, Log-Likelihood, Kolmogorov-Smirnov (K-S)distances and p-value are presented as (7.5976, -146.1937, 0.09563, 0.3198). It shows that one parameter generalized exponential distribution fits the data set 1. Furthermore, the plot of the empirical cumulative distribution and the fitted one parameter generalized exponential distribution for data set 1 as shown in Figure 4.

For Real Data analysis given in Table 8, power estimate and reliability estimate under Type II censoring scheme, true estimate (1.0000), UMVUE (0.9828, -0.0172, 0.0003), MLE (0.9830, -0.0170, 0.0003) and true estimate (0.0821), UMVUE (0.0810, -0.0692, 0.0048), MLE (0.0801, -0.0689, 0.0047). Similarly, power estimate and reliability estimate under Type I censoring scheme, true estimate (1.0000), UMVUE (1.0264, 0.0264, 0.0007), MLE (1.0203, 0.0203, 0.0004) and true estimate (0.1206), UMVUE (0.1035, -0.0170, 0.0003), MLE (0.1033, -0.0173, 0.0003). Similarly, for data set 2 which is given in Table 9, is describe the stress rupture life of kevlar 49 per epoxy stands failure at 90 percent stress level used by Andrews and Herzberg (1985) and Barlow et al. (1984). GED has fit well as the MLE of unknown parameter $(\widetilde{\theta})$, Log-Likelihood, Kolmogorov-Smirnov (K-S)distances and p-value are presented as (0.9289, -103.2288, 0.09152, 0.2390). Graphical representation of data set 2 is shown in Figure 5. For Real Data analysis given in Table 9, power estimate and reliability estimate under Type II censoring scheme, true estimate (1.0000), UMVUE (1.0013, 0.0013, 0.0003), MLE (1.0029, 0.0029, 0.0002) and true estimate (0.0821), UMVUE (0.0832, 0.0011, 0.0003), MLE (0.0855, 0.0034, 0.0002). Similarly, power estimate and reliability estimate under Type I censoring scheme, true estimate (1.0000), UMVUE (0.9177, -0.0823, 0.0068), MLE (0.9155, -0.0845, 0.0071) and true estimate (0.1206), UMVUE (0.1143, -0.0062, 0.1e-03), MLE (0.1141, -0.0063, 1e-04).

7. Summary of Results

The analysis of various estimators has been considered in this study which is based on the MSEs. The Table 1 depicts that for lesser values of t, the result of UMVUE is preforming better than the MLE of R(t) under Type II censoring scheme. This is also analyzed that the bigger values of t MLE is giving better results than UMVUE of R(t) under Type II censoring scheme, while this also analyzed that in case of larger value of r both estimators have same performance. Table 1 shows that with the increase in the value of r the value of MSEs with respect to both estimators is decreasing. As describe in Table 2 the performance of UMVUE is not better as compare to the results of MLE for different values of (r, s). As far as the results of Type I censoring scheme are concerned, a very important sampling scheme is introduced in this censoring scheme known as Bartholomew (1963). According to this Type I censoring scheme as depicted from Table 3 for lesser values of t and different values of to (termination time), UMVUE gives better results as that of MLE. Similarly, for lesser values of to i.e. termination time and different values of t. UMVUE and MLE are equally efficient. It is also shown from Table 3 that when the to is equal to one and for different values of t, the bias and MSEs showing far better result than other values of to. Hence proves the significance of Bartholomew (1963) scheme. In Table 4 it is inferred that at the larger values of too and to, the MSEs shows better result for Stress-strength reliability for UMVUE and MLE, when n= 50 and the value of m is small. Also in Table 5 same results have been discussed that at the larger values of too and to, the MSEs shows better result for Stress-strength reliability for UMVUE and MLE, when value of n is small and m = 50.

8. Conclusion

The contribution of this paper deals with the the estimation and testing procedures for two reliability functions viz., R(t) and $P = P(X > Y)$ of one parameter of Generalized Exponential Distribution(GED)under Type II and Type I censoring scheme. As far as point estimation is concerned, the two reliability measures namely as R(t) and $P = P(X > Y)$ are derived for UMVUE and MLE under Type II and I censoring scheme.The exact confidence interval are obtained under Type II censoring scheme.Developing different hypothesis procedures under both types of censoring scheme. In this study an estimators are derived from UMVUE and MLE under Type II and Type I censoring scheme and the comparison has been made among these estimators using Manto Carlo simulation. The validity of various hypothesis procedures has been done and real data study is analyzed.

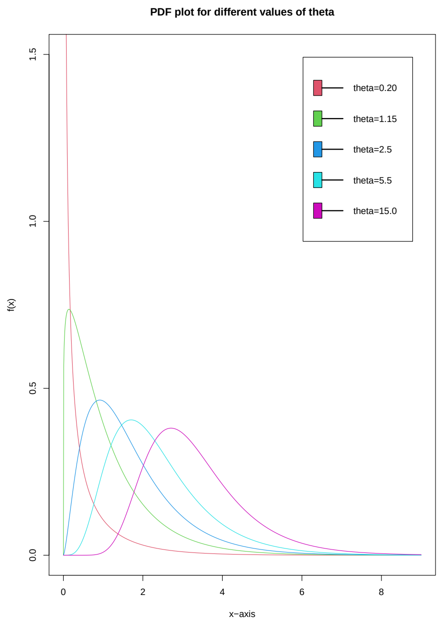

Figure 1 PDF plot for different values of theta

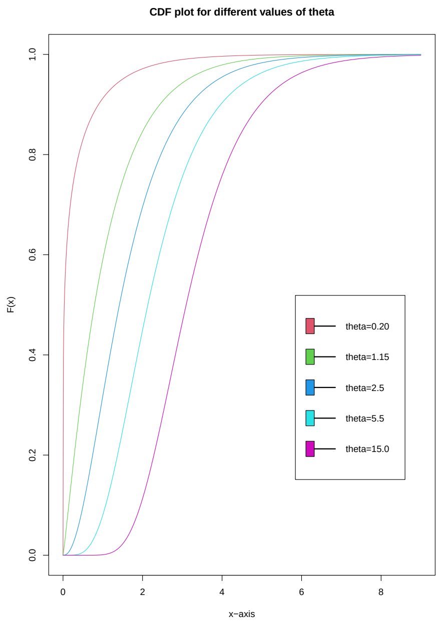

Figure 2 CDF plot for different values of theta

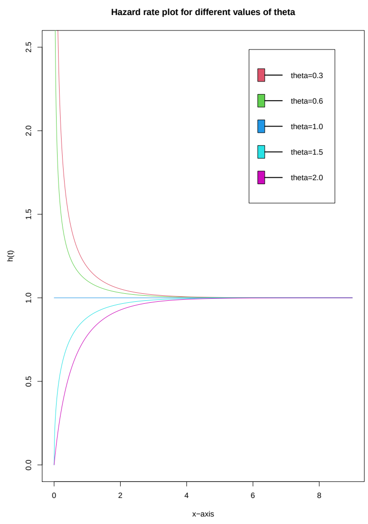

Figure 3 Hazard rate plot for different values of theta.

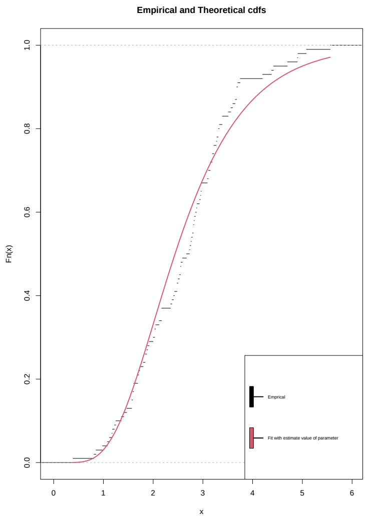

Figure 4 Maximum distance between ECDF and fitted CDF of One parameter GED Plots based on data set 1

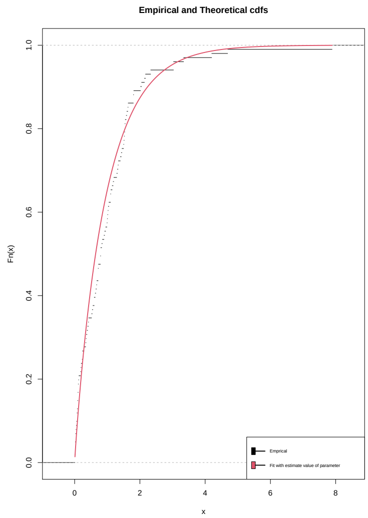

Figure 5 Maximum distance between ECDF and fitted CDF of One parameter GED Plots based on data set 2

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