Thailand Statistician
April 2023; 21(2): 383-396
http://statassoc.or.th
Contributed paper

Availability and Performance Analysis of Primary Treatment Unit of Sewage Plant

Monika Saini, Drishty Goyal and Ashish Kumar*
Department of Mathematics & Statistics, Manipal University Jaipur, Jaipur, India
*Corresponding author; e-mail: ashishbrak2020@gmail.com

Received: 5 May 2020
Revised: 14 June 2021
Accepted: 19 June 2021

Abstract

The main objective of present study is to analyze availability and performance of primary treatment unit of sewage plant (STP) with component wise redundancy. For this purpose, a novel mathematical model for availability and performance analysis of primary unit of sewage treatment plant has been proposed introducing the concept of redundancy and constant failure rates. Primary unit of sewage treatment system is a serial process system in which five subsystems comprises in series structure. The system has been analyzed using Markov birth-death process, Chapman-Kolmogorov differential equation and supplementary variable technique. All the repair rates considered as arbitrarily distributed. The needful data has been taken with the help of the maintenance personnel of the plant. Numerical and graphical results of the study have been appended for a particular case and the results also shared with the plant personnel.

Keywords: Availability, exponential distribution, performance analysis, supplementary variable technique, Markov birth-death process.

1. Introduction

We as human being depends on natural resources for our basic needs and survivals. These resources included are water, food, air, oil, shelters, minerals and such other things. Increment in population affects the number of natural resources because these natural resources are limited in nature. With these limited resources explosion raise in demands cannot fulfil sufficiently. Due to industrialization, tourism, domestic waste harmful gases and waste include in air, rivers and in natural resources which makes our environment polluted. India is in a drastic situation in the context of insufficient water resources. This situation can handle by reuses of treated sewage water, which can reduce the water problems and fulfil the demand of fresh water. The sewage treatment plant can help in preventing the environment from waste produced by the human beings. Sewage treatment plant helps in reducing water problem and energy. We can reuse treated water and compost waste as fertilizer and for generating energy. In such type of industry and process some redundant systems use for increase in productivity and maintenance. By redundancy we mean duplication of critical components used as backup so the system can perform without any interruption.

The literature reflects that reliability and availability has been analyzed by several methods and approaches. Adhikary et al. (2012) analyzed RAM for coal fired thermal power plants and proposed prevented maintenance scheduling based on reliability. Aggarwal et al. (2014) proposed a method to analyze the availability for butter oil production system on the bases of time interval and optimized its performance by the help of genetic algorithm. For this purpose, Markov birth-death process is used to develop Chapman–Kolmogorov differential equations by considering failure and repair rates as exponentially distributed. Ardakan and Hamadani (2014) simplified the component allocation problem for a standby redundant system. Dash and Behera (2010) used nonparametric data envelopment analysis for determining efficiency and performance Analysis for Indian coal fired power plants.

Gandhi et al. (2003) presented the methodology to estimate the failure causes in tribo-mechanical systems by using a di graphical approach. Gowid et al. (2014) suggested a methodology to obtain the reliability and maintenance in the liquefaction system FLNG terminals with the help of Markov modeling. Singh and Mahajan (1999) and Gupta et al. (2005) examined the parameters of reliability in butter oil production & utensils production plant in series process by considering exponential distributed failure rates. Reliability of the system is determined by using Markovian approach and solved the differential equations by using fourth order Runge-Kutta method. Gupta et al. (2008) developed model for decision support system in the feed water unit of the thermal power plant by using Markov birth-death process. The performance of the system is analyzed on the bases of making variation in failure and repair rate. It is helpful in planning maintenance strategy to increase the performance level of feed water unit in the thermal power plant.

Jesus and Francisco (2009) suggested a method to study the Gas Turbines availability in the electric power plants. The study is based on functional tree development and studied the consequences of failure rate in the system. Kumar et al. (1989) studded the maintenance planning of pulping system to enhance the performance in paper industry by consisting subsystem as in series. Kumar and Tewari (2017) deals with the analysis of CSDGB filling system's operation in the beverage plant with the help of particle swarm method of optimization. Kumar et al. (2017) investigated optimization of casting process through reliability approach. Kumar et al. (2007) presented the method for availability in CO2 cooling system of fertilizer plant by using Markov modeling analysis. Marseguerra et al. (2006) discussed about the GA optimization with RAMS applications and formulated the multi-objective optimization problem and illustrated the alternative approaches.

Yu et al. (2007) suggested a method to analyze the reliability in the redundant system. Mishra et al. (2016) conducted availability for Brake Drum production system by the Markovian approaches and conduct the failure rate and repair rate as constant for developing system model. Modgil et al. (2013) analyzed the model for performance in the shoe manufacturing unit by using Markov birth-death process and conduct the availability (TDSA) based on certain time interval with the help of long-term availability. Various expressions for availability and MTBF for redundant and non-redundant systems are derived. Khanduja et al. (2011) deals with the optimization and conduct a model to analyze the performance in stocks preparation unit for maintenance strategy in the paper plant with the help of genetic algorithm plant by using Markov birth-death process. This model helps in planning the maintenance strategy to enhance the performance of the bleaching system of a paper plant unit. Sabouhi et al. (2016) focused on the reliability and availability of power plants and developed a model for gas and steam turbine power plants and evaluate system's reliability.

Sachdeva et al. (2008a) developed a model to study the paper plant system by using generalized stochastic petri nets (GSPN) and reliability parameters evaluated by using Markovian approach.

Shahrzad et al. (2014) presented a component design model for availability analysis in multi-state weighted as k-out-of-n system. Singh (1989) discussed utility of reliability in performance enhancement of the fertilizer production using standby components. Ram and Kumar (2015), Sinha (2016), Pandey et al. (2018), Niwas and Garg (2018) and Saini and Kumar (2019) developed a lot of reliability and performance models for serial process of various industries by adopting various techniques.

Recently, Goyal et al. (2019) conducted reliability, maintainability and sensitivity analysis of physical processing unit of sewage treatment plants. Kumar et al. (2020) performed RAM analysis of soft water treatment and supply plant to enhance the operational availability. Gupta et al. (2020) investigated the operational availability of generators in steam turbine power plants. Gupta et al. (2021) carried out RAM analysis of generator unit of steam turbine power plants. Sinwar et al. (2021) developed a mathematical model for performance optimization of physical processing unit in sewage treatment plant using genetic algorithm and particle swarm optimization. The literature revealed that most of the work focused on the analysis of whole system no attention is paid to analyze the system at component level. So, here an effort has been made. The manuscript is divided into nine sections including introduction, system description, model development and conclusion.

2. System Description

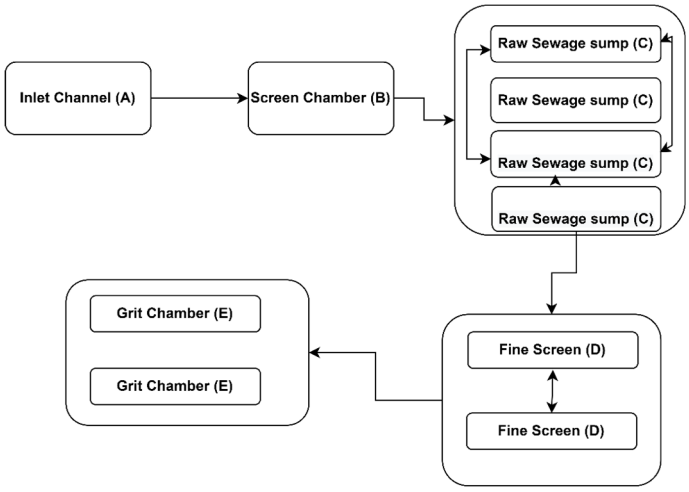

Figure 1 Configuration diagram of primary unit

In sewage water treatment plant (Figure 1) raw sewage collects in inlet channel from chambers after commencement it transfers to raw sewage sump which transfer it to the fine screen with the help of vacuum pressure for screening which removes all the solid waste from water, then it transfers to the grit chambers to remove grit from it. As describe above, sewage treatment consists of five subsystems in its primary section. These five subsystems are describing as follows:

i) Subsystem "A" (Inlet Channel)

In subsystem A, Inlet channel works in series, any failure in this subsystem can causes of failure in complete system. In this step raw sewage water collects here then it is transfer to the screen chamber.

ii) Subsystem "B" (Screen Chamber)

In subsystem B, Screen chamber works in series with subsystem A. Any failure in subsystem B can causes of failure in complete system.

iii) Subsystem "C" (Raw Sewage sump)

In subsystem C, it consists for raw sewage sump from this it works on 3-out-of-3 with one standby condition. Raw sewage transfers here from inlet channel then it transfers to the fine screen chamber with the help of vacuum pressure for screening.

iv) Subsystem "D" (Fine Screen)

In subsystem D, it consists of one manual and one mechanical fine screen. In this one is operative and other keeps on standby. Failure in one screen can reduce the capacity of the system. It removes all the solid waste from raw sewage water. These screens are inclined in the 45- degree angle. After removal of all the solid waste it transfers to the grit chambers.

v) Subsystem "E" (Grit Chamber)

In Subsystem E, it consists of two grit chambers from this it on works 2-out-of-2 conditions. Grit chamber removes all the grit such a sand, eggshells, small partials which cannot removed by fine screen chambers.

3. Assumptions

a) Proper facility for replacement and repair, not waiting time between failure and repair. Sufficient repairmen available in the plant to do repair activities.

b) Distribution for the failure rate consider as exponential while repair is arbitrarily distributed. The failure rate of various subsystems follows exponential distribution.

c) System performs as new system with full capacity after the repair.

4. Notations

A, B, C, C_1, D, E	:	All subsystems are working with full capacity
D_1	:	D_1 is working as cold standby
a, b, c, d, d_1, e	:	Subsystem has failed
$\lambda_i \ (1 \le i \le 6)$	:	Respectively failure rates in subsystems A, B, C, C_1, D, E, D_1
$\gamma_i(x) (1 \le i \le 6)$	:	Respectively repair rate in subsystems A, B, C, C_1, D, E, D_1
$P_0(t)$	:	Probability that system is working with full capacity
$P_i(x,t), (i = 1,...,19)$	:	Probability of subsystem on i^{th} state at time t with repair time x
$S_i, (i = 1,2,...,19)$	:	State of the subsystem

5. Mathematical Modeling of the System

The mathematical model of the sewage treatment plant using Markov birth death process and supplementary variable technique associated with state transition diagram (Figure -2) is developed as follows:

$$P_0(t+\Delta t) = (1-\lambda_1\Delta t-\lambda_2\Delta t-3\lambda_3\Delta t-\lambda_4\Delta t-2\lambda_5\Delta t)P_0(t)+\int_0^\infty \gamma_1(x)P_4(x,t)dx\Delta t+\int_0^\infty \gamma_2(x)P_5(x,t)dx\Delta t$$
$$+\int_0^\infty \gamma_3(x)P_1(x,t)dx\Delta t+\int_0^\infty \gamma_4(x)P_3(x,t)dx\Delta t+\int_0^\infty \gamma_5(x)P_6(x,t)dx\Delta t$$

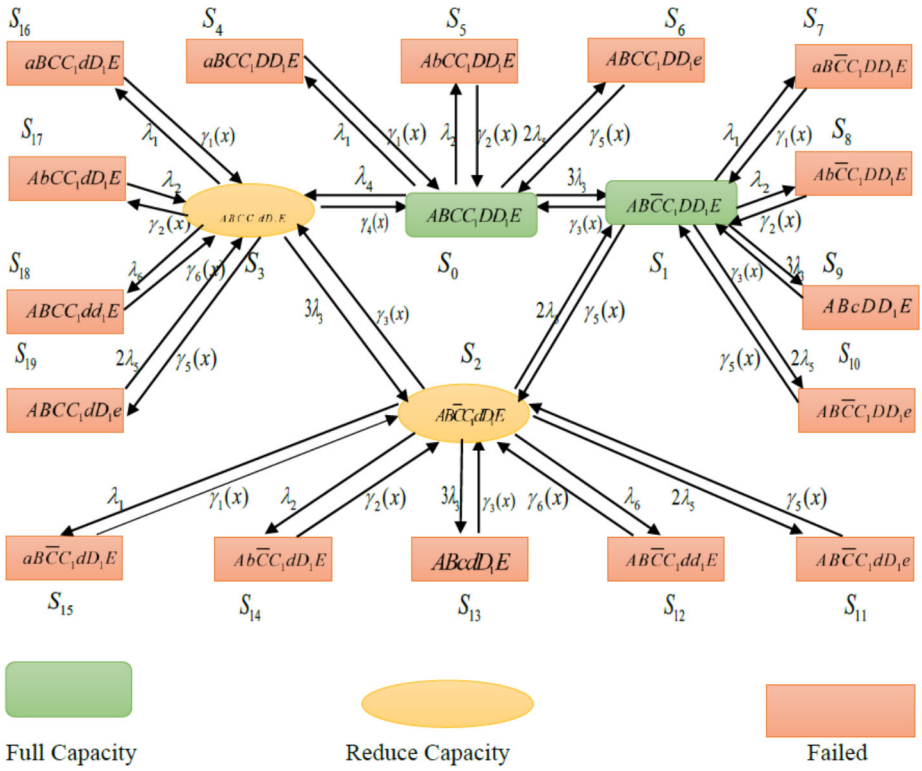

Figure 2 State changeover of the system

$$P_0(t+\Delta t)-P_0(t)=[(-\lambda_1-\lambda_2-3\lambda_3-\lambda_4-2\lambda_5)P_0(t)+\int_0^\infty \gamma_1(x)P_4(x,t)dx+\int_0^\infty \gamma_2(x)P_5(x,t)dx$$
$$+\int_0^\infty \gamma_3(x)P_1(x,t)dx+\int_0^\infty \gamma_4(x)P_3(x,t)dx+\int_0^\infty \gamma_5(x)P_6(x,t)dx]$$

Dividing both side by Δt and limit $\Delta t \to \infty$,

$$\lim_{\Delta t\to 0}\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t}=\lim_{\Delta t\to 0}[(-\lambda_1-\lambda_2-3\lambda_3-\lambda_4-2\lambda_5)P_0(t)+\int_0^\infty \gamma_1(x)P_4(x,t)dx$$
$$+\int_0^\infty \gamma_2(x)P_5(x,t)dx+\int_0^\infty \gamma_3(x)P_1(x,t)dx+\int_0^\infty \gamma_4(x)P_3(x,t)dx$$
$$+\int_0^\infty \gamma_5(x)P_6(x,t)dx]$$

$$\left[\frac{dP_0(t)}{dt}\right]+(\lambda_1+\lambda_2+3\lambda_3+\lambda_4+2\lambda_5)P_0(t)=\int_0^\infty \gamma_1(x)P_4(x,t)dx+\int_0^\infty \gamma_2(x)P_5(x,t)dx+\int_0^\infty \gamma_3(x)P_1(x,t)dx$$
$$+\int_0^\infty \gamma_4(x)P_3(x,t)dx+\int_0^\infty \gamma_5(x)P_6(x,t)dx,$$
$$\left[\frac{d}{dt}+\lambda_0\right]P_0(t)=\delta_0. \tag{1}$$

Similarly,

$$\left[\frac{d}{dt}+\frac{d}{dx}+\lambda_i(x)\right]P_i(x,t)=\delta_i(x,t),\ i=1,2,3, \tag{2}$$

where

$\alpha_0 = \lambda_1 + \lambda_2 + 3\lambda_3 + \lambda_4 + 2\lambda_5,$

$\beta_0 = \int_0^\infty \gamma_1(x)P_4(x,t)dx + \int_0^\infty \gamma_2(x)P_5(x,t)dx + \int_0^\infty \gamma_3(x)P_1(x,t)dx + \int_0^\infty \gamma_4(x)P_3(x,t)dx + \int_0^\infty \gamma_5(x)P_6(x,t)dx,$

$\alpha_1(x) = \alpha_0 + \gamma_3(x),$

$\beta_1(x,t) = \gamma_1(x)P_7(x,t) + \gamma_2(x)P_8(x,t) + \gamma_3(x)P_9(x,t) + \gamma_4(x)P_2(x,t) + \gamma_5(x)P_{10}(x,t) + 3\lambda_3(x)P_0(t),$

$\alpha_2(x) = \alpha_0 - \lambda_4 + \lambda_6 + \gamma_3(x) + \gamma_4(x),$

$\beta_1(x,t) = \gamma_1(x)P_{15}(x,t) + \gamma_2(x)P_{14}(x,t) + \gamma_3(x)P_{13}(x,t) + \gamma_6(x)P_{12}(x,t) + \gamma_5(x)P_{11}(x,t) + 3\lambda_3(x)P_3(x,t) + \lambda_4(x)P_1(x,t),$

$\alpha_3(x) = \alpha_0 - \lambda_4 + \lambda_6 + \gamma_4(x),$

$\beta_3(x,t) = \gamma_1(x)P_{16}(x,t) + \gamma_2(x)P_{17}(x,t) + \gamma_3(x)P_2(x,t) + \gamma_6(x)P_{18}(x,t) + \gamma_5(x)P_{19}(x,t) + \lambda_4(x)P_0(t),$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_1(x)\right]P_4(x,t) = \lambda_1 P_0(t), \tag{3}$$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_2(x)\right]P_5(x,t) = \lambda_2 P_0(t), \tag{4}$$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_5(x)\right]P_6(x,t) = 2\lambda_5 P_0(t), \tag{5}$$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_1(x)\right]P_7(x,t) = \lambda_1 P_1(x,t), \tag{6}$$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_2(x)\right]P_8(x,t) = \lambda_2 P_1(x,t), \tag{7}$$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_3(x)\right]P_9(x,t) = 3\lambda_3 P_1(x,t), \tag{8}$$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_5(x)\right]P_{10}(x,t) = 2\lambda_5 P_1(x,t), \tag{9}$$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_5(x)\right]P_{11}(x,t) = 2\lambda_5 P_2(x,t), \tag{10}$$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_6(x)\right]P_{12}(x,t) = \lambda_6 P_2(x,t) \tag{11}$$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_3(x)\right]P_{13}(x,t) = 3\lambda_3 P_2(x,t), \tag{12}$$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_2(x)\right]P_{14}(x,t) = \lambda_2 P_2(x,t), \tag{13}$$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_1(x)\right]P_{15}(x,t) = \lambda_1 P_2(x,t), \tag{14}$$

$$\left[\frac{d}{dt} + \frac{d}{dx} + \gamma_1(x)\right]P_{16}(x,t) = \lambda_1 P_3(x,t), \tag{15}$$

$$\left[\frac{d}{dt}+\frac{d}{dx}+\gamma_2(x)\right]P_{17}(x,t)=\lambda_2 P_3(x,t), \tag{16}$$

$$\left[\frac{d}{dt}+\frac{d}{dx}+\gamma_6(x)\right]P_{18}(x,t)=\lambda_6 P_3(x,t), \tag{17}$$

$$\left[\frac{d}{dt}+\frac{d}{dx}+\gamma_5(x)\right]P_{19}(x,t)=2\lambda_5 P_3(x,t). \tag{18}$$

The associated boundary conditions are as follows:

$P_1(0,t)=3\lambda_3 P_0(t),\ P_2(0,t)=3\lambda_3 P_3(t)+\lambda_4 P_1(t),\ P_3(0,t)=\lambda_4 P_0(t),\ P_4(0,t)=\lambda_1 P_0(t),$
$P_5(0,t)=\lambda_2 P_0(t),\ P_6(0,t)=2\lambda_5 P_0(t),\ P_7(0,t)=\lambda_1 P_1(t),\ P_8(0,t)=\lambda_2 P_1(t),\ P_9(0,t)=3\lambda_3 P_1(t),$
$P_{10}(0,t)=2\lambda_5 P_1(t),\ P_{11}(0,t)=2\lambda_5 P_2(t),\ P_{12}(0,t)=\lambda_6 P_2(t),\ P_{13}(0,t)=3\lambda_3 P_2(t),\ P_{14}(0,t)=\lambda_2 P_2(t),$
$P_{15}(0,t)=\lambda_1 P_2(t),\ P_{16}(0,t)=\lambda_1 P_3(t),\ P_{17}(0,t)=\lambda_2 P_3(t),\ P_{18}(0,t)=\lambda_6 P_3(t),\ P_{19}(0,t)=2\lambda_5 P_3(t).$

The initial conditions are as follows: $P_0(0)=1$ and $P_i(0)=0,\ i=1,...,19$. This system of all the differential equations with considering boundary and initial conditions known as Chapman-Kolmogorov differential difference equations.

6. Particular Case

By considering all repair time distribution as exponentially distributed, the mathematical model reduced in the following expression:

$$\left[\frac{d}{dt}+\lambda_1+\lambda_2+3\lambda_3+\lambda_4+2\lambda_5\right]P_0(t)=\gamma_1 P_4(t)+\gamma_2 P_5(t)+\gamma_3 P_1(t)+\gamma_4 P_3(t)+\gamma_5 P_6(t), \tag{19}$$

$$\left[\frac{d}{dt}+\lambda_1+\lambda_2+3\lambda_3+\lambda_4+2\lambda_5+\gamma_3\right]P_1(t)=\gamma_1 P_7(t)+\gamma_2 P_8(t)+\gamma_3 P_9(t)+\gamma_4 P_2(t)+\gamma_5 P_{10}(t)+3\lambda_3 P_0(t), \tag{20}$$

$$\left[\frac{d}{dt}+\lambda_1+\lambda_2+3\lambda_3+\lambda_6+2\lambda_5+\gamma_3+\gamma_4\right]P_2(t)=\gamma_1 P_{15}(t)+\gamma_2 P_{14}(t)+\gamma_3 P_{13}(t)+\gamma_5 P_{11}(t)+\gamma_6 P_{12}(t)+3\lambda_3 P_3(t)+\lambda_4 P_1(t), \tag{21}$$

$$\left[\frac{d}{dt}+\lambda_1+\lambda_2+3\lambda_3+\lambda_6+2\lambda_5+\gamma_4\right]P_3(t)=\gamma_1 P_{16}(t)+\gamma_2 P_{17}(t)+\gamma_3 P_2(t)+\gamma_5 P_{19}(t)+\gamma_6 P_{18}(t)+\lambda_4 P_0(t), \tag{22}$$

$$\left[\frac{d}{dt}+\gamma_j\right]P_i(t)=k\lambda_j P_0(t); i=4,5,6; j=1,2,5; k=1,1,2, \tag{23}$$

$$\left[\frac{d}{dt}+\gamma_j\right]P_i(t)=k\lambda_j P_1(t); i=7,8,9,10; j=1,2,3,5; k=1,1,3,2, \tag{24}$$

$$\left[\frac{d}{dt}+\gamma_j\right]P_i(t)=k\lambda_j P_2(t); i=11,12,13,14,15; j=5,6,3,2,1; k=2,1,3,1,1, \tag{25}$$

$$\left[\frac{d}{dt}+\gamma_j\right]P_i(t)=k\lambda_j P_3(t); i=16,17,18,19; j=1,2,6,5; k=1,1,1,2. \tag{26}$$

Initial Conditions: $P_0(0)=1$ and $P_i(0)=0, i=1,...,19$. To calculate long run availability, we can take $\frac{d}{dt}=0$ as $t\to\infty$ and $P_i(t)=P_i$. From Equations (19)-(26), steady state probabilities are

$$P_1=\frac{3\lambda_3}{\gamma_3}P_0,\ P_2=\frac{3\lambda_3}{\gamma_3}\frac{\lambda_4}{\gamma_4}P_0,\ P_3=\frac{\lambda_4}{\gamma_4}P_0,\ P_4=\frac{\lambda_1}{\gamma_1}P_0,\ P_5=\frac{\lambda_2}{\gamma_2}P_0,\ P_7=\frac{3\lambda_3}{\gamma_3}\frac{\lambda_1}{\gamma_1}P_0,$$

$$P_9=\frac{3\lambda_3}{\gamma_3}\frac{3\lambda_3}{\gamma_3}P_0,\ P_{10}=\frac{3\lambda_3}{\gamma_3}\frac{2\lambda_5}{\gamma_5}P_0,\ P_{11}=\frac{3\lambda_3}{\gamma_3}\frac{2\lambda_5}{\gamma_5}\frac{\lambda_4}{\gamma_4}P_0,\ P_{12}=\frac{3\lambda_3}{\gamma_3}\frac{\lambda_6}{\gamma_6}\frac{\lambda_4}{\gamma_4}P_0,\ P_{13}=\frac{3\lambda_3}{\gamma_3}\frac{3\lambda_3}{\gamma_3}\frac{\lambda_4}{\gamma_4}P_0,$$

$$P_{14}=\frac{3\lambda_3}{\gamma_3}\frac{\lambda_2}{\gamma_2}\frac{\lambda_4}{\gamma_4}P_0,\ P_{15}=\frac{3\lambda_3}{\gamma_3}\frac{\lambda_1}{\gamma_1}\frac{\lambda_4}{\gamma_4}P_0,\ P_{16}=\frac{\lambda_1}{\gamma_1}\frac{\lambda_4}{\gamma_4}P_0,\ P_{17}=\frac{\lambda_2}{\gamma_2}\frac{\lambda_4}{\gamma_4}P_0,\ P_{18}=\frac{\lambda_6}{\gamma_6}\frac{\lambda_4}{\gamma_4}P_0,$$

$$P_{19}=\frac{2\lambda_5}{\gamma_5}\frac{\lambda_4}{\gamma_4}P_0.$$

Using normalized condition $\sum P_i=1$,

$$P_0=\left[1+\left(1+\frac{3\lambda_3}{\gamma_3}\right)\left(\frac{\lambda_1}{\gamma_1}+\frac{\lambda_2}{\gamma_2}+\frac{3\lambda_3}{\gamma_3}+\frac{\lambda_4}{\gamma_4}+\frac{2\lambda_5}{\gamma_5}\right)+\frac{\lambda_4}{\gamma_4}\left(\frac{\lambda_1}{\gamma_1}+\frac{\lambda_2}{\gamma_2}+\frac{2\lambda_5}{\gamma_5}+\frac{\lambda_6}{\gamma_6}\right)\left(1+\frac{3\lambda_3}{\gamma_3}\right)+\frac{3\lambda_3}{\gamma_3}\frac{3\lambda_3}{\gamma_3}\frac{\lambda_4}{\gamma_4}\right]^{-1},$$

long run availability (A_v),

$$A_v=P_0+P_1+P_2+P_3$$

$$=\left[1+\frac{3\lambda_3}{\gamma_3}+\frac{3\lambda_3}{\gamma_3}\frac{\lambda_4}{\gamma_4}+\frac{\lambda_4}{\gamma_4}\right]\left[1+\left(1+\frac{3\lambda_3}{\gamma_3}\right)\left(\frac{\lambda_1}{\gamma_1}+\frac{\lambda_2}{\gamma_2}+\frac{3\lambda_3}{\gamma_3}+\frac{\lambda_4}{\gamma_4}+\frac{2\lambda_5}{\gamma_5}\right)+\frac{\lambda_4}{\gamma_4}\left(\frac{\lambda_1}{\gamma_1}+\frac{\lambda_2}{\gamma_2}+\frac{2\lambda_5}{\gamma_5}+\frac{\lambda_6}{\gamma_6}\right)\left(1+\frac{3\lambda_3}{\gamma_3}\right)+\frac{3\lambda_3}{\gamma_3}\frac{3\lambda_3}{\gamma_3}\frac{\lambda_4}{\gamma_4}\right]^{-1}. \quad (27)$$

7. Performance Analysis

In this section, a formula for performance analysis is proposed in which K_1 represent the total revenue taken as per unit time, A_v represent the long run availability which we have calculated above and K_2 represent the total repair cost.

$$\text{Performance}=K_1A_v-K_2$$

$$=K_1\left[1+\frac{3\lambda_3}{\gamma_3}+\frac{3\lambda_3}{\gamma_3}\frac{\lambda_4}{\gamma_4}+\frac{\lambda_4}{\gamma_4}\right]\left[1+\left(1+\frac{3\lambda_3}{\gamma_3}\right)\left(\frac{\lambda_1}{\gamma_1}+\frac{\lambda_2}{\gamma_2}+\frac{3\lambda_3}{\gamma_3}+\frac{\lambda_4}{\gamma_4}+\frac{2\lambda_5}{\gamma_5}\right)+\frac{\lambda_4}{\gamma_4}\left(\frac{\lambda_1}{\gamma_1}+\frac{\lambda_2}{\gamma_2}+\frac{2\lambda_5}{\gamma_5}+\frac{\lambda_6}{\gamma_6}\right)\left(1+\frac{3\lambda_3}{\gamma_3}\right)+\frac{3\lambda_3}{\gamma_3}\frac{3\lambda_3}{\gamma_3}\frac{\lambda_4}{\gamma_4}\right]^{-1}-K_2$$

$$=8000\left[1+\frac{3\lambda_3}{\gamma_3}+\frac{3\lambda_3}{\gamma_3}\frac{\lambda_4}{\gamma_4}+\frac{\lambda_4}{\gamma_4}\right]\left[1+\left(1+\frac{3\lambda_3}{\gamma_3}\right)\left(\frac{\lambda_1}{\gamma_1}+\frac{\lambda_2}{\gamma_2}+\frac{3\lambda_3}{\gamma_3}+\frac{\lambda_4}{\gamma_4}+\frac{2\lambda_5}{\gamma_5}\right)+\frac{\lambda_4}{\gamma_4}\left(\frac{\lambda_1}{\gamma_1}+\frac{\lambda_2}{\gamma_2}+\frac{2\lambda_5}{\gamma_5}+\frac{\lambda_6}{\gamma_6}\right)\left(1+\frac{3\lambda_3}{\gamma_3}\right)+\frac{3\lambda_3}{\gamma_3}\frac{3\lambda_3}{\gamma_3}\frac{\lambda_4}{\gamma_4}\right]^{-1}-800. \quad (28)$$

8. Numerical Analysis and Discussion

In this section, effect of various subsystem's failure rates is analyzed on availability of the system with respect to failure rate λ_5 ranges from 0.009 to 0.018. The values of the failure rates are changes as follows: $\lambda_1=0.0008$ to 0.008, $\lambda_2=0.002$ to 0.02, $\lambda_3=0.03$ to 0.3, $\lambda_4=0.0004$ to 0.004 and $\lambda_6=0.006$ to 0.06 by considering failure rate of subsystems follows exponential distribution and repair rates are $\gamma_1=0.2$, $\gamma_2=0.72$, $\gamma_3=1.5$, $\gamma_4=0.08$, $\gamma_5=0.99$, $\gamma_6=0.82$. Availability in this system decrease 3.41% approximately when failure rate λ_1 increase from 0.0008 to 0.008. Availability in this system decrease 2.39% approximately when failure rate λ_2 increase from 0.002 to 0.02. Availability of system decrease with 7.07% approximately when failure rate λ_3 increase from 0.03 to

0.3. Availability decreases approximately 0.076% when failure rate λ_4 increase from 0.0004 to 0.004. Availability shows 0.32% decline with increase of failure rate λ_6 from 0.006 to 0.06.

Table 1 Effect of variation in failure rates on system's availability w.r.t. failure rate λ_5

λ_5	$S0=\lambda_1=0.000$ $\lambda_2=0.002$ $\lambda_3=0.03$, $\lambda_4=0.000$ $\lambda_5=0.009$ $\lambda_6=0.006$	$S1=\lambda_1=0.008$, $\lambda_2=0.002$, $\lambda_3=0.03$, $\lambda_4=0.0004$, $\lambda_5=0.009$, $\lambda_6=0.006$	$S2=\lambda_1=0.0008$, $\lambda_2=0.02$, $\lambda_3=0.03$, $\lambda_4=0.0004$, $\lambda_5=0.009$, $\lambda_6=0.006$	$S3=\lambda_1=0.0008$, $\lambda_2=0.002$, $\lambda_3=0.3$, $\lambda_4=0.0004$, $\lambda_5=0.009$, $\lambda_6=0.006$	$S4=\lambda_1=0.0008$, $\lambda_2=0.002$, $\lambda_3=0.03$, $\lambda_4=0.004$, $\lambda_5=0.009$, $\lambda_6=0.006$	$S6=\lambda_1=0.0008$, $\lambda_2=0.002$, $\lambda_3=0.03$, $\lambda_4=0.004$, $\lambda_5=0.009$, $\lambda_6=0.06$
0.009	0.982699	0.9491217	0.9591354	0.913168	0.9819517	0.9823826
0.010	0.981665	0.9481570	0.9581502	0.9120087	0.9808801	0.9813491
0.011	0.980633	0.9471942	0.9571670	0.9108524	0.9798107	0.9803178
0.012	0.979603	0.9462334	0.9561859	0.9096989	0.9787437	0.9792887
0.013	0.978575	0.9452745	0.9552068	0.9085484	0.9776790	0.9782617
0.014	0.977550	0.9443176	0.9542296	0.9074008	0.9766166	0.9772368
0.015	0.976526	0.9433626	0.9532545	0.9062561	0.9755565	0.9762141
0.016	0.975505	0.9424095	0.9522814	0.9051142	0.9744988	0.9751936
0.017	0.974486	0.9414584	0.9513102	0.9039753	0.9734433	0.9741752
0.018	0.973469	0.9405092	0.9503410	0.9028392	0.9723901	0.9731589

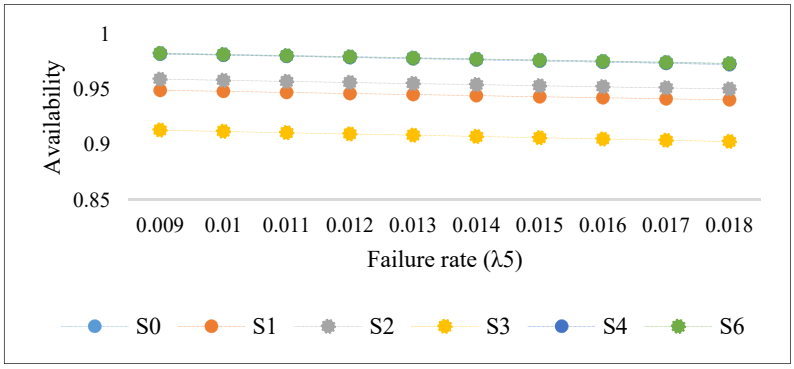

Figure 3 Effect of variation in failure rates on system's availability w.r.t. failure rate λ_5

Effect of variation in various repair rates on system availability is studied herewith by assuming all failure and repair time random variables as exponential distributed with parameters: $\lambda_1=0.2$ to 0.8, $\lambda_2=0.72$ to 1.5, $\lambda_3=0.03$ to 2.0, $\lambda_4=0.08$ to 0.25 and $\lambda_6=0.82$ to 2.5 by considering repair rate $\lambda_5=0.99$ to 1.08 failure rates are $\lambda_1=0.0008$, $\lambda_2=0.002$, $\lambda_3=0.03$, $\lambda_4=0.0004$, $\lambda_5=0.009$ and $\lambda_6=0.006$. Availability of system increase 0.29% approximately when repair rate γ_1 increase from 0.2 to 0.8. Availability of the system shows increment of 0.14% approximately when repair rate γ_2 increase from 0.72 to 1.5. Availability of the system increase 0.061% approximately when repair rate γ_3 increase from 1.5 to 2.0. Availability also shows increment 0.0061% and 0.0023% respectively with the variation of repair rates γ_4 increase from 0.08 to 0.25 and γ_6 increase from 0.82 to 2.5.

Table 2 Effect of variation in repair rates on system's availability w.r.t. repair rate γ_5

γ_5	$P0=\gamma_1=0.2,$ $\gamma_2=0.72,$ $\gamma_3=1.5,$ $\gamma_4=0.08,$ $\gamma_5=0.99,$ $\gamma_6=0.82$	$P1=\gamma_1=0.8,$ $\gamma_2=0.72,$ $\gamma_3=1.5,$ $\gamma_4=0.08,$ $\gamma_5=0.99,$ $\gamma_6=0.82$	$P2=\gamma_1=0.2,$ $\gamma_2=1.5,$ $\gamma_3=1.5,$ $\gamma_4=0.08,$ $\gamma_5=0.99,$ $\gamma_6=0.82$	$P3=\gamma_1=0.2,$ $\gamma_2=0.72,$ $\gamma_3=2.0,$ $\gamma_4=0.08,$ $\gamma_5=0.99,$ $\gamma_6=0.82$	$P4=\gamma_1=0.2,$ $\gamma_2=0.72,$ $\gamma_3=1.5,$ $\gamma_4=0.25,$ $\gamma_5=0.99,$ $\gamma_6=0.82$	$P6=\gamma_1=0.2,$ $\gamma_2=0.72,$ $\gamma_3=1.5,$ $\gamma_4=0.08,$ $\gamma_5=0.99,$ $\gamma_6=2.5$
0.99	0.982699	0.9856046	0.9840958	0.9832917	0.9827581	0.9827225
1.00	0.982792	0.9856983	0.9841892	0.9833838	0.9828511	0.9828157
1.01	0.982883	0.9857902	0.9842809	0.9834741	0.9829422	0.9829071
1.02	0.982973	0.9858803	0.9843707	0.9835627	0.9830315	0.9829967
1.03	0.983061	0.9859687	0.9844588	0.9836495	0.9831191	0.9830845
1.04	0.983147	0.9860554	0.9845452	0.9837347	0.983205	0.9831707
1.05	0.983232	0.9861404	0.9846300	0.9838183	0.9832893	0.9832553
1.06	0.983315	0.9862239	0.9847132	0.9839003	0.983372	0.9833382
1.07	0.983396	0.9863058	0.9847949	0.9839808	0.9834532	0.9834197
1.08	0.983476	0.9863862	0.9848751	0.9840598	0.983533	0.9834996

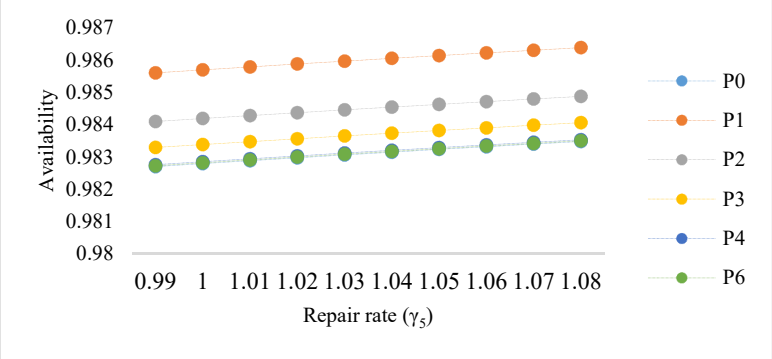

Figure 4 Effect of variation in repair rates on system's availability w.r.t. repair rate γ_5

Table 3 Effect of variation in failure rates on system's performance w.r.t. failure rate λ_5

λ_5	$S0=\lambda_1=0.0008,$ $\lambda_2=0.002,$ $\lambda_3=0.03,$ $\lambda_4=0.0004,$ $\lambda_5=0.009,$ $\lambda_6=0.006$	$S1=\lambda_1=0.008,$ $\lambda_2=0.002,$ $\lambda_3=0.03,$ $\lambda_4=0.0004,$ $\lambda_5=0.009,$ $\lambda_6=0.006$	$S2=\lambda_1=0.0008,$ $\lambda_2=0.02,$ $\lambda_3=0.03,$ $\lambda_4=0.0004,$ $\lambda_5=0.009,$ $\lambda_6=0.006$	$S3=\lambda_1=0.0008,$ $\lambda_2=0.002,$ $\lambda_3=0.3,$ $\lambda_4=0.0004,$ $\lambda_5=0.009,$ $\lambda_6=0.006$	$S4=\lambda_1=0.0008,$ $\lambda_2=0.002,$ $\lambda_3=0.03,$ $\lambda_4=0.004,$ $\lambda_5=0.009,$ $\lambda_6=0.006$	$S6=\lambda_1=0.0008,$ $\lambda_2=0.002,$ $\lambda_3=0.03,$ $\lambda_4=0.004,$ $\lambda_5=0.009,$ $\lambda_6=0.06$
0.009	7839.271	7544.081	7632.115	7122.714	7833.065	7836.490
0.010	7829.434	7534.88	7622.727	7112.197	7822.931	7826.660
0.011	7819.619	7525.698	7613.358	7101.707	7812.819	7816.850
0.012	7809.824	7516.535	7604.009	7091.244	7802.729	7807.061
0.013	7800.049	7507.391	7594.679	7080.807	7792.661	7797.293
0.014	7790.295	7498.265	7585.367	7070.396	7782.615	7787.545
0.015	7780.562	7489.157	7576.075	7060.012	7772.590	7777.817
0.016	7770.849	7480.068	7566.802	7049.654	7762.588	7768.11
0.017	7761.156	7470.997	7557.548	7039.322	7752.607	7758.423
0.018	7751.483	7461.944	7548.312	7029.016	7742.647	7748.757

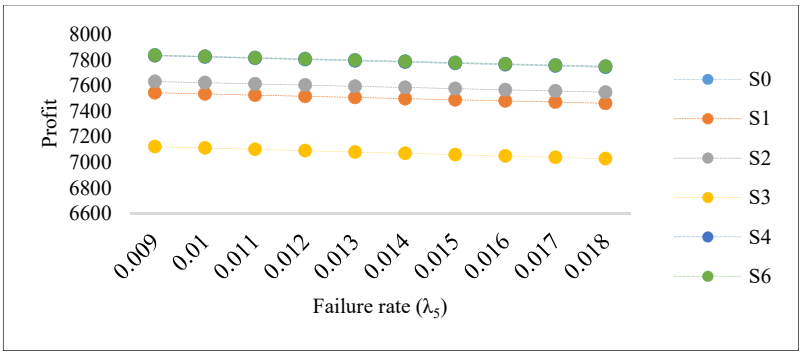

Figure 5 Effect of variation in failure rates on system's performance w.r.t. failure rate λ_5

Effect of various subsystem's failure rates are analyzed on profit of the system with respect to failure rate λ_5 ranges from 0.009 to 0.018. The values of the failure rates are changes as follows: $\lambda_1 = 0.0008$ to $0.008, \lambda_2 = 0.002$ to $0.02, \lambda_3 = 0.03$ to 0.3, $\lambda_4 = 0.0004$ to 0.004 and $\lambda_6 = 0.006$ to 0.06 by considering failure rate of subsystems follows exponential distribution and repair rates are $\gamma_1 = 0.2$, $\gamma_2 = 0.72$, $\gamma_3 = 1.5$, $\gamma_4 = 0.08$, $\gamma_5 = 0.99$, $\gamma_6 = 0.82$. Performance of system decrease 3.77% approximately when failure rate λ_1 increase from 0.0008 to 0.008. Performance of the system decrease 2.64% approximately when failure rate λ_2 of screen chamber subsystem increase from 0.002 to 0.02. Raw sewage sump shows 9.14% decline when failure rate λ_3 increase from 0.03 to 0.3. The impact of failure rates on system failure rates on availability and profit are shown in Figure 3 and Figure 5. Fine screen and Grit chamber also decrease the performance 0.08% and 0.035% approximately when failure rates increase respectively λ_4 from 0.0004 to 0.004 and λ_6 from 0.006 to 0.06.

Table 4 Effect of variation in repair rates on system's performance w.r.t. repair rate γ_5

γ_5	$P0 = \gamma_1 = 0.2$, $\gamma_2 = 0.72$, $\gamma_3 = 1.5$, $\gamma_4 = 0.08$, $\gamma_5 = 0.99$, $\gamma_6 = 0.82$	$P1 = \gamma_1 = 0.8$, $\gamma_2 = 0.72$, $\gamma_3 = 1.5$, $\gamma_4 = 0.08$, $\gamma_5 = 0.99$, $\gamma_6 = 0.82$	$P2 = \gamma_1 = 0.2$, $\gamma_2 = 1.5$, $\gamma_3 = 1.5$, $\gamma_4 = 0.08$, $\gamma_5 = 0.99$, $\gamma_6 = 0.82$	$P3 = \gamma_1 = 0.2$, $\gamma_2 = 0.72$, $\gamma_3 = 2.0$, $\gamma_4 = 0.08$, $\gamma_5 = 0.99$, $\gamma_6 = 0.82$	$P4 = \gamma_1 = 0.2$, $\gamma_2 = 0.72$, $\gamma_3 = 1.5$, $\gamma_4 = 0.25$, $\gamma_5 = 0.99$, $\gamma_6 = 0.82$	$P6 = \gamma_1 = 0.2$, $\gamma_2 = 0.72$, $\gamma_3 = 1.5$, $\gamma_4 = 0.08$, $\gamma_5 = 0.99$, $\gamma_6 = 2.5$
0.99	7839.271	7864.815	7851.551	7845.147	7839.762	7839.478
1.00	7840.157	7865.707	7852.440	7846.025	7840.647	7840.365
1.01	7841.026	7866.580	7853.311	7846.885	7841.513	7841.233
1.02	7841.878	7867.437	7854.165	7847.729	7842.363	7842.086
1.03	7842.713	7868.278	7855.003	7848.557	7843.197	7842.921
1.04	7843.533	7869.102	7855.825	7849.369	7844.015	7843.741
1.05	7844.337	7869.911	7856.632	7850.166	7844.817	7844.545
1.06	7845.126	7870.705	7857.423	7850.947	7845.604	7845.334
1.07	7845.901	7871.484	7858.200	7851.715	7846.377	7846.109
1.08	7846.661	7872.248	7858.962	7852.468	7847.135	7846.869

Effect of variation in various repair rates on system performance is studied herewith by assuming all failure and repair time random variables as exponential distributed with parameters: $\lambda_1 = 0.2$ to 0.8, $\lambda_2 = 0.72$ to 1.5, $\lambda_3 = 0.03$ to 2.0, $\lambda_4 = 0.08$ to 0.25 and $\lambda_6 = 0.82$ to 2.5 by considering repair rate $\gamma_5 = 0.99$ to 1.08 failure rates are $\lambda_1 = 0.0008$, $\lambda_2 = 0.002$, $\lambda_3 = 0.03$, $\lambda_4 = 0.0004$, $\lambda_5 = 0.009$, $\lambda_6 = 0.006$. Performance of the system increase 0.33% approximately when repair rate γ_1 increase from 0.2 to 0.8. System's performance increase approximately 0.16% when repair rate γ_2 increase from 0.72 to 1.5. System's performance have respective increments 0.075%, 0.0063% and 0.0026% with changing the repair rates γ_3 increase from 1.5 to 2.0, γ_4 increase from 0.08 to 0.25 and γ_6 increase from 0.82 to 2.5. The impact of repair rates on availability and profit of system is shown in Figure 4 and Figure 6, respectively.

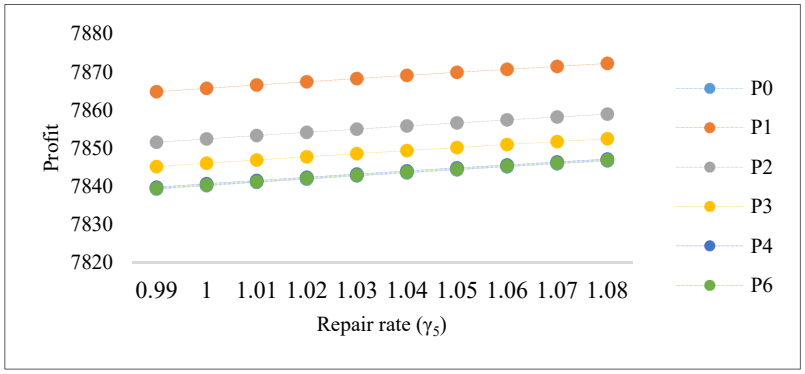

Figure 6 Effect of variation in repair rates on system's profit w.r.t. repair rate γ_5

9. Conclusion

The proposed model is helpful to system developers as it provides the availability and performance of the sewage treatment plant. Detailed analysis of numerical results revealed that failure rate and repair rate have high impact on availability and performance of primary unit of sewage treatment plant. It is observed that there is a huge variation from 0.076% to 7.07% variation in availability with respect to variation in various failure rates. Raw sewage sump subsystem's failure rate is highly sensitive and causes a high variation in availability and performance. The performance of model decreases approximately 9.04% if the failure rates of raw sewage sump increase. The repair rate of the subsystems is less sensitive, and variation varies in the range 0.0029% to 0.33%. Inlet chamber and screen chamber are other components which have significant effect on availability and performance of system. The outcomes of the study are shared with the plant employees and found the results significant to improve the plant performance.

References

Adhikary DD, Bose GK, Chattopadhyay S, Mitra S. RAM investigation of coal fired thermal power plants: a case study. Int J Ind Eng Comput. 2012; 3: 423-434.

Aggarwal AK, Singh V, Kumar S. Availability analysis and performance optimization of a butter oil production system: a case study. Int J Syst Assur Eng Manag. 2014; 8: 538-554.

Ardakan MA, Hamadani AZ. Reliability- redundancy allocation problem with cold standby redundancy strategy. Simul Model Pract Theory. 2014; 42(3):107-118.

Behera SK, Dash AP. Performance analysis of coal fired power plants in India. Proceedings of the International Conference on Industrial Engineering and Operations Management, Dhaka, Bangladesh. 2010; 1-6.

Gandhi OP, Sehgal R, Angra S. Failure cause identification of tribo-mechanical system. Reliab Eng Syst Saf. 2003; 65: 259-270.

Gowid S, Dixon R, Ghani S. Optimization of reliability and maintenance of liquefaction system on FLNG terminals using Markov modeling. Int J Qual Reliab Manage. 2014; 31(3): 293-310.

Gupta P, Lal AK, Sharma RK, Singh J. Numerical analysis of reliability and availability of the serial processes in butter-oil processing plant. Int J Qual Reliab Manage. 2005; 22(3): 303-316.

Gupta N, Kumar A, Saini M. Reliability and maintainability investigation of generator in steam turbine power plant using RAMD analysis. J Phys Conf Ser. 2021; 1714 (1), https://doi.org/10.1088/1742-6596/1714/1/012009.

Gupta S, Tewari PC, Sharma AK. Performance modeling and decision support system of feed water unit of a thermal power plant. South African J Ind Eng. 2008; 19(2): 125-134.

Gupta N, Saini M, Kumar A. Operational availability analysis of generators in steam turbine power plants. SN Appl Sci. 2020; 2(4), https://doi.org/10.1007/s42452-020-2520-y.

Goyal D, Kumar A, Saini M, Joshi H. Reliability, maintainability and sensitivity analysis of physical processing unit of sewage treatment plant. SN Appl Sci. 2019; 1(11), https://doi.org/10.1007/s42452-019-1544-7.

Jesus S, Francisco G. Availability analysis of gas turbines used in power plants. Int J Thermodyn. 2009; 12: 28-37.

Khanduja R, Tewari PC, Kumar D. Steady state behavior and maintenance planning of bleaching system in a paper plant. Int J Indust Eng. 2012; 7(12): 39-44.

Khanduja R, Tewari PC, Chauhan RS. Performance modeling and optimization for the stock preparation unit of a paper plant using genetic algorithm. Int J Qual Reliab Manage. 2011; 28 (6): 688-703.

Kumar D, Singh I, Pandey PC. Maintenance planning of pulping system in paper industry. Reliab Eng Syst Saf. 1989; 25(4): 293-302.

Kumar A, Singh R, Saini M, Dahiya O. Reliability, availability and maintainability analysis to improve the operational performance of soft water treatment and supply plant. J Eng Sci Technol Rev. 2020; 13(5): 183-192.

Kumar P, Tewari PC. Performance analysis and optimization for CSDGB filling system of a beverage plant using particle swarm optimization. Int J Ind Eng Comput. 2017; 8: 303-314.

Kumar S, Tewari PC, Sharma R. Simulated availability of CO_2 cooling system in a fertilizer plant. Industr Eng J. 2007; 36(10): 19-23.

Kumar A, Ram M, Rawat RS. Optimization of casting process through reliability approach. Int J Qual Reliab Manage. 2017; 34(6): 833-848.

Marseguerra M, Zio E, Martorell S. Basics of genetic algorithms (GA) optimization for RAMS applications. Reliab Eng Syst Saf. 2006; 91(9): 977-991.

Mishra S, Bhardwaj P, Bhadauria N. Optimal availability analysis of brake drum manufacturing system by using Markov approach. Int J Eng Technol Manage Appl Sci. 2016; 4: 147-154.

Modgil V, Sharma SK, Singh J. Performance modeling and availability analysis of shoe upper manufacturing unit. Int J Qual Reliab Manage. 2013; 30(8): 816-831.

Niwas R, Garg H. An approach for analyzing the reliability and profit of an industrial system based on the cost-free warranty policy. J Braz Soc Mech Sci Eng. 2018; 40, https://doi.org/10.1007/s40430-018-1167-8.

Pandey P, Mukhopadhyay AK, Chattopadhyaya S. Reliability analysis and failure rate evaluation for critical subsystems of the dragline. J Braz Soc Mech Sci Eng. 2018; 40:50.

Ram M, Kumar A. Performability analysis of a system under 1-out-of-2: G scheme with perfect reworking. J Braz Soc Mech Sci Eng. 2015; 37:1029.

Sabouhi H, Abbaspour A, Firuzabad M, Dehghanian P. Reliability Modeling and Availability Analysis of Combined Cycle Power Plants. Int J Elec Power. 2016; 79: 108-119.

Sachdeva A, Kumar D, Kumar P. Reliability analysis of pulping system using Petri nets. Int J Qual Reliab Manage. 2008; 25(8): 860-877.

Shahrzad FR, Xie Min, Kien MN, Richard CMY Dynamic availability assessment and optimal component design of multi-state weighted k-out-of-n systems. Reliab Eng Syst Saf. 2014; 123(3): 57-62.

Saini M, Kumar A. Performance analysis of evaporation system in sugar industry using RAMD analysis. J Braz Soc Mech Sci Eng. 2019; 41(4), https://doi.org/10.1007/s40430-019-1681-3.

Singh J. A warm stand-by redundant system with common cause failures. Reliab Eng Syst Saf. 1989; 26: 135-141.

Singh J, Mahajan P. Reliability of utensils manufacturing plant-a case study. Oper Res. 1999; 36(3): 260-269.

Sinha RS, Mukhopadhyay AK. Failure analysis of jaw crusher and its components using ANOVA. J Braz Soc Mech Sci Eng. 2016; 38(2): 665-678.

Sinwar D, Saini M, Singh D, Goyal D, Kumar A. Availability and performance optimization of physical processing unit in sewage treatment plant using genetic algorithm and particle swarm optimization. Int J Syst Assur Eng Manage. 2021; 12(1): 1-12.

Yu H, Chu C, Châtelet E, Yalaoui F. Reliability optimization of a redundant system with failure dependencies. Reliab Eng Syst Saf. 2007; 92: 1627-1634.