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Efficient Class of Estimators of Population Mean under Double Sampling

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Abstract

In this article, an efficient class of estimators of population mean is opined under double sampling scheme using auxiliary information which admits various existing estimators like the usual mean estimator, classical ratio estimator, Singh (2011) estimator and Bhushan and Gupta (2019) estimators. The first order approximation expressions of mean square error of the opined estimator are established and the efficiency conditions are derived. Further, the soundness of the efficiency conditions is studied by an empirical study employing different real populations. The findings of the empirical study show the ascendancy of the opined class of estimators over the estimators existing till date.

Keywords: Bias, efficiency, mean square error, double sampling.

1. Introduction

In survey sampling, the utilization of supplementary information furnishes a better improvement over the efficiency of estimators constructed for the estimation of unknown population parameters. The literature describes several ratio, regression and exponential methods utilizing the auxiliary variable at the estimation stage. Many prominent authors developed several modified and improved ratio, regression and exponential type estimators by using the population information of the auxiliary variable x. However, the information about the population mean of the auxiliary variable is not always available. In the aforesaid environment, the most popular sampling scheme is the double (two-phase) sampling scheme which was first established by Neyman (1938) to accumulate information on the strata sizes in stratified sampling. It is customarily acquired when the accumulation of information on study variable is very costly but relatively cheaper to accumulate information on auxiliary variables that are correlated with the study variables. Due to these reasons, the double sampling (DS) becomes a powerful and cost-effective scheme for obtaining the authentic estimate in the first phase sample for the unknown parameters of the auxiliary variable. In literature, Sukhatme (1962) investigated the classical ratio estimator in DS. Following Srivenkataraman (1980), Kumar and Bahl (2006) envisaged a class of dual to exponential type ratio estimator using DS. Singh and Vishwakarma (2007) investigated Bahl and Tuteja (1991) exponential ratio and product estimators of population mean under DS. Singh (2011) provided Prasad (1989) estimator under DS. Ozgul and Cingi (2014) developed a class of exponential regression cum ratio estimator in DS. Kalita et al. (2016) suggested exponential ratio-cum-exponential dual to ratio estimators using DS. Following Kumar and Bahl (2006) and Kalita et al. (2016), Bazad and Bazad (2019) developed some classes of dual to ratio exponential type estimators. Bhushan and Gupta (2019) provided some log type estimators of population mean using DS. Zaman (2020) suggested generalized exponential estimators of population mean using an auxiliary attribute. Zaman and Kadilar (2021a) introduced a new class of exponential estimators for finite population mean in DS whereas Zaman and Kadilar (2021b) examined an exponential ratio and product estimators of population mean under stratified two-phase sampling. Zaman (2021) considered an efficient exponential estimator of the mean under stratified random sampling $(StRS)$. Bhushan et al. (2021a) suggested some efficient classes of estimators under $StRS$. Bhushan et al. (2021b) developed some efficient classes of estimators under DS. Recently, Bhushan and Kumar (2023) suggested a new efficient class of estimators of population mean using DS.

In this study, we develop an efficient class of estimators for population mean using auxiliary information in DS. The rest of the paper is organized as follows. In Section 2, we have reviewed the existing estimators under DS suggested till date. In Section 3, the suggested class of estimators is given along with its properties. In Section 4, the efficiency conditions are derived. In Section 5, an empirical study is carried out using some real data sets. Finally, the conclusion is given in Section 6.

2. Brushup of Relevant Works

Consider a finite population $\kappa = (\kappa_1, \kappa_2, ..., \kappa_N)$ of N identifiable units. Let y_i and x_i possess the values of the study and auxiliary variables for the i^{th} unit κ_i. Also, let the sample means $\bar{y} = n^{-1}\sum_{i=1}^{n} y_i$ and $\bar{x} = n^{-1}\sum_{i=1}^{n} x_i$ of the study and auxiliary variables be an unbiased estimator of population means $\bar{Y}$ and $\bar{X}$ respectively, obtained from the second sample of size n and $\bar{x}' = {n'}^{-1}\sum_{i=1}^{n'} x_i$ is sample mean obtained from the first sample of size n'. Let C_x and C_y be the coefficient of variation of auxiliary variable x and study variable y respectively. We consider two cases to draw the required sample under DS as discussed below:

Case I when the second phase sample of size n is a sub-sample of the first-phase sample of size n', and

Case II when the second phase sample of size n is drawn independently of the first-phase sample of size n', refer to Bose (1943).

The unbiased estimator of population mean $\bar{Y}$ of the study variable y is given as

$$\bar{y}_m = \bar{y}.$$

The variance of the estimator $\bar{y}_m$ is given by

$$V(\bar{y}_m) = f_n \bar{Y}^2 C_y^2,$$

where $f_n = (n^{-1} - N^{-1})$. In DS, Sukhatme (1962) suggested the conventional ratio estimator using auxiliary variable as

$$\bar{y}_r = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right).$$

The MSE expressions of the estimator $\bar{y}_r$ in case I and case II are given by

$$MSE(\bar{y}_r)_I = \bar{Y}^2[f_n C_y^2 + f_{nn'}(C_x^2 - 2\rho_{xy}C_xC_y)], \tag{1}$$

$$MSE(\bar{y}_r)_{II} = \bar{Y}^2[f_n C_y^2 + (f_n + f_{n'})C_x^2 - 2f_n\rho_{xy}C_xC_y], \tag{2}$$

where $f_n' = ({n'}^{-1} - N^{-1})$, $f_{nn'} = f_n - f_{n'}$ and C_x is the population coefficient of variation of auxiliary variable x and ρ_{xy} is the population coefficient of correlation between study variable and auxiliary variable.

The classical regression estimator using auxiliary information under DS is defined as

$$\bar{y}_{lr} = \bar{y} + b\left(\bar{x}' - \bar{x}\right),$$

where b is the regression coefficient of y on x. The MSE expressions of the estimator $\bar{y}_{lr}$ in Case I and Case II are given by

$$MSE(\bar{y}_{lr})_I = \bar{Y}^2\left[f_nC_y^2 + (f_n - f_{n'})\left(\frac{b}{R}\right)^2 C_x^2 - 2\left(\frac{b}{R}\right)(f_n - f_{n'})\rho_{xy}C_xC_y\right],$$

$$MSE(\bar{y}_{lr})_{II} = \bar{Y}^2\left[f_nC_y^2 + (f_n + f_{n'})\left(\frac{b}{R}\right)^2 C_x^2 - 2\left(\frac{b}{R}\right)f_n\rho_{xy}C_xC_y\right],$$

where $R = \bar{Y}/\bar{X}$. The minimum MSE of the estimator $\bar{y}_{lr}$ at optimum value of $b = R\rho_{xy}(C_y/C_x)$ in Case I and at optimum value of $b = (f_n/(f_n + f_{n'}))R\rho_{xy}(C_y/C_x)$ in Case II is given by

$$minMSE(\bar{y}_{lr})_I = \bar{Y}^2C_y^2[f_n - f_{nn'}\rho_{xy}^2], \tag{3}$$

$$minMSE(\bar{y}_{lr})_{II} = \bar{Y}^2f_nC_y^2\left[1 - \left(\frac{f_n}{f_n + f_{n'}}\right)\rho_{xy}^2\right]. \tag{4}$$

Kumar and Bahl (2006) suggested a class of dual to exponential type ratio estimator for finite population mean $\bar{Y}$ in DS as

$$\bar{y}_{kb} = \bar{y}\left(\frac{\bar{x}_d^*}{\bar{x}'}\right),$$

where $\bar{x}_d^* = [(1+g)\bar{x}' - g\bar{x}]$, $g = n/(n' - n)$.

The MSE expressions of the estimator $\bar{y}_{kb}$ in Case I and Case II are given by

$$MSE(\bar{y}_{kb})_I = \bar{Y}^2\left[f_nC_y^2 + \frac{n}{(n'-n)}f_{nn'}\left(\frac{n}{(n'-n)}C_x^2 - 2\rho_{xy}C_xC_y\right)\right], \tag{5}$$

$$MSE(\bar{y}_{kb})_{II} = \bar{Y}^2\left[f_nC_y^2 + \frac{n}{(n'-n)}\left(\frac{n}{(n'-n)}(f_n + f_{n'})C_x^2 - 2f_n\rho_{xy}C_xC_y\right)\right]. \tag{6}$$

On the lines of Bahl and Tuteja (1991), Singh and Vishwakarma (2007) evoked the ratio exponential estimator under DS as

$$\bar{y}_{sv} = \bar{y}\exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right).$$

The MSE of the estimator $\bar{y}_{sv}$ in Case I and Case II is given by

$$MSE(\bar{y}_{sv})_I = \bar{Y}^2\left[f_nC_y^2 + (f_n - f_{n'})\frac{C_x^2}{4} - (f_n - f_{n'})\rho_{xy}C_xC_y\right], \tag{7}$$

$$MSE(\bar{y}_{sv})_{II} = \bar{Y}^2\left[f_nC_y^2 + (f_n + f_{n'})\frac{C_x^2}{4} - f_n\rho_{xy}C_xC_y\right]. \tag{8}$$

Motivated by Prasad (1989), Singh (2011) developed the following ratio type estimator under DS as

$$\bar{y}_k = k\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)$$

where k is a scalar. The minimum MSE of the estimator $\bar{y}_k$ at optimum value of $k = [1 + f_{nn'}(C_x^2 - \rho_{xy}C_xC_y)]/[(1 + f_nC_y^2 + f_{nn'}(C_x^2 - 2\rho_{xy}C_xC_y) + 2f_{nn'}(C_x^2 - \rho_{xy}C_xC_y))]$ in Case I and at optimum value of $k = [1 + f_n\{C_x^2 - \rho_{xy}C_xC_y\}]/[\{1 + f_nC_y^2 + (f_n + f_{n'})C_x^2 - 2f_n\rho_{xy}C_xC_y\} + 2f_n(C_x^2 - \rho_{xy}C_xC_y)]$ in Case II is given by

$$minMSE(\bar{y}_k)_I = \bar{Y}^2\left[\begin{array}{l}(k-1)^2 + k^2\left\{f_nC_y^2 + f_{nn'}(C_x^2 - 2\rho_{xy}C_xC_y)\right\} \\ +2k(k-1)f_{nn'}(C_x^2 - \rho_{xy}C_xC_y)\end{array}\right], \tag{9}$$

$$minMSE(\bar{y}_k)_{II} = \bar{Y}^2\left[\begin{array}{l}(k-1)^2 + k^2\{f_nC_y^2 + (f_n + f_{n'})C_x^2 - 2f_n\rho_{xy}C_xC_y\} \\ +2k(k-1)(f_nC_x^2 - f_n\rho_{xy}C_xC_y)\end{array}\right]. \tag{10}$$

Ozgul and Cingi (2014) introduced a class of exponential regression cum ratio estimator under DS as

$$\bar{y}_{oc} = \{k_1\bar{y} + k_2(\bar{x}' - \bar{x})\} \exp\left(\frac{\bar{x}^{'*} - \bar{x}^*}{\bar{x}^{'*} + \bar{x}^*}\right),$$

where k_1, k_2 are suitably chosen scalars, $\bar{x}^{'*} = a\bar{x}' + b$ and $\bar{x}^* = a\bar{x} + b$ such that $a(\neq 0)$, b are either real values or function of some known parameters of auxiliary variable x namely, standard deviation S_x, coefficient of variation C_x, coefficient of skewness $\beta_1(x)$, coefficient of kurtosis $\beta_2(x)$ and known parameter of auxiliary variable x and study variable y as coefficient of correlation ρ_{xy}.

The minimum MSE of the estimator $\bar{y}_{oc}$ at $k_{1(opt)} = 1 - \{(2 - f_{nn'}\theta^2C_x^2)/(1 + f_n - f_{nn'}\rho_{xy}^2)\}$, $k_{2(opt)} = R\{(\theta - 1) + (1 - k_{1(opt)})(2\theta - \rho_{xy}(C_y/C_x))\}$ in case I and at $k_{1(opt)} = 1 - \{(2 - (f_n + f_{n'})\theta^2C_x^2)/(1 + f_n - f_n\rho_{xy}^2)\}$, $k_{2(opt)} = R\{(\theta - 1) + (1 - k_{1(opt)})(2\theta - \rho_{xy}(C_y/C_x))\}$ in case II is given by

$$MSE(\bar{y}_{oc})_I = \bar{Y}^2 \left[\frac{MSE(\bar{y}_{lr})_I(1 - f_{nn'}\theta^2C_x^2) - \left(\frac{f_{nn'}^2\theta^4C_x^4}{4}\right)}{\bar{Y}^2 + MSE(\bar{y}_{lr})_I}\right] \quad (11)$$

$$MSE(\bar{y}_{oc})_{II} = \bar{Y}^2 \left[\frac{MSE(\bar{y}_{lr})_{II}\{1 - (f_n + f_{n'})\theta^2C_x^2\} - \left(\frac{(f_n+f_{n'})^2\theta^4C_x^4}{4}\right)}{\bar{Y}^2 + MSE(\bar{y}_{lr})_{II}}\right] \quad (12)$$

where $\theta = a\bar{X}/2(a\bar{X} + b)$. Kalita et al. (2016) introduced the following exponential ratio-cum-exponential dual to ratio estimators in DS as

$$\bar{y}_{ks} = \bar{y}\left[\alpha \exp\left\{\frac{(\bar{x}' - \bar{x})}{(\bar{x}' + \bar{x})}\right\} + \beta \exp\left\{\frac{(\bar{x}_d^* - \bar{x}')}{(\bar{x}_d^* + \bar{x}')}\right\}\right]$$

where α and β are unknown constant such that $\alpha + \beta = 1$. The MSE of the estimator $\bar{y}_{ks}$ at optimum values of $\alpha = [\{2\rho_{xy}C_y/(1 - g)C_x\} - \{g/(1 - g)\}]$, $\beta = 1 - \alpha$ in Case I and at optimum values of $\alpha = [\{2f_n\rho_{xy}C_y/(f_n + f_{n'})(1 - g)C_x\} - \{g/(1 - g)\}]$, $\beta = 1 - \alpha$ in Case II is given by

$$MSE(\bar{y}_{ks})_I = \bar{Y}^2C_y^2\left[f_n - f_{nn'}\rho_{xy}^2\right], \quad (13)$$

$$MSE(\bar{y}_{ks})_{II} = \bar{Y}^2 f_n C_y^2\left[1 - \left(\frac{f_n}{f_n + f_{n'}}\right)\rho_{xy}^2\right]. \quad (14)$$

Following Kumar and Bahl (2006) and Kalita et al. (2016), Bazad and Bazad (2019) developed general class of dual to ratio exponential type estimators for population mean $\bar{Y}$ using auxiliary variable x in DS as

$$\bar{y}_{bb_1} = \bar{y}\left[\alpha \exp\left\{\frac{(\bar{x}^{'*} - \bar{x}^*)}{(\bar{x}^{'*} + \bar{x}^*)}\right\} + \beta \exp\left\{\frac{(\bar{x}_d^* - \bar{x}^{'*})}{(\bar{x}_d^* + \bar{x}^{'*})}\right\}\right].$$

The MSE expressions of the estimator $\bar{y}_{bb_1}$ in Case I and Case II is given by

$$MSE(\bar{y}_{bb_1})_I = \bar{Y}^2C_y^2\left[f_n - f_{nn'}\rho_{xy}^2\right], \quad (15)$$

$$MSE(\bar{y}_{bb_1})_{II} = \bar{Y}^2 f_n C_y^2\left[1 - \left(\frac{f_n}{f_n + f_{n'}}\right)\rho_{xy}^2\right]. \quad (16)$$

Bazad and Bazad (2019) suggested another class of dual to ratio exponential type estimator under DS as

$$\bar{y}_{bb_2} = \bar{y}\left[\alpha \exp\left\{\frac{(\bar{x}^{'*} - \bar{x}^*)}{(\bar{x}^{'*} + \bar{x}^*)}\right\} + \beta \exp\left\{\frac{(\bar{x}_d^* - \bar{x}^{'*})}{(\bar{x}_d^* + \bar{x}^{'*})}\right\} + \gamma \exp\left\{\frac{(\bar{x}_d^* - \bar{x}^*)}{(\bar{x}_d^* + \bar{x}^*)}\right\}\right],$$

where $\alpha+\beta+\gamma=1$. The MSE expressions of the estimator $\bar{y}_{bb_2}$ in case I and case II is given by

$$MSE(\bar{y}_{bb_2})_I = \bar{Y}^2 C_y^2 \left[f_n - f_{nn'}\rho_{xy}^2\right], \tag{17}$$

$$MSE(\bar{y}_{bb_2})_{II} = \bar{Y}^2 f_n C_y^2 \left[1 - \left(\frac{f_n}{f_n + f_{n'}}\right)\rho_{xy}^2\right]. \tag{18}$$

Bhushan and Gupta (2019) suggested log type estimators under DS as

$$\bar{y}_{g_1} = \bar{y}\left[1 + \log\left(\frac{\bar{x}}{\bar{x}'}\right)\right]^{\alpha_1},$$

$$\bar{y}_{g_2} = \bar{y}\left[1 + \alpha_2 \log\left(\frac{\bar{x}}{\bar{x}'}\right)\right],$$

where α_1 and α_2 are suitably chosen scalars. The MSE expressions of the estimator $\bar{y}_{g_i},\ i=1,2$ in Case I and Case II are given by

$$MSE(\bar{y}_{g_i})_I = \bar{Y}^2\left[f_n C_y^2 + (f_n - f_{n'})\alpha_i^2 C_x^2 + 2\alpha_i (f_n - f_{n'})\rho_{xy} C_x C_y\right]$$

$$MSE(\bar{y}_{g_i})_{II} = \bar{Y}^2\left[f_n C_y^2 + (f_n + f_{n'})\alpha_i^2 C_x^2 + 2\alpha_i f_n \rho_{xy} C_x C_y\right].$$

The minimum MSE of estimators $\bar{y}_{g_i},\ i=1,2$ at $\alpha_{i(opt)} = -\rho_{xy}(C_y/C_x)$ in Case I and at $\alpha_{i(opt)} = -(f_n/(f_n+f_{n'}))\rho_{xy}(C_y/C_x)$ in Case II is given by

$$minMSE(\bar{y}_{g_i})_I = \bar{Y}^2 C_y^2 [f_n - f_{nn'}\rho_{xy}^2], \tag{19}$$

$$minMSE(\bar{y}_{g_i})_{II} = \bar{Y}^2 f_n C_y^2 \left[1 - \left(\frac{f_n}{f_n + f_{n'}}\right)\rho_{xy}^2\right]. \tag{20}$$

It is seen that Kalita et al. (2016) estimator $\bar{y}_{ks}$, Bazad and Bazad (2019) estimators $\bar{y}_{bb_1}$, $\bar{y}_{bb_2}$ and Bhushan and Gupta (2019) estimators $\bar{y}_{g_i},\ i=1,2$ attain the minimum MSE of the classical regression estimator under DS.

3. Suggested Class of Estimators

Motivated by Bhushan et al. (2021c), we suggest an efficient class of estimators for estimating the population mean $\bar{Y}$ of study variable y using information on auxiliary variable x under DS as

$$T_p = \left[w_1\bar{y} + w_2\bar{y}\left(\frac{\bar{x}'^*}{\theta\bar{x}^* + (1-\theta)\bar{x}'^*}\right)^g\right]\left[1 + \log\left(\frac{\bar{x}^*}{\bar{x}'^*}\right)\right]^{\alpha_1},$$

where w_1, w_2 and α_1 are suitably chosen scalars and θ, g are real constants which assume real values to design different estimators. The value of α_1 can be obtained from Bhushan and Gupta (2019) estimator $\bar{y}_{g_1}$. We would like to note that:

(i). for (w_1, w_2, α_1)=(1,0,0); $T_p \to \bar{y}_m$ (Usual mean estimator)

(ii). for $(w_1, w_2, g, \theta, a, b, \alpha_1)$=(0,1,1,1,1,0,0); $T_p \to \bar{y}_r$ (Classical ratio estimator)

(iii). for $(w_1, w_2, g, \theta, a, b, \alpha_1)$=(0,k,1,1,1,0,0); $T_p \to \bar{y}_k$ (Singh (2011) estimator)

(iv). for (w_1, w_2, α_1)=(1,0,α_1); $T_p \to \bar{y}_{g_1}$ (Bhushan and Gupta (2019) estimator)

Several other estimators of different parameters can be generated from the envisaged estimator T_p for appropriate values of scalars $(w_1, w_2, g, \theta, a, b, \alpha_1)$.

Case I To derive the MSE of the suggested family of estimators, let us assume that $\bar{y} = \bar{Y}(1+e_0)$, $\bar{x} = \bar{X}(1+e_1)$ and $\bar{x}' = \bar{X}(1+e_1')$ such that $E(e_0) = E(e_1) = E(e_1') = 0$,

$E(e_0^2) = f_n C_y^2, \quad E(e_1^2) = f_n C_x^2, \quad E(e_1'^2) = f_{n'} C_x^2, \quad E(e_0, e_1) = f_n \rho C_x C_y,$
$E(e_1 e_1') = f_{n'} C_x^2, \quad E(e_0 e_1') = f_{n'} \rho C_x C_y.$
where $C_y = S_y/\bar{Y}$, $C_x = S_x/\bar{X}$, $\rho_{xy} = S_{xy}/S_x S_y$, $S_y^2 = \sum_{i=1}^N (y_i - \bar{Y})^2/(N-1)$, $S_x^2 = \sum_{i=1}^N (x_i - \bar{X})^2/(N-1)$ and $S_{yx}^2 = \sum_{i=1}^N (y_i - \bar{Y})(x_i - \bar{X})/(N-1)$.
The suggested estimator T_p is expressed in terms of $e's$ as

$$T_p = \begin{array}{c} \bar{Y}\left[w_1(1+e_0) + w_2(1+e_0)\left\{1+\theta(\nu e_1 - \nu e_1' + \nu^2 e_1'^2 - \nu^2 e_1 e_1')\right\}^{-g}\right] \\ \times\left[1+\nu e_1 - \nu e_1' - \frac{\nu^2 e_1^2}{2} + \frac{\nu^2 e_1'^2}{2}\right]^{\alpha_1} \end{array},$$

$$T_p - \bar{Y} = \bar{Y}\left[\begin{array}{l} w_1\left\{\begin{array}{l} 1+e_0+\alpha_1\left(\nu e_1 - \nu e_1' - \nu^2 e^2 + \nu e_0 e_1 - \nu e_0 e_1' + \nu^2 e_1 e_1'\right) \\ +\frac{\alpha_1^2}{2}\left(\nu^2 e_1^2 + \nu^2 e_1'^2 - 2\nu^2 e_1 e_1'\right) \end{array}\right\} \\ +w_2\left\{\begin{array}{l} 1+e_0+\alpha_1\left(\nu e_0 e_1 + \nu^2 e_1 e_1' - \nu e_0 e_1' - \nu^2 e_1^2\right) \\ -g\theta\left(\nu e_1 - \nu e_1' + \nu^2 e_1'^2 - \nu^2 e_1 e_1' + \nu e_0 e_1 - \nu e_0 e_1'\right) \\ +\frac{g(g+1)}{2}\theta^2\left(\nu^2 e_1^2 + \nu^2 e_1'^2 - 2\nu^2 e_1 e_1'\right) \\ +\frac{\alpha_1^2}{2}\left(\nu^2 e_1^2 + \nu^2 e_1'^2 - 2\nu^2 e_1 e_1'\right) \\ -g\theta\alpha_1\left(\nu^2 e_1^2 + \nu^2 e_1'^2 - 2\nu^2 e_1 e_1'\right) \end{array}\right\} - 1 \end{array}\right]. \quad (21)$$

Squaring and taking expectation both sides of (21), we get the MSE of the suggested estimator up to first order approximation as

$$MSE(T_p) = \bar{Y}^2\left[1 + w_1^2 E_1 + w_2^2 E_2 + 2w_1 w_2 E_3 - 2w_1 E_4 - 2w_2 E_5\right], \quad (22)$$

where

$$E_1 = [1 + f_n C_y^2 + (2\alpha_1^2\nu^2 - 2\alpha_1\nu^2)(f_n - f_{n'})C_x^2 + 4\alpha_1\nu(f_n - f_{n'})\rho_{xy}C_y C_x]$$
$$E_2 = \left[\begin{array}{l} 1 + f_n C_y^2 + \left\{2\alpha_1^2\nu^2 - 2\alpha_1\nu^2 + g^2\theta^2\nu^2 - 4\alpha_1 g\theta\nu^2 + g(g+1)\theta^2\nu^2\right\}(f_n - f_{n'})C_x^2 \\ +4\nu(\alpha_1 - g\theta)(f_n - f_{n'})\rho_{xy}C_y C_x \end{array}\right]$$
$$E_3 = \left[\begin{array}{l} 1 + f_n C_y^2 + \left\{2\alpha_1^2\nu^2 - 2\alpha_1 g\theta\nu^2 - 2\alpha_1\nu^2 + \frac{g(g+1)}{2}\theta^2\nu^2\right\}(f_n - f_{n'})C_x^2 \\ +2\nu(2\alpha_1 - g\theta)(f_n - f_{n'})\rho_{xy}C_y C_x \end{array}\right]$$
$$E_4 = \left[1 + \left\{\frac{\alpha_1^2}{2}\nu^2 - \alpha_1\nu^2\right\}(f_n - f_{n'})C_x^2 + \alpha_1\nu(f_n - f_{n'})\rho_{xy}C_y C_x\right]$$
$$E_5 = \left[\begin{array}{l} 1 + \left\{\frac{\alpha_1^2}{2}\nu^2 - \alpha_1\nu^2 - \alpha_1 g\theta\nu^2 + \frac{g(g+1)}{2}\theta^2\nu^2\right\}(f_n - f_{n'})C_x^2 \\ +(\alpha_1\nu - g\theta\nu)(f_n - f_{n'})\rho_{xy}C_y C_x \end{array}\right]$$
$$\nu = \frac{a\bar{X}}{(a\bar{X} + b)}.$$

The optimum values of w_1 and w_2 can be obtained by minimizing (22) w.r.t. w_1 and w_2 as

$$w_{1(opt)} = \frac{(E_2E_4 - E_3E_5)}{(E_1E_2 - E_3^2)},$$
$$w_{2(opt)} = \frac{(E_1E_5 - E_3E_4)}{(E_1E_2 - E_3^2)}.$$

Now, the minimum MSE can be obtained by putting optimum values of w_1 and w_2 in (22) as

$$minMSE(T_p) = \bar{Y}^2\left[1 - \frac{(E_1E_5^2 + E_2E_4^2 - 2E_3E_4E_5)}{(E_1E_2 - E_3^2)}\right] \quad (23)$$

Case II To obtain the MSE of the suggested class of estimators, let
$\bar{y} = \bar{Y}(1+e_0)$, $\bar{x} = \bar{X}(1+e_1)$ and $\bar{x}' = \bar{X}(1+e_1')$ such that $E(e_0) = E(e_1) = E(e_1') = 0$,

$E(e_0^2) = f_n C_y^2, \quad E(e_1^2) = f_n C_x^2, \quad E(e_1'^2) = f_{n'} C_x^2, \quad E(e_0, e_1) = f_n \rho C_x C_y,$
$E(e_1 e_1') = 0, \quad E(e_0 e_1') = 0.$

Squaring and taking expectation both sides of (21), we get the MSE of the suggested class of estimators up to first order approximation as

$$MSE(T_p) = \bar{Y}^2 \left[1 + w_1^2 F_1 + w_2^2 F_2 + 2w_1 w_2 F_3 - 2w_1 F_4 - 2w_2 F_5\right], \tag{24}$$

where

$$F_1 = [1 + f_n C_y^2 + (2\alpha_1^2\nu^2 - 2\alpha_1\nu^2)(f_n + f_{n'})C_x^2 + 4\alpha_1\nu f_n \rho_{xy} C_y C_x]$$

$$F_2 = \begin{bmatrix} 1 + f_n C_y^2 + \left\{2\alpha_1^2\nu^2 - 2\alpha_1\nu^2 + g^2\theta^2\nu^2 - 4\alpha_1 g\theta\nu^2 + g(g+1)\theta^2\nu^2\right\}(f_n + f_{n'})C_x^2 \\ +4\nu(\alpha_1 - g\theta) f_n \rho_{xy} C_y C_x \end{bmatrix}$$

$$F_3 = \begin{bmatrix} 1 + f_n C_y^2 + \left\{2\alpha_1^2\nu^2 - 2\alpha_1 g\theta\nu^2 - 2\alpha_1\nu^2 + \frac{g(g+1)}{2}\theta^2\nu^2\right\}(f_n + f_{n'})C_x^2 \\ +2\nu(2\alpha_1 - g\theta) f_n \rho_{xy} C_y C_x \end{bmatrix}$$

$$F_4 = \left[\ 1 + \left\{\frac{\alpha_1^2}{2}\nu^2 - \alpha_1\nu^2\right\}(f_n + f_{n'})C_x^2 + \alpha_1\nu f_n \rho_{xy} C_y C_x\ \right]$$

$$F_5 = \left[\ 1 + \left\{\frac{\alpha_1^2}{2}\nu^2 - \alpha_1\nu^2 - \alpha_1 g\theta\nu^2 + \frac{g(g+1)}{2}\theta^2\nu^2\right\}(f_n + f_{n'})C_x^2 + (\alpha_1\nu - g\theta\nu) f_n \rho_{xy} C_y C_x\ \right].$$

The optimum values of w_1 and w_2 can be obtained by minimizing (24) w.r.t. w_1 and w_2 as

$$w_{1(opt)} = \frac{(F_2F_4 - F_3F_5)}{(F_1F_2 - F_3^2)},$$
$$w_{2(opt)} = \frac{(F_1F_5 - F_3F_4)}{(F_1F_2 - F_3^2)}.$$

Now, the minimum MSE can be obtained by putting optimum values of w_1 and w_2 in (24) as

$$minMSE(T_p) = \bar{Y}^2 \left[1 - \frac{(F_1F_5^2 + F_2F_4^2 - 2F_3F_4F_5)}{(F_1F_2 - F_3^2)}\right]. \tag{25}$$

Corollary 1 *The suggested class of estimators perform better in Case II as compare to Ccase I, iff*

$$\frac{(E_1E_5^2 + E_2E_4^2 - 2E_3E_4E_5)}{(E_1E_2 - E_3^2)} > \frac{(F_1F_5^2 + F_2F_4^2 - 2F_3F_4F_5)}{(F_1F_2 - F_3^2)}, \tag{26}$$

otherwise, equally efficient in both the cases if equality holds in (26).

Proof: On comparing the minimum MSE of the suggested estimators obtained in Case I and Case II from (23) and (25), we obtain (26).

4. Efficiency Conditions

This section considers the efficiency comparison of the suggested class of estimators with the existing estimators in Case I and Case II.

Case I On comparing the minimum MSE of the suggested class of estimators T_p from (23)

1. with the variance of usual mean estimator $\bar{y}_m$ from (2), we get

$$\frac{(E_1E_5^2 + E_2E_4^2 - 2E_3E_4E_5)}{(E_1E_2 - E_3^2)} > 1 - f_n C_y^2.$$

2. with the MSE of classical ratio estimator $\bar{y}_r$ from (1), we get

$$\frac{(E_1E_5^2 + E_2E_4^2 - 2E_3E_4E_5)}{(E_1E_2 - E_3^2)} > 1 - f_n C_y^2 - (f_n - f_{n'})(C_x^2 - 2\rho_{xy} C_x C_y).$$

Table 1 Some members of the proposed class of estimator T_p

Estimators	a	b
$T_{p_{(1)}}$	1	0
$T_{p_{(2)}}$	1	C_x
$T_{p_{(3)}}$	1	$\beta_2(x)$
$T_{p_{(4)}}$	C_x	$\beta_2(x)$
$T_{p_{(5)}}$	$\beta_2(x)$	C_x
$T_{p_{(6)}}$	1	ρ_{xy}
$T_{p_{(7)}}$	C_x	ρ
$T_{p_{(8)}}$	ρ_{xy}	C_x
$T_{p_{(9)}}$	$\beta_2(x)$	ρ_{xy}
$T_{p_{(10)}}$	ρ_{xy}	$\beta_2(x)$

3. with the MSE of classical regression estimator $\bar{y}_{lr}$ from (3), we get

$$\frac{(E_1E_5^2+E_2E_4^2-2E_3E_4E_5)}{(E_1E_2-E_3^2)}>1-C_y^2[f_n-f_{nn'}\rho_{xy}^2].$$

4. with the MSE of Kumar and Bahl (2006) estimator $\bar{y}_{kb}$ from (5), we get

$$\frac{(E_1E_5^2+E_2E_4^2-2E_3E_4E_5)}{(E_1E_2-E_3^2)}>1-\left[f_nC_y^2+\frac{n}{(n'-n)}f_{nn'}\left(\frac{n}{(n'-n)}C_x^2-2\rho_{xy}C_xC_y\right)\right].$$

5. with the MSE of Singh and Vishwakarma (2007) estimator $\bar{y}_{sv}$ from (7), we get

$$\frac{(E_1E_5^2+E_2E_4^2-2E_3E_4E_5)}{(E_1E_2-E_3^2)}>1-\left[f_nC_y^2+(f_n-f_{n'})\frac{C_x^2}{4}-(f_n-f_{n'})\rho_{xy}C_xC_y\right].$$

6. with the MSE of Singh (2011) estimator $\bar{y}_k$ from (9), we get

$$\frac{(E_1E_5^2+E_2E_4^2-2E_3E_4E_5)}{(E_1E_2-E_3^2)}>1-\left[\begin{array}{l}(k-1)^2+k^2\left\{f_nC_y^2+f_{nn'}(C_x^2-2\rho_{xy}C_xC_y)\right\}\\+2k(k-1)f_{nn'}(C_x^2-\rho C_xC_y)\end{array}\right].$$

7. with the MSE of Ozgul and Cingi (2014) estimator $\bar{y}_{oc}$ from (11), we get

$$\frac{(E_1E_5^2+E_2E_4^2-2E_3E_4E_5)}{(E_1E_2-E_3^2)}>1-\left[\frac{MSE(\bar{y}_{lr})_I(1-f_{nn'}\theta^2C_x^2)-\left(\frac{f_{nn'}^2\theta^4C_x^4}{4}\right)}{\bar{Y}^2+MSE(\bar{y}_{lr})_I}\right].$$

8. with the MSE of Kalita et al. (2016) estimator $\bar{y}_{ks}$ from (13), we get

$$\frac{(E_1E_5^2+E_2E_4^2-2E_3E_4E_5)}{(E_1E_2-E_3^2)}>1-C_y^2\left[f_n-f_{nn'}\rho_{xy}^2\right].$$

9. with the MSE of Bazad and Bazad (2019) estimators $\bar{y}_{bb_i},\ i=1,2$ from (15) and (17), we get

$$\frac{(E_1E_5^2+E_2E_4^2-2E_3E_4E_5)}{(E_1E_2-E_3^2)}>1-C_y^2\left[f_n-f_{nn'}\rho_{xy}^2\right].$$

10. with the MSE of Bhushan and Gupta (2019) estimators $\bar{y}_{g_i},\ i=1,2$ from (19) and (20), we get

$$\frac{(E_1E_5^2+E_2E_4^2-2E_3E_4E_5)}{(E_1E_2-E_3^2)} > 1-C_y^2\left[f_n-f_{nn'}\rho_{xy}^2\right].$$

Case II On comparing the minimum MSE of the suggested class of estimators T_p from (25)

11. with the variance of usual mean estimator $\bar{y}_m$ from (2), we get

$$\frac{(F_1F_5^2+F_2F_4^2-2F_3F_4F_5)}{(F_1F_2-F_3^2)} > 1-f_nC_y^2.$$

12. with the MSE of classical ratio estimator $\bar{y}_r$ from (1), we get

$$\frac{(F_1F_5^2+F_2F_4^2-2F_3F_4F_5)}{(F_1F_2-F_3^2)} > 1-f_nC_y^2-(f_n+f_{n'})C_x^2+2f_n\rho_{xy}C_xC_y.$$

13. with the MSE of classical regression estimator $\bar{y}_{lr}$ from (4), we get

$$\frac{(F_1F_5^2+F_2F_4^2-2F_3F_4F_5)}{(F_1F_2-F_3^2)} > 1-f_nC_y^2\left[1-\left(\frac{f_n}{f_n+f_{n'}}\right)\rho_{xy}^2\right].$$

14. with the MSE of Kumar and Bahl (2006) estimator $\bar{y}_{kb}$ from (6), we get

$$\frac{(F_1F_5^2+F_2F_4^2-2F_3F_4F_5)}{(F_1F_2-F_3^2)} > 1-\left[f_nC_y^2+\frac{n}{(n'-n)}\left(\frac{n}{(n'-n)}(f_n+f_{n'})C_x^2-2f_n\rho_{xy}C_xC_y\right)\right].$$

15. with the MSE of Singh and Vishwakarma (2007) estimator $\bar{y}_{sv}$ from (8), we get

$$\frac{(F_1F_5^2+F_2F_4^2-2F_3F_4F_5)}{(F_1F_2-F_3^2)} > 1-\left[f_nC_y^2+(f_n+f_{n'})\frac{C_x^2}{4}-f_n\rho_{xy}C_xC_y\right].$$

16. with the MSE of Singh (2011) estimator $\bar{y}_k$ from (10), we get

$$\frac{(F_1F_5^2+F_2F_4^2-2F_3F_4F_5)}{(F_1F_2-F_3^2)} > 1-\left[\begin{array}{l}(k-1)^2+k^2\{f_nC_y^2+(f_n+f_{n'})C_x^2-2f_n\rho_{xy}C_xC_y\}\\ +2k(k-1)(f_nC_x^2-f_n\rho_{xy}C_xC_y)\end{array}\right].$$

17. with the MSE of Ozgul and Cingi (2014) estimator $\bar{y}_{oc}$ from (12), we get

$$\frac{(F_1F_5^2+F_2F_4^2-2F_3F_4F_5)}{(F_1F_2-F_3^2)} > 1-\left[\frac{MSE(\bar{y}_{lr})_{II}\{1-(f_n+f_{n'})\theta^2C_x^2\}-\left(\frac{(f_n+f_{n'})^2\theta^4C_x^4}{4}\right)}{\bar{Y}^2+MSE(\bar{y}_{lr})_{II}}\right].$$

18. with the MSE of Kalita et al. (2016) estimator $\bar{y}_{ks}$ from (14), we get

$$\frac{(F_1F_5^2+F_2F_4^2-2F_3F_4F_5)}{(F_1F_2-F_3^2)} > 1-f_nC_y^2\left[1-\left(\frac{f_n}{f_n+f_{n'}}\right)\rho_{xy}^2\right].$$

19. with the MSE of Bazad and Bazad (2019) estimators $\bar{y}_{bb_i},\ i=1,2$ from (16) and (18), we get

$$\frac{(F_1F_5^2+F_2F_4^2-2F_3F_4F_5)}{(F_1F_2-F_3^2)} > 1-f_nC_y^2\left[1-\left(\frac{f_n}{f_n+f_{n'}}\right)\rho_{xy}^2\right].$$

20. with the MSE of Bhushan and Gupta (2019) estimators $\bar{y}_{g_i},\ i=1,2$ from (20), we get

$$\frac{(F_1F_5^2+F_2F_4^2-2F_3F_4F_5)}{(F_1F_2-F_3^2)} > 1-f_nC_y^2\left[1-\left(\frac{f_n}{f_n+f_{n'}}\right)\rho_{xy}^2\right].$$

Under these conditions, the proposed class of estimators dominate the estimators reviewed in previous section. Further, whether these conditions hold in practice is verified through an empirical study.

5. Empirical study

To illustrate the merits of the suggested class of estimators, we have accomplished an empirical study over four real populations which are described below:

Population 1: [Source: Cingi et al. (2007)]
Y=the number of teachers, X=the number of student in both primary and secondary school for 923 districts, $\bar{Y}$=436.3, $\bar{X}$=11440.50, C_x=1.86, C_y=1.72, ρ_{xy}=0.955, n=200, n'=400 and N=923.

Population 2: [Source: Kadilar and Cingi (2006)]
Y=level of apple production, X=number of apple trees, $\bar{Y}$=625.37, $\bar{X}$=13.93, C_x=1.653, C_y=1.866, ρ_{xy}=0.865, n=20, n'=40 and N=104.

Population 3: [Source: Murthy (1967)]
Y=output, X=fixed capital, $\bar{Y}$=51.826, $\bar{X}$=11.265, C_x=0.751, C_y=0.354, ρ_{xy}=0.9413, n=20, n'=40 and N=80.

Population 4: [Source: Kumar and Bahl (2006), p. 324]
Y=number of cultivators in the village, X=population of village, $\bar{Y}$=449.846, $\bar{X}$=2909.105, C_x=0.7696, C_y=0.8871, ρ_{xy}=0.8818, n=20, n'=95 and N=487.

Using the above data sets, we have computed the MSE and percent relative efficiency (PRE) of the existing and suggested estimators. The PRE of different estimators T=($\bar{y}_m$, $\bar{y}_r$, $\bar{y}_{lr}$, $\bar{y}_{kb}$, $\bar{y}_{sv}$, $\bar{y}_k$, $\bar{y}_{oc}$, $\bar{y}_{bb_1}$, $\bar{y}_{bb_2}$, $\bar{y}_{g_1}$, $\bar{y}_{g_2}$ and T_p) is calculated w.r.t. usual mean estimator $\bar{y}_m$ using the following formula.

$$PRE = \frac{V(\bar{y}_m)}{MSE(T)} \times 100.$$

The empirical results for population 1 to population 4 are summarized in Table 2 to Table 3. These results show that the suggested class of estimators fares better in terms of lesser MSE and greater PRE than the other existing estimators discussed in this study.

6. Conclusions

In this study, we have opined an efficient class of estimators of population mean $\bar{Y}$ of study variable y using the information on an auxiliary variable under double sampling. The usual mean estimator $\bar{y}_m$, classical ratio estimator $\bar{y}_r$ envisaged by Sukhatme (1962), Singh (2011) estimator $\bar{y}_k$ and Bhushan and Gupta (2019) estimator $\bar{y}_{g_1}$ are identified as the members of the suggested class of estimators. The MSE expression of the proposed class of estimator is obtained to the first order of approximation. The theoretical results of the suggested class of estimators have been obtained and illustrated by an empirical study carried out over four real data sets. From the perusal of the empirical results of Table 2 and Table 3, we have the following observations:

(i). the suggested class of estimators T_p performs better than the usual mean estimator $\bar{y}_m$, classical ratio and regression estimators $\bar{y}_r$ & $\bar{y}_{lr}$, Kumar and Bahl (2006) estimator $\bar{y}_{kb}$, Singh and Vishwakarma (2007) estimators $\bar{y}_{sv}$, Singh (2011) estimator $\bar{y}_k$, Ozgul and Cingi (2014) estimator$\bar{y}_{oc}$, Kalita et al. (2016) estimator $\bar{y}_{kc}$, Bazad and Bazad (2019) estimators $\bar{y}_{bb_i},\ i = 1, 2$ and Bhushan and Gupta (2019) estimators $\bar{y}_{g_i},\ i = 1, 2$ in terms of the minimum MSE and maximum PRE.

(ii). the proposed class of estimators performs better in the case II as compare to the case I in each populations.

Thus, we firmly advocate to consider the suggested class of estimators in the usual practice. Further, the proposed class of estimators can also be established using information on auxiliary attribute and it is authors future direction of work.

Table 2 MSE and PRE of different estimators

	Population 1				Population 2			
	Case I		Case II		Case I		Case II	
Estimators	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
$\bar{y}_m$	2205.63	100	2205.63	100	54993.75	100	54993.75	100
$\bar{y}_r$	944.10	233.62	1162.18	189.78	29536.17	186.19	30310.14	181.43
$\bar{y}_{lr}$	921.60	239.32	728.35	302.82	29521.36	186.28	25197.14	218.25
$\bar{y}_{kb}$	944.10	233.62	1162.18	189.78	29536.17	186.19	30310.14	181.43
$\bar{y}_{sv}$	1163.26	189.60	805.85	273.70	35586.14	154.53	27753.03	198.15
$\bar{y}_k$	941.16	234.35	1162.15	189.78	27662.89	198.79	29939.74	183.68
$\bar{y}_{oc}$	909.23	242.58	712.19	309.69	25574.27	215.03	20064.75	274.08
$\bar{y}_{ks}$	921.60	239.32	728.35	302.82	29521.36	186.28	25197.14	218.25
$\bar{y}_{bb_1}$	921.60	239.32	728.35	302.82	29521.36	186.28	25197.14	218.25
$\bar{y}_{bb_2}$	921.60	239.32	728.35	302.82	29521.36	186.28	25197.14	218.25
$\bar{y}_{g_i},\ i=1,2$	921.60	239.32	728.35	302.82	29521.36	186.28	25197.14	218.25
T_p^d	**905.89**	**243.47**	**701.39**	**314.46**	**25151.42**	**218.65**	**18509.44**	**297.11**

Table 3 MSE and PRE of different estimators

	Population 3				Population 4			
	Case I		Case II		Case I		Case II	
Estimators	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
$\bar{y}_m$	12.62	100	12.62	100	7635.38	100	7635.38	100
$\bar{y}_r$	16.88	74.74	37.95	33.25	2748.77	277.77	2715.40	281.18
$\bar{y}_{lr}$	5.16	244.31	4.23	298.09	2747.50	277.90	2589.93	294.80
$\bar{y}_{kb}$	16.88	74.74	37.95	33.25	5407.08	141.21	5001.00	152.67
$\bar{y}_{sv}$	5.28	238.77	6.35	198.69	4009.29	190.44	3484.84	219.10
$\bar{y}_k$	16.87	74.77	37.89	33.30	2710.73	281.67	2700.07	282.78
$\bar{y}_{oc}$	5.08	248.29	4.10	307.24	2647.32	288.41	2471.75	308.90
$\bar{y}_{ks}$	5.16	244.31	4.23	298.09	2747.50	277.90	2589.93	294.80
$\bar{y}_{bb_1}$	5.16	244.31	4.23	298.09	2747.50	277.90	2589.93	294.80
$\bar{y}_{bb_2}$	5.16	244.31	4.23	298.09	2747.50	277.90	2589.93	294.80
$\bar{y}_{g_i},\ i=1,2$	5.16	244.31	4.23	298.09	2747.50	277.90	2589.93	294.80
T_p	**5.04**	**250.33**	**3.99**	**315.99**	**2620.07**	**291.41**	**2419.51**	**315.57**

References

Bahl S, Tuteja RK. Ratio and product type exponential estimators. J Inf Optim Sci. 1991; 12(1): 159-164.

Bazad R, Bazad P. Two phase sampling for estimation of population mean with auxiliary information using exponential type of estimator. J Stat Manag Syst. 2019; DOI: 10.1080/09720510.2019.1592858.

Bhushan S, Gupta R. Some new log type class of double sampling estimators. Int J Agric Stat Sci. 2019; 14(1): 31-39.

Bhushan S, Kumar A, Singh S. Some efficient classes of estimators under stratified sampling. Commun Stat - Theory Methods. 2021a; DOI: 10.1080/03610926.2021.1939052.

Bhushan S, Kumar A, Kumar S. Singh S. On Some efficient classes of estimators under double sampling. Int J Agric Stat Sci. 2021b; 17(2): 519-524.

Bhushan S, Kumar A, Singh S, Kumar S. An improved class of estimators of population mean under simple random sampling. Philipp Stat. 2021c; 70(1): 33-47.

Bhushan S, Kumar A. New efficient class of estimators of population mean using two-phase sampling. Int J Math Oper Res. 2023; 24(2): 155-172.

Bose C. Note on the sampling error in the method of double sampling. Sankhya: Indian J Stat. 1943; 6: 330.

Cingi H, Kadilar C, Kocberber G. Determination of the opportunities of the primary and the secondary schools in Turkey and suggestions to solve the determined problems. 2007; (TUBITAK, SOBAG106K077 (in Turkish).

Kadilar C, Cingi H. An Improvement in estimating the population mean by using the correlation coefficient. Hacettepe J Math Stat. 2006; 35(1): 103-109.

Kalita D, Singh BK, Choudhury S. Exponential ratio-cum-exponential dual to ratio estimator in double sampling. Forging Connections between Computational Mathematics and Computational Geometry. Proc Math Stat., Springer, 2016; 124: 211-220. DOI:10.5176/2251-1911_CMCGS14.05.

Kumar M, Bahl S. A class of dual to ratio estimator in double sampling. Stat Pap. 2006; 47: 319-326.

Murthy MN. Sampling theory and methods, Calcutta: Statistical Publishing Society; 1967.

Neyman J. Contribution to the theory of sampling human populations. J Am Stat Assoc. 1938; 33: 101-116.

Ozgul N, Cingi H. A new class of exponential regression cum ratio estimator in two phase sampling. Hacettepe J Math Stat. 2014; 43(1): 131-140.

Prasad B. Some improved ratio type estimators of population mean and ratio in finite population sample surveys. Commun Stat - Theory Methods. 1989; 18: 379-392.

Singh HP, Vishwakarma G. Modified exponential ratio and product estimators for finite population mean in double sampling. Austrian J Stat. 2007; 36(3): 217-225.

Singh B. A new ratio estimator for finite population mean in double sampling. Int J Agric Stat Sci. 2011; 7(2): 651-656.

Srivenkataramana T. A dual to ratio estimator in sample surveys. Biometrika. 1980; 67: 199-204.

Sukhatme BV. Some ratio-type estimators in two-phase sampling. J Am Stat Assoc. 1962; 57: 628-632.

Zaman T. Generalized exponential estimators for the finite population mean. Stat Transit. 2020; 21(1): 159-168.

Zaman T, Kadilar C. New class of exponential estimators for finite population mean in two-phase sampling. Commun Stat - Theory Methods. 2021a; 50(4): 874-889.

Zaman T, Kadilar C. Exponential ratio and product type estimators of the mean in stratified two-phase sampling. AIMS Math. 2021b; 6(5): 4265-4279.

Zaman T. An efficient exponential estimator of the mean under stratified random sampling. Math Popul Stud. 2021; 28(2): 104-121.