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Remodeling Multivariate Control Chart by Using Spatial Signed Rank for Detecting Mean Shift in Normal and Non-Normal Processes

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Abstract

This research aimed to modify the traditional multivariate control charts by using the multivariate spatial signed rank under the normal distribution, the t distribution, and the gamma distribution. The performance of the modern multivariate control charts is measured based on the average run length (ARL). The ARL is computed using a Monte Carlo simulation. The Monte Carlo approach is applied to simulate the circumstances via MATLAB software. The spatial signed-rank multivariate exponentially weighted moving average (SSRM) control chart is found to be the most appropriate approach to detect the small mean shifts $(\delta \le 0.5)$ and the large smoothing parameters $(\lambda \ge 0.35)$ of all three distributions. Besides, SSRM is a robust tool for detecting waste and is suitable for most industrial processes.

Keywords: Multivariate exponentially weighted moving average (MEWMA), statistical process control (SPC), average run length (ARL), detection of nonconforming product, correlation of quality characteristics.

1. Introduction

Products that are manufactured using the same machine might still yield different results. Most products are made according to detailed specifications. However, some products may not be according to the specifications, which are called nonconforming products or wastes. Therefore, nonconforming products lead to a negative effect on a company not only through the loss of raw materials but also through the cost of man-hours, scrap, and rework, which are called internal failure costs. Internal failure costs are incurred when products, components, materials, and services fail to meet quality requirements. Statistical process control (SPC) is a method for monitoring and controlling a process in order to improve the process's performance while reducing the variability in key parameters (Montgomery 2009). Moreover, it is used to monitor and reduce variability, enabling processes to return quickly to a state of control (Montgomery 2013). Univariate control charts are used to monitor and control a single quality characteristic. When several correlated quality characteristics must be monitored, some independent control charts might be used incorrectly to monitor and control each quality characteristic separately. Consequently, using separate control charts to monitor and control multiple correlated quality characteristics is not an efficient way to monitor correlated characteristics due to the fact that the correlation among these characteristics weakens the performance of the single control charts and also results in a higher false alarm risk. Thus, the multivariate control chart has become an issue and has played an important role at the present time.

Shamma and Shamma (1992) studied the development and evaluation of control charts using double exponentially weighted moving average (dEWMA). The dEWMA control chart performed much for small and moderate shifts in the process mean better than a Shewhart control chart. The properties were similar to an exponentially weighted moving average (EWMA) control chart, but the dEWMA had smaller variability and had more smoothing of the data with no compromise in the sensitivity of detecting the shifts in the process mean. It also had an optimal average run length (ARL) for a larger smoothing parameter when compared with the EWMA control chart which the properties were more desirable for some industrial processes. Alkahtani (2013) studied the robustness of the dEWMA versus EWMA control charts to non-normal processes. The EWMA and dEWMA charts were more robust to the t distribution. The dEWMA was more robust to non-normality for larger smoothing parameters. Furthermore, Shamma and Shamma (1992) pointed out that Baxley (1990) found a simulated industrial process required a larger λ $(\lambda = 0.35)$ but the optimal EWMA control chart required $\lambda = 0.05.$ The dEWMA control chart was more sensitive to larger smoothing parameters than the EWMA. Alkahtani and Schaffer (2012) studied a double multivariate exponentially weighted moving average (dMEWMA) control chart for process location monitoring. The dMEWMA outperformed the MEWMA and Hotelling's χ^2 control charts for small and large shifts. In comparison to the MEWMA control chart, the dMEWMA chart was optimal for larger smoothing parameters (λ) and performs much better for very small shifts in the process mean. Tiengket et al. (2020) studied the construction of bivariate copulas on Hotelling's T^2 control chart, and the bivariate copulas approach can be fitted to Hotelling's T^2 control chart. Sukparungsee et al. (2021) studied the effects of constructed bivariate copulas on multivariate control chart effectiveness, and the performance of Hotelling's T^2 control chart is superior to the MCUSUM control chart for all shifts in the mean vector of process. Furthermore, by applying the presented control chart to two sets of real data, data set of the strength of 1.5 cm glass fibers measured at the National Physical Laboratory, England, and a data set of the strength of a glass of the aircraft window, it was found that for a small shift $(\delta \le 0.1),$ the MCUSUM control chart is better than Hotelling's T^2 control chart. Tiengket et al. (2022) studied the efficiency of constructed bivariate copulas for MEWMA and Hotelling's T^2 control charts, and the performances of the MEWMA and Hotelling's T^2 control charts were similar for small shifts $(\delta \le 0.01)$ but the MEWMA control chart showed higher performance for moderate to large shifts.

Kvam and Vidakovic (2007) reported that the actual various situations were usually non-normal distributions. A nonparametric multivariate control chart could be an alternative and would play an important role in the quality control of future products. In the case where analysts do not know the underlying distribution of the data, the appropriate statistical techniques are called non-parametric or distribution-free methods. In addition, Zeinab (2013) conducted a study of an affine invariant signed-rank multivariate exponentially weighted moving average (SRMEWMA) control chart for process location monitoring. The SRMEWMA's performance was superior to the MEWMA and Hotelling's T^2 control charts, but its detection of small shifts did respond very well to small smoothing parameters in the process mean of the skewed distributions. Chakraborty and Chaudhuri (1998) reported the spatial signed-rank method as a transformation-retransformation technique with inner standardization that achieved affine invariant and equivariant properties (Nevalainen et al. 2018).

In this research, the MEWMA and dMEWMA control charts are integrated with the spatial signed rank under the normal distribution, the t distribution, and the gamma distribution. The performance measure is the ARL. The new multivariate control chart can quickly detect small shifts (δ is less than or equal to 0.5) at a larger smoothing parameter (λ is greater than or equal to 0.35) in the process mean under a non-normal distribution.

2. Materials and Methods

2.1. Initial parameters

In this research, let $\mathbf{x}_{ij}$, $i = 1, 2, ..., n$ and $j = 1, 2, ..., p$ be the random $n \times p$ matrix with the size of the sample data being that n is equal to 15,000 units and the quality characteristics (p) are equal to 2. The $n \times p$ matrices are randomized in three situations for the normal distribution, the t distribution, and the gamma distribution. The distributions are simulated with a mean matrix (μ_0), and variance-covariance matrix (Σ_0) as shown in Table 1. These are used in the simulation instead of the real history data of the process under the non-centrality parameter $(\delta = 0)$ is the in-control process, the study condition on the ARL_0 is equal to 370, and the smoothing parameter. (λ = 0.05, 0.1, 0.2, 0.3, 0.35, 0.4, 0.5 and 0.8). Then is simulated under the non-centrality parameter. (δ = 0.1, 0.25, 0.5, 1.0, 1.5 and 2.5) are the out-of-control process. For the study selection of the λ and δ values got the idea from several researchers as follows: Montgomery (2005) states that if small shifts (roughly 0.5 standard deviations or less) are of primary concern, the typical recommendation is to choose a small λ to say equal to 0.01, 0.025, or 0.05; if moderate shifts (roughly between 0.5 and 1.5 standard deviations) are of greater concern choose $\lambda = 0.10$, whereas if larger shifts (roughly 1.5 standard deviations or more) are of concern, choose $\lambda = 0.20$ (Graham et al. 2011). Baxley RV (1990) reported that, when drifts are present, using the correct value for λ has distinct advantages for reduction of process variability because both feedback adjustments are shown in Figure 1, a plot of the forecast error sigma versus λ over the range from 0 to 1, which is seen to be relatively flat in the region around the optimal value of 0.338.

Consequently, the case studies ($p \times \delta \times \text{ARL}_0 \times \lambda \times \text{Dist}^n$) of this research are equal to 168 scenarios.

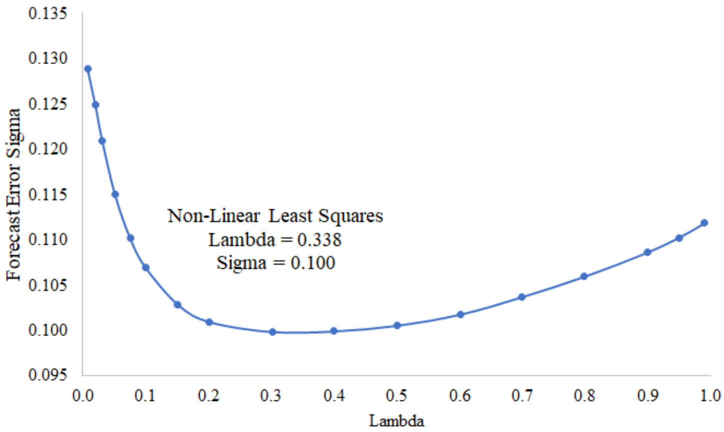


Figure 1 EWMA forecast error versus lambda for the Monsanto data (Baxley RV 1990)

Table 1 Mean and variance-covariance of the initial history data of each distribution

Dist^n	Variable $(\mathbf{x}_{ij})$	Mean (μ_0)	VCV (Σ_0)
Normal	$N_{p=2}(\mu,\Sigma)$	$(0,0,\ldots,0)'$	$\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$
t	$t_{p=2}(\upsilon=5)$	$(0,0,\ldots,0)'$	$\begin{bmatrix} 1.7 & 0.85 \\ 0.85 & 1.7 \end{bmatrix}$
Gamma	$Gam_{p=2}(\alpha=3,\ \beta=1)$, $\rho=0.5$	$(3,3,\ldots,3)'$	$\begin{bmatrix} 3 & 1.5 \\ 1.5 & 3 \end{bmatrix}$

Dist^n is distribution, VCV is variance-covariance.

2.2. Traditional models

In this research, the symbols (i) and (ii) refer to MEWMA and dMEWMA.

1) MEWMA control chart

The MEWMA equation (Alkahtani and Schaffer 2012) is written as equation number (1):

$$\mathbf{y}_i = \Lambda\mathbf{x}_i + (\mathbf{I}-\Lambda)\mathbf{y}_{i-1}, \tag{1}$$

where $\mathbf{y}_i$, $i=1,2,...,n$ is the mean vector of each distribution calculated from the random observation vector of $\mathbf{x}_i$. There are the normal distribution, the t distribution, and the gamma distribution, respectively. $\mathbf{y}_0$ is the mean vector of the historical data that $\mathbf{y}_0=\mu_0$. Λ is the $p\times p$ diagonal smoothing parameter matrix and equal to diag $(\lambda_1,\lambda_2,\ldots,\lambda_p)$ with $0<\lambda_j\le 1$, $j=1,2,...,p$. $\lambda=0.05$, 0.1, 0.2, 0.3, 0.35, 0.4, 0.5 and 0.8 and $\mathbf{I}$ is the identity matrix.

$$\Sigma_{\mathbf{y}_i} = \left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]\Sigma_0, \tag{2}$$

where $\Sigma_{\mathbf{y}_i}$ is the exact variance-covariance matrix of $\mathbf{y}_i$

$$T_i^2 = \mathbf{y}_i'\Sigma_{\mathbf{y}_i}^{-1}\mathbf{y}_i, \tag{3}$$

where T_i^2 is the MEWMA control statistics. If $T_i^2 > \mathrm{UCL}_{(i)}$, the signal is shown as an out-of-control state.

2) dMEWMA control chart

The dMEWMA equation (Alkahtani and Schaffer 2012) is written as equation number (4):

$$\mathbf{z}_i = \Lambda \mathbf{y}_i + (\mathbf{I} - \Lambda)\mathbf{z}_{i-1}, \tag{4}$$

where $\mathbf{z}_i$, $i = 1, 2, ..., n$ is the mean vector of each distribution calculated from the mean vector of $\mathbf{y}_i$. $\mathbf{z}_0$ is the mean vector of the historical data that $\mathbf{z}_0 = \mathbf{y}_0 = \mu_0$.

$$\Sigma_{\mathbf{z}_i} = \left\{ \frac{\lambda^4}{\left[1-(1-\lambda)^2\right]^3} \right\} \begin{bmatrix} 1+(1-\lambda)^2 - (i+1)^2(1-\lambda)^{2i} \\ +(2i^2+2i-1)(1-\lambda)^{2i+2} \\ -i^2(1-\lambda)^{2i+4} \end{bmatrix} \Sigma_0, \tag{5}$$

where $\Sigma_{\mathbf{z}_i}$ is the exact variance-covariance matrix of $\mathbf{z}_i$ and

$$T_{di}^2 = \mathbf{z}_i{}' \Sigma_{\mathbf{z}_i}{}^{-1} \mathbf{z}_i, \tag{6}$$

where T_{di}^2 is the dMEWMA control statistics. If $T_{di}^2 > UCL_{(ii)}$, the signal is shown as an out-of-control state.

2.3. Multivariate spatial signed-rank

The multivariate spatial signed rank is modified from the spatial sign as $\mathbf{U}(\mathbf{x}) = \|\mathbf{x}\|^{-1}\mathbf{x}$, when $\mathbf{x} \neq 0$. $\|\mathbf{x}\| = (\mathbf{x}'\mathbf{x})^{1/2}$ as the Euclidean length of a vector $\mathbf{x}_i = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)'$, $i = 1, 2, ..., n$ and $\mathbf{U}(\mathbf{x}) = 0$, when $\mathbf{x} = 0$. Then, the next step is to modify them again by the spatial ranks as $\mathbf{R}(\mathbf{x}) = \mathbf{AVE}\{\mathbf{U}(\mathbf{x} - \mathbf{x}_i)\}$ to be the multivariate spatial signed rank. The R package named "SpatialNP" has functions to compute spatial signs, ranks, and signed ranks, that were created by Sirkia et al. (2018) and are used to transform the data to be spatial signed-rank data. The function is written as equation number (7):

$$\mathbf{Q}(\mathbf{x}) = \frac{1}{2}\left[\mathbf{R}_{\mathbf{x}}(\mathbf{x}) + \mathbf{R}_{-\mathbf{x}}(\mathbf{x})\right], \tag{7}$$

where $\mathbf{R}(\mathbf{x}) = \mathbf{R}_{\mathbf{x}}(\mathbf{x})$, $\mathbf{Q}(\mathbf{x}) = \mathbf{Q}_{\mathbf{x}}(\mathbf{x})$, and $\mathbf{Q}(-\mathbf{x}) = -\mathbf{Q}(\mathbf{x})$ that is also odd (Oja 2010).

The $\hat{\mathbf{Q}}_i$, $i = 1, 2, ..., n$ is called the standardized spatial signed rank based on the inner standardization then the $Q^2(\mathbf{XS}^{-1/2})$ is used to test the statistics for the affine invariant. When the $Q^2(\mathbf{XS}^{-1/2})$ has a small value, it fails to reject $H_0 : \mu = 0$. Thus, the $\hat{\mathbf{Q}}_i$ is the affine invariant. The R package named "MNM" is tested for a multivariate location using different score functions, that were created by Nordhausen et al. (2018) and is used to the statistical testing of the spatial signed-rank dataset for the affine invariant test, which is written as equation number (8):

$$Q^2(\mathbf{XS}^{-1/2}) = \mathbf{1}_n' \hat{\mathbf{Q}} (\hat{\mathbf{Q}}'\hat{\mathbf{Q}})^{-1} \hat{\mathbf{Q}}' \mathbf{1}_n = np. \frac{\left|\mathbf{AVE}\left\{\hat{\mathbf{Q}}_i\right\}\right|^2}{\mathbf{AVE}\left\{\left|\hat{\mathbf{Q}}_i\right|^2\right\}}, \tag{8}$$

where $Q^2(\mathbf{XS}^{-1/2})$ is the multivariate spatial signed-rank test statistics.

2.4. Performance measurement

The average run length (ARL) is used to measure the control chart performance. The ARL is the average number of points that must be plotted before a point indicates an out-of-control condition. In this research, the ARL is calculated using the sum of the sample data from the 15,000 units of consecutive points on an in-control state before signaling the first out-of-control state and repeating m times, where m is equal to 5,000 (NCSS 2018). The ARL is written as equation number (9):

$$ARL = \frac{\sum_{k=1}^{5,000} RL_k}{5,000}, \tag{9}$$

where RL_k is the k^{th} run length, $k = 1, 2, ..., m.$ RL_k is the number of observations used to monitor before out-of-control in the simulation's k^{th} round.

The ARL_0 is an in-control process that should be large enough, while the ARL_1 is an out-of-control process that should be small (Sukparungsee et al. 2017). In this research, the study of the condition of the ARL_0 is equal to 370. All of the ARLs are calculated using the Monte Carlo method. The ARL_1 is used to measure the performance of the MEWMA, dMEWMA, SSRM, and SSRdM, respectively. The ARL depends on the smoothing parameter (λ), mean (μ), and variance-covariance (Σ) through a non-centrality parameter (δ), which is used to measure the multivariate distance between the mean shift (μ_{shift}) and the target mean (μ_{target}). The state of an in-control process is δ equal to 0 but the out-of-control process is δ equal to 0.1, 0.25, 0.5, 1, 1.5, and 2.5, respectively. The δ is written as equation number (10):

$$\delta = \left[(\mu_{shift} - \mu_{target})' \Sigma_0^{-1} (\mu_{shift} - \mu_{target}) \right]^{1/2}, \tag{10}$$

where Σ_0 is the variance-covariance matrix of the historical data.

2.5. Performance measurement

The transformation of the multivariate control chart was performed by using a spatial signed rank via the Monte Carlo simulation approach. The Monte Carlo simulation is easy to understand and popular to use, that is computed via MATLAB software (Thongrong et al. 2016). MATLAB is a proprietary multi-paradigm programming language and numeric computing environment. The simulation procedure consists of three main phases, as shown in Figure 2.

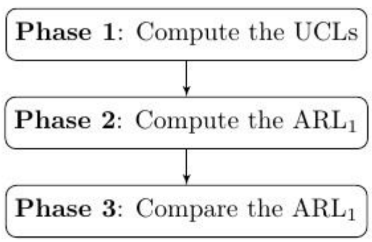


Figure 2 Simulation procedures

Phase 1: Compute the UCLs for each control chart and the distribution of the ARL_0 to be equal to 370.

1.1) Create a random $n \times p$ matrix $\mathbf{x}_{ij}$, 15,000 units for the normal distribution, the t distribution, and the gamma distribution.

1.2) Transformation of $\mathbf{x}_{ij}$ is standardized using a spatial signed rank based on inner standardization $\mathbf{Q}(\mathbf{x}_{ij})$.

1.3) For each distribution, apply 1.1), and 1.2) by separating them from the random vector for a traditional multivariate control chart $(\mathbf{x}_i)$, and transforming with a multivariate spatial signed rank $[\mathbf{Q}(\mathbf{x}_i)]$.

1.4) Apply 1.3) by placing the $\mathbf{x}_i$ vector to compute the MEWMA, and dMEWMA control statistics by using Equations (3) and (6), and by putting the $\mathbf{Q}(\mathbf{x}_\mathrm{i})$ vector to compute the SSRM, and SSRdM control statistics by using Equations (13) and (16).

1.5) Use the control statistics of T_i^2, T_{di}^2, Q_i^{SSRM}, and Q_i^{SSRdM} are obtained from 1.4) to be compared with the random initial UCLs value. If the control statistics are greater than the UCLs, the signal indicates an out-of-control state. Compute the ARL_0 using Equation (9) on each condition of p and λ with the bisection method, and follow by adjusting the UCLs value until calculating the ARL_0 is equal to 370.

1.6) The UCLs are recorded as $UCL_{(i)}$ for MEWMA, $UCL_{(ii)}$ for dMEWMA, $UCL_{(iii)}$ for SSRM, and $UCL_{(iv)}$ for SSRdM, respectively.

Phase 2: Compute the ARL_1 for each multivariate control chart and distribution at the non-centrality parameter. δ is equal to 0.1, 0.25, 0.5, 1, 1.5, and 2.5, respectively.

2.1) Calculate the μ_{shift} by using Equation (10) which moves out of the μ_{target} for each condition of δ.

2.2) The μ_{shift} is obtained from the 2 and add to the first column of $\mathbf{x}_i$ and $\mathbf{Q}(\mathbf{x}_\mathrm{i})$ in the 1.3).

2.3) Compute the control statistics of T_i^2, T_{di}^2, Q_i^{SSRM}, and Q_i^{SSRdM} as in 1.4), then compare with the UCLs from 1.6). Calculate the ARL_1 by using Equation (9) on each condition of p and λ.

2.4) The ARL_1 are recorded.

Phase 3: Compare the ARL_1 between the multivariate control charts of each distribution to summarize the highest efficiency. The conclusion of the highest efficiency multivariate control chart by using the ARL_1 in 2.4). These are tested for normal distribution properties by the Kolmogorov-Smirnov (K-S) test statistics.

3.1) If the results indicate normally distributed, then summarize via randomized complete block designs.

3.2) If the results indicate non-normally distributed, then summarize via a non-parametric method using the Friedman test statistics.

3. New Structural Models

In this research, the symbols (iii) and (iv) refer to SSRM and SSRdM.

3.1. SSRM control chart

The SSRM is modified from the MEWMA and written as equation number (11):

$$\mathbf{Q}(\mathbf{y}_i) = \Lambda\mathbf{Q}(\mathbf{x}_i) + (\mathbf{I} - \Lambda)\mathbf{Q}(\mathbf{y}_{i-1}), \tag{11}$$

where $\mathbf{Q}(\mathbf{y}_i),\ i = 1, 2, ..., n$ is the mean vector calculated from the spatial signed rank of $\mathbf{Q}(\mathbf{x}_i),$ which is the observation vector $(\mathbf{x}_i)$ then transformed by the function of the multivariate spatial signed rank method. $\mathbf{Q}(\mathbf{y}_0)$ is the mean vector of the historical data that is equal to $(0, 0, \ldots, 0)'.$

$$\Sigma_{\mathbf{Q}(\mathbf{y}_i)} = \left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]\Sigma_{\mathbf{Q}(\mathbf{x}_i)}, \tag{12}$$

where $\sum_{\mathbf{Q}(\mathbf{y}_i)}$ is the exact variance- covariance matrix of $\mathbf{Q}(\mathbf{y}_i),$ and $\sum_{\mathbf{Q}(\mathbf{x}_i)}$ is the variance-covariance matrix of $\mathbf{Q}(\mathbf{x}_i).$

$$Q_i^{SSRM} = \mathbf{Q}(\mathbf{y}_i)'\Sigma_{\mathbf{Q}(\mathbf{y}_i)}^{-1}\mathbf{Q}(\mathbf{y}_i), \tag{13}$$

where Q_i^{SSRM} is the SSRM control statistics. If $Q_i^{SSRM} > UCL_{(iii)},$ the signal is shown as an out-of-control state.

3.2. SSRdM control chart

The SSRdM is modified from the dMEWMA and written as equation number (14):

$$\mathbf{Q}(\mathbf{z}_i) = \Lambda\mathbf{Q}(\mathbf{y}_i) + (\mathbf{I} - \Lambda)\mathbf{Q}(\mathbf{z}_{i-1}), \tag{14}$$

where $\mathbf{Q}(\mathbf{z}_i),\ i = 1, 2, ..., n$ is the mean vector of each distribution calculated from the mean vector of $\mathbf{Q}(\mathbf{y}_i).$ $\mathbf{Q}(\mathbf{z}_0)$ is the mean vector of the historical data that is equal to $(0, 0, \ldots, 0)'.$

$$\Sigma_{\mathbf{Q}(\mathbf{z}_i)} = \left\{\frac{\lambda^4}{\left[1-(1-\lambda)^2\right]^3}\right\}\begin{bmatrix} 1+(1-\lambda)^2-(i+1)^2(1-\lambda)^{2i} \\ +(2i^2+2i-1)(1-\lambda)^{2i+2} \\ -i^2(1-\lambda)^{2i+4} \end{bmatrix}\Sigma_{\mathbf{Q}(\mathbf{x}_i)}, \tag{15}$$

where $\sum_{\mathbf{Q}(\mathbf{z}_i)}$ is the exact variance-covariance matrix of $\mathbf{Q}(\mathbf{z}_i).$

$$Q_i^{SSRdM} = \mathbf{Q}(\mathbf{z}_i)'\Sigma_{\mathbf{Q}(\mathbf{z}_i)}^{-1}\mathbf{Q}(\mathbf{z}_i), \tag{16}$$

where Q_i^{SSRdM} is the SSRdM control statistics. If $Q_i^{SSRdM} > UCL_{(iv)},$ the signal is shown as an out-of-control state.

4. Results and Discussion

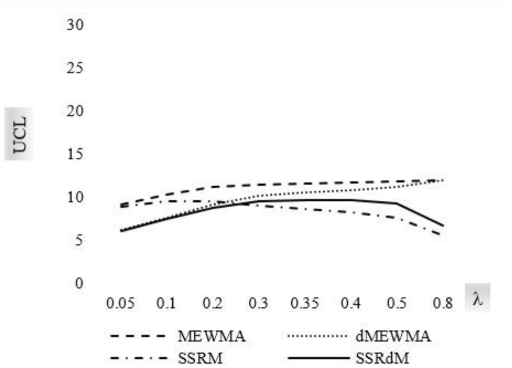
4.1. Simulation results

This research uses 15,000 random samples per situation. The three situations are the normal distribution, the t distribution, and the gamma distribution. The random samples are modified by using the spatial signed rank based on inner standardization. The researchers give the symbols (i), (ii), (iii), and (iv) that refer to MEWMA, dMEWMA, SSRM, and SSRdM, respectively. A good UCL for the control charts gives narrow control limits or small values. In this research, the $\lambda < 0.3$ refers to the small smoothing parameters. However, $\lambda \geq 0.3$ refers to the large smoothing parameters (Baxley 1990).

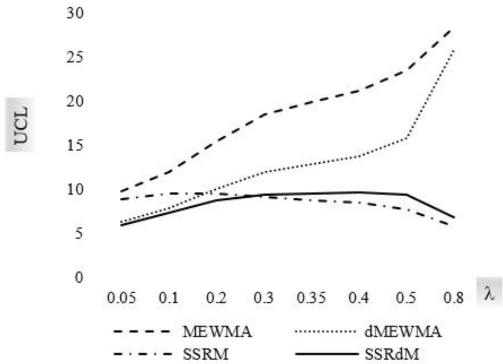
When under consideration the $\lambda \geq 0.3$, the $\text{UCL}_{(iii)}$ and $\text{UCL}_{(iv)}$ control charts are less than the $\text{UCL}_{(i)}$ and $\text{UCL}_{(ii)}$ control charts for all distributions, thus the efficiency of the (iii) and (iv) control charts in detecting the mean shift is better than the (i) and (ii) control charts for all distributions. Both the normal distribution and the t distribution make the $\text{UCL}_{(iii)}$ control chart to be less than the $\text{UCL}_{(i)}$, $\text{UCL}_{(ii)}$ and $\text{UCL}_{(iv)}$ control charts, thus the efficiency of the (iii) control chart in detecting the mean shift is better than the (i), (ii), and (iv) control charts for the normal distribution and the t distribution. However, the gamma distribution makes the $\text{UCL}_{(iii)}$ control chart to be less than the $\text{UCL}_{(i)}$, $\text{UCL}_{(ii)}$ and $\text{UCL}_{(iv)}$ control charts for all λ's, thus the efficiency of the (iii) control chart in detecting the mean shift is better than the (i), (ii), and (iv) control charts for all λ's and the gamma distribution. Therefore, the efficiency of the (iii) control chart in detecting the mean shift is the most effective for $\lambda \geq 0.3$ and all distributions.

When under consideration the $\lambda \leq 0.2$, the $\text{UCL}_{(ii)}$ and $\text{UCL}_{(iv)}$ control charts are less than the $\text{UCL}_{(i)}$ and $\text{UCL}_{(iii)}$ control charts for the normal distribution and the t distribution, thus the efficiency of the (ii) and (iv) control charts in detecting the mean shift is better than the (i) and (iii) control charts for the normal distribution and the t distribution. However, the gamma distribution makes the $\text{UCL}_{(iii)}$ control chart to be less than the $\text{UCL}_{(i)}$, $\text{UCL}_{(ii)}$, and $\text{UCL}_{(iv)}$ control charts for $\lambda \leq 0.2$, thus the efficiency of the (iii) control chart in detecting the mean shift is better than the (i), (ii), and (iv) control charts for $\lambda \leq 0.2$ and the gamma distribution. Therefore, the efficiency of the (iv) control chart in detecting the mean shift is most effective for $\lambda \leq 0.2$ and both the normal distribution and the t distribution. However, the efficiency of the (iii) control chart in detecting the mean shift is the most effective for $\lambda \leq 0.2$ and the gamma distribution.

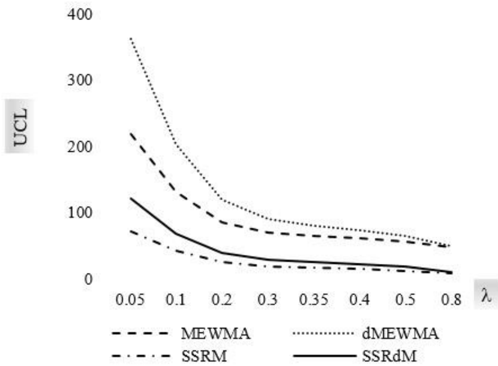
When taking these into consideration the $\lambda < 0.3$, Both the normal distribution and the t distribution make the $\text{UCL}_{(iv)}$ control chart to be less than the $\text{UCL}_{(i)}$, $\text{UCL}_{(ii)}$ and $\text{UCL}_{(iii)}$ control charts, thus the efficiency of the (iv) control chart in detecting the mean shift is better than the (i), (ii), and (iii) control charts for the normal distribution and the t distribution. However, the gamma distribution makes the $UCL_{(iii)}$ control chart to be less than the $\text{UCL}_{(i)}$, $\text{UCL}_{(ii)}$ and $\text{UCL}_{(iv)}$ control charts for $\lambda < 0.3$, thus the efficiency of the (iii) control chart in detecting the mean shift is better than the (i), (ii), and (iv) control charts for $\lambda < 0.3$ and the gamma distribution. Therefore, the efficiency of the (iii) control chart is rather close to the (iv) control chart in detecting the mean shift is the most effective for $0.2 < \lambda < 0.3$ and both the normal distribution and the t distribution. However, the efficiency of the (iii) control chart in detecting the mean shift is most effective for $0.2 < \lambda < 0.3$ and the gamma distribution, as shown in Figure 3 and Table 2.



(a) The normal distribution



(b) The t distribution



(c) The gamma distribution

Figure 3 Comparisons of the UCLs under the different λ values and distributions

Table 2 Comparisons of the UCLs for the $\mathrm{ARL}_0 \cong 370$ and $p = 2$ under the different λ values and distributions

Dist^n	MCC	λ							
		0.05	0.1	0.2	0.3	0.35	0.4	0.5	0.8
Normal	(i)	9.10	10.21	11.05	11.40	11.51	11.62	11.74	11.86
	(ii)	6.11	7.52	9.09	10.07	10.44	10.72	11.14	11.81
	(iii)	8.75	9.50	9.50	**8.94**	**8.57**	**8.20**	**7.45**	**5.42**
	(iv)	**5.97**	**7.37**	**8.72**	9.37	9.54	9.56	9.23	6.56
t	(i)	9.73	11.88	15.29	18.41	19.76	21.07	23.44	28.24
	(ii)	6.20	7.76	9.95	11.81	12.73	13.66	15.71	25.74
	(iii)	8.76	9.49	9.49	**9.01**	**8.69**	**8.36**	**7.67**	**5.68**
	(iv)	**5.91**	**7.28**	**8.72**	9.37	9.49	9.51	9.29	6.81
Gamma	(i)	217.25	131.23	85.19	69.05	64.36	60.73	55.60	46.73
	(ii)	360.60	202.54	118.50	89.03	80.19	73.57	64.10	49.81
	(iii)	**71.47**	**41.55**	**24.62**	**18.07**	**15.99**	**14.35**	**11.84**	**7.35**
	(iv)	121.08	67.25	38.30	27.67	24.39	21.77	17.65	9.33

(i). MEWMA; (ii). dMEWMA; (iii). SSRM; (iv). SSRdM.
MCC is multivariate control chart.

The λ is a smoothing parameter. $\lambda < 0.3$ refers to the small smoothing parameters. However, $\lambda \geq 0.3$ mean the large smoothing parameters. δ is the non-centrality parameter, which is a value indicating the level of shifts in the mean from the target mean. In this research, there are no shift $(\delta = 0)$, small shifts ($\delta \leq 0.5$), moderate shift $(\delta = 1)$, and large shifts ($\delta \geq 1.5$) (Graham et al. 2011).

The ARL_1 values are rounded up in practical applications that are usually integers, as shown in Appendix A. The ARL_1 values of the (i), (ii), (iii), and (iv) control charts are tested by using the Kolmogorov-Smirnov (K-S) test statistics, which provide the p-values to be equal to 0.00 at a significance level of 0.05. Thus, the ARL_1 values of all distributions have been non-normally distributed at a significance level of 0.05 and then summarized the effectiveness of the control charts via a non-parametric method using the Friedman test statistics. The mean ranks of ARL_1 values obtained from the Friedman test statistics are shown in the row as [*] in Table 10 to Table 15, which show a significant difference at level 0.05 and provide the p-values to be equal to 0.00, and then the mean ranks in Appendix A are used to conclude the effectiveness of the best control charts under an unknown distribution, as shown in Table 3.

Appendix A is summarized in Table 3. At $\delta > 0.5$, both the normal and t distributions cause the performance trend of the (iv) control chart to decrease continuously until it reaches 0.3. The (iv) control chart is most effective when the data has moderate and large mean shifts and small smoothing parameters. When considering all δ's and $\lambda \geq 0.3$, the performance trend of the (iii) control chart is increased continuously and replaces the performance of the (iv) control chart. When the data has small, moderate, and large mean shifts and large smoothing parameters, the (iii) control chart is the most effective. For the t distribution, the (ii) control chart at $\delta = 2.5$ and $\lambda = 0.05$ is more effective. The (ii) control chart is most effective when the data has large mean shifts and only has a $\lambda = 0.05$. For $\delta \leq 0.5$ and all λ's, the (iii) control chart is dominant and the most effective (Haanchumpol et al. 2019). When considering the gamma distribution of all δ's and λ's, the (iii) control chart is dominant (Haanchumpol et al. 2020). The study's results are in accordance with Baxley (1990), i.e., the industrial processes require a smoothing parameter, and λ is larger than 0.35 under the non-normally distributed process.

4.2. Results of real practice

This topic practically proposes the implementation of the newly discovered multivariate control chart in real factory processes. The researcher defines the symbols (i), (ii), (iii), and (iv) that refer to MEWMA, dMEWMA, SSRM, and SSRdM, respectively. The real factory examples consist of two industries including the tensile strength of steel production represents the normally distributed process, and blade wheel production of the blower housing represents the non-normally distributed process.

1) Case Study 1: the tensile strength of steel production for the normally distributed process

The tensile strength of steel has three important quality characteristics that consist of 1.0% yield load (KN), breaking load (KN), and elongation (PCT). The sample sizes used are equal to 100 units. The first group of data is equal to 60 units, which represent the historical data. It is separated into 6 vectors, with each of the vectors consisting of 10 units. These 6 vectors are used for the simulation and calibrate the necessary parameters for calculating the ARL_0 and UCLs of the multivariate control charts in Phase 1 (Zou and Tsung 2011). The distribution of the remaining group with the size of 40 units via the Kolmogorov-Smirnov (K-S) test statistics is not significant at the 0.05 level. Thus, they are inferred as a normal distribution. It is separated into 2 vectors, with each of the vectors consisting of 20 units to repeat 2 times. It is represented by the data that are used to calculate the control statistics of Phase 2. The conditions of the λ are equal to 0.35, 0.4, 0.5, 0.6, 0.7, and 0.8. The δ shifts are equal to 0.1, 0.25, and 0.5.

Table 3 Performance of the best control charts using the Friedman test statistics for the $\text{ARL}_0 \cong 370$ and $p = 2$ under the values of λ, δ and distributions

Dist^n	δ	λ							
		0.05	0.1	0.2	0.3	0.35	0.4	0.5	0.8
Normal	0.1	*	*	*	*	*	*	*	*
	0.25	*	*	*	*	*	*	*	*
	0.5	*	*	*	*	*	*	*	*
	1	(iv)	*	*	*	*	*	*	*
	1.5	(iv)	(iv)	*	*	*	*	*	*
	2.5	(iv)	(iv)	(iv)	*	*	*	*	*
t	0.1	*	*	*	*	*	*	*	*
	0.25	*	*	*	*	*	*	*	*
	0.5	*	*	*	*	*	*	*	*
	1	(iv)	*	*	*	*	*	*	*
	1.5	(iv)	(iv)	*	*	*	*	*	*
	2.5	(ii)	(iv)	(iv)	*	*	*	*	*
Gamma	0.1	*	*	*	*	*	*	*	*
	0.25	*	*	*	*	*	*	*	*
	0.5	*	*	*	*	*	*	*	*
	1	*	*	*	*	*	*	*	*
	1.5	*	*	*	*	*	*	*	*
	2.5	*	*	*	*	*	*	*	*

(i). MEWMA; (ii). dMEWMA; (iii). SSRM; (iv). SSRdM.
* SSRM is dominant.

In Appendix B, the researcher presented the $\text{UCL}_{(i)}, \text{UCL}_{(ii)}, \text{UCL}_{(iii)}$ and $\text{UCL}_{(iv)}$ in phase 1 and the control statistics of the (i), (ii), (iii), and (iv) control charts in Phase 2 for detecting three variables of the tensile stress of steel for a normally distributed process under the λ to be equal to 0.35, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively. Only δ is equal to 0.25 when repeated two times. The results can be read in Figure 4 to Figure 9 as follows:

At $\lambda = 0.35$ and $\delta = 0.25$ are generated the $\text{UCL}_{(i)}, \text{UCL}_{(ii)}, \text{UCL}_{(iii)}$ and $\text{UCL}_{(iv)}$ control charts, that are equal to 199.97, 272.96, 10.98, and 13.40, respectively. Then the control statistics for MCCs in Phase 2 are compared with the UCLs from Phase 1, as shown in Figure 4. The first repeat found that the (ii) control chart give the first signal as "out-of-control" at the 67^{th} observation, then the number of responses is saved to be equal to 1 for the (ii) control chart. However, the (iv) control chart signal is "out-of-control" at the 71^{st} observation, and the (i) and (iii) control chart signals are "out-of-control" at the 74^{th} observation, indicating that they are slowly detected. For the second repeat found that the (ii) control chart gives the first signal as "out-of-control" at the 88^{th} observation, then the number of responses is saved to be equal to 1 for the (ii) control chart. However, the (i), (iii), and (iv) control chart signals are "out-of-control" at the 89^{th} observation, indicating that they are slowly detected.

At $\lambda = 0.4$ and $\delta = 0.25$ are generated the $UCL_{(i)}, UCL_{(ii)}, UCL_{(iii)}$ and $UCL_{(iv)}$ control charts, that are equal to 189.48, 261.40, 10.41, and 12.85, respectively. Then the control statistics for MCCs in Phase 2 are compared with the UCLs from Phase 1, as shown in Figure 5. The first repeat found that the (ii) control chart give the first signal as "out-of-control" at the 67^{th} observation, then the number of responses is saved to be equal to 1 for the (ii) control chart. However, the (iii) and (iv) control chart signals are "out-of-control" at the 74^{th} observation, and the (i) control chart signal is "out-of-control" at the 79^{th} observation, indicating that they are slowly detected. For the second repeat found that the (ii), (iii), and (iv) control charts give the first signal as "out-of-control" at the 89^{th} observation, then the number of responses is saved to be equal to 1 for the (ii), (iii), and (iv) control charts. However, the (i) control chart signal is "out-of-control" at the 90^{th} observation, indicating that it is slowly detected.

At $\lambda = 0.5$ and $\delta = 0.25$ are generated the $UCL_{(i)}, UCL_{(ii)}, UCL_{(iii)}$ and $UCL_{(iv)}$ control charts, that are equal to 166.04, 230.88, 9.60, and 12.63, respectively. Then the control statistics for MCCs in Phase 2 are compared with the UCLs from Phase 1, as shown in Figure 6. The first repeat found that the (ii), (iii), and (iv) control charts give the first signal as "out-of-control" at the 74^{th} observation, then the number of responses is saved to be equal to 1 for the (ii), (iii), and (iv) control charts. However, the (i) control chart remains "in-control". For the second repeat found that the (ii) and (iii) control charts give the first signal as "out-of-control" at the 89^{th} observation, then the number of responses is saved to be equal to 1 for the (ii) and (iii) control charts. However, the (iv) control chart signal is "out-of-control" at the 90^{th} observation, indicating that it is slowly detected, and the (i) control chart remains "in-control".

At $\lambda = 0.6$ and $\delta = 0.25$ are generated the $UCL_{(i)}, UCL_{(ii)}, UCL_{(iii)}$ and $UCL_{(iv)}$ control charts, that are equal to 141.76, 202.48, 8.56, and 11.04, respectively. Then the control statistics for MCCs in Phase 2 are compared with the UCLs from Phase 1, as shown in Figure 7. The first repeat found that the (iii) and (iv) control charts give the first signal as "out-of-control" at the 74^{th} observation, then the number of responses is saved to be equal to 1 for the (iii) and (iv) control charts. However, the (i) and (ii) control charts remain "in-control". For the second repeat found that the (iii) control chart gives the first signal as "out-of-control" at the 89^{th} observation, then the number of responses is saved to be equal to 1 for the (iii) control chart. However, the (iv) control chart signal is "out-of-control" at the 90^{th} observation, indicating that it is slowly detected, and the (i) and (ii) control charts remain "in-control".

At $\lambda = 0.7$ and $\delta = 0.25$ are generated the $UCL_{(i)}, UCL_{(ii)}, UCL_{(iii)}$ and $UCL_{(iv)}$ control charts, that are equal to 118.94, 169.98, 7.48, and 9.76, respectively. Then the control statistics for MCCs in Phase 2 are compared with the UCLs from Phase 1, as shown in Figure 8. The first repeat found that the (iii) and (iv) control charts give the first signal as "out-of-control" at the 74^{th} observation, then the number of responses is saved to be equal to 1 for the (iii) and (iv) control charts. However, the (i) and (ii) control charts remain "in-control". For the second repeat found that the (iii) control chart gives the first signal as "out-of-control" at the 89^{th} observation, then the number of responses is saved to be equal to 1 for the (iii) control chart. However, the (iv) control chart signal is "out-of-control" at the 90^{th} observation, indicating that it is slowly detected, and the (i) and (ii) control charts remain "in-control".

At $\lambda = 0.8$ and $\delta = 0.25$ are generated the $UCL_{(i)}, UCL_{(ii)}, UCL_{(iii)}$ and $UCL_{(iv)}$ control charts, that are equal to 98.88, 132.28, 6.59, and 7.99, respectively. Then the control statistics for MCCs in Phase 2 are compared with the UCLs from Phase 1, as shown in Figure 9. The first repeat found that the (iii) and (iv) control charts give the first signal as "out-of-control" at the 74^{th} observation, then the number of responses is saved to be equal to 1 for the (iii) and (iv) control charts. However, the (i) and (ii) control charts remain "in-control". For the second repeat found that the (iii) and (iv) control charts give the first signal as "out-of-control" at the 89^{th} observation, then the number of responses is saved to be equal to 1 for the (iii) and (iv) control charts. However, the (i) and (ii) control charts remain "in-control".

Consequently, the (iii) control chart provides the number of responses quicker than the (i), (ii), and (iv) control charts for detecting small mean shifts $(\delta \le 0.5)$ and the large smoothing parameters $(\lambda \ge 0.35)$ under the normally distributed process, as shown in Table 4 that is summarized from Appendix B.

Table 4 Comparisons of the number of responses for detecting small mean shifts $(\delta \le 0.5)$ and the large smoothing parameters $(\lambda \ge 0.35)$ under the normally distributed process

δ	MCC	λ						$\lambda \ge 0.35$
		0.35	0.4	0.5	0.6	0.7	0.8	
0.1	(i)							
	(ii)	2	2	2				6
	(iii)			1	2	2	1	6
	(iv)				1	1	2	4
0.25	(i)							
	(ii)	2	2	2				6
	(iii)		1	2	2	2	2	9
	(iv)		1	1	1	1	2	6
0.5	(i)							
	(ii)	1	1	1				3
	(iii)	1	1	2	2	1	1	8
	(iv)	1		1	2	2	2	8
$\delta \le 0.5$	(i)							
	(ii)	5	5	5				15
	(iii)	1	2	5	6	5	4	23
	(iv)	1	1	2	4	4	6	18

MCC is multivariate control chart.

Table 5 shows the results of the mean ranks of the multivariate control charts are repeated 2 times via the Friedman test statistics. The mean ranks are classified by λ and δ. The λ is equal to 0.35, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively. Each of the λ is separated into three sections of the δ that are equal to 0.1, 0.25, and 0.5, respectively. The distribution of the multivariate control charts in columns under the small mean shifts $(\delta \leq 0.5)$ and the large values of the smoothing parameter $(\lambda \geq 0.35)$ via the Kolmogorov-Smirnov (K-S) test statistics are significant at the 0.05 level. Thus, they are inferred as a non-normal distribution.

Table 5 The mean ranks of MCCs by Friedman test statistics at $p = 3$ for repeat 2 times under the normally distributed process

λ	δ	Mean ranks			
		(i)	(ii)	(iii)	(iv)
0.35	0.1	2.50	**1.00**	3.25	3.25
	0.25	3.25	**1.00**	3.25	2.50
	0.5	3.75	**2.00**	2.25	**2.00**
0.4	0.1	4.00	**1.00**	2.50	2.50
	0.25	4.00	**1.50**	2.25	2.25
	0.5	3.50	**1.75**	2.00	2.75
0.5	0.1	4.00	**1.25**	2.00	2.75
	0.25	4.00	**1.75**	**1.75**	2.50
	0.5	4.00	2.00	**1.50**	2.50
0.6	0.1	3.50	3.50	**1.25**	1.75
	0.25	3.50	3.50	**1.25**	1.75
	0.5	3.50	3.50	**1.50**	**1.50**
0.7	0.1	3.50	3.50	**1.25**	1.75
	0.25	3.50	3.50	**1.25**	1.75
	0.5	3.50	3.50	1.75	**1.25**
0.8	0.1	3.25	3.25	2.25	**1.25**
	0.25	3.50	3.50	**1.50**	**1.50**
	0.5	3.50	3.50	1.75	**1.25**

Comparing the performance of multivariate control charts for the small mean shifts $(\delta \leq 0.5)$ and the large smoothing parameter $(\lambda \geq 0.35)$ will use Friedman's test statistics. It is used to compare the mean ranks between the (i), (ii), (iii), and (iv) control charts. The p-value of the mean ranks is significant at the 0.05 level. The performances of the (i), (ii), (iii), and (iv) control charts show the significant differences at the 0.05 level. The (iii) control chart provides a mean rank lower than the (i), (ii), and (iv) control charts. Consequently, the (iii) control chart is the most efficient for detecting small mean shifts $(\delta \leq 0.5)$ and the large smoothing parameters $(\lambda \geq 0.35)$ under the normally distributed process, as shown in Table 6.

Table 6 The ranks of mean ranks of MCCs under the normally distributed process

MCCs	Symbols	Mean ranks
MEWMA	(i)	3.611
dMEWMA	(ii)	2.361
SSRM	**(iii)**	**1.917**
SSRdM	(iv)	2.111

2) Case Study 2: blade wheel production of the blower housing for the non-normally distributed process

The blade wheel of the blower has three important quality characteristics that consist of diagonal length (mm), edge welding (mm), and blade wheel length (mm). The sample sizes used are equal to 140 units. The first group of data is equal to 80 units, which represent the historical data. It is separated into 8 vectors, with each of the vectors consisting of 10 units. These 8 vectors are used for the simulation and calibrate the necessary parameters for calculating the ARL_0 and UCLs of the multivariate control charts in Phase 1 (Zou and Tsung 2011). The distribution of the remaining group with the size of 60 units via the Kolmogorov-Smirnov (K-S) test statistics is significant at the 0.05 level. Thus, they are inferred as a non-normal distribution. It is separated into 3 vectors, with each of the vectors consisting of 20 units to repeat 3 times. It is represented by the data that are used to calculate the control statistics of Phase 2. The conditions of the λ are equal to 0.35, 0.4, 0.5, 0.6, 0.7, and 0.8. The δ shifts are equal to 0.1, 0.25, and 0.5.

In Appendix C, the researcher presented the $\text{UCL}_{(i)}, \text{UCL}_{(ii)}, \text{UCL}_{(iii)}$ and $\text{UCL}_{(iv)}$ in phase 1 and the control statistics of the (i), (ii), (iii), and (iv) control charts in phase 2 for detecting three variables of the blade wheel of blower for a non-normally distributed process under the λ to be equal to 0.35, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively. Only δ is equal to 0.25 when repeated two times. The results can be read in Figure 10 to Figure 15 as follows:

At $\lambda = 0.35$ and $\delta = 0.25$ are generated the $\text{UCL}_{(i)}, \text{UCL}_{(ii)}, \text{UCL}_{(iii)}$ and $\text{UCL}_{(iv)}$ control charts, that are equal to 25.54, 29.36, 14.30, and 18.29, respectively. Then the control statistics for MCCs in Phase 2 are compared with the UCLs from Phase 1, as shown in Figure 10. The first repeat found that the (iii) and (iv) control charts give the first signal as “out-of-control” at the 100^{th} observation, then the number of responses is saved to be equal to 1 for the (iii) and (iv) control charts. However, the (i) and (ii) remain “in-control”. For the second repeat found that the (iii) control chart gives the first signal as “out-of-control” at the 103^{rd} observation, then the number of responses is saved to be equal to 1 for the (iii) control chart. However, the (iv) signal is “out-of-control” at the 105^{th} observation. The (ii) signal is “out-of-control” at the 114^{th} observation, indicating that they are slowly detected, and the (i) remains “in-control”. For the third repeat found that the (iv) control chart gives the first signal as “out-of-control” at the 126^{th} observation, then the number of responses is saved to be equal to 1 for the (iv) control chart. However, the (ii) and (iii) signals are “out-of-control” at the 127^{th} observation, and the (i) signal is “out-of-control” at the 138^{th} observation, indicating that they are slowly detected.

At $\lambda = 0.4$ and $\delta = 0.25$ are generated the $\text{UCL}_{(i)}, \text{UCL}_{(ii)}, \text{UCL}_{(iii)}$ and $\text{UCL}_{(iv)}$ control charts, that are equal to 23.72, 27.94, 13.37, and 17.78, respectively. Then the control statistics for MCCs in Phase 2 are compared with the UCLs from Phase 1, as shown in Figure 11. The first repeat found that the (iii) and (iv) control charts give the first signal as “out-of-control” at the 100^{th} observation, then the number of responses is saved to be equal to 1 for the (iii) and (iv) control charts. However, the (i) and (ii) remain "in-control". For the second repeat found that the (iii) control chart gives the first signal as "out-of-control" at the 102^{nd} observation, then the number of responses is saved to be equal to 1 for the (iii) control chart. However, the (iv) control chart signal is "out-of-control" at the 103^{rd} observation. The (i) control chart signal is "out-of-control" at the 108^{th} observation, and the (ii) control chart signal is "out-of-control" at the 119^{th} observation, indicating that they are slowly detected. For the third repeat found that the (iv) control chart gives the first signal as "out-of-control" at the 126^{th} observation, then the number of responses is saved to be equal to 1 for the (iv) control chart. However, the (ii) control chart signal is "out-of-control" at the 127^{th} observation. The (iii) control chart signal is "out-of-control" at the 131^{st} observation, and the (i) control chart signal is "out-of-control" at the 138^{th} observation, indicating that they are slowly detected.

At $\lambda = 0.5$ and $\delta = 0.25$ are generated the $\text{UCL}_{(i)}, \text{UCL}_{(ii)}, \text{UCL}_{(iii)}$ and $\text{UCL}_{(iv)}$ control charts, that are equal to 21.85, 24.18, 11.58, and 15.56, respectively. Then the control statistics for MCCs in Phase 2 are compared with the UCLs from Phase 1, as shown in Figure 12. The first repeat found that the (iii) and (iv) control charts give the first signal as "out-of-control" at the 100^{th} observation, then the number of responses is saved to be equal to 1 for the (iii) and (iv) control charts. However, the (i) and (ii) remain "in-control". For the second repeat found that the (iii) control chart gives the first signal as "out-of-control" at the 102^{nd} observation, then the number of responses is saved to be equal to 1 for the (iii) control chart. However, the (iv) control chart signal is "out-of-control" at the 103^{rd} observation. The (i) control chart signal is "out-of-control" at the 108^{th} observation, indicating that they are slowly detected, and the (ii) control chart remains "in-control". For the third repeat found that the (iv) control chart gives the first signal as "out-of-control" at the 126^{th} observation, then the number of responses is saved to be equal to 1 for the (iv) control chart. However, the (ii) control chart signal is "out-of-control" at the 127^{th} observation. The (iii) control chart signal is "out-of-control" at the 131^{st} observation, indicating that they are slowly detected, and the (i) remains "in-control".

At $\lambda = 0.6$ and $\delta = 0.25$ are generated the $\text{UCL}_{(i)}, \text{UCL}_{(ii)}, \text{UCL}_{(iii)}$ and $\text{UCL}_{(iv)}$ control charts, that are equal to 19.54, 22.43, 10.03, and 13.30, respectively. Then the control statistics for MCCs in Phase 2 are compared with the UCLs from Phase 1, as shown in Figure 13. The first repeat found that the (i) control chart give the first signal as "out-of-control" at the 94^{th} observation, then the number of responses is saved to be equal to 1 for the (i) control chart. However, the (iii) and (iv) control chart signals are "out-of-control" at the 100^{th} observation, indicating that they are slowly detected, and the (ii) control chart remains "in-control". For the second repeat found that the (iii) control chart gives the first signal as "out-of-control" at the 102^{nd} observation, then the number of responses is saved to be equal to 1 for the (iii) control chart. However, the (iv) control chart signal is "out-of-control" at the 103^{rd} observation. The (i) control chart signal is "out-of-control" at the 108^{th} observation, indicating that they are slowly detected, and the (ii) control chart remains "in-control". For the third repeat found that the (iv) control chart gives the first signal as "out-of-control" at the 126^{th} observation, then the number of responses is saved to be equal to 1 for the (iv) control chart. However, the (iii) control chart signal is "out-of-control" at the 131^{st} observation. The (i) control chart signal is "out-of-control" at the 133^{rd} observation, and the (ii) control chart signal is "out-of-control" at the 138^{th} observation, indicating that they are slowly detected.

At $\lambda = 0.7$ and $\delta = 0.25$ are generated the $UCL_{(i)}, UCL_{(ii)}, UCL_{(iii)}$ and $UCL_{(iv)}$ control charts, that are equal to 17.59, 20.99, 8.72, and 11.29, respectively. Then the control statistics for MCCs in Phase 2 are compared with the UCLs from Phase 1, as shown in Figure 14. The first repeat found that the (i) and (iii) control charts give the first signal as "out-of-control" at the 94^{th} observation, then the number of responses is saved to be equal to 1 for the (i) and (iii) control charts. However, the (ii) and (iv) control chart signals are "out-of-control" at the 100^{th} observation, indicating that they are slowly detected. For the second repeat found that the (iii) and (iv) control charts give the first signal as "out-of-control" at the 102^{nd} observation, then the number of responses is saved to be equal to 1 for the (iii) and (iv) control charts. However, the (i) and (ii) control chart signals are "out-of-control" at the 108^{th} observation, indicating that they are slowly detected. For the third repeat found that the (iii) control chart gives the first signal as "out-of-control" at the 130^{th} observation, then the number of responses is saved to be equal to 1 for the (iii) control chart. However, the (iv) control chart signal is "out-of-control" at the 131^{st} observation. The (i) control chart signal is "out-of-control" at the 133^{rd} observation, indicating that they are slowly detected, and the (ii) control chart remains "in-control".

At $\lambda = 0.8$ and $\delta = 0.25$ are generated the $UCL_{(i)}, UCL_{(ii)}, UCL_{(iii)}$ and $UCL_{(iv)}$ control charts, that are equal to 15.65, 17.56, 7.61, and 9.42, respectively. Then the control statistics for MCCs in Phase 2 are compared with the UCLs from Phase 1, as shown in Figure 15. The first repeat found that the (i), (ii), (iii), and (iv) control charts give the first signal as "out-of-control" at the 94^{th} observation, then the number of responses is saved to be equal to 1 for the (i), (ii), (iii), and (iv) control charts. For the second repeat found that the (iii) and (iv) control charts give the first signal as "out-of-control" at the 102^{nd} observation, then the number of responses is saved to be equal to 1 for the (iii) and (iv) control charts. However, the (i) and (ii) control chart signals are "out-of-control" at the 108^{th} observation, indicating that they are slowly detected. For the third repeat found that the (iii) control chart gives the first signal as "out-of-control" at the 130^{th} observation, then the number of responses is saved to be equal to 1 for the (iii) control chart. However, the (iv) control chart signal is "out-of-control" at the 131^{st} observation. The (i) and (ii) control chart signals are "out-of-control" at the 133^{rd} observation, indicating that they are slowly detected.

Consequently, the (iii) control chart provides the number of responses quicker than the (i), (ii), and (iv) control charts for detecting small mean shifts $(\delta \le 0.5)$ and the large smoothing parameters $(\lambda \ge 0.35)$ under the normally distributed process, as shown in Table 7 that is summarized from Appendix C.

Table 8 shows the results of mean ranks of the multivariate control charts are repeated 3 times via the Friedman test statistics. The mean ranks are classified by λ and δ. The λ is equal to 0.35, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively. Each of the λ is separated into three sections of the δ that are equal to 0.1, 0.25, and 0.5, respectively. The distribution of the multivariate control charts in columns under the small mean shifts $(\delta \le 0.5)$ and the large values of the smoothing parameter $(\lambda \ge 0.35)$ via the Kolmogorov-Smirnov (K-S) test statistics are significant at the 0.05 level. Thus, they are inferred as a non-normal distribution.

Table 7 Comparisons of the number of responses for detecting small mean shifts $(\delta \leq 0.5)$ and the large smoothing parameters $(\lambda \geq 0.35)$ under the non-normally distributed process

δ	MCC	λ						$\lambda \geq 0.35$
		0.35	0.4	0.5	0.6	0.7	0.8	
0.1	(i)				1	1	1	3
	(ii)						1	1
	(iii)	2	2	2	1	3	3	13
	(iv)	2	2	2	1	1	1	9
0.25	(i)				1	1	1	3
	(ii)						1	1
	(iii)	2	2	2	1	3	3	13
	(iv)	2	2	2	1	1	2	10
0.5	(i)							
	(ii)							
	(iii)	3	2	2	2	3	3	15
	(iv)	2	2	1	2	1	3	11
$\delta \leq 0.5$	(i)				2	2	2	6
	(ii)						2	2
	(iii)	7	6	6	4	9	9	41
	(iv)	6	6	5	4	3	6	30

MCC is multivariate control chart

Table 8 The mean ranks of MCCs by Friedman test statistics at $p = 3$ for repeat 3 times under the non-normally distributed process

λ	δ	Mean ranks			
		(i)	(ii)	(iii)	(iv)
0.35	0.1	3.50	3.33	1.67	**1.50**
	0.25	3.83	3.00	1.67	**1.50**
	0.5	3.67	3.33	**1.33**	1.67
0.4	0.1	3.50	3.17	1.83	**1.50**
	0.25	3.50	3.17	1.83	**1.50**
	0.5	3.67	3.33	**1.50**	**1.50**
0.5	0.1	3.17	3.83	**1.50**	**1.50**
	0.25	3.50	3.17	1.83	**1.50**
	0.5	3.33	3.33	**1.67**	**1.67**
0.6	0.1	2.50	3.83	**1.83**	**1.83**
	0.25	2.33	4.00	**1.83**	**1.83**
	0.5	2.67	4.00	**1.50**	1.83
0.7	0.1	2.67	3.83	**1.33**	2.17
	0.25	2.67	3.67	**1.33**	2.33
	0.5	2.83	4.00	**1.17**	2.00
0.8	0.1	3.00	3.00	**1.50**	2.50
	0.25	3.17	3.17	**1.67**	2.00
	0.5	3.50	3.50	**1.50**	**1.50**

Comparing the performance of multivariate control charts for the small mean shifts $(\delta \le 0.5)$ and the large smoothing parameter $(\lambda \ge 0.35)$ will use Friedman's test statistics. It is used to compare the mean ranks between the (i), (ii), (iii), and (iv) control charts. The p-value of the mean ranks is significant at the 0.05 level. The performances of the (i), (ii), (iii), and (iv) control charts show significant differences at the 0.05 level. The (iii) control chart provides a mean rank lower than the (i), (ii), and (iv) control charts. Consequently, the (iii) control chart is the most efficient for detecting small mean shifts $(\delta \le 0.5)$ and the large smoothing parameters $(\lambda \ge 0.35)$ under the non-normally distributed process, as shown in Table 9.

Table 9 The ranks of mean ranks of MCCs under the non-normally distributed process

MCCs	Symbols	Mean ranks
MEWMA	(i)	3.500
dMEWMA	(ii)	3.500
SSRM	**(iii)**	**1.444**
SSRdM	(iv)	1.556

5. Conclusions and Recommendation

This research studied the development of traditional control charts using a spatial signed-rank method based on inner standardization via the Monte Carlo simulation approach. The random samples comprised three situations, i.e., normal distribution, the t distribution, and the gamma distribution. In this research, the (i), (ii), (iii), and (iv) referred to multivariate exponentially weighted moving average (MEWMA), double multivariate exponentially weighted moving average (dMEWMA, spatial signed-rank multivariate exponentially weighted moving average (SSRM), and spatial signed-rank double multivariate exponentially weighted moving average (SSRdM), respectively.

When considering the efficiency of all control charts under the UCLs, found that the efficiency of the (iii) control chart in detecting the mean shift was the most effective for $\lambda \ge 0.3$ and all distributions. However, the efficiency of the (iii) control chart in detecting the mean shift was the most effective for $\lambda \le 0.2$ and the gamma distribution. Then the efficiency of the (iii) control chart was rather close to the (iv) control chart in detecting the mean shift is the most effective for $0.2 < \lambda < 0.3$ and both the normal distribution and the t distribution. However, the efficiency of the (iii) control chart in detecting the mean shift was most effective for $0.2 < \lambda < 0.3$ and the gamma distribution. Then the efficiency of the (iv) control chart in detecting the mean shift was the most effective for $\lambda \le 0.2$ and both the normal distribution and the t distribution. As a result, concluded that the efficiency of the (iii) control chart was most effective in detecting the mean shift for large smoothing parameters $(\lambda \ge 0.3)$ and for the normal, t, and gamma distributions, respectively.

Simulation results, the Friedman test statistics was used to compare the best performance for the control charts regardless of the data distribution. The Friedman test was used to rank the mean of the ARL_1 between the (i), (ii), (iii), and (iv) control charts. The normal distribution and the t distribution at $\delta > 0.5$ and $\lambda < 0.3,$ the (iv) control chart, were most effective when the data was moderate and large mean shifts, and small smoothing parameters. For the normal distribution, the t distribution, and the gamma distribution at $\delta \le 0.5$ and all λ's, the (iii) control chart was the most effective when the data was small mean shifts, and the small and large smoothing parameter.

Practical results, the real factory examples consist of two industries including the tensile strength of steel production represents the normally distributed process and blade wheel production of the blower housing represents the non-normally distributed process. In both processes, the (iii) control chart provides the mean rank lower than the (i), (ii), and (iv) control charts. Consequently, the (iii) control chart is most efficient for detecting small mean shifts ($\delta \leq 0.5$) and the large smoothing parameters ($\lambda \geq 0.35$) under normal and non-normal distribution processes.

Further study shall generate the gamma distribution for the quality characteristic variables (p) to be greater than 2, that we could not find a program to generate the gamma distribution for p greater than 2. In this research, the results could not be compared between the normal distribution, the t distribution, and the gamma distribution because the researcher did not determine the mean and variance-covariance values of each random variable distribution were equal. In future research, the researcher can compare the three quality characteristics in the same distributions under the same mean and variance, whereby the researcher can transform the t distribution and the gamma distribution into a normal distribution and compare them. The research can be extended to the Weibull distribution or other continuous distributions that are continuous variables. The Weibull distribution is one of the gamma distribution families widely used to study maintenance engineering, quality engineering, and product reliability analysis. Thus, future studies of the performance of non-parametric multivariate control charts can be brought to compare between the gamma distribution, the Weibull distribution, and other continuous distributions. In this research, the performance measure was ARL only. Thus, the average run length (ARL) and the standard deviation of the run length (SDRL) have been traditionally used as measures of a control chart's performance. The run-length distribution is highly skewed to the right, especially for an in-control process or when the shift is small, so the value of the SDRL is quite high. Even if the ARL exists, it is, in most cases, associated with a high SDRL, which is undesirable. Extremely large values for any of the run-length characteristics, mean that those run-length characteristics can't be computed within a practical time, i.e., using the ARL as a performance measure can be misleading. The skewness is higher for smaller shifts in the location. Thus, the median run length (MRL) is a more credible measure of a chart's performance since it is less affected by the skewness of the run-length distribution (Gan 1993, Maravelakis et al. 2005, Teoh et al. 2013). The use of the MRL ensures better control over the false alarm rate (FAR) in the sense that no more than 50% of the false alarms are guaranteed to be realized before the MRL (Graham et al. 2014).

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Appendix A: Comparisons of the ARL_1

Table 10 Comparisons of the ARL_1 for detecting the mean shifts of $\lambda = 0.05,\ 0.1,\ 0.2,$ and 0.3 when the $\text{ARL}_0 \cong 370$ and $p = 2$ under the normal distribution

δ	λ															
	0.05				0.1				0.2				0.3			
	UCL															
	9.10	6.11	8.75	5.97	10.21	7.52	9.50	7.37	11.05	9.09	9.50	8.72	11.40	10.07	8.94	9.37
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
0	370	370	370	370	371	371	370	371	371	371	370	370	370	370	370	371
0.1	241	211	**231**	209	276	243	**255**	243	304	283	**276**	266	321	302	**281**	282
	[2.41]	[2.69]	**[2.26*]**	[2.64]	[2.43]	[2.69]	**[2.25*]**	[2.64]	[2.45]	[2.72]	**[2.28*]**	[2.55]	[2.50]	[2.68]	**[2.29*]**	[2.53]
0.25	83	67	**79**	67	111	86	**96**	86	153	118	**114**	107	185	147	**117**	122
	[2.55]	[2.59]	**[2.33*]**	[2.53]	[2.64]	[2.59]	**[2.27*]**	[2.50]	[2.74]	[2.69]	**[2.16*]**	[2.41]	[2.80]	[2.76]	**[2.05*]**	[2.40]
0.5	26	22	**25**	22	32	26	**29**	26	46	33	**33**	31	62	43	**33**	36
	[2.59]	[2.56]	**[2.37*]**	[2.48]	[2.71]	[2.55]	**[2.26*]**	[2.48]	[2.92]	[2.60]	**[2.09*]**	[2.39]	[3.05]	[2.74]	**[1.88*]**	[2.33]
1.0	8	7	7	**6**	9	8	**8**	8	10	9	**8**	8	13	10	**8**	9
	[2.72]	[2.46]	[2.53]	**[2.30*]**	[2.72]	[2.54]	**[2.32*]**	[2.43]	[2.91]	[2.60]	**[2.06*]**	[2.43]	[3.11]	[2.68]	**[1.84*]**	[2.37]
1.5	4	3	4	**3**	4	4	4	**4**	5	4	**4**	4	5	4	**4**	4
	[2.83]	[2.32]	[2.63]	**[2.22*]**	[2.78]	[2.44]	[2.45]	**[2.33*]**	[2.85]	[2.60]	**[2.15*]**	[2.40]	[2.99]	[2.71]	**[1.89*]**	[2.40]
2.5	1	1	1	**1**	2	1	1	**1**	2	2	1	**1**	2	2	**1**	2
	[2.80]	[2.26]	[2.69]	**[2.25*]**	[2.79]	[2.34]	[2.57]	**[2.31*]**	[2.79]	[2.48]	[2.37]	**[2.36*]**	[2.82]	[2.60]	**[2.20*]**	[2.39]

The one without the bracket [ ] is ARL_1

[*] are the mean ranks of the ARL_1 of multivariate control charts (MCCs) by the Friedman test, which showed the type of MCCs used for detecting mean shifts led to statistically significant differences.

Table 11 Comparisons of the ARL_1 for detecting the mean shifts of $\lambda = 0.35$, 0.4, 0.5 and 0.8 when the $ARL_0 \cong 370$ and $p = 2$ under the normal distribution

δ	λ															
	0.35				0.4				0.5				0.8			
	UCL															
	11.51	10.44	8.57	9.54	11.62	10.72	8.20	9.56	11.74	11.14	7.45	9.23	11.86	11.81	5.42	6.56
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
0	371	370	371	371	370	370	371	371	370	370	371	371	370	371	370	370
0.1	327	306	**272**	288	334	317	**269**	290	340	324	**259**	286	357	349	**197**	241
	[2.52]	[2.67]	**[2.27*]**	[2.54]	[2.55]	[2.66]	**[2.26*]**	[2.54]	[2.59]	[2.65]	**[2.25*]**	[2.51]	[2.76]	[2.76]	**[2.09*]**	[2.39]
0.25	199	160	**115**	129	218	172	**110**	133	237	191	**99**	129	288	261	**59**	83
	[2.83]	[2.78]	**[1.97*]**	[2.41]	[2.88]	[2.80]	**[1.90*]**	[2.42]	[2.96]	[2.86]	**[1.83*]**	[2.35]	[3.20]	[3.13]	**[1.60*]**	[2.08]
0.5	71	48	**33**	38	79	53	**32**	39	101	65	**29**	39	161	132	**16**	24
	[3.12]	[2.80]	**[1.77*]**	[2.31]	[3.17]	[2.85]	**[1.70*]**	[2.29]	[3.28]	[2.95]	**[1.58*]**	[2.20]	[3.42]	[3.26]	**[1.42*]**	[1.90]
1.0	14	10	**8**	9	16	11	**8**	9	21	13	**7**	9	43	30	**4**	6
	[3.19]	[2.73]	**[1.74*]**	[2.33]	[3.27]	[2.79]	**[1.65*]**	[2.29]	[3.39]	[2.91]	**[1.53*]**	[2.17]	[3.54]	[3.22]	**[1.43*]**	[1.81]
1.5	5	4	**3**	4	6	5	**3**	4	7	5	**3**	4	14	9	**2**	3
	[3.07]	[2.75]	**[1.80*]**	[2.38]	[3.16]	[2.79]	**[1.72*]**	[2.33]	[3.30]	[2.88]	**[1.61*]**	[2.21]	[3.51]	[3.15]	**[1.55*]**	[1.78]
2.5	2	2	**1**	2	2	2	**1**	2	2	2	**1**	1	3	2	**1**	1
	[2.84]	[2.65]	**[2.13*]**	[2.39]	[2.89]	[2.68]	**[2.07*]**	[2.37]	[2.97]	[2.75]	**[1.99*]**	[2.30]	[3.19]	[2.96]	**[1.85*]**	[2.00]

The one without the bracket [ ] is ARL_1

[*] are the mean ranks of the ARL_1 of multivariate control charts (MCCs) by the Friedman test, which showed the type of MCCs used for detecting mean shifts led to statistically significant differences.

Table 12 Comparisons of the ARL_1 for detecting the mean shifts of $\lambda = 0.05$, 0.1, 0.2 and 0.3 when the $ARL_0 \cong 370$ and $p = 2$ under the t distribution

δ	λ															
	0.05				0.1				0.2				0.3			
	UCL															
	9.73	6.20	8.76	5.91	11.88	7.76	9.49	7.28	15.29	9.95	9.49	8.72	18.41	11.81	9.01	9.37
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
0	370	370	370	371	371	371	371	371	371	371	371	370	370	370	370	371
0.1	258	220	**236**	207	304	255	**260**	244	346	306	**271**	274	362	333	**281**	286
	[2.46]	[2.72]	**[2.30*]**	[2.53]	[2.49]	[2.69]	**[2.26*]**	[2.55]	[2.50]	[2.74]	**[2.25*]**	[2.52]	[2.52]	[2.73]	**[2.28*]**	[2.47]
0.25	98	71	**81**	68	154	94	**99**	85	255	146	**111**	110	312	204	**117**	126
	[2.77]	[2.59]	**[2.25*]**	[2.39]	[2.96]	[2.60]	**[2.17*]**	[2.28]	[3.01]	[2.75]	**[1.99*]**	[2.25]	[3.00]	[2.85]	**[1.92*]**	[2.23]
0.5	29	23	**25**	22	45	27	**29**	26	106	41	**33**	31	193	67	**35**	37
	[2.83]	[2.53]	**[2.29*]**	[2.36]	[3.20]	[2.46]	**[2.11*]**	[2.24]	[3.48]	[2.63]	**[1.83*]**	[2.06]	[3.54]	[2.82]	**[1.66*]**	[1.98]
1.0	8	7	8	**7**	11	8	**8**	8	19	10	**9**	8	44	12	**9**	9
	[2.96]	[2.40]	[2.46]	**[2.19*]**	[3.25]	[2.44]	**[2.15*]**	[2.17]	[3.64]	[2.54]	**[1.78*]**	[2.04]	[3.78]	[2.73]	**[1.58*]**	[1.91]
1.5	4	3	4	**3**	5	4	4	**4**	7	4	**4**	4	12	5	**4**	4
	[3.04]	[2.26]	[2.58]	**[2.13*]**	[3.23]	[2.36]	[2.28]	**[2.14*]**	[3.61]	[2.52]	**[1.83*]**	[2.05]	[3.80]	[2.67]	**[1.60*]**	[1.93]
2.5	2	**1**	1	1	2	1	1	**1**	2	2	1	**1**	3	2	**1**	2
	[2.95]	**[2.17*]**	[2.68]	[2.21]	[3.09]	[2.24]	[2.49]	**[2.17*]**	[3.39]	[2.40]	[2.12]	**[2.10*]**	[3.63]	[2.53]	**[1.85*]**	[1.99]

The one without the bracket [ ] is ARL_1

[*] are the mean ranks of the ARL_1 of multivariate control charts (MCCs) by the Friedman test, which showed the type of MCCs used for detecting mean shifts led to statistically significant differences.

Table 13 Comparisons of the ARL_1 for detecting the mean shifts of $\lambda = 0.35$, 0.4, 0.5 and 0.8 when the $\mathrm{ARL}_0 \cong 370$ and $p = 2$ under the t distribution

δ	λ															
	0.35				0.4				0.5				0.8			
	UCL															
	19.76	12.73	8.69	9.49	21.07	13.66	8.36	9.51	23.44	15.71	7.67	9.29	28.24	25.74	5.68	6.81
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
0	371	370	370	370	370	370	371	370	370	371	370	371	371	371	371	370
0.1	362	341	**279**	289	364	350	**280**	288	364	355	**275**	291	369	369	**216**	253
	[2.52]	[2.73]	**[2.28*]**	[2.48]	[2.53]	[2.72]	**[2.29*]**	[2.45]	[2.59]	[2.66]	**[2.30*]**	[2.45]	[2.75]	[2.74]	**[2.14*]**	[2.37]
0.25	324	232	**116**	131	332	258	**114**	132	346	295	**108**	130	364	356	**67**	95
	[2.99]	[2.89]	**[1.89*]**	[2.23]	[2.99]	[2.93]	**[1.87*]**	[2.21]	[3.02]	[2.98]	**[1.82*]**	[2.17]	[3.18]	[3.16]	**[1.63*]**	[2.03]
0.5	224	86	**35**	39	251	109	**34**	40	286	163	**32**	40	336	319	**19**	28
	[3.52]	[2.91]	**[1.60*]**	[1.97]	[3.52]	[2.97]	**[1.57*]**	[1.95]	[3.48]	[3.09]	**[1.51*]**	[1.92]	[3.41]	[3.36]	**[1.40*]**	[1.83]
1.0	66	15	**8**	9	91	19	**8**	9	142	33	**8**	9	256	207	**5**	7
	[3.81]	[2.81]	**[1.50*]**	[1.88]	[3.82]	[2.88]	**[1.45*]**	[1.85]	[3.82]	[2.98]	**[1.39*]**	[1.81]	[3.57]	[3.37]	**[1.38*]**	[1.69]
1.5	17	6	**4**	4	25	6	**3**	4	51	9	**3**	4	162	102	**2**	3
	[3.84]	[2.76]	**[1.52*]**	[1.88]	[3.87]	[2.83]	**[1.47*]**	[1.84]	[3.89]	[2.93]	**[1.41*]**	[1.77]	[3.66]	[3.29]	**[1.43*]**	[1.62]
2.5	4	2	**1**	2	4	2	**1**	2	7	2	**1**	1	43	16	**1**	1
	[3.72]	[2.60]	**[1.75*]**	[1.94]	[3.78]	[2.67]	**[1.67*]**	[1.88]	[3.86]	[2.80]	**[1.57*]**	[1.78]	[3.79]	[3.15]	**[1.48*]**	[1.57]

The one without the bracket [ ] is ARL_1

[*] are the mean ranks of the ARL_1 of multivariate control charts (MCCs) by the Friedman test, which showed the type of MCCs used for detecting mean shifts led to statistically significant differences.

Table 14 Comparisons of the ARL_1 for detecting the mean shifts of $\lambda = 0.05$, 0.1, 0.2 and 0.3 when the $ARL_0 \cong 370$ and $p = 2$ under the gamma distribution

δ	λ															
	0.05				0.1				0.2				0.3			
	UCL															
	217.25	360.60	71.47	121.08	131.23	202.54	41.55	67.25	85.19	118.50	24.62	38.30	69.05	89.03	18.07	27.67
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
0	370	370	370	371	371	371	371	371	370	371	371	371	370	370	370	371
0.1	254	246	**135**	143	268	247	**128**	125	289	261	**133**	128	307	285	**134**	136
	[2.96]	[3.64]	**[1.37*]**	[2.02]	[2.91]	[3.52]	**[1.58*]**	[1.99]	[2.87]	[3.34]	**[1.75*]**	[2.05]	[2.89]	[3.21]	**[1.81*]**	[2.09]
0.25	154	151	**64**	79	164	142	**49**	53	199	157	**44**	45	230	185	**43**	45
	[3.17]	[3.69]	**[1.06*]**	[2.09]	[3.29]	[3.59]	**[1.26*]**	[1.87]	[3.31]	[3.51]	**[1.42*]**	[1.76]	[3.30]	[3.45]	**[1.47*]**	[1.78]
0.5	84	92	**35**	52	81	71	**23**	30	107	73	**17**	19	138	91	**15**	16
	[3.09]	[3.80]	**[1.00*]**	[2.11]	[3.38]	[3.56]	**[1.03*]**	[2.04]	[3.51]	[3.43]	**[1.19*]**	[1.87]	[3.53]	[3.41]	**[1.28*]**	[1.78]
1.0	42	57	**18**	31	32	34	**11**	17	35	26	**7**	9	48	28	**5**	7
	[2.93]	[3.99]	**[1.00*]**	[2.08]	[3.19]	[3.77]	**[1.00*]**	[2.04]	[3.60]	[3.38]	**[1.01*]**	[2.01]	[3.68]	[3.29]	**[1.08*]**	[1.94]
1.5	27	42	**11**	21	18	23	**7**	11	16	15	**4**	6	20	13	**3**	4
	[2.95]	[4.00]	**[1.00*]**	[2.05]	[3.00]	[3.97]	**[1.00*]**	[2.02]	[3.45]	[3.54]	**[1.00*]**	[2.02]	[3.69]	[3.29]	**[1.04*]**	[1.98]
2.5	15	26	**6**	11	9	14	**3**	6	7	8	**2**	3	6	6	**1**	2
	[2.98]	[4.00]	**[1.00*]**	[2.02]	[2.98]	[4.00]	**[1.00*]**	[2.02]	[3.06]	[3.92]	**[1.02*]**	[2.01]	[3.44]	[3.54]	**[1.13*]**	[1.90]

The one without the bracket [ ] is ARL_1

[*] are the mean ranks of the ARL_1 of multivariate control charts (MCCs) by the Friedman test, which showed the type of MCCs used for detecting mean shifts led to statistically significant differences.

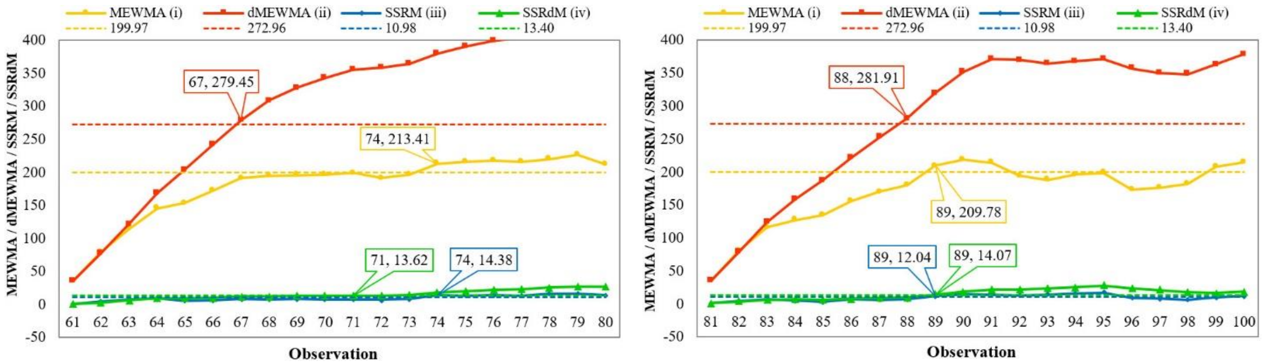
Table 15 Comparisons of the ARL_1 for detecting the mean shifts of $\lambda = 0.35$, 0.4, 0.5 and 0.8 when the $\text{ARL}_0 \cong 370$ and $p = 2$ under the gamma distribution

δ	λ															
	0.35				0.4				0.5				0.8			
	UCL															
	64.36	80.19	15.99	24.39	60.73	73.57	14.35	21.77	55.60	64.10	11.84	17.65	46.73	49.81	7.35	9.33
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
0	371	370	370	371	371	371	371	371	371	371	371	371	371	370	370	371
0.1	316	289	**133**	140	323	296	**132**	143	329	307	**126**	141	344	331	**103**	120
	[2.90]	[3.15]	**[1.82*]**	[2.13]	[2.92]	[3.10]	**[1.84*]**	[2.15]	[2.97]	[3.05]	**[1.85*]**	[2.13]	[3.07]	[3.06]	**[1.82*]**	[2.05]
0.25	242	196	**43**	46	251	210	**42**	47	271	228	**40**	46	302	285	**29**	38
	[3.30]	[3.41]	**[1.48*]**	[1.81]	[3.30]	[3.38]	**[1.49*]**	[1.83]	[3.33]	[3.33]	**[1.51*]**	[1.83]	[3.38]	[3.34]	**[1.47*]**	[1.81]
0.5	152	99	**14**	16	163	111	**13**	15	189	135	**12**	15	241	214	**8**	11
	[3.54]	[3.39]	**[1.31*]**	[1.76]	[3.54]	[3.39]	**[1.33*]**	[1.75]	[3.55]	[3.37]	**[1.35*]**	[1.74]	[3.51]	[3.41]	**[1.39*]**	[1.69]
1.0	57	30	**5**	6	65	34	**4**	6	85	45	**4**	5	143	110	**3**	3
	[3.70]	[3.27]	**[1.13*]**	[1.90]	[3.70]	[3.27]	**[1.17*]**	[1.85]	[3.70]	[3.27]	**[1.25*]**	[1.78]	[3.59]	[3.38]	**[1.39*]**	[1.64]
1.5	23	13	**3**	4	27	14	**3**	3	38	17	**2**	3	79	55	**1**	2
	[3.73]	[3.25]	**[1.09*]**	[1.94]	[3.76]	[3.22]	**[1.14*]**	[1.89]	[3.78]	[3.20]	**[1.22*]**	[1.80]	[3.64]	[3.34]	**[1.40*]**	[1.63]
2.5	7	6	**1**	2	7	5	**1**	2	9	5	**1**	1	23	14	**1**	1
	[3.57]	[3.40]	**[1.16*]**	[1.87]	[3.66]	[3.32]	**[1.19*]**	[1.84]	[3.75]	[3.22]	**[1.27*]**	[1.76]	[3.69]	[3.27]	**[1.44*]**	[1.59]

The one without the bracket [ ] is ARL_1

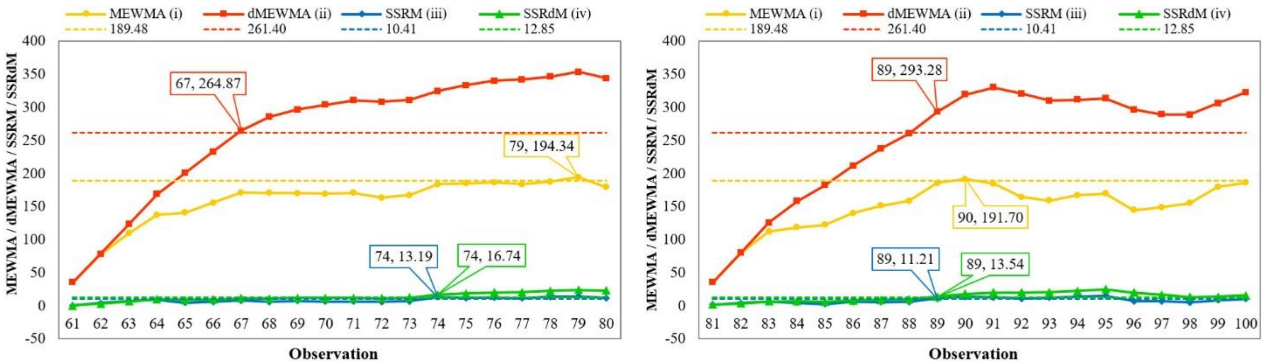
[*] are the mean ranks of the ARL_1 of multivariate control charts (MCCs) by the Friedman test, which showed the type of MCCs used for detecting mean shifts led to statistically significant differences.

Appendix B: UCLs and control statistics in Phases 1 and 2 for a normally distributed process



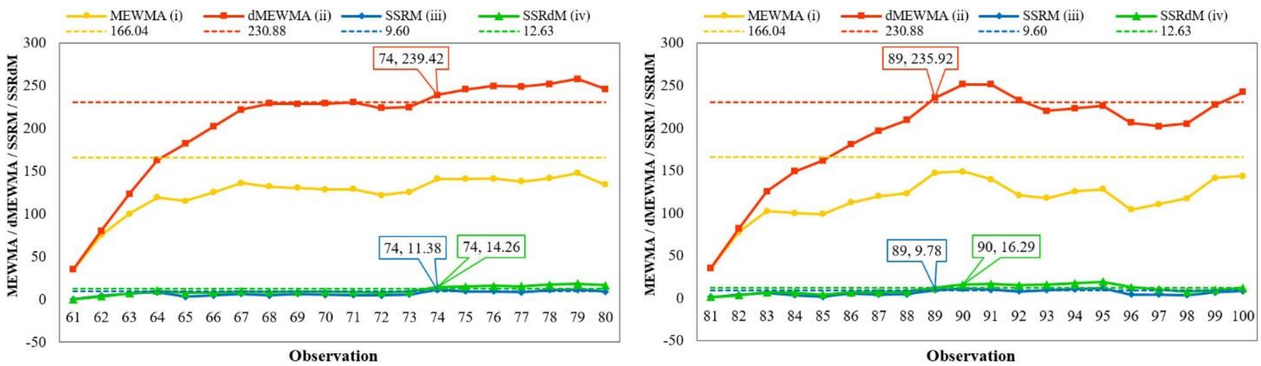
(a) The first repeat (b) The second repeat

Figure 4 UCLs and control statistics in Phases 1 and 2 of the (i), (ii), (iii), and (iv) control charts for detecting of three variables of the tensile stress of steel for a normally distributed process under $\lambda = 0.35$ and $\delta = 0.25$



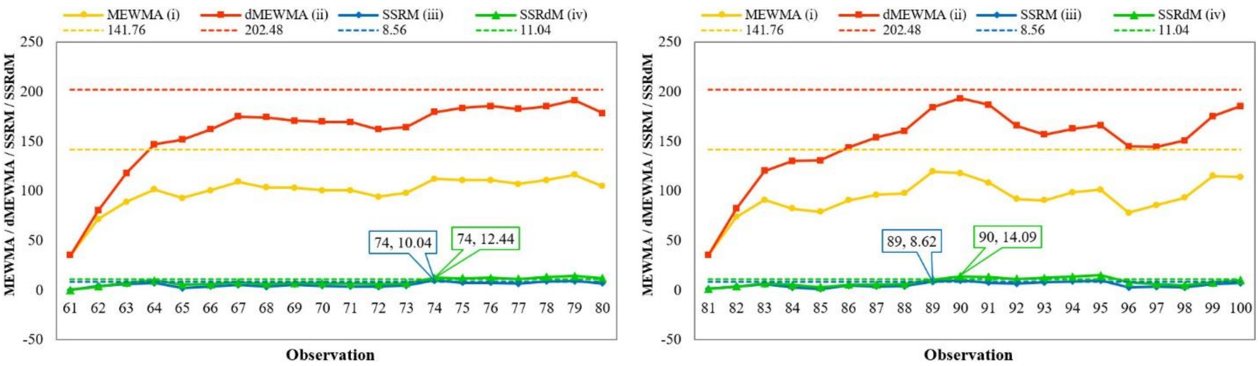
(a) The first repeat (b) The second repeat

Figure 5 UCLs and control statistics in Phases 1 and 2 of the (i), (ii), (iii), and (iv) control charts for detecting of three variables of the tensile stress of steel for a normally distributed process under $\lambda = 0.4$ and $\delta = 0.25$



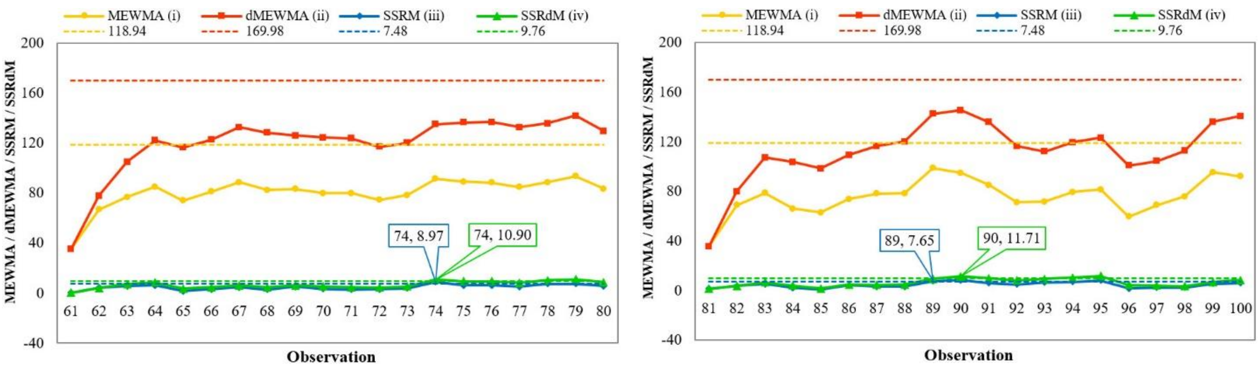
(a) The first repeat (b) The second repeat

Figure 6 UCLs and control statistics in Phases 1 and 2 of the (i), (ii), (iii), and (iv) control charts for detecting of three variables of the tensile stress of steel for a normally distributed process under $\lambda = 0.5$ and $\delta = 0.25$



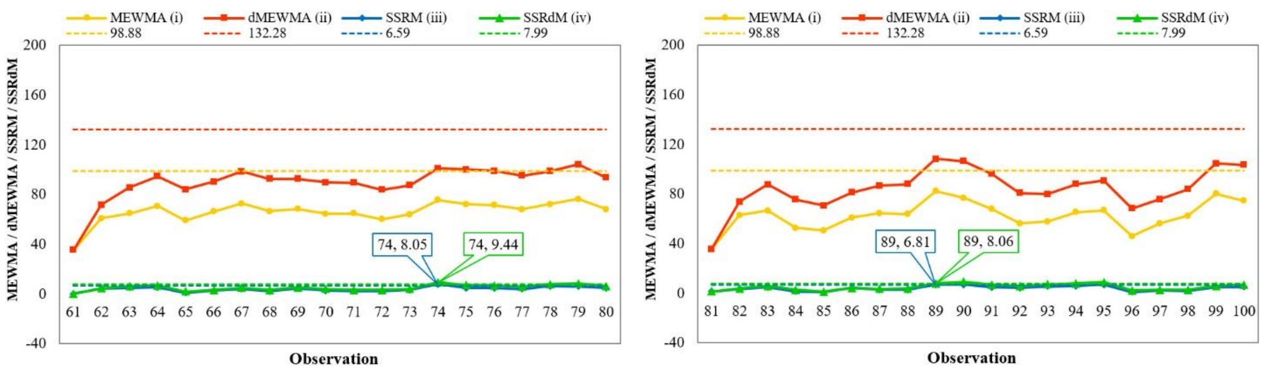
(a) The first repeat (b) The second repeat

Figure 7 UCLs and control statistics in Phases 1 and 2 of the (i), (ii), (iii), and (iv) control charts for detecting of three variables of the tensile stress of steel for a normally distributed process under $\lambda = 0.6$ and $\delta = 0.25$



(a) The first repeat (b) The second repeat

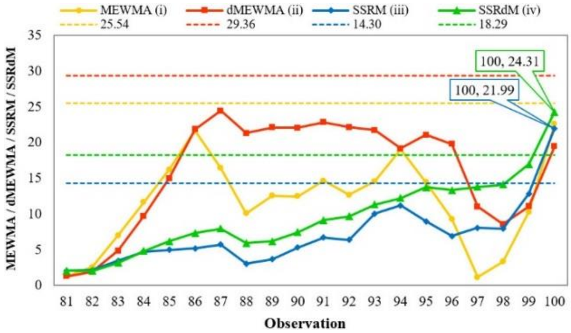
Figure 8 UCLs and control statistics in Phases 1 and 2 of the (i), (ii), (iii), and (iv) control charts for detecting of three variables of the tensile stress of steel for a normally distributed process under $\lambda = 0.7$ and $\delta = 0.25$



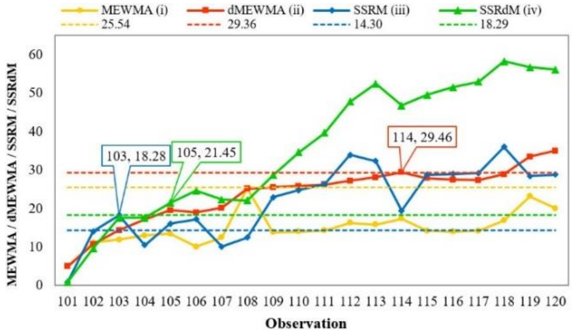
(a) The first repeat (b) The second repeat

Figure 9 UCLs and control statistics in Phases 1 and 2 of the (i), (ii), (iii), and (iv) control charts for detecting of three variables of the tensile stress of steel for a normally distributed process under $\lambda = 0.8$ and $\delta = 0.25$

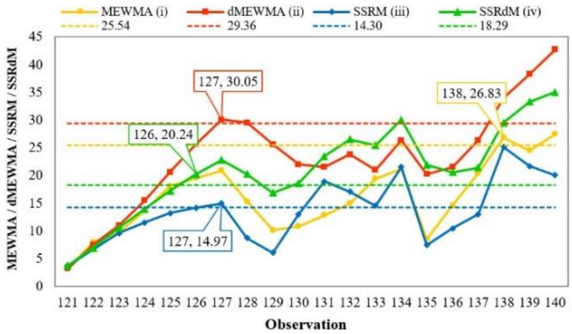
Appendix C: UCLs and control statistics in Phases 1 and 2 for a non-normally distributed process



(a) The first repeat

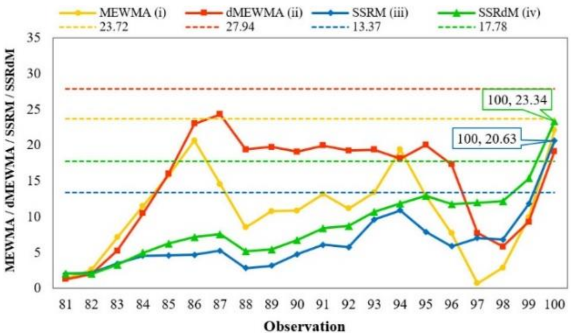


(b) The second repeat

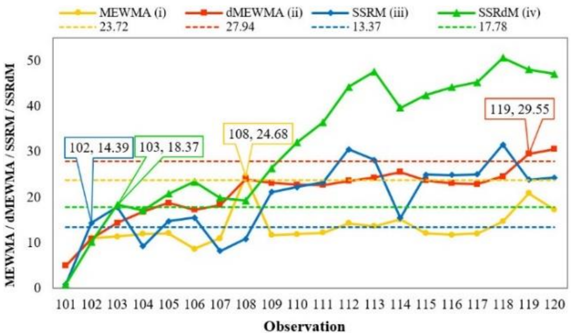


(c) The third repeat

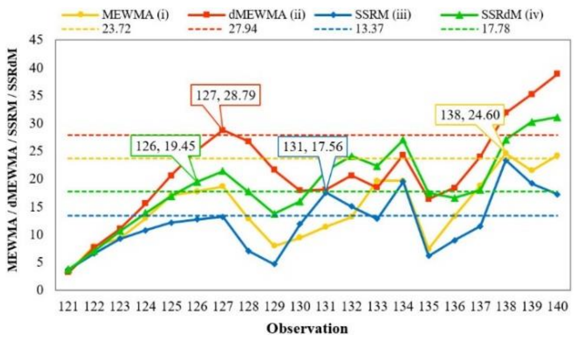
Figure 10 UCLs and control statistics in Phases 1 and 2 of the (i), (ii), (iii), and (iv) control charts for detecting of three variables of the blade wheel of blower for a non-normally distributed process under $\lambda = 0.35$ and $\delta = 0.25$



(a) The first repeat

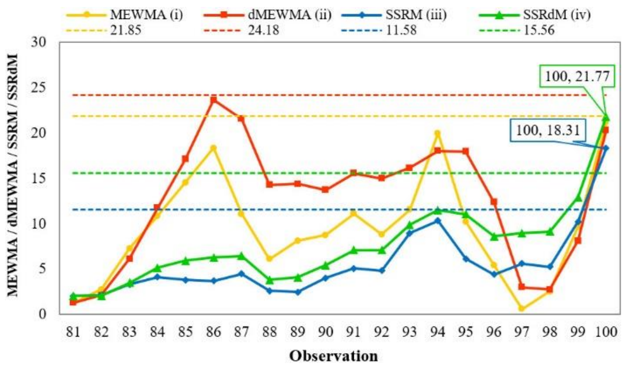


(b) The second repeat

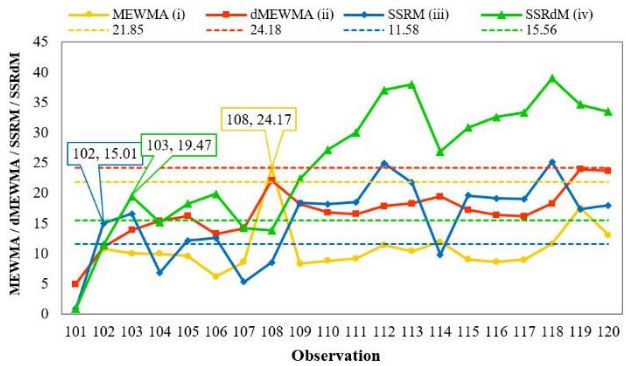


(c) The third repeat

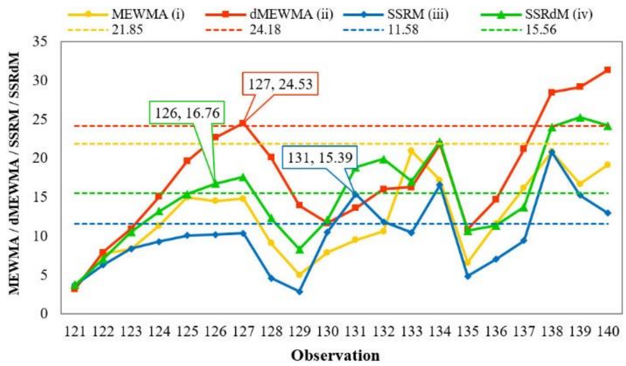
Figure 11 UCLs and control statistics in Phases 1 and 2 of the (i), (ii), (iii), and (iv) control charts for detecting of three variables of the blade wheel of blower for a non-normally distributed process under $\lambda = 0.4$ and $\delta = 0.25$



(a) The first repeat

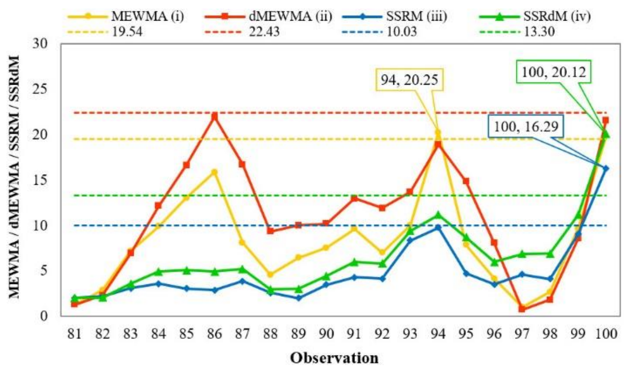


(b) The second repeat

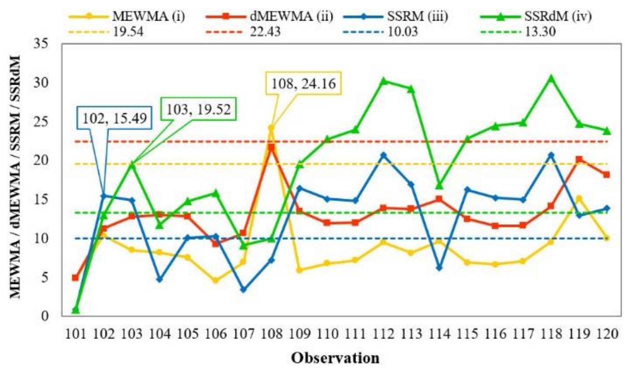


(c) The third repeat

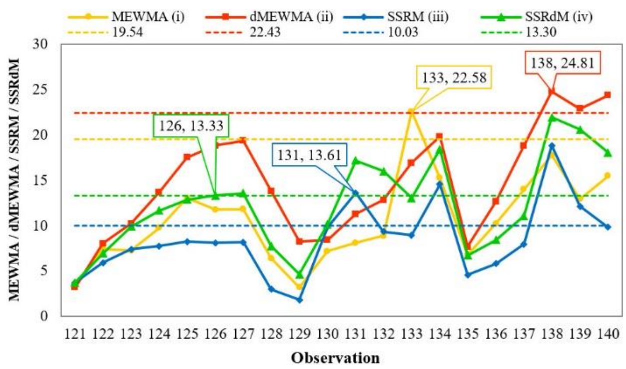
Figure 12 UCLs and control statistics in Phases 1 and 2 of the (i), (ii), (iii), and (iv) control charts for detecting of three variables of the blade wheel of blower for a non-normally distributed process under $\lambda = 0.5$ and $\delta = 0.25$



(a) The first repeat

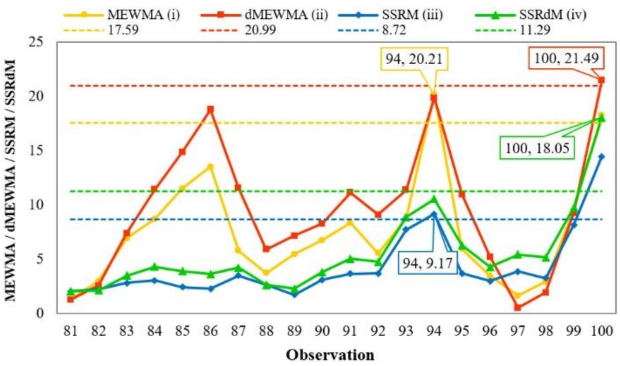


(b) The second repeat

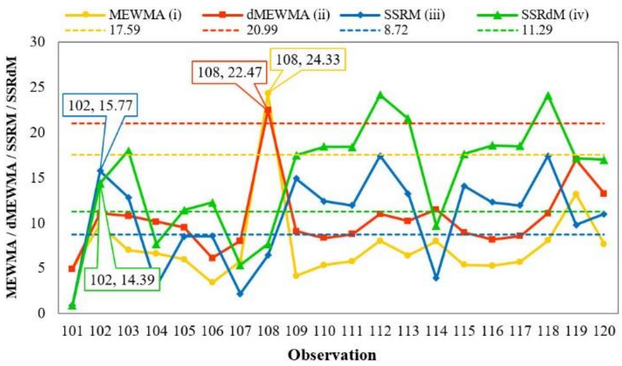


(c) The third repeat

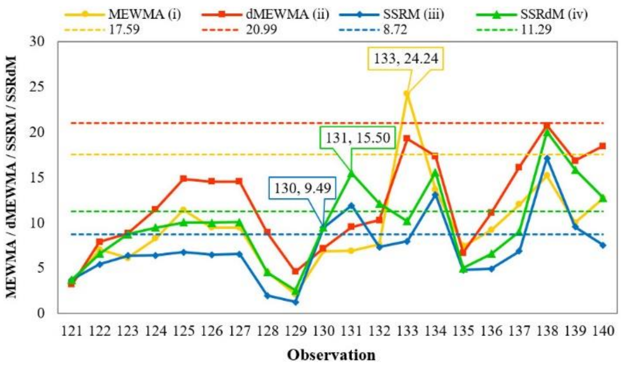
Figure 13 UCLs and control statistics in Phases 1 and 2 of the (i), (ii), (iii), and (iv) control charts for detecting of three variables of the blade wheel of blower for a non-normally distributed process under $\lambda = 0.6$ and $\delta = 0.25$



(a) The first repeat

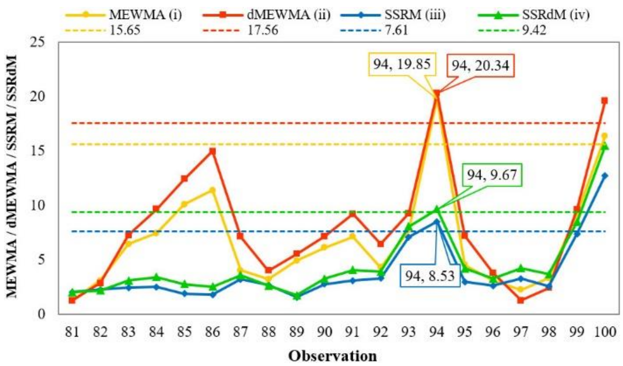


(b) The second repeat

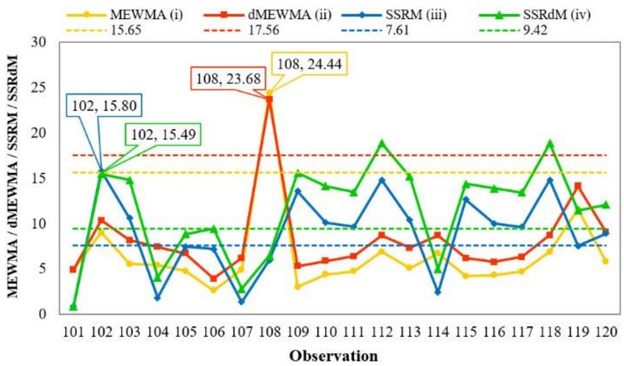


(c) The third repeat

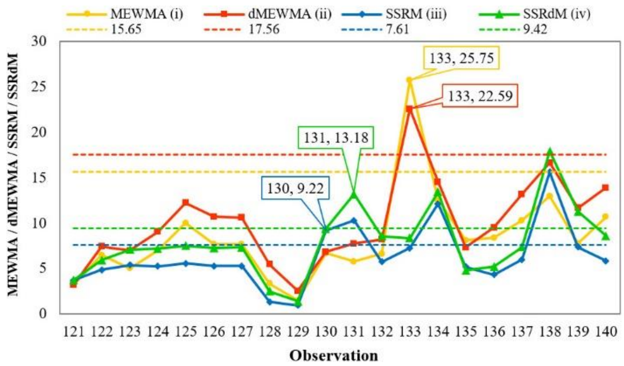
Figure 14 UCLs and control statistics in phase 1 and 2 of the (i), (ii), (iii), and (iv) control charts for detecting of three variables of the blade wheel of blower for a non-normally distributed process under $\lambda = 0.7$ and $\delta = 0.25$



(a) The first repeat



(b) The second repeat



(c) The third repeat

Figure 15 UCLs and control statistics in phase 1 and 2 of the (i), (ii), (iii), and (iv) control charts for detecting of three variables of the blade wheel of blower for a non-normally distributed process under $\lambda = 0.8$ and $\delta = 0.25$