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On GARCH Models and Applications: Foreign Exchange Rate Volatility and a Price Index

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Abstract

This paper studies the benefit of GARCH models, and presents two applications on the exchanging of the rate volatility of Algerian Dinar against the Euro and US Dollar and CAC40French index. The goal of this work is to add another benefit of univariate GARCH models which this work contains an application relates to exchange rates between the Algerian Dinar (DZD), the American dollar (USD) and the Euro from 1/01/2014 to 15/05/2019. Also, this paper applies analytical solutions from Zeghdoudi et al. (2014) to price a swap on the volatility of the CAC40 French index for five years (from October 2013 to April 2018).

Keywords: Volatility swaps, Heston model, GARCH, ARCH, forecasting.

1. Introduction

In 1982 Engle proposed the autoregressive conditional heteroscedastic (ARCH) model with an estimation of the parameters. Thus, the model allowed the data to determine the best weights and to use in forecasting the variance. A useful generalization of this model consists of the ARCH parameterization introduced by Bollerslev (1986). Moreover, generalized autoregressive conditional heteroscedastic (GARCH) models have been developed for modeling the volatility of financial data. To this end, it plays an important role in financial decisions. Volatility is one of the principal parameters employed to describe and measure the fluctuations of asset prices. It plays an important role in the modern financial analysis of which risk management, option valuation and asset allocation. There are different types of volatility: implied volatility, local volatility and stochastic volatility.

Volatility swaps allow investors to directly negotiate and control the volatility of an asset that is typically used for an exchange rate, but could also be a unique name index. However, the variance swap is reliable in the index market as it can be replicated with a linear combination of options and a dynamic position on futures.

The goal of this paper is to add another benefit of univariate GARCH models which this work contains an application relates to exchange rates between the Algerian Dinar (DZD), the American dollar (USD) and the Euro from 1/01/2014 to 15/05/2019. The DZD for one Euro and for one Dollar (USD). Also, we apply analytical solutions from Zeghdoudi et al. (2014) to price a swap on the volatility of the CAC40 French index for five years (from October 2013 to April 2018). Moreover, there are several studies confirm this results. As mentioned before early to this topic are Ezzebsa et al. (2013) Zeghdoudi et al. (2013, 2014), Zeghdoudi and Bouseba (2015), and Priviledge (2016).

Recently, Amrani and Zeghdoudi (2021) presented the advantages of multivariate GARCH models in the oil market and the energy market, which we found that the assembly performance of the BEKK-GARCH form is better than that of other models.

The structure of the paper is as follows. Section 2 provides econometric methodology and a preliminary analysis of GARCH models. Section 3 displays and discusses some definitions and notations of swap, stock's volatility, stock's volatility swap and variance swap, while Section 4 provides our applications and conclusions.

2. Analytical Framework of GARCH Family Models

Autoregressive (AR), moving average (MA) and the mixed autoregressive moving average (ARMA) models are often very useful in modelling general time series. However, they all have the assumption of homoscedasticity (or equal variance) for the errors; this is not appropriate when dealing with the financial market variables such as the stock price indices or currency exchange rates. These financial market variables typically have three characteristics which general time series models have failed to consider.

1. The unconditional distribution of financial time series such as the stock price returns X_t has heavier tails than the normal distribution.

2. Values of X_t do not have much correlation, but values of X_t^2 are highly correlated.

3. The changes in X_t tend to cluster. Large (small) changes in X_t tend to be followed by large (small) changes, as documented by Mandelbrot (1963).

One of the earliest time series models allowing for heteroscedasticity is the ARCH model introduced by Engle (1982). The ARCH models have the ability to capture all the above characteristics in financial market variables. Bollerslev (1986) extended this idea into GARCH models which give more parsimonious results than ARCH models, similar to the situation where ARMA models are preferred over AR models.

In the following subsection, we shall present several GARCH models. For details, we can see (Zakoian 1991, 1994). According to Engel (1982) model, conditional heteroscedasticity in a stock return (SRET) denoted by X_t.

Definition 1 Denote a financial asset with price S_t at time t (t is an integer) and price S_{t-1} at $t-1$, the return is defined as $X_t = \ln \dfrac{S_t}{S_{t-1}}$.

2.1. GARCH (*p*, *q*) model

A process X_t satisfies a representation GARCH (*p, q*) if

$$X_t = z_t \sqrt{h_t},$$

where $h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i X_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}$, $z_t \to N\left(0, \sigma^2\right)$ with $\alpha_0 > 0, \alpha_i \geq 0$, $\forall i = 1,...,q$ and $\beta_i \geq 0$, $\forall i = 1,..., p$. The X_t admit the conditional moments:

$$E(X_t \mid X_{t-1}) = 0,$$

$$V(X_t \mid X_{t-1}) = h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i X_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}.$$

The GARCH (1,1) is the most popular model in the empirical literature:

$$X_t = z_t \sqrt{h_t}$$

where $h_t = \alpha_0 + \alpha_1 X_{t-1}^2 + \beta_1 h_{t-1}.$

2.2. GARCH (*p*,*q*) with errors model

We consider a linear regressive model in the following form:

$$X_t = E(X_t \mid X_{t-1}) + \varepsilon_t,$$

where $\varepsilon_t \to N\left(0,\sigma^2\right)$ and $E(\varepsilon_t \varepsilon_s) = 0$ if $s \neq t,$ which are satisfying the condition with difference of martingale $E(\varepsilon_t \mid \varepsilon_{t-1}) = 0.$ It is always supposed that the process ε_t can be written in the form

$$\varepsilon_t = z_t \sqrt{h_t}, \text{ where } z_t \to N\left(0,\sigma^2\right), \text{ and } h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i X_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i},$$

with $\alpha_0 > 0, \alpha_i > 0, \forall i = 1,...,q, \beta_i \geq 0, \forall i = 1,..., p.$

We seek to model the conditional volatility of the noise process X_t. For take into account the dynamics observed on $X_t^{\,2}$, we may have to impose a value high of the q parameter in the ARCH(q) modeling which can cause problems estimation. This is a difficulty similar to that encountered in the modeling of conditional expectation: if Wold's theorem ensures that any stationary series has a representation of type MA, it is possible that for a given series, the order of this MA is particularly high, even infinite. In that case, Box and Jenkins propose to regain sparingly using a representation of type AR(p) or ARMA(p,q).

2.3. ARMA-GARCH models

It is Weiss (1986) who introduced into the conditional variance additional effects explained variables. Indeed, GARCH modeling can be applied to the initial process, but to the innovation process. This then allows introducing various additional effects of explanatory variables either in the conditional average, either in the conditional variance. For example, we can consider a model of linear regression with GARCH errors:

$$Y_t = X_t b + \varepsilon_t; \;\; \varepsilon_t \to \text{GARCH}(p,q).$$

We can also consider a model ARMA with GARCH errors:

$$\Phi(L)Y_t = \Theta(L)\varepsilon_t; \quad \varepsilon_t \to \text{GARCH}(p,q),$$

where $\Phi(L)$ and $\Theta(L)$ are linear operators (polynomials in L).This model is called ARMA-GARCH model. Then, we can design an ARMA model in which the non-conditional variance of Y_t can have an effect on the conditional variance:

$$\Phi(L)Y_t = \Theta(L)\varepsilon_t,$$

$$E(\varepsilon_t \mid \varepsilon_{t-1}) = 0, V(\varepsilon_t \mid \varepsilon_{t-1}) = c + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-J}^{\;2} + \gamma_0 [E(Y_t \mid Y_{t-1})]^2 + \sum_{j=1}^{p} \gamma_j Y_{t-j}^2.$$

2.4. GARCH-M models

Engle et al. (1987) proposed the general autoregressive conditional heteroscedasticity in mean (GARCH-M) models, where the conditional variance is an explanatory variable of the conditional mean. These processes seem more adapted thus to a description of the influence of volatility on the output of the titles.

Definition 2 The writing of GARCH-M model relates to non-stationary of its process of conditional variance and by an infinite non conditional variance. Let Y_t be a process with $E(Y_t)=0,$ this process can be written in the following form

$$Y_t = X_t b + \delta ht + z_t \sqrt{h_t} = X_t b + \delta V(\varepsilon_t / \varepsilon_{t-1}) + z_t \sqrt{h_t}, \varepsilon_t = z_t \sqrt{h_t},$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i},$$

$$E(\varepsilon_t / \varepsilon_{t-1}) = 0,$$

$$V(\varepsilon_t / \varepsilon_{t-1}) = V(Y_t / Y_{t-1}) = h_t.$$

Besides the linear form of the writing of Y_t above, we can consider various alternatives of the relation between the dependent variable Y_t and the conditional variance. For example, we can consider the following cases:

Log-Linear form: $X_t = X_t b + \delta \log h_t + \varepsilon_t,$

Square Root form: $Y_t = X_t b + \delta \sqrt{h_t} + \varepsilon_t.$

2.5. Forecasting foreign exchange rate

Exchange rate is an important variable for the financial decision. Also, it gives information on the internal balance and the foreign equilibrium of the economics of one country. Another way is that the exchange rates have very important influence on the investors' correct investment strategies choice.

Therefore, the scientific researchers and practices in this field seek the act descriptions and predictions about exchange rate volatility, purchasing-power parity (PPP) theory, interest rate parity theory, international payments theory etc.

By rapid development of global economic and statistics, there are several linear and nonlinear models of time series that modeling the exchange which the autoregressive integrated moving average (ARMA) model, random walk model and GARCH models. Also, the GARCH model prediction in RMB exchange rate time series is feasible. West and Cho (1995) uses time's series models to compare the forecasting efficiency which the GARCH is the best model. On the other hand, in the short-term forecasting; ARMA model's forecasting effect is inferior. Moreover, there are several studies confirm this results. As mentioned before early to these topics are Hopper (1997), Engle and Jeffrey (1997). Brooks (1997), Brooks and Burke (1998) and Zeghdoudi et al. (2013).

3. Volatility Swaps

In this section, we give some definitions and notations of swap, stock's volatility, stock's volatility swap and variance swap.

Definition 3 Swaps were introduced in the 1980s and is an agreement between two parties to exchange cash flows at one or several future dates as defined in Tuckman and Serrat (1996). In this contract one party agrees to pay a fixed amount to a counterpart who in turn honors the agreement by paying a floating amount, which depends on the level of some specific underlying.

Definition 4 A stocks' volatility is the simplest measure of its risk less or uncertainty. Formally, the volatility $\sigma_R(S)$ is the annualized standard deviation of the stock's returns during the period of interest, where the subscript R denotes the observed or "realized" volatility for the stock S.

Definition 5 (Demeterfi et al. 1999) A stock volatility swap is a forward contract on the annualized volatility. Its payoff at expiration is equal to $N(\sigma_R(S)-K_{vol})$, where $\sigma_R(S)=\sqrt{\frac{1}{T}\int_0^T \sigma_s^2 ds}$, σ_t is a stochastic stock volatility, K_{vol} is the annualized volatility delivery price and N is the notional amount of the swap in Euro annualized volatility point.

Definition 6 (Demeterfi et al. 1999) A variance swap is a forward contract on annualized variance, the square of the realized volatility. Its payoff at expiration is equal to $N(\sigma_R^2(S)-K_{\text{var}})$ where K_{var} is the annualized volatility delivery price, and N is the notional amount of the swap in Euros per annualized volatility point squared.

Notation 1 We note that $\sigma_R^2(S)=V$.

Using the Brockhaus and Long (2000) and Javaheri (2004) approximation which is used the second order Taylor formula for $\sqrt{V}$, we have $E(\sqrt{V})\approx\sqrt{E(V)}-\frac{Var(V)}{8E^{\frac{3}{2}}(V)}$, where $\frac{Var(V)}{8E^{\frac{3}{2}}(V)}$ is the convexity adjustment. Namely, the last formula is necessary for computing the volatility swap.

3.1. Volatility swaps for Heston (1993) model

Let (Ω, F, F_t, P) be a probability space with filtration F_t, $t\in[0;T]$. We consider the risk-neutral Heston stochastic volatility model for the price S_t and variance follow the following model:

$$\begin{cases} dS_t = r_t S_t dt + \sigma_t S_t dw_1(t), \\ d\sigma_t^2 = k(\theta^2-\sigma_t^2)dt + \xi\sigma_t dw_2(t), \end{cases}$$

where r_t is deterministic interest rate, $\sigma_0>0$ and $\theta>0$ are short and long volatility, $k>0$ is a reversion speed, $\xi>0$ is a volatility of volatility parameter, $w_1(t)$ and $w_2(t)$ are two standard Brownian motions.

3.2. Explicit expression and properties of σ_t^2

In this section, we give an explicit expression and properties of σ_t^2 which we used in section 4. Consider the following function $\Phi(t)=\xi^{-2}\int_0^t (e^{k\Phi(s)}(\sigma_0^2-\theta^2+\tilde{w}_2(t)+\theta^2 e^{2k\Phi(s)})^{-1}ds.$

Definition 7 We define $B(t)=\tilde{w}_2(\Phi_t^{-1})$, $\tilde{\mathbf{F}}_t=\mathbf{F}_{\Phi_t^{-1}}$ and $t\wedge s=\min(t,s)$, where Φ_t^{-1} is an inverse function of Φ_t. The properties of $B(t)$ are the following:

a) $\tilde{\mathbf{F}}_t -$martingale and $E\left(B(t)=0\right)$,

b) $E(B^2(t))=\xi^2\left(\frac{e^{kt}-1}{k}(\sigma_0^2-\theta^2)+\frac{e^{2kt}-1}{2k}\theta^2\right)$,

c) $E(B(s)(B(t))=\xi^2\left(\frac{e^{k(t\wedge s)}-1}{k}(\sigma_0^2-\theta^2)+\frac{e^{2k(t\wedge s)}-1}{2k}\theta^2\right)$.

Lemma1 Let as above. Then,

$$\sigma_t^2=e^{-kt}(\sigma_0^2-\theta^2+B(t))+\theta^2,$$

$$E(\sigma_t^2)=e^{-kt}\left(\sigma_0^2-\theta^2\right)+\theta^2,$$

$$E(\sigma_s^2\sigma_t^2)=\xi^2e^{-k(t+s)}\left(\frac{e^{k(t\wedge s)}-1}{k}(\sigma_0^2-\theta^2)+\frac{e^{2k(t\wedge s)}-1}{2k}\theta^2\right)+e^{-k(t+s)}(\sigma_0^2-\theta^2)^2$$
$$+e^{-kt}(\sigma_0^2-\theta^2)\theta^2+e^{-ks}(\sigma_0^2-\theta^2)\theta^2+\theta^4.$$

Theorem 1 *We have*

a) $E(V)=\frac{1-e^{-kT}}{kT}(\sigma_0^2-\theta^2)+\theta^2$,

b) $Var(V)=\frac{\xi^2e^{-2kT}}{2k^3T^2}\left((2e^{2kT}-4kTe^{kT}-2)(\sigma_0^2-\theta^2)+(2kTe^{2kT}-3e^{2kT}+4e^{kT}-1)\theta^2\right)$.

Proof: see Zeghdoudi et al. (2014).

4. Applications and Discussion

In this section, we used Eviews software.

4.1. First application

Figures 1 and 2 displays the history of exchange rates between the Algerian Dinar (DZD), the American Dollar (USD), and the Euro (EUR) from 1/01/2014 to 15/05/2019.

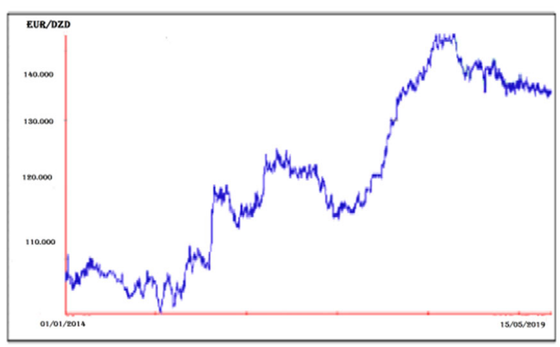

Figure 1 History of exchange rates between DZD and EUR

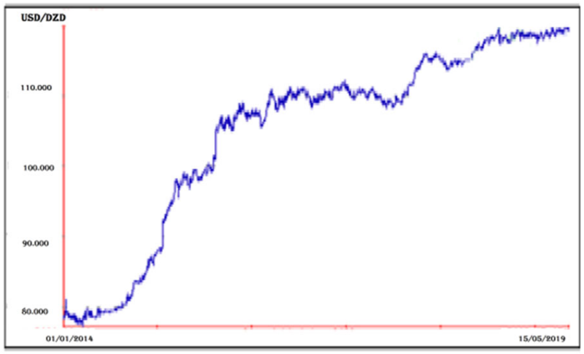

Figure 2 History of exchange rates between DZD and USD

The visual reading of the series of exchange rate EUR/DZD and USD/DZD displays not stationarity in mean and in variance. In this case, it is necessary to use a transformation of the series of exchange rate. For example, we get $Y_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$, employ $Y_t' = \frac{S_t - S_{t-1}}{S_{t-1}}$, where S_t is the exchange rate volatility because the unit root test of Dickey-Fuller (DF) and Philips Péron (PP) tests confirm the stationarity of the series.

4.1.1 Unit root tests and descriptive analysis

In this section, we summarize unit root tests and descriptive analysis results of $S_{t(Dollar)}$, $S_{t(Euro)}$, $Y_{t(Dollar)}$, $Y'_{t(Dollar)}$, $Y_{t(Euro)}$ and $Y'_{t(Euro)}$, respectively.

Table 1 Unit root tests of $S_{t(Dollar)}$, $Y_{t(Dollar)}$, $Y'_{t(Dollar)}$, $S_{t(Euro)}$, $Y_{t(Euro)}$ and $Y'_{t(Euro)}$ of $\alpha = 1\%$

Test	DF	PP
$S_{t(Dollar)}$	−2.388	−2.300
$Y_{t(Dollar)}$	−18.914	−19.062
$Y'_{t(Dollar)}$	−19.029	−19 .327
$S_{t(Euro)}$	−3.331	−2.976
$Y_{t(Euro)}$	−13.129	−17.066
$Y'_{t(Euro)}$	−13.534	−17.718

Table 1 show that $Y'_{t(Dollar)}$, $Y'_{t(Euro)}$ series are more stationary compared to the others series. Also, we notice that the exchange rate USD/DZD is more stationary then the exchange rate Euro/DZD. Table 2 shows the descriptive analysis results of $S_{t(Dollar)}$, $S_{t(Euro)}$, $Y_{t(Dollar)}$, $Y'_{t(Dollar)}$, $Y_{t(Euro)}$ and $Y'_{t(Euro)}$, respectively. Tables 3 and 4 show the comparison study with several models.

Table 2 Descriptive statistics of $S_{t(Dollar)}$, $S_{t(Euro)}$, $Y_{t(Dollar)}$, $Y'_{t(Dollar)}$, $Y_{t(Euro)}$ and $Y'_{t(Euro)}$

Test	Mean	Median	Std.Dev	Skewness	Kurtosis	Jarque-Bera
$S_{t(Dollar)}$	79.472	79.255	0.746	0.198	2.223	5.700
$Y_{t(Dollar)}$	2.40E06	−6.87E05	0.0037	0.3395	4.361	16.214
$Y'_{t(Dollar)}$	2.40E05	0.0001	0.004	0.386	4.443	16.273
$S_{t(Euro)}$	104.237	104.067	0.998	0.605	0.933	9.248
$Y_{t(Euro)}$	5.97E05	6.42E05	0.0045	–0.080	4.613	16.218
$Y'_{t(Euro)}$	8.78E06	0.0001	0.0044	–0.039	4.658	16.846

Table 3 Comparison $Y'_{t(Dollar)}$ with several models

Models	Adj R^2	SEE	BIC	RMSE	MAE	MAPE
MA(19)	0.3486	0.00323	−8.2264	0.00388	0.002612	1.1673
ARMA(8,15)	0.3477	0.00314	−8.1117	0.00352	0.002387	1.4467
ARCH(2)	1.0000	8.04E-15	−62.876	7.82E-15	5.42E-15	3.34E-15
GARCH(2,1)	1.0000	1.30E-15	−66.489	1.23E-15	8.46E-16	4.95E-16

Table 4 Comparison $Y'_{t(Euro)}$ with several models

Models	Adj R^2	SEE	BIC	RMSE	MAE	MAPE
MA(2,3)	0.2637	1.824437	4.772	2.1445	1.7515	1.3412
ARMA(10,13)	0.3334	1.721631	4.431	2.0114	1.5975	2.6360
ARCH(2)	1.0000	2.22E-16	−70.761	2.24E-16	1.26E-16	8.42E-19
GARCH(2,1)	1.0000	2.2E-17	−74.643	2.15E-17	2.8E-18	2.57E-18

Remark 1 The criterions to judge for the best models are relatively small then BIC (Schwartz criterion) is measured by $n\ln(SEE)+k\ln(n)$, SEE, RMSE, MAE, MAPE and relatively high Adj R^2.

4.1.2 Comparison by the duration ratio (short and long terms)

In Table 5, we summarized the profit in LV, with modeling of the conditional variance, compared to modeling ARMA with ARCH, GARCH, IGARCH, TGARCH, PARCH and EGARCH models, these results are made over 7 days (short-term) before making a comparison with the results of long term (1/01/2014 to 15/05/2019).

Table 5 Profit in likelihood (short-term)

Processes	ARCH	GARCH	IGARCH	TGARCH	PARCH	EGARCH
Euro	0.3582900%	0.3438560%	0.3549363%	0.3467922%	0.3771933%	0.4420363%
USD	0.2281921%	0.2075854%	0.2290%	0.2403422%	0.23675%	0.23196%

Table 5 shows that the conditional variance of each model is stripped for the two series in the period of 7 days (short term) compared to that which it has been calculated in the period of long term; i.e. the period has an influence on persistence of the volatility, and that the results in the short term are better than the results obtained by the long-term, and well on, thanks to the decrease of its conditional variance.

Remark 2 The change of the period for the symmetrical models remains better than the asymmetrical models thanks to its minimal variance and more precisely, the GARCH model is the best among the symmetrical and asymmetrical models thanks to the same reason. In end we can conclude that the best period in our modeling is the short-term period, thanks to minimal variance.

4.2. Second application

In this section, we apply analytical solutions from Section 3 to price a swap on the volatility of the CAC40 French index for five years (from October 2013 to April 2018). The first step of this application is to study the stationary of the series. To this end, we used the unit root test of DF and PP.

4.2.1 Unit root tests and descriptive analysis

In this section, we present unit root tests and descriptive analysis results of S_{cac}.

Table 6 Unit root tests of S_{cac} for $\alpha = 1\%$

Test	DF	PP
S_{cac}	–36.6454	–36.0107

In Table 6 the unit root test confirms the stationary of the series.

Table 7 Descriptive statistics of S_{cac}

Test	Mean	Median	Std.Dev	Skewness	Kurtosis	Jarque-Bera
S_{cac}	0.00005	0.0000	0.0159	–0.089	7.456	812.821

Table 7 shows all statistic parameters of CAC40 French index, mean of time series is 0.0000531, median is 0, standard deviation is 0.0159 and skewness is –0.089, so it is negative and the kurtosis is larger than 3 are called leptokurtic, indicating higher plead and fatter tails than the normal distribution. The Jarque-Bera statistic is 812.821 and so we can forecast an uptrend.

Table 8 Comparison S_{cac} with several models

Models	Adj R^2	SEE	BIC	RMSE	MAE	MAPE
ARCH(2)	0.9899	0.0075	–2.7267	0.0146	0.0097	3.6112
ARCH(4)	0.9899	0.0072	–2.8110	0.0108	0.0074	3.4681
GARCH(2,1)	0.9924	0.0031	–7.8946	0.0027	0.0028	2.9455
GARCH(1,1)	1.0000	0.0025	–8.9944	0.0026	0.0019	2.7424

Table 8 confirm that GARCH(1,1) model is clearly the best performing models as they receive the lowest score on fitting metrics whiles representation the lowest MAE, RMSE, MAPE, SEE and BIC among all models. Now, we use the following relationship to calculate the discrete GARCH(1,1) parameters:

$$\begin{cases} V = \dfrac{C}{1-\alpha-\beta}, \\ \theta = \dfrac{V}{\Delta t_L}, \sigma_0 = \dfrac{V}{\Delta t_S}, \\ k = \dfrac{1-\alpha-\beta}{\Delta t}, \\ \xi^2 = \dfrac{\alpha^2(k-1)}{\Delta t}, \end{cases}$$

Where $\Delta t_L = \frac{1}{252}$, 252 trading days in any given year, $\Delta t_S = \frac{1}{63}$, 63 trading days in any given three months, $C = 2.03\times10^{-7}$ (the coefficient of variance equation of GARCH(1,1)), $\alpha = -0.008422$, $\beta = 0.98042$, $K = 7.2562$.

4.3. Numerical Applications

We set $V = 72.239422\times10^{-7}; \theta = 0.00182; \sigma_0 = 0.000455; k = 7.08145; \xi^2 = 0.1115$, and we use the relations (a) and (b) of Theorem1 for a swap maturity $T = 0.9$ year, we find

$$E(V) = 2.8341\times10^{-6},\ Var(V) = 5.0982\times10^{-9}.$$

The convexity adjustment $\dfrac{Var(V)}{8E^{\frac{3}{2}}(V)} = 0.1406$ and $E(\sqrt{V}) \approx -0.1302$. If the non-adjusted strike is equal to 20%, then the adjusted strike is equal to $20\% - 14.06\% = 5.94\%$. This is the fixed leg of the volatility swap for a maturity $T = 0.9$. Repeating this approach for a series of maturities up to five years we obtain the following table.

Table 9 Relationship of the non-adjusted and adjusted volatility for the same series of maturities

non-adjusted strike	20%	15%	10%	8%	5%
Adjusted strike	5.94%	4.64%	3.15%	2.67%	2.04%
Maturity T	0.9	1.9	2.9	3.9	4.9

According to Table 9 non-adjusted strike and adjusted strike are decreasing exponentially up to five years.

5. Conclusions and Perspectives

As a conclusion the new approach provides an improved technique for carrying out foreign exchange rates forecasting and index price. The scope of the work has an application on the Algerian foreign exchange rates as against two currencies and on the volatility of the CAC40 French index, an improvement of the work can be made by extending the work to other country foreign exchange rates, other index price and energy markets.

In this work, we can conclude that the ARCH/GARCH models are simple and easy to handle. Moreover, the forecast accuracy of nonlinear ARCH/GARCH models is compared to the traditional MA, ARMA linear models. However, nonlinear time's series models performed better than linear time's series models, especially GARCH.Also, we can conclude that the exchange rate of dollar is more stable than exchange rate of Euro.

The results also show that GARCH(1,1) model improves the forecasting performance. This result later further implies that the GARCH(1,1) model might be more useful than the other models ARCH(2), ARCH(4), GARCH(2,1) when implementing risk management strategies for CAC40 French index. We think there is still much work to be done, especially regarding the relationship between oil and stock prices and could be useful to investors and other market participants, such as, financial managers, analysts and firms in order to manage their investments and minimize their portfolio risks. In addition, this work can be generalized on data relating to natural gas and electricity. We can also use other multivariate GARCH models and COGARCH.

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