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On Designing Sampling Inspection Plans by Variables Based on Skew-Normal Distribution

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Abstract

Acceptance sampling, also termed as sampling inspection, is an essential quality control technique which describes the rules and procedures for deciding whether the manufactured lot of commodities can be accepted or rejected based on the inspection of one or more samples. Sampling inspection by the method of variables is one of the categories of sampling inspection plans in which the discretion of acceptance or rejection is made hinge on some specific rule which is framed according to the measurement of a quality characteristic under study. In this scenario, the quality feature under study is considered to be a continuous random variable that can be demonstrated using any continuous type statistical distribution. Application of skew-normal distribution is considered in this paper for establishing acceptance sampling inspection plans for variables based on the examination of units from a single sample.

Keywords: Sampling inspection plans, variable sampling, skew-normal distribution, non-normality, skewness.

1. Introduction

Sampling inspection plans or acceptance sampling plans play a cardinal role in determining the quality of products by reviewing the sample of units taken from the lot of products. More often, the product control methodologies are grouped into two categories, namely, sampling inspection by the method of attributes and sampling inspection by the method of variables. In sampling inspection by the method of attributes, selection of one or more samples is made from the manufactured lot of items and the sampled units are classified into defective (non-conforming) or non-defective (conforming) according to some prescribed decision criteria. The choice of accepting or rejecting the lot is taken based on some explicit criteria.

In sampling inspection by the method of variables, a sample of manufactured commodities is picked from the lot, and then the quality feature of interest is measured and recorded. The decision about the lot acceptance or lot disposition is interpreted according to some specific criteria, which are framed based on the measurement of the quality variable under study. This method is applicable to the cases where the quality feature under study is a continuous random variable, which is quantifiable on a continuous scale.

A peculiar aspect of variable sampling is that the quality feature under study is a continuous random variable which can be demonstrated using any continuous type probability distribution from the literature of statistical theory. A noteworthy advantage of variable sampling is that the decision is made about the lot quality in accordance with the exact measurements of the quality variable. It provides more accurate information about the product quality than compared to the sampling inspection by attributes. Variable sampling plans can be used with same level of protection as attribute sampling plans, but with lesser number of sampling units, showing its efficiency with respect to the sample size.

Under normality assumption of the quality feature, many researchers have devised the variable sampling plans. When the normality assumption is sheered or when the quality variable shows some kind of skewness, the standard variable sampling plans based on normal distribution are no longer applicable. In such situations, the need for the application of non-normal distributions or skewed distributions arises. Many researchers have initiated the works on variable sampling plans for non-normal populations. Zimmer and Burr (1963) considered Burr distribution for constructing variable sampling plans based on the measures of skewness and kurtosis. Takagi (1972) designed sampling procedures for variable inspection based on non-normal distributions when the population variance is unknown. Geetha and Vijayaraghavan (2013) considered Pareto distribution for the construction of variable sampling plans. Seifi and Nezhad (2017) constructed variable sampling plan for resubmitted lots under Bayesian approach. Other major developments in variables sampling plans include the works of Srivastava (1961), Owen (1969), Duncan (1986), Guenther (1972, 1985), Yeh (1990, 1994), Yeh et.al. (2006), Aslam et.al. (2013), Balamurali and Usha (2013), Yen and Cheng (2009), Wu et.al. (2008, 2012, 2018), Geetha and Pavithra (2019), and Rao et.al. (2021).

The aspects of construction and assessment of variable sampling plans are quite easy when the quality feature under study stick to follow normality assumption. In industrial applications, the normality assumption of the quality variable may be sheered or the quality feature may exhibit non-normal patterns. In such situations, designing of sampling inspection plans by the method of variables becomes unwieldy.

In this paper, the designing procedure of sampling inspection plans by the method of variables based on a single sample is examined when the distribution of the quality feature of the manufactured commodity under study shows a similar pattern of skew-normal distribution.

2. Skew-Normal Distribution (SND)

It is the universal trend among statisticians to move towards statistical or probabilistic models that are more flexible to represent the data to incorporate all the features of data as adequately as possible. The origin of skew-normal distribution (SND) is one of the most suitable forms of statistical distributions for modelling data in real time situations. O'Hagan and Leonard (1976) proposed skew-normal distribution for the Bayesian estimation of the location parameter of normal distribution as a prior. Azzalini (1985, 1986) addressed the significant properties of SND and presented several generalizations and multivariate expansions.

SND is a prolongation of normal distribution, which includes a supplementary parameter to represent asymmetry. It permits an ongoing fluctuation from the symmetric feature of normality to the asymmetric feature of non-normality, which is effectual in numerous real life scenarios. The benefit of SND can be thought for two possible reasons. On theoretical perspective, it relishes a plenty of conventional properties, which correspond to those of normal distribution. From applied stand point, it is pertinent for patterning the data that shows a uni-modal shape with little skewness, a situation most commonly arises in real life situations.

Let Z be a random variable representing the quality characteristic of a manufactured item and follows SND with asymmetry parameter $\gamma \in R.$ The probability density function of SND is as follows:

$$f(z) = 2\phi(z)\Phi(\gamma z), \tag{1}$$

where φ and Φ denote the density function and distribution function of $N(0,1),$ respectively.

The probability density function of SND given in (1) can be considered as a normal density multiplied by a power element $2\Phi(\gamma z),$ which is dependent on the skewness argument $\gamma.$ If $\gamma < 0,$ more load will contribute to the negative values of $z,$ which results in a left-skewed distribution, whereas $\gamma > 0,$ leads to positive skewness.

A notable feature of SND is that it incorporates the normal distribution as a specific form when $\gamma = 0.$ That is, normal distribution is not a limiting form of SND whereas it is a limiting case for most of the other distributions. This property makes SND a prevalent model to address the skewness which beholds in many of the economic and financial data. The distribution function of SND is as follows

$$F(z) = 2\int_{-\infty}^{z}\int_{-\infty}^{\gamma z} \phi(t)\phi(u)\; dudt,$$

which comes closer to the distribution function of a bivariate normal distribution.

Now, consider the transformation $x = \xi + \omega z,$ where $z \sim SND(0, 1, \gamma)$ or $z \sim SND(\gamma).$ Then,

$$z = \frac{x-\xi}{\omega};\ \ \frac{dz}{dx} = \frac{1}{\omega},\ \text{and}\ f(x) = 2\phi\left(\frac{x-\xi}{\omega}\right)\Phi\left(\gamma\frac{x-\xi}{\omega}\right)\left|\frac{dz}{dx}\right| = \frac{1}{\pi\omega}e^{-\frac{1}{2}\left(\frac{x-\xi}{\omega}\right)^2}\int_{-\infty}^{\gamma\left(\frac{x-\xi}{\omega}\right)} e^{-\frac{t^2}{2}}dt,$$

which is the density of three parameter SND and is denoted by $x \sim SND(\xi, \omega^2, \gamma),$ where the parameters $\xi,$ ω and γ represent location, scale and shape, respectively. The moment generating function of $SND(\xi, \omega^2, \gamma)$ is given by

$$M_x(t) = 2e^{t\xi + \frac{t^2\omega^2}{2}}\Phi(\delta\omega t),$$

where $\delta = \dfrac{\gamma}{\sqrt{1+\gamma^2}}.$

The moments of skew-normal distribution can be obtained expanding the moment generating function. The mean and the variance of SND are, then, determined as

$$E(X) = \mu = \sqrt{\frac{2}{\pi}}\delta\omega + \xi\ \text{ and }\ V(X) = \omega^2\left[1 - \frac{2}{\pi}\delta^2\right].$$

The skewness of SND is given by

$$\alpha_3 = \frac{4-\pi}{2}\frac{\left(\delta\sqrt{2/\pi}\right)^3}{\left(1-2\delta^2/\pi\right)^{3/2}}$$

and kurtosis is expressed as

$$\alpha_4 = 2(\pi - 3)\frac{\left(\delta\sqrt{2/\pi}\right)^4}{\left(1 - 2\delta^2/\pi\right)^2},$$

where $\delta = \dfrac{\gamma}{\sqrt{1+\gamma^2}}$ is a function of the asymmetry parameter γ. From the above expressions, it can be observed that skewness as well as kurtosis of SND are the functions of δ alone.

3. Estimation of the IVT Distribution Parameters

Acceptance sampling plans for variables based on a single sample can be stated based on the conditions mentioned hereafter.

i. The random variable depicting the quality feature is quantifiable on a continuous scale that is having a recognized model of statistical distribution.

ii. Each single item subjected to quality monitoring has provided with a single specification limit say, lower specification limit (LSL), L, or upper specification limit (USL), U. If a measurement on a particular item goes beyond the specification, the item is considered as an unsatisfactory component. The working process of an acceptance sampling plan based on variable inspection for a single sample is described in the following manner:

Step 1: Randomly select a small group of elements of size n from the submitted population of manufactured items. Measure the quality variable of interest for each unit in the selected group and record the measurements.

Step 2: Approve the submitted population of manufactured commodities as accepted, if $\bar{x} + k\sigma \le U$ or $\bar{x} - k\sigma \ge L$; and reject, otherwise. The conditions are chosen according to the given specification. If σ is unknown, s, an unbiased estimate of population standard deviation, is used in the place of σ.

The sample size n and acceptance constant k constitute the parameters of the acceptance sampling plans for variables based on a single sample.

4. Operating Characteristic Function

The effectiveness of any acceptance sampling plan can be evaluated using an important measure, called operating characteristic (OC) function. It gives the probability of acceptance of a lot with a specified proportion of faulty items. It is denoted by $P_a(p)$, where p is the fraction of defective or nonconforming items in the bunch of commodities. When USL is provided, the proportion of nonconforming items and probability of accepting the lot are, respectively, given by

$$p = P(X > U \mid \mu)$$

and

$$P_a(p) = P(\bar{x} + k\sigma \le U \mid \mu), \tag{2}$$

when σ is unknown, the sample standard deviation, s, which is an unbiased estimate of σ, is used. A plot of probability of acceptance against the proportion of defective or nonconforming items is known as the operating characteristic (OC) curve, which is commonly used for comparing the efficiencies of sampling plans. A common procedure for designing a sampling plan is described by specifying two points on the OC curve, viz., $(p_0, 1-\alpha)$ and (p_1, β), p_0 and p_1 denote the acceptable quality level (AQL) and limiting quality level (LQL), respectively; α and β are producer's risk and consumer's risk, respectively. The AQL and LQL are, respectively, defined by

$$AQL = p_0 = P(X > U \mid \mu_0), \tag{3}$$

and

$$LQL = p_1 = P(X > U \mid \mu_1), \tag{4}$$

where μ_0 and μ_1 are the means of probability distribution of the quality variable under study which results in AQL and LQL. The producer's risk and consumer's risk can be obtained using the given requirements of AQL and LQL as

$$\alpha = P(\overline{x} + k\sigma > U \mid \mu_0) \tag{5}$$

and

$$1-\beta = P(\overline{x} + k\sigma > U \mid \mu_1). \tag{6}$$

when σ is unknown, the estimate s is used for the evaluation of α and β.

The probability of acceptance and the proportion of nonconforming items are defined based on the underlying distribution of the quality feature under study. They are obtained using the cumulative probability distribution function of SND.

5. Designing Procedure of Variable Single Sampling Plan

In most of the practical scenarios, the quality feature of the product under study will show some kind of deviations from the normality assumption. In such situations, the standard variable sampling plans cannot be utilized. It is, also, the fact that population parameters of the statistical distribution, which is used to model the quality variable, are unidentifiable in most of the cases and hence, they are estimated using the sample statistics. If the distribution of quality variable under consideration is deviating from normality, the development phase of unknown σ sampling plans becomes more cumbersome.

Let $F_x(x;\xi,\omega,\gamma)$ be the cumulative distribution function of SND, which is also considered as the distribution function of the quality feature. From (3), it is easily observed that the acceptable quality level p_0 can be found using the cumulative distribution function of quality parameter, which can be expressed as

$$AQL = p_0 = 1 - F_x(x;\mu_0),$$

where μ_0 is the mean of SND, which results in an acceptable quality level. Similarly, from equation (4) the limiting quality level p_1 can be expressed as

$$LQL = p_1 = 1 - F_x(x;\mu_1),$$

where μ_1 is the mean of SND, which results in limiting quality level.

Also, from equations (5) and (6), the producer's risk and consumer's risk can be obtained using the cumulative distribution function $F_x(x;\xi,\omega,\gamma)$ as

$$\alpha = 1 - F_x(\overline{x};\mu_0) \text{ and } 1-\beta = 1 - F_x(\overline{x};\mu_1).$$

When the underlying distribution of quality parameter is normal, the designing of variable sampling plan includes the process of determining the standard normal deviate value K_p^*. For designing a variable sampling plan for non-normal distributions, the value of K_p^* corresponds to the deviate value of underlying distribution. Hence, for obtaining the parameters of a variable sampling plan under SND, the deviate values are obtained as follows

$$K_{p_0}^* = \frac{x_{p_0} - M}{S}, \tag{7}$$

where x_{p_0} is the value of x for which the upper tail area of SND is p_0 and K_p^* is the standardized value of x_{p_0}. Here, M and S denote, respectively, the mean and standard deviation of the SND.

Similarly, the deviate value corresponding to the limiting quality level is obtained as

$$K_{p_1}^* = \frac{x_{p_1} - M}{S}, \tag{8}$$

where x_{p_1} is the value of x for which the upper tail area of SND is p_1 and K_p^* is the standardized value of x_{p_1}. The optimum values of the parameters of a variable sampling plan for non-normal populations is given by Zimmer and Burr (1963) as

$$n_U = \left[\frac{K_\alpha + K_\beta}{K_{p_0}^* - K_{p_1}^*}\right]^2, \tag{9}$$

and

$$k_U = \frac{K_\alpha K_{p_1}^* + K_\beta K_{p_0}^*}{K_\alpha + K_\beta}. \tag{10}$$

Similarly, the constants of a variable sampling plan when the lower specification limit is specified, can be determined as

$$n_L = \left[\frac{K_\alpha + K_\beta}{K_{1-p_0}^* - K_{1-p_1}^*}\right]^2, \text{ and } k_L = \frac{K_\alpha K_{1-p_1}^* + K_\beta K_{1-p_0}^*}{K_\alpha + K_\beta}.$$

In 1972, Takagi introduced an approach for obtaining the sample size and the acceptance constant of single sampling inspection plans by variables based on a wide range of non-normal distributions and proposed an expansion component in the context of the measures of skewness and kurtosis. According to Takagi (1972), when σ is unknown, $\bar{x} \pm ks$ will follow normal distribution asymptotically with parameters, $\mu_y = \mu \pm k\sigma$ and $\sigma_y^2 = \frac{\sigma^2}{n}\left[1 + \frac{k^2}{4}(\alpha_4 - 1) \pm k\alpha_3\right]$, where μ, σ, α_3 and α_4 represent the mean, standard deviation, skewness and kurtosis of the underlying probability distribution, respectively. The operating characteristic function given in (2) can be rewritten, approximately, as

$$P_a(p) = P\left(Z \le \frac{U - \mu_y}{\sigma_y} \mid \mu\right),$$

where μ_y and σ_y denote the mean and standard deviation of $\bar{x} \pm ks$ and Z is the standardized variable.

Let us denote $K_p^* = (U - \mu)/\sigma$. Then, for the designated points of acceptable and limiting quality levels, one may have

$$\mu_0 = U - K_{p_0}^* \sigma \text{ and } \mu_1 = U - K_{p_1}^* \sigma.$$

Let α and β denote the producer's risk and consumer's risk, respectively. Then, the normal deviates corresponding to the specified risks are given by

$$K_\alpha = \frac{U - \mu_{x0}}{\sigma_x} == \frac{U - (\mu_0 + k\sigma)}{\sqrt{(\sigma^2/n)\left[1 + (k^2/4)(\alpha_4 - 1) + k\alpha_3\right]}} = \frac{U - (\mu_0 + k_U\sigma)}{\sigma\sqrt{e_U/n_U}},$$

and

$$-K_\beta = \frac{U-\mu_{x1}}{\sigma_x} = \frac{U-(\mu_1+k\sigma)}{\sqrt{(\sigma^2/n)\left[1+(k^2/4)(\alpha_4-1)+k\alpha_3\right]}} = \frac{U-(\mu_1+k_U\sigma)}{\sigma\sqrt{e_U/n_U}}.$$

Using the expressions of μ_0, μ_1, K_α and K_β one can obtain the following:

$$K^*_{p_0} = k_U + K_\alpha\sqrt{\frac{e_U}{n_U}}, \tag{11}$$

$$K^*_{p_1} = k_U - K_\beta\sqrt{\frac{e_U}{n_U}}. \tag{12}$$

Solving (11) and (12), the explicit expressions for the parameters of sampling inspection plan can be obtained as

$$n_U = e_U\left[\frac{K_\alpha + K_\beta}{K^*_{p_0} - K^*_{p_1}}\right]^2 \tag{13}$$

and

$$k_U = \frac{K_\alpha K^*_{p_1} + K_\beta K^*_{p_0}}{K_\alpha + K_\beta}, \tag{14}$$

where K_α and K_β denote the well-known standard normal deviates exceeding the probabilities α and β, respectively, and $e_U = 1+(k_U^2/4)(\alpha_4-1)+k_U\alpha_3$ is known as the expansion factor. The expansion factor is utilized in order to gain information about the parameters of known σ plans with the relations $n'_U = n_U / e_U$ and $k'_U = k_U$. In a similar manner, the constants of variable sampling plan, when the lower specification limit is specified, can be determined as

$$n_L = e_L\left[\frac{K_\alpha + K_\beta}{K^*_{1-p_0} - K^*_{1-p_1}}\right]^2 \text{ and } k_L = \frac{K_\alpha K^*_{1-p_1} + K_\beta K^*_{1-p_0}}{K_\alpha + K_\beta},$$

where $e_L = 1+(k_L^2/4)(\alpha_4-1)+k_L\alpha_3$, making use of which the parameters of a known σ plan can be obtained as $n'_L = n_L / e_L$ and $k'_L = k_L$.

6. Numerical Illustration 1

Let X be the quality variable which follows SND having the parameters, $\xi = 0, \omega^2 = 1$ and $\gamma = 1.5$. The mean, variance, skewness and kurtosis are fixed as $\mu = 0.6638, \sigma^2 = 0.5593, \alpha_3 = 0.3$ and $\alpha_4 = 0.176$, respectively. For the specified $\gamma = 1.5$, the value of δ is obtained as 0.832. Assume that the upper specification limit, U, for the quality feature is specified as 2.5. For the specified quality levels, say $p_0 = 0.01$ and $p_1 = 0.065$, and the associated producer's and consumer's risks, $\alpha = 0.05$ and $\beta = 0.10$, one can obtain the parameters of the sampling plan by variables using the expressions (9) and (10). Corresponding to $p_0 = 0.01$ and $p_1 = 0.065$, the deviate values $K^*_{p_0}$ and $K^*_{p_1}$ are obtained, from (7) and (8), as 2.556647 and 1.579227, respectively. Corresponding to the specified $\alpha = 0.05$ and $\beta = 0.10$, the values of K_α and K_β are obtained as 1.644854 and 1.281552. Given the upper specification limit and other requirements, the parameters of single sampling plan by variables are obtained from (9) and (10) as $n = 8.964105$, which when rounded becomes 9 and $k = 2.007266$.

Also, the value of e_U is obtained as 0.772182 and the parameters of the unknown standard deviation sampling plan by variables are determined from (13) and (14) as $n' = 6.92192,$ which when rounded becomes 7 and $k = 2.007266.$

A simulation study is carried out for comparing the results arrived in the above illustration. Assume that the standard deviation is known. The simulated results are based on 1,000 runs using R programming. By simulation based on 1,000 runs, the randomly generated sample of 9 observations from SND having the specified parameters $\xi = 0, \omega^2 = 1$ and $\gamma = 1.5$ yields the sample mean, viz., $\bar{x} = 0.66155.$

It is known that the acceptance criterion under a variable sampling plan is given as $\bar{x} + k\sigma \leq U.$ It can be seen that $\bar{x} + k\sigma = 0.66155 + 2.007266 \times 0.7478 = 2.16266,$ which is less than the upper specification limit, i.e., $U = 2.5.$ Hence, the lot would be accepted.

7. Numerical Illustration 2

A manufacturing industry produces various types of LCD that can be used in making different peripherals. For a particular model of LCD, the target value of thickness is set as 0.70 mm. In the purchasing contract, the upper specification limit was specified as $U = 0.75$ mm. If the measurement of quality variable does not fall within the upper specification, it would be considered as nonconforming or defective.

From the history, it was ascertained that the thickness of the LCD follows SND having the parameters $\xi = 0.7, \omega = 0.02$ and $\gamma = 1.135.$ The mean and standard deviation were determined as $\mu = 0.7119$ and $\sigma = 0.016.$ Under the given conditions, for the specified values of $p_0 = 0.02, p_1 = 0.06, \alpha = 0.05$ and $\beta = 0.10,$ it is required to obtain a single sampling plan by variables. Corresponding to the specified quality levels and the risks, one obtains the following: $K^*_{p_0} = 2.15605, K^*_{p_1} = 1.596599, K_\alpha = 1.644854$ and $K_\beta = 1.281552.$ Thus, the optimum values of the parameters n and k of the sampling plan are determined from (9) and (10) as $n = 27.36$ or 28 and $k = 1.841598.$

In order to make the comparison of the results obtained in the illustration, data have been simulated based on 1,000 runs using R programming. The simulated data of 28 observations from SND having the specified parameters provide the sample statistics, viz, average and standard deviation which are found to be $\bar{x} = 0.7141429$ and $s = 0.01627.$

It can be observed that $\bar{x} + k\sigma = 0.7141429 + 1.841598 \times 0.016 = 0.74364$ which falls below the upper specification limit, $U = 0.75.$ Hence, the lot would be accepted.

8. Conclusion

Normal distribution and its wide range of properties play a prominent role in the theory and applications of statistics. Acceptance sampling plans for variable inspection consider normal distribution as an important model for the quality variable. Even though applications of normal distribution are a plenty in various domains, there are situations, where non-normality arises in the real life data. In this paper, a skew-normal distribution (SND) is considered for designing an acceptance sampling plan by variables based on the information acquired from a single sample. The strategy for the choice of variable single sampling inspection plans when the quality feature shows the behavior of SND is discussed, when the producer's and consumer's requirements are specified. The numerical illustrations are given to demonstrate how the proposed plan could be executed in practical application. The simulation based on 1,000 runs for generating samples from SND has been done utilizing R programming and the simulated results are compared with the results arrived in the illustrations.

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