Proportion Based Dual Unbiased Exponential Type Estimators of Population Mean

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Abstract

Prior to the concept of auxiliary information, the estimator used for estimating the finite population mean was the sample mean estimator. By making use of auxiliary information, some new estimators of population mean were proposed from time to time to get more precise estimates. The importance of these estimators lies in that they estimate the population mean of the study variable precisely by making use of the information of some other variable called the auxiliary variable whose information is readily available. Besides ratio, product, regression, and difference estimators, the exponential estimators were also proposed which may be employed even when there is a low degree of correlation between the study and auxiliary variable. This paper proposes dual unbiased exponential-type estimators in simple random sampling when the auxiliary information is qualitative in nature. The theoretical expressions of bias and mean square error (MSE) have been evaluated. A comparative study of MSE expressions of the estimators shows that the proposed estimators produce more efficient results than the existing sample mean estimator, the estimators of Naik and Gupta (1996), exponential estimators of Singh *et al.* (2007), and the family of exponential estimators proposed by Zaman & Kadilar (2019). To support the claim of gain in efficiency, a numerical study is also carried out using the data of two populations.

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Keywords: Auxiliary attribute, dual estimator, unbiased estimator, bias, mean square error

1. Introduction

The main aim of sampling is to learn about the population on the basis of part of the population called the sample drawn from it. The sample can be drawn using various techniques like simple random sampling (SRS), stratified random sampling, systematic sampling, etc. depending on the population from which the sample is drawn. When the population is homogeneous, the simple SRS technique is used in which each and every available unit of the population has an equal chance of being selected in the sample. There are two techniques of selecting a simple random sample viz. lottery method and use of random number tables. In SRS, a sample can be selected from the population either by using simple random sampling without replacement (SRSWOR) procedure or by using simple random sampling with replacement (SRSWR) procedure. If a unit is selected and noted,

returned to the population before the next drawing is made it is called SRSWR. If the selected unit is not replaced before the next drawing, it is called SRSWOR. The sample is taken in order to infer about the population parameters and to strengthen this inference auxiliary information is used. So the use of auxiliary information in sampling theory is having paramount importance to produce efficient estimators. The origin of using auxiliary information can be traced back to the origin of sampling theory. The use of auxiliary information by Cochran (1940) proposed a ratio estimator that is more efficient than the sample mean. This estimator can be employed when the study and auxiliary variables have a high degree of positive correlation. Sometimes the study and auxiliary variables are highly negatively correlated, therefore the work of Robson (1957) proposed a product type estimator for the population mean. Later several researchers such as Sisodia and Dwivedi (1981), Singh and Tailor (2003), Khoshnevisan *et al.* (2007), Yadav and Kadilar (2013), Hafeez and Shabbir (2015), Hafeez *et al.* (2020), Hussain *et al.* (2021) etc. in their work modified these classical ratio and product type estimators and proposed some new modified estimators. The newly proposed estimators were found to estimate the population means more precisely than the conventional ones. Sometimes the study variable is quantitative while the auxiliary information is qualitative in nature, for example, the height of a person may depend on sex, the efficiency of a dog may depend on its breed, the yield of a wheat crop may depend on its variety, amount of milk produced by a cow may depend on its breed, etc. In all these situations neither the classical ratio and product estimators nor their modified versions can be used. Therefore taking advantage of point biserial correlation and the knowledge of some population parameters of auxiliary attribute such as the population proportion (P), coefficient of variation (C_p), coefficient of kurtosis $(\beta_2(\phi))$, etc. Authors such as Naik and Gupta (1996), Singh *et al.* (2007), Abd-Elfattah *et al.* (2010), and Zaman & Kadilar (2019), etc. proposed ratio and product type estimators that can be employed in these situations.

The estimators discussed above are all biased. Although the problem of bias can be dealt with by increasing the sample size, jackknifing, bootstrapping, etc. However the authors Yadav *et al.* (2012) and Singh *et al.* (2014) proposed the estimators of population mean which are almost unbiased and therefore do not require any bias reduction methods. The ratio cum product estimators of population mean proposed by Singh *et al.* (2009), Sharma and Tailor (2010), and Tailor *et al.* (2012), etc. can be employed for both positively and negatively correlated variables.

By keeping an eye on the above discussion, it can be concluded that the estimators of population mean which are efficient, unbiased, work equally efficiently for both positively and negatively correlated variables, and can be employed for low degrees of correlation too will have a greater scope. The present paper is an attempt to work in this direction and therefore the objective of the paper is to propose proportion-based dual exponential estimators of population mean which may be unbiased.

2.   Material and Methods

Draw a sample of size n from the population containing a total of N units using the SRSWOR procedure. Let Y_i and ϕ_i denote the ith unit $(i = 1, 2, , N)$ associated with the study variable and the auxiliary attribute respectively. Let us suppose that there is a complete dichotomy in the population with respect to the presence or absence of an attribute ϕ (*say*) and it is also assumed that this attribute takes two values 0 and 1, as

$$\phi_i = \begin{cases} 1, & \text{if the } i^{th} \text{ unit of the population possesses the given attribute } \phi. \\ 0, & \text{otherwise.} \end{cases}$$

Let,

(a) The number of units possessing the attribute ϕ in the population, $A = \sum_{i=1}^{N} \phi_i$. Therefore $P = \frac{A}{N}$ is the proportion of units possessing the given attribute ϕ in the population.

(b) The number of units possessing the attribute ϕ in the sample, $a = \sum_{i=1}^{n} \phi_i$. Therefore $p = \frac{a}{n}$ is the proportion of units possessing the given attribute ϕ in the sample.

Some of the important formulae used in the paper are presented as
$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ are the population mean and sample mean of the study variable respectively. $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$ and $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ are the population mean square and sample mean square of the study variable respectively. $S_\phi^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\phi_i - P)^2$ and $s_\phi^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\phi_i - p)^2$ are the population mean square and the sample mean square respectively for auxiliary attribute. $C_y = \frac{S_y}{\bar{Y}}$ and $C_p = \frac{S_\phi}{P}$ are the coefficient of variation of the study variable and auxiliary attribute respectively. $S_{y\phi} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i\phi_i - NP\bar{Y})^2$ and $s_{y\phi} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i\phi_i - np\bar{y})^2$ are the population and sample covariance between the study variable and auxiliary attribute respectively. $\rho_{pb} = \frac{S_{y\phi}}{S_y S_\phi}$ is the correlation between Y and ϕ. $\gamma = \frac{1-f}{n}$, where $f = \frac{n}{N}$ is the sampling fraction.

3. Some Existing Estimators of Population Mean

The estimator used for estimating the finite population mean when there is no auxiliary information available is the sample mean estimator as

$$t_1 = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

The Bias and MSE of the estimator t_1 are as

$$Bias(t_1) = 0 \tag{1}$$

$$MSE(t_1) = \gamma \bar{Y}^2 C_y^2. \tag{2}$$

The ratio (t_2) and product (t_3) type estimators of the population mean proposed by Naik and Gupta (1996), when the auxiliary variable is qualitative in nature are as

$$t_2 = \bar{y}\left[\frac{P}{p}\right].$$

The Bias and MSE expressions of estimator t_2 are as

$$Bias(t_2) = \gamma \bar{Y}(C_p^2 - C_{yp}) \tag{3}$$

$$MSE(t_2) = \gamma \bar{Y}^2(C_y^2 + C_p^2 - 2C_{yp}). \tag{4}$$

$$t_3 = \bar{y}\left[\frac{p}{P}\right].$$

With Bias and MSE as

$$Bias(t_3) = \gamma \bar{Y} C_{yp} \tag{5}$$

$$MSE(t_3) = \gamma \bar{Y}^2(C_y^2 + C_p^2 + 2C_{yp}). \tag{6}$$

If the auxiliary variable is an attribute, there is a point biserial correlation between the study and the auxiliary variable. Therefore Singh *et al.* (2007) proposed exponential ratio (t_4) and product (t_5) estimators as

$$t_4 = \bar{y}\exp\left[\frac{P-p}{P+p}\right].$$

The expressions of Bias and MSE of t_4 are as

$$Bias(t_4) = \gamma \bar{Y}\left(\frac{1}{8}C_p^2 - \frac{1}{2}C_{yp}\right) \tag{7}$$

$$MSE(t_4) = \gamma \bar{Y}^2\left(C_y^2 + \frac{1}{4}C_p^2 - C_{yp}\right). \tag{8}$$

$$t_5 = \bar{y}\exp\left[\frac{p-P}{P+p}\right].$$

With the Bias and MSE as

$$Bias(t_5) = \gamma\bar{Y}\left(\frac{1}{8}C_p^2 + \frac{1}{2}C_{yp}\right) \tag{9}$$

$$MSE(t_5) = \gamma\bar{Y}^2\left(C_y^2 + \frac{1}{4}C_p^2 + C_{yp}\right). \tag{10}$$

Zaman and Kadilar (2019) proposed a novel family of ratio exponential estimators by using the information of auxiliary attributes as

$$t_i = \bar{y}\exp\left[\frac{(kP+l)-(kp+l)}{(kP+l)+(kp+l)}\right], \qquad i = 6,7,...,14$$

where k ($\neq 0$) and l are either real numbers or the functions of some known parameters of auxiliary attribute such as coefficient of variation (C_p), coefficient of kurtosis ($\beta_2(\phi)$) and point biserial correlation (ρ_{pb}) or the combination of two. The Bias and MSE of t_i are as

$$Bias(t_i) = \gamma\bar{Y}^2(\theta_i^2 C_p^2 - \theta_i C_{yp}). \tag{11}$$

$$MSE(t_i) = \gamma\bar{Y}^2(C_y^2 + \theta_i^2 C_p^2 - 2\theta_i C_{yp}), \tag{12}$$

where,

$$\theta_6 = \frac{P}{2(P+\beta_2(\phi))}; \qquad \theta_7 = \frac{P}{2(P+C_p)}; \qquad \theta_8 = \frac{P}{2(P+\rho_{pb})};$$
$$\theta_9 = \frac{P\beta_2(\phi)}{2(P\beta_2(\phi)+C_p)}; \qquad \theta_{10} = \frac{PC_p}{2(PC_p+\beta_2(\phi))}; \qquad \theta_{11} = \frac{PC_p}{2(PC_p+\rho_{pb})};$$
$$\theta_{12} = \frac{P\rho_{pb}}{2(P\rho_{pb}+C_p)}; \qquad \theta_{13} = \frac{P\beta_2(\phi)}{2(P\beta_2(\phi)+\rho_{pb})}; \qquad \theta_{14} = \frac{P\rho_{pb}}{2(P\rho_{pb}+\beta_2(\phi))}.$$

4. Proposed Estimators

The proposed proportion based unbiased dual exponential estimators are as

$$t_{de1} = \bar{y}\left[\omega\exp\left(\frac{P-p}{aP}\right) + (1-\omega)\exp\left(\frac{p-P}{aP}\right)\right] \tag{13}$$

$$t_{de2} = \bar{y}\left[\mu\exp\left(\frac{P-p}{bp}\right) + (1-\mu)\exp\left(\frac{p-P}{bp}\right)\right], \tag{14}$$

where $a(\neq 0)$, $b(\neq 0)$, ω and μ are constants. The values of a and b are chosen such that the proposed estimators are unbiased and the values of ω & μ are chosen such that MSE (t_{dej}) $(j = 1,2)$ is minimum.

To obtain the Bias and MSE expressions for the estimators t_{de1} and t_{de2} up to the first order of approximation, let

$$e_y = \bar{Y}^{-1}(\bar{y}-\bar{Y}) \qquad \text{and} \qquad e_\phi = P^{-1}(p-P).$$

The expected values of quantities e_y, e_ϕ, e_y^2, e_ϕ^2, and $e_y e_\phi$ are obtained as

$$E(e_y) = E(e_\phi) = 0, \qquad E(e_y^2) = \gamma C_y^2, \qquad E(e_\phi^2) = \gamma C_p^2, \qquad E(e_y e_\phi) = \gamma C_{yp}.$$

On expressing the estimators t_{dej} $(j = 1,2)$ in terms of e_y and e_ϕ, the expressions obtained are as

$$t_{de1} = \bar{Y}(1+e_y)\left[\omega\exp(-a^{-1}e_\phi) + (1-\omega)\exp(a^{-1}e_\phi)\right]. \tag{15}$$

$$t_{de2} = \bar{Y}(1+e_y)\left[\mu\exp\left(-e_\phi(b+be_\phi)^{-1}\right) + (1-\mu)\exp\left(e_\phi(b+be_\phi)^{-1}\right)\right]. \tag{16}$$

After solving Eqns. (15) and (16) and neglecting the terms containing higher powers (greater than two) of $e^{,s}$, the expressions obtained are as

$$t_{de1} = \bar{Y}\left(1 + e_y + (1-2\omega)\frac{e_\phi}{a} + (1-2\omega)\frac{e_y e_\phi}{a} + \frac{e_\phi^2}{2a^2}\right)$$

$$\Rightarrow t_{de1} - \bar{Y} = \bar{Y}\left(e_y + (1-2\omega)\frac{e_\phi}{a} + (1-2\omega)\frac{e_y e_\phi}{a} + \frac{e_\phi^2}{2a^2}\right) \tag{17}$$

$$t_{de2} = \bar{Y}\left[1 + e_y + (1-2\mu)\frac{e_\phi}{b} + (1-2\mu)\frac{e_y e_\phi}{b} + \left(\frac{1}{2b} + 2\mu - 1\right)\frac{e_\phi^2}{b}\right]$$

$$\Rightarrow t_{de2} - \bar{Y} = \bar{Y}\left[e_y + (1-2\mu)\frac{e_\phi}{b} + (1-2\mu)\frac{e_y e_\phi}{b} + \left(\frac{1}{2b} + 2\mu - 1\right)\frac{e_\phi^2}{b}\right]. \tag{18}$$

Taking the expectation on both sides of Eqns. (17) and (18), the Bias of the proposed dual estimators t_{dej} $(j = 1, 2)$ upto O (n^{-1}) is obtained as

$$Bias(t_{de1}) = \gamma\bar{Y}\left[\frac{1}{2a^2}C_p^2 + (1-2\omega)\frac{C_{yp}}{a}\right] \tag{19}$$

$$Bias(t_{de2}) = \gamma\bar{Y}\frac{1}{b}\left[\left(\frac{1}{2b} + 2\mu - 1\right)C_p^2 + (1-2\mu)C_{yp}\right]. \tag{20}$$

Now squaring Eqns. (17) and (18) after that taking expectation both sides, the resulting expressions will give the MSE of the estimators t_{dej} $(j = 1, 2)$ upto O (n^{-1}) as

$$MSE(t_{de1}) = \gamma\bar{Y}^2\left(C_y^2 + (1-2\omega)^2\frac{C_p^2}{a^2} + 2(1-2\omega)\frac{C_{yp}}{a}\right) \tag{21}$$

$$MSE(t_{de2}) = \gamma\bar{Y}^2\left(C_y^2 + (1-2\mu)^2\frac{C_p^2}{b^2} + 2(1-2\mu)\frac{C_{yp}}{b}\right). \tag{22}$$

The estimators t_{de1} and t_{de2} are found unbiased, if

$$a = \frac{C_p}{2(2\omega - 1)\rho_{pb}C_y} \qquad \text{and} \qquad b = \frac{C_p}{2(2\omega - 1)(\rho_{pb}C_y - C_p)} \text{ respectively.}$$

Using the values of a and b in Eqns. (21) and (22) respectively, the expressions obtained are as follows

$$MSE(t_{de1}) = \gamma\bar{Y}^2\left[C_y^2 + 4(1-2\omega)^4\rho_{pb}^2 C_y^2 - 4(1-2\omega)^2\rho_{pb}^2 C_y^2\right] \tag{23}$$

$$MSE(t_{de2}) = \gamma\bar{Y}^2\left[C_y^2 + 4(1-2\mu)^4(\rho_{pb}C_y - C_p)^2 - 4(1-2\mu)^2(\rho_{pb}C_y - C_p)\rho_{pb}C_y\right]. \tag{24}$$

Differentiating Eqn. (23) partially with respect to ω and equating to zero as

$$\frac{\partial}{\partial\omega}[MSE(t_{de1})] = 0$$

$$\Rightarrow 32(1-2\omega)^3\rho_{pb}^2 C_y^2 = 16(1-2\omega)\rho_{pb}^2 C_y^2$$

$$\Rightarrow 2(1-2\omega)^2 = 1$$

$$\Rightarrow \omega_{opt} = 0.146, \ 0.854.$$

Similarly differentiating Eqn. (24) partially with respect to μ and equating to zero as

$$\frac{\partial}{\partial \mu}[MSE(t_{de2})] = 0$$

$$\Rightarrow 32(1-2\mu)^3(\rho_{pb}C_y - C_p)^2 = 16(1-2\mu)(\rho_{pb}C_y - C_p)\rho_{pb}C_y$$

$$\Rightarrow (1-2\mu)^2 = \frac{\rho_{pb}C_y}{2(\rho_{pb}C_y - C_p)}$$

$$\Rightarrow \mu_{opt} = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\rho_{pb}C_y}{2(\rho_{pb}C_y - C_p)}}, \ \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\rho_{pb}C_y}{2(\rho_{pb}C_y - C_p)}}.$$

The quantities a, b, μ_{opt} involve the unknown constant C_y. The value of this constant is assumed to be known in advance, its value may either be obtained from past experience or from the pilot survey (See Reddy (1974), Singh & Kumar (2008)). Now substituting the values of ω_{opt} and μ_{opt} in Eqns. (23) and (24) respectively, the minimum value of MSE up to O (n^{-1}) for the estimators t_{dej} $(j = 1, 2)$ is obtained as

$$MSE_{min}(t_{dej}) = \gamma \bar{Y}^2 C_y^2 (1 - \rho_{pb}^2). \tag{25}$$

5.   Theoretical Efficiency Comparison

In this section the MSE expression of proposed estimators t_{dej} $(j = 1, 2)$ are compared with the MSE expressions of the estimators t_1, t_2, t_3, t_4, t_5 and $t_i (i = 6, 7, ..., 14)$. Therefore the conditions under which the efficiency of the proposed estimators outperform the efficiency of existing estimators are obtained by comparing Eqns. (2), (4), (6), (8), (10), (12) and (25). The conditions obtained are as

(i) $MSE(t_{dej}) < MSE(t_1), if \ \rho_{pb}^2 > 0.$

(ii) $MSE(t_{dej}) < MSE(t_2), if \ (C_p - \rho_{pb}C_y)^2 > 0.$

(iii) $MSE(t_{dej}) < MSE(t_3), if \ (C_p + \rho_{pb}C_y)^2 > 0.$

(iv) $MSE(t_{dej}) < MSE(t_4), if \ (C_p - 2\rho_{pb}C_y)^2 > 0.$

(v) $MSE(t_{dej}) < MSE(t_5), if \ (C_p + \rho_{pb}C_y)^2 > 0.$

(vi) $MSE(t_{dej}) < MSE(t_i), if \ (\theta_i C_p - \rho_{pb}C_y)^2 > 0.$

The conditions (i) to (vi) always hold, thus the proposed dual estimators t_{dej} $(j = 1, 2)$ are always more efficient than the existing estimators considered.

6.   Numerical Study

The numerical study of the work done in this paper is carried out by taking the data of two populations P1 and P2 where the auxiliary information is qualitative in nature. The population P1 is from Sukhatme and Sukhatme (1970), where the study variable (Y) is the number of villages in the circles and the auxiliary attribute (ϕ) is a circle consisting of more than five villages. The population P2 is from Mukhopadhyay (2000), in which the study variable (Y) is the household size and the auxiliary attribute (ϕ) is a household that availed an agricultural loan from a bank. Different constants of the populations P1 and P2 are given in Table 1 as

Table 1 Constants of population data sets.

Population	N	$\bar{Y}$	P	ρ_{pb}	C_y	C_p	$\beta_2(\phi)$	C_{yp}
P1	89	3.36	0.124	0.766	0.60	2.678	3.492	1.230
P2	25	9.44	0.400	−0.387	0.442	1.250	−1.976	− 0.215

Table 2 MSE, Bias and Percent relative efficiency (PRE) of various ratio estimators.

Estimator	Population P1					
	$n = 20$			$n = 40$		
	MSE	Bias	PRE	MSE	Bias	PRE
t_1	0.165	0.000	100.000	0.069	0.000	100.000
t_2	1.468	0.414	11.240	0.477	0.101	14.465
t_4	0.243	0.012	67.901	0.074	0.001	93.243
t_6	0.058	0.001	284.483	0.065	0.000	106.154
t_7	0.059	0.003	279.661	0.021	0.000	328.571
t_8	0.105	0.008	157.143	0.034	0.003	202.941
t_9	0.388	0.164	42.526	0.080	0.002	86.250
t_{10}	0.354	0.151	46.610	0.048	0.002	143.750
t_{11}	0.058	0.001	284.483	0.022	0.001	313.630
t_{12}	0.066	0.007	250.000	0.023	0.002	300.000
t_{13}	0.061	0.007	270.492	0.073	0.001	94.521
t_{14}	0.061	0.005	270.492	0.067	0.000	102.985
t_{dej}	0.057	0.000	289.474	0.020	0.000	345.000

Table 3 MSE, Bias and Percent relative efficiency (PRE) of various product estimators.

Estimator	Population P2					
	$n = 5$			$n = 10$		
	MSE	Bias	PRE	MSE	Bias	PRE
t_1	4.557	0.000	100.000	0.838	0.000	100.000
t_3	4.268	0.160	106.771	1.780	0.050	47.079
t_5	3.765	0.044	121.036	0.762	0.003	109.974
t_{dej}	3.757	0.000	121.293	0.661	0.000	126.770

Table 2 and Table 3 clearly show that the percent relative efficiency (PRE) of the proposed proportion based estimators t_{dej} $(j = 1, 2)$ is maximum among all the ratio and product type estimators considered for both the populations P1 & P2 and are also unbiased. Further, the said tables reveal that on increasing the sample size, the PRE also increases meaning that the proposed estimators t_{dej} $(j = 1, 2)$ begin to estimate population mean precisely on increasing sample size.

7. Conclusion

- The estimators proposed in the study are as

$$t_{de1} = \bar{y} \left[\omega \exp \left(\frac{P - p}{aP} \right) + (1 - \omega) \exp \left(\frac{p - P}{aP} \right) \right]$$

$$t_{de2} = \bar{y} \left[\mu \exp \left(\frac{P - p}{bp} \right) + (1 - \mu) \exp \left(\frac{p - P}{bp} \right) \right].$$

- The proposed estimators are always more efficient than the sample mean, the estimators of Naik and Gupta (1996), the exponential estimators of Singh *et al.* (2007), and Zaman & Kadilar (2019).

- The proposed estimators t_{de1} and t_{de2} are unbiased and can be used for both positively and negatively correlated variables equally efficiently.

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