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Modeling Daily Return Volatility Through GJR(1,1) Model and Realized Volatility Measure

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Abstract

The Glosten–Jagannathan–Runkle (GJR) model is an extension of the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model by adding an asymmetric term to allow conditional volatility to respond differently to the past returns based on their signs. Given the high-frequency data, the Realized Volatility (RV) measures have received great attention. This study investigates the fitting performance of the conventional GJR(1,1) model and two extended GJR models incorporating the Realized Kernel as an RV component, namely the GJR-X(1,1) and RealGJR(1,1), with two different return error distributions, namely Normal and Student-t, on the Financial Times Stock Exchange 100 (FTSE100) index for the daily period from January 2000 to December 2017. The study begins by evaluating the estimation ability of the Excel's Solver's GRG Non-Linear method, which is a simple and easy tool for financial practitioner, and the ARWM (Adaptive Random Walk Metropolis) method requiring computer programming knowledge. This study found that the Excel's Solver's GRG Non-Linear method is an easy and accurate estimation tool as the estimated values are relatively close to the ARWM results. To select the best fit model amongst competing models, the Akaike Information Criterion (AIC) statistical method was used. On the basis of AIC, this study found the empirical merit of the RealGJR(1,1) model with Student-t distribution, that is its potential to provide the best fit model at any sample sizes. The class of RealGJR(1,1) model exhibits lowest risk than the others so that investors intend to keep the stocks and is most capable of capturing rapid changes in the volatility level. These results are highly recommended to the financial practitioners and analysts dealing with high frequency financial data and GJR volatility modeling.

Keywords: ARWM, GJR model, Excel's Solver's GRG non-linear, realized kernel, student-t distribution

1. Introduction

Volatility is a statistical measure of fluctuation in the value of asset return (price changes) during a given period of time. Volatility is usually measured by the standard deviation of log (with the natural base) returns from that same asset. One of the well-known parametric volatility models is the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model introduced by Bollerslev (1986). This model is performed using daily returns only and states that the current conditional variance is not only influenced by past returns but also by past variances.

Recently, the Realized Volatility (RV) measure has quickly gained substantial attention as a measure of daily volatility (Andersen, 2018). This measure is a non-parametric measure and upon the high-frequency intraday returns. By adding the RV as an exogenous component into the conditional variance process, Engle (2002) proposed a so called GARCH-X model. This model was shown to have an empirically significant advantage in the data fitting. Furthermore, Hansen et al. (2012) developed the GARCH-X model to be Realized GARCH (RealGARCH) by expressing the exogenous component as an equation depending on some unknown parameter. Thus, the RealGARCH model fully specifies the dynamic specifications for both return and the RV measure.

Motivated by the above studies, this study discusses the models that are similar to Engle (2002) and Hansen et al. (2012) but focuses on the Glosten–Jagannathan–Runkle (GJR) model introduced by Glosten et al. (1993). The GJR model is an extension of the GARCH model which allows the current conditional variances to have different effects to the past positive and negative returns. The GJR model was demonstrated by Nugroho et al. (2019a) to provide a better data fit than the GARCH model. On the basis of author's knowledge, there are no studies in the literature that have discussed joining the GJR model and the RV measure.

Due to the presence of heavy tails in financial time-series, the assumption of a Student-*t* distribution instead of Normal distribution for return error in the context of GARCH-type models was suggested by some literatures. For example, see Nugroho and Susanto (2017), Nugroho et al. (2019b), and Kusumawati et al. (2020).

Furthermore, the estimation of model is carried out by using the Generalized Reduced Gradient (GRG) Non-Linear method in the Excel's Solver and the Adaptive Random Walk Metropolis (ARWM) method which is implemented in the Matlab software by making own code. Nugroho et al. (2019a,b) and Kusumawati et al. (2020) showed that the Excel's Solver's GRG Non-Linear method is easy to use for financial practitioner and has a good ability to estimate the GARCH-type models. Meanwhile, the ARWM method was shown by Nugroho (2018) as a statistically efficient method in estimating the GARCH(1,1) model with Normal and Student-*t* distributions.

Therefore, this study investigates the accuracy of the Excel's Solver's GRG Non-Linear method for estimating of the proposed models. The accuracy refers to how closely the estimated values of the model parameters to the ARWM results. The study then compares the fitting performance of the GJR(1,1), GJR-X(1,1), and RealGJR(1,1) models based on daily stock index of the Financial Times Stock Exchange 100 (FTSE100). The FTSE100 stock index is an index of 100 publicly traded companies in London (United Kingdom) with the highest market capitalization. This stock index is one of the stock market indices of the top 10 countries which constitute 66% of the world Gross Domestic Product (GDP) (Chaudhary et al., 2020). Thus the FTSE100 stock index can be taken as proxy to represent the performance of the overall stock market.

The remainder of the paper is organized as follows. Section 2 describes the proposed models, estimation methods, and evaluation method. Section 3 presents the description of observed data, implementation of estimation methods, and empirical results of estimation of the proposed models. Section 4 concludes.

2. Materials and Methods

2.1. GJR model

Consider a return time series

$$R_t = \varepsilon_t, \text{ where } \varepsilon_t \sim N(0, \sigma_t^2),$$

in which R_t denotes the return of financial asset, σ_t denotes the volatility (standard deviation) of returns at time t, and ε_t is an independent identically distributed (i.i.d) random variables Normally distributed with mean 0 and variance σ_t^2. The returns are simply computed as the first differences of the logarithm of asset prices sampled at the same unit time interval.

Differently from the original GARCH model, the GJR model proposed by Glosten et al. (1993) allows for asymmetric effects, that is the sign and the size of the past residuals or the past return shocks (if occurred) would influence current volatility differently. Mathematically, the conditional variance equation of GJR model contains an unknown parameter specified as the asymmetric parameter. In particular, the conditional variance equation of GJR(1,1) is defined as

$$\sigma_t^2 = \omega + (\alpha_1 + \alpha_2 I_{t-1})R_{t-1}^2 + \beta\sigma_{t-1}^2,$$

where the parameter α_2 drives the presence/absence of asymmetric effect, I is an indicator function which is defined as

$$I_{t-1} = \begin{cases} 0 & \text{if } R_{t-1} \geq 0 \\ 1 & \text{otherwise.} \end{cases}$$

In order to ensure the positivity and the stationarity of the conditional variance σ_t^2, the parameters must satisfy the following conditions:

$$\omega \geq 0, \alpha_1 \geq 0, \beta \geq 0, \alpha_1 + \alpha_2 \geq 0, \alpha_1 + 0.5\alpha_2 + \beta < 1.$$

The sum of α_1 and α_2 coefficients reflects a reaction of volatility to new information (return). In this case, volatility is very sensitive to different values of α_2 (Hill et al., 2018). When $\alpha_2 = 0$, the model reduces to the GARCH(1,1) model. This means that positive and negative returns of equal strength at the past period have the same effect on the current conditional variance. When $\alpha_2 \neq 0$, the positive and negative returns of the same size at the past period have different impact on the current conditional variance. When $\alpha_2 > 0$, negative return has a greater effect than positive return and this phenomenon is commonly known as "leverage effect". Conversely, when $\alpha_2 < 0$, positive return has a greater effect than negative return.

2.2. GJR-X model

Similar to Engle (2002), the GJR-X model can be constructed from the GJR model by adding an intra-daily data component. The conditional variance equation of GJR-X(1,1) particularly can be expressed as follows:

$$\sigma_t^2 = \omega + (\alpha_1 + \alpha_2 I_{t-1})R_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma x_{t-1} \tag{1}$$

with $\gamma > 0$ for ensuring the positivity and the stationarity of the conditional variance. In Eqn. (1), x_t is an exogenous component in the form of RV data. RV, according to Nugroho and Morimoto (2014), is a measure of the volatility of high frequency data in a day (intraday high frequency data). RV measure has various versions, including bipower variation, two scales realized volatility, and Realized Kernel (RK). This study uses the Realized Kernel (RK) of Barndorff-Nielsen et al. (2008) which has been shown to be unbiased and convergent at a faster rate than other measures.

2.3. RealGJR model

Following the construction of the RealGARCH(1,1) model of Hansen et al. (2012), the RealGJR(1,1) model is obtained from the GJR-X(1,1) model by expressing the exogenous component as a following equation:

$$x_t = \xi + \varphi\sigma_t^2 + u_t, \text{ where } u_t \sim N\left(0, s_u^2\right).$$

The conditions for the parameters are similar to Gerlach and Wang (2016):

$$0 < \alpha_1 + 0.5\alpha_2 + \beta + \gamma\varphi < 1 \text{ and } \omega + \gamma\xi > 0$$

to ensure the long-run variance is finite and positive. Further, it is sufficient that ω, α_1, $\alpha_1 + \alpha_2$, β, and γ are all positive to ensure positivity of each σ_t^2.

2.4. Models with normal and student-t distributions

Suppose $R = \{R_1, R_2, \ldots, R_T\}$ denotes the vector of returns, $X = \{x_1, x_2, \ldots, x_T\}$ represents the vector of RV, and $\sigma^2 = \left\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_T^2\right\}$ represents the vector of the conditional variance, where T is the number of observation. The total log-likelihood function for the GJR-X(1,1) model which assumes Normal distribution for the returns error ε_t is given by:

$$\mathscr{L}\left(R|\sigma^2\right) = -\frac{1}{2}\sum_{t=1}^{T}\left[\log(2\pi) + \log\left(\sigma_t^2\right) + \frac{R_t^2}{\sigma_t^2}\right]. \tag{2}$$

In the case of Student-t distribution for the returns error ε_t, the total log-likelihood function for the GJR-X(1,1) model with Student-t distribution, called as GJRt-X(1,1) model, is given by [in accordance with Asimow and Maxwell (2010)]:

$$\begin{aligned}\mathscr{L}\left(R|\sigma^2,\nu\right) = T&\left[\log\Gamma\left(\frac{\nu+1}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right) - \log\Gamma\left(\frac{1}{2}\right)\right] \\ &- \frac{1}{2}\sum_{t=1}^{T}\left[\log\left(\sigma_t^2(\nu-2)\right) + (1+\nu)\log\left(1+\frac{R_t^2}{\sigma_t^2(\nu-2)}\right)\right], \end{aligned} \tag{3}$$

where $2 < \nu < \infty$ denotes the degrees of freedom which is estimated together with the other parameters in the model and $\Gamma(\cdot)$ is the gamma function. The parameter ν measures the tail thickness of the Student-t distribution. The smaller the value of ν, the thicker the tail and the greater the value of ν towards infinity, the tail is close to the Normal distribution [see in Ramachandran (2021)].

For the RealGJR(1,1) models, the log-likelihood function can be obtained as the sum of the log-likelihood functions of the return and RV equations. The total log-likelihood function for a Normally distributed RV with mean $\xi + \varphi\sigma_t^2$ and variance s_u^2 is given by:

$$\mathscr{L}\left(X|\xi,\varphi,s_u^2,\sigma^2\right) = -\frac{1}{2}\sum_{t=1}^{T}\left[\log\left(2\pi s_u^2\right) + \frac{\left(x_t - \xi - \varphi\sigma_t^2\right)^2}{s_u^2}\right]. \tag{4}$$

Therefore, when ε_t is Normally distributed, the total log-likelihood function for the RealGJR(1,1) model is the sum of Eqns. (2) and (4):

$$\mathscr{L}(R,X|\theta) = \mathscr{L}\left(R|\sigma^2\right) + \mathscr{L}\left(X|\xi,\varphi,s_u^2,\sigma^2\right),$$

where θ is the vector of model parameters. Meanwhile, when ε_t is Student-t distributed, the total log-likelihood function for the RealGJR(1,1) model with Student-t distribution, called as RealGJRt(1,1) model, is the sum of Eqns. (3) and (4):

$$\mathscr{L}(R,X|\theta) = \mathscr{L}\left(R|\sigma^2,\nu\right) + \mathscr{L}\left(X|\xi,\varphi,s_u^2,\sigma^2\right).$$

2.5. Estimation methods

The purpose of statistical modeling estimation is to maximize the likelihood or its logarithm. There are many software that can be used to estimate the GARCH-type models, including Eviews, SAS, GAUSS, TSP, MATLAB, and R. However, these softwares require a fairly good knowledge of computer programming. As the Microsoft Excel is perhaps the most widely used software programs in the world, this study uses Excel's Solver to estimate the proposed models by maximizing the log-likelihood. In particular, the GRG Non-Linear approach in the Excel's Solver is chosen as a method to estimate the model parameters that maximize the log-likelihood of the model. Following Maia et al. (2017) and Rothwell (2017), the process of GRG Non-Linear approach can be described as follows. The first step is to choose initial values that satisfy the parameters conditions. These initial values are considered as the initial solutions and would change slightly to improve the parameter and objective values. As the objective is to maximize the log-likelihood function, this function value will gradually “increase” in the search direction.

To see the accuracy of the result of Excel’s Solver, the models are also estimated by using the ARWM method in the Markov Chain Monte Carlo (MCMC) algorithm, which is implemented in the Matlab program by making own code. The ARWM method is used to generate a Markov chain, which is the first step in the MCMC algorithm. Then, the statistical values of the Markov chain, such as mean, standard deviation, and Bayesian interval, are calculated based on the Monte Carlo approach and taken as the MCMC output (the second step).

The ARWM steps can be seen in Salim et al. (2016) and summarized as follows. At the i-th iteration, the parameter θ is generated as follows:

$$\theta^{(i)} = \theta^{(i-1)} + z^{(i)}, \text{ where } z^{(i)} \sim N(0, \Delta^{(i)}),$$

where $\Delta^{(i)}$ is the step-width. On the basis of Bayesian approach, the posterior distribution for the parameter θ conditional on the data y is defined by:

$$\log p(\theta|y) = \mathscr{L}(y|\theta) + \log p(\theta),$$

where $p(\theta)$ is the prior distribution for θ. Candidate $\theta^{(i)}$ is accepted if $\frac{p\left(\theta^{(i)}|y\right)}{p\left(\theta^{(i-1)}|y\right)} > u$, where the variable random u has a Uniform distribution $(0,1)$.

For example, the MCMC algorithm to estimate the parameters of the RealGARCHt(1,1) model is summarized as follows. Initial values are chosen, and an ARWM method is then applied in the first step of MCMC algorithm to produce successive draws of ω, α_1, α_2, β, γ, ξ, φ, s_u^2, and ν via the respective conditional posteriors.

Step 1: Generate Markov chains.

(i) Draw ω from $p\left(\omega|\alpha_1, \alpha_2, \beta, \gamma, \xi, \varphi, s_u^2, \nu\right)$

(ii) Draw α_1 from $p\left(\alpha_1|\omega, \alpha_2, \beta, \gamma, \xi, \varphi, s_u^2, \nu\right)$

(iii) Draw α_2 from $p\left(\alpha_2|\omega, \alpha_1, \beta, \gamma, \xi, \varphi, s_u^2, \nu\right)$

(iv) Draw β from $p\left(\beta|\omega, \alpha_1, \alpha_2, \gamma, \xi, \varphi, s_u^2, \nu\right)$

(v) Draw γ from $p\left(\gamma|\omega, \alpha_1, \alpha_2, \beta, \xi, \varphi, s_u^2, \nu\right)$

(vi) Draw ξ from $p\left(\xi|\omega, \alpha_1, \alpha_2, \beta, \gamma, \varphi, s_u^2, \nu\right)$

(vii) Draw φ from $p\left(\varphi|\omega, \alpha_1, \alpha_2, \beta, \gamma, \xi, s_u^2, \nu\right)$

(viii) Draw s_u^2 from $p\left(s_u^2|\omega, \alpha_1, \alpha_2, \beta, \gamma, \xi, \varphi, \nu\right)$

(ix) Draw ν from $p\left(\nu|\omega, \alpha_1, \alpha_2, \beta, \gamma, \xi, \varphi, s_u^2\right)$

Step 2: Obtain the statistical values of MCMC draws each: posterior mean and Highest Posterior Density Interval.

In comparison to the Excel’s Solver’s GRG Non-Linear method, the MCMC scheme has the advantage of being able to estimate the Bayesian interval, which is used to determine the significance of parameter. One of the Bayesian intervals is Highest Posterior Density (HPD) which was introduced in 1999 by Ming-Hui Chen and Qi-Man Shao. Given a set of MCMC draws with length M for the parameter θ, the algorithm to estimate the HPD interval at the significance level α is constructed as follows (Le et al., 2020):

(1) Compute $M_{cut} = [\alpha \times M]$ and $M_{span} = M - M_{cut}$, where $[x]$ represents the standard rounding function.

(2) Order the estimated values such that $\theta_1 \leq \theta_2 \leq \cdots \leq \theta_M$.

(3) Calculate $\theta^* = \{\theta_j\}_{j=M_{span}}^{M} - \{\theta_j\}_{m=1}^{M_{cut}}$.

(4) Find the index m^*, where $\theta^*(m^*)$ is the minimum value.

(5) The HPD interval: $\left(\theta_{m^*}, \theta_{m^*+M_{span}}\right)$.

2.6. Evaluation method

In selecting a model that gives the best data fit, this study uses Akaike Information Criterion (AIC) which was introduced in 1973 by Hirotugu Akaike in his seminal paper "Information Theory and an Extension of the Maximum Likelihood Principle". This criterion compares a set of models that do not have to be nested. Two models are said to be nested if one model is a special case of the other model. The statistical values of the AIC is defined by (Cavanaugh and Neath, 2019):

$$AIC = 2(k - \mathscr{L}),$$

where k represents the number of parameters in the estimated model and $\mathscr{L}$ denotes the log-likelihood value of the estimated model. The criterion is that the model with the smallest AIC value being considered the best.

3. Data Description and Empirical Results

3.1. Observed data

Data set used to analysis consists of daily returns and RK of FTSE100 index over the period from January 2000 to December 2017, which was taken from the "Oxford-Man Institute's realized library" at https://realized.oxford-man.ox.ac.uk/data/download. Both data sets consist of 4503 observations, excluding weekends and national holidays, and are measured in percentage.

To further understand the distributional properties of the returns, Table 1 presents summary statistics for the return and RK series. The negative mean for returns indicates loss in the stock market during the observation period. The positive standard deviation for returns shows the dispersion of returns from its mean and high level of variability of log price changes in the stock market for the trading period under observation. The kurtosis value for returns significantly exceeds 3, which indicates the existence of heavy-tails in the return series. Moreover, the Jarque–Bera (JB) normality test rejects the normality for return series, which is indicated by the JB statistic of 3869.6 with p-value less than 0.05. Therefore, the assumption of Student-t distribution for return error in the proposed models would be more appropriate than the Normal distribution. Notice that although the distribution of RK has clearly heavy tail characteristic, the Normal distribution is used for the RK distribution in Eqn. (2) as in most studies.

Table 1 Descriptive statistics of returns and RK

Statistics	Returns	RK
Mean	–0.035	0.766
Standard Deviation	0.930	0.484
Maximum	7.044	5.707
Minimum	–5.760	0.197
Kurtosis	7.53	15.20
JB Stat.	3870	33200
p-value	0.001	0.001

The percentage daily log returns (hereinafter simply called daily returns) as well as the percentage RK series plotted against time are displayed in Figure 1. Time series plot of the daily returns show that mean and variance of the series are constant over time with absence of trend, indicating that the return series follow the random walk model and is thus weakly stationary.

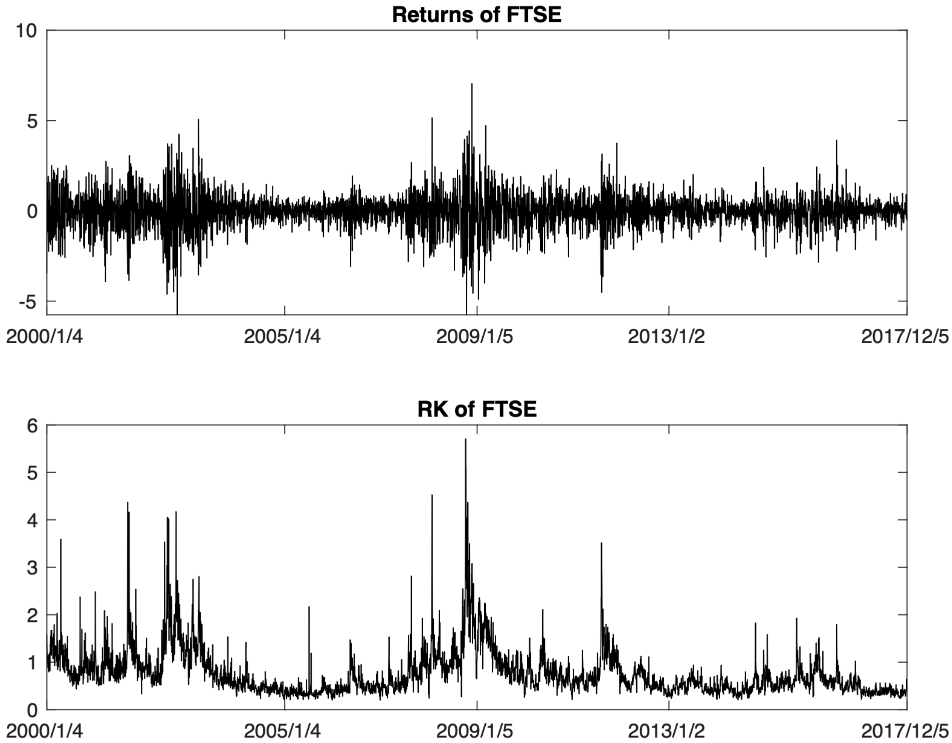

Figure 1 Time series plots of the return and RK on FTSE100.

3.2. Implementation of the estimation method

All calculations are done by using both Solver in Microsoft Excel 2019 and Matlab R2020b. In the case of Excel's Solver the estimation steps follow Nugroho et al. (2018), while in the case of ARWM the Matlab code is modified from Nugroho et al. (2019a).

As the ARWM method is carried out in the MCMC algorithm, the parameter estimation is based on the joint posterior density obtained from the Bayes' theorem. In this study the model specification is completed by the prior distributions of all parameters of interest. The prior distributions for the parameter ω, α_1, α_2, β, γ, ξ, φ, and s_u^2 are specified by $N(0,1000)$ as in Ardia and Hoogerheide (2010) in the context of GARCH model and the parameter ν is specified by $\exp(0,01)$ as in Deschamps (2006). The initial values for the model parameters are set as follows:

$$\omega = 0.005, \alpha_1 = 0.01, \alpha_2 = 0.1, \beta = 0.9, \gamma = 0.1,$$
$$\xi = 0.01, \varphi = 0.5, s_u^2 = 0.02, \nu = 10.$$

Since the Excel's Solver's GRG Non-Linear method was found by Christoffersen (2012) and Nugroho (2018) to sensitive to the initial value, the initial values of all unknown parameters in the Excel's Solver are set to close the ARWM results. The initial values for the parameters in the GJR-X(1,1) models are

$$\omega = 0.005, \alpha_1 = 0.01, \alpha_2 = 0.1, \beta = 0.9, \gamma = 0.1, \nu = 10.$$

and in the RealGJR(1,1) models are

$$\omega = 0.0005, \alpha_1 = 0.005, \alpha_2 = 0.05, \beta = 0.6, \gamma = 0.3,$$
$$\xi = 0.1, \varphi = 0.9, s_u^2 = 0.05, \nu = 10.$$

3.3. Estimation results

First, we evaluate the efficiency of ARWM sampling method via trace plot of parameter draws. Inspecting the path of the posterior samples via trace plot as well as the empirical autocorrelation function (Vehtari et al., 2021) gives good visual indicators of mixing property, which indicates how fast the chain of posterior samples converges to the target distribution. For example, Figure 2 plots the MCMC output for all parameters in the RealGJR(1,1) model with Student-*t* distribution, using 6000 MCMC samples and discard the first 1000 values as burn-in. The figure shows that the chains of γ, ξ, and φ move slowly through their parameter spaces, while the others display better good mixing properties. In particular, the values of Integrated Autocorrelation Time (IACT, see in Sokal (1997)) for parameter γ, ξ, and φ are estimated to about 174, 202, and 175, respectively. Meanwhile, the IACTs for the others are less than 80. Therefore the ARWM method can be considered efficient in estimating the proposed models.

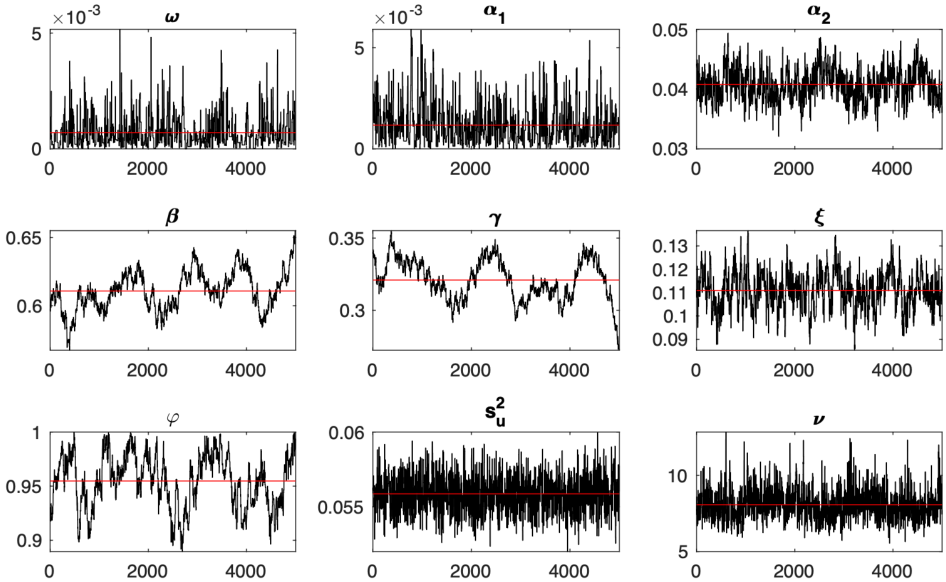

Figure 2 Trace plots of MCMC output for the parameters of RealGJR(1,1) model with Student-*t* distribution. The red line indicates the mean of the posterior samples.

Table 2 presents the point estimates for each of the parameters estimated by the Excels's Solver's GRG Non-Linear and ARWM methods. First, the conditions for ω in the GJR-X(1,1) model and for ω and α_1 in the RealGJR(1,1) model were violated by the Excels's Solver's GRG Non-Linear method. The Solver produces zero for both parameters since the estimated value is very close to zero referring to the ARWM results. However, the violations do not influence the estimation results for the other parameters. Ignoring the the parameters ω and α_1, both methods produce the estimation values that can be considered not significantly different, indicated by the relative errors are about 0.13 for α_1 in the GJR*t*-X (1,1), 0.11 for α_2 in the RealGJR(1,1), 0.28 for ν in the RealGJR*t*(1,1), and less than 0.10 for the others. This result suggests that the Excels's Solver's GRG Non-Linear has a good ability to estimate the proposed models.

Table 2 Estimation result of FTSE100 data parameter

Parameter	GJR(1,1)		GJR-X(1,1)		RealGJR(1,1)	
	Normal	Student-t	Normal	Student-t	Normal	Student-t
Excel's Solver's GRG Non-Linear						
ω	0.0061	0.0048	0.0000	0.0000	0.0000	0.0000
α_1	0.0222	0.0149	0.0162	0.0111	0.0000	0.0000
α_2	0.1033	0.1070	0.1212	0.1252	0.0459	0.0417
β	0.9139	0.9217	0.8757	0.8870	0.6213	0.6141
γ	-	-	0.0391	0.0324	0.3228	0.3166
ξ	-	-	-	-	0.1237	0.1127
φ	-	-	-	-	0.9039	0.9602
s_u^2	-	-	-	-	0.0559	0.0556
ν	-	9.39	-	10.15	-	10.27
$\mathscr{L}$	–5095.42	–5050.63	–5073.53	–5036.39	–5085.98	–5029.99
ARWM						
ω	0.0063	0.0050	0.0005	0.0005	0.0005	0.0006
α_1	0.0244	0.0156	0.0170	0.0128	0.0011	0.0010
α_2	0.1038	0.1070	0.1238	0.1238	0.0414	0.0412
β	0.9116	0.9208	0.8728	0.8815	0.6158	0.6046
γ	-	-	0.0395	0.0333	0.3149	0.3264
ξ	-	-	-	-	0.1134*	0.1115*
φ	-	-	-	-	0.9463	0.9541
s_u^2	-	-	-	-	0.0557	0.0558
ν	-	9.69	-	10.51	-	8.03
$\mathscr{L}$	–5097.13	–5052.87	–5076.80	–5039.78	–5087.53	–5032.85

Note: * means that parameter is significant at 5%

We found that the estimates of α_2 is positive and statistically significant in terms of HPD interval at the 1% level, indicating the presence of the leverage effect. The result reflects that the past volatility of the FTSE stock index tends to increase when stock index prices drop. In comparison to GJR models, the estimates of α_2 are larger (more positive) for GJR-X(1,1) models and smaller (closer to zero) for RealGJR models. This result suggests a higher level of asymmetry in the leverage effect produced by GJR-X(1,1) models and a lower level produced by RealGJR(1,1) models. Since the higher leverage increases the risk of the equity stake, the GJR-X(1,1) models can increase investor's confidence to sell more of the stocks to achieve greater benefits. On other words, the RealGJR(1,1) models can increase investor's confidence to keep the stocks.

Consistent with the common finding in most of the empirical studies, we found that volatility produced by GJR(1,1) model to be highly persistent, which is indicated by the estimates of β about 0.92. Meanwhile, the GJR-X(1,1) and RealGJR(1,1) models, respectively, have intermediate and low degrees of persistence of volatility according to Bauwens and Rombouts (2007). Lower volatility in leads to smaller variations of return, hence the RealGJR(1,1) models exhibit lowest risk, consistent with the previous result on leverage effect. This result suggests shorter-lasting periods of high volatility of the models incorporating RK data, see Figure 3 in the case of models with Student-t distribution.

Regarding the coefficient on x_{t-1}, the estimates of γ in the GJR-X(1,1) and RealGJR(1,1) models are larger than the estimate of α_1 in the GJR(1,1) model. According to Hansen and Huang (2016), the result reflects the fact that the RK measure offers a much stronger signal on the future volatility than does the squared returns. Therefore, both GJR-X(1,1) and RealGJR(1,1) models are able to capture rapid changes in the volatility level. Table 2 shows that the strongest signal is offered by the RealGJR(1,1) models.

In the case of RealGJR(1,1) models, the parameter ξ is included to adjust the potential biases of the realized measures due to the microstructure noise and non-tradings hours (Yamauchi and Omori, 2020). We found the significant and positive values for ξ in terms of the 99% HPD interval. It falls within the range from 0.093 to 0.136 in the case of Normal model and from 0.093 to 0.135 in the

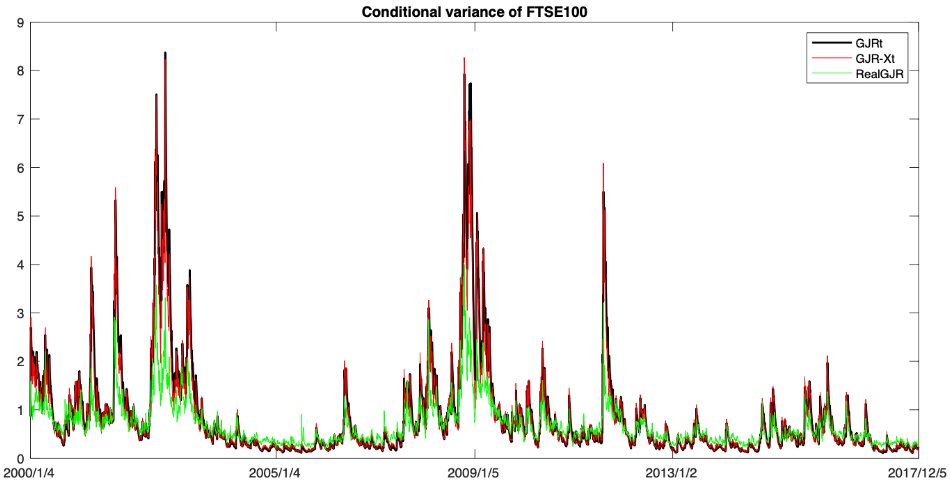

Figure 3 Time plots of daily conditional variance.

case of Student-*t* model. The results indicate the presence of microstructure noise; a deviation of the fundamental price of a security (Romero, 2016). It reflects the fact that the RK data are contamined with this type noise at high observation frequencies; in other words, the data contain an "observation error" component.

Next, the point estimates of φ are close to one, which suggest that the RK data is roughly proportional to the conditional variance. This empirical result suggests that the FTSE100 volatility during the observation time period amounts to about 95% of daily volatility.

Furthermore, the models with Student-*t* distribution have approximately 10 degrees of freedom suggesting that the distribution of return errors has a heavier tail than a Normal distribution. This empirical result indicates that the FTSE100 data are modeled in superior manner when Student-*t* distribution is applied to the returns error as the model is better at capturing the excess kurtosis in the data compared to the model with Normal distribution.

3.4. Best fit model selection

To select the best fit models with appropriate distributional assumption, AIC is employed in conjunction with log-likelihood. This criterion does not require nested models and considers the model with the smallest information criteria value as the best model. Per distribution and overall, Table 3 ranks the proposes models according to their AIC values. The AIC suggests that the Student-*t* distribution might more appropriate than Normal distribution for modeling the FTSE100 data. The AIC gives preference to the GJR-X(1,1) and RealGJR(1,1) models over plain GJR(1,1) model with respect to data fit in each distribution. In particular, we note that the RealGJR(1,1) model is a better fit than the GJR-X(1,1) when the Student-*t* distribution is applied only. In general, the AIC selects the RealGJR(1,1) model with Student-*t* distribution as the best model to model the daily FTSE100 return volatility.

Finally, we investigate the fitting performance of the competing GJR-type models at several sample sizes. In this case, the samples of a given size are taken by treating the carried-forward data from the last observation. For example, the sample of 200 observations contains data covering the period from 22 February 2017 to 5 December 2017, the sample of 400 observations contains data covering the period from 11 May 2016 to 5 December 2017, etc. Figure 4 presents plots of AIC values for the competing models at sample sizes with an interval of 200 and Table 3 lists AIC values at four small sample sizes and four large sample sizes only. For the smallest size (200 last observations) and largest sizes (4500 observations), the samples present different fitting performance from the other

Table 3 Sample size vs AIC value

Sample Size	GJR(1,1)		GJR-X(1,1)		RealGJR(1,1)	
	Normal	Student-*t*	Normal	Student-*t*	Normal	Student-*t*
200	206.76^{5}	203.20^{3}	207.48^{6}	204.38^{4}	-304.08^{2}	-308.83^{1}
400	621.23^{6}	591.60^{4}	608.45^{5}	585.24^{3}	-51.56^{2}	-82.22^{1}
600	1108.56^{6}	1061.28^{4}	1083.67^{5}	1050.22^{3}	402.05^{2}	365.43^{1}
800	1515.65^{6}	1456.11^{4}	1486.59^{5}	1444.48^{3}	639.87^{2}	596.20^{1}
4000	8683.14^{6}	8589.76^{4}	8645.70^{5}	8565.72^{3}	8154.86^{2}	8029.61^{1}
4200	9319.38^{6}	9233.16^{4}	9278.90^{5}	9207.61^{3}	9194.99^{2}	9080.77^{1}
4400	9846.56^{6}	9759.63^{4}	9805.43^{5}	9732.40^{3}	9726.07^{2}	9615.30^{1}
4500	10199.58^{6}	10113.44^{3}	10159.62^{4}	10089.30^{2}	10190.12^{5}	10081.86^{1}

Note: Superscript denotes the rank of model.

sizes, except for the RealGJR(1,1) model with Student-*t* distribution. In other words, the sample sizes of 400 to 4400 last daily observations are ideal to analyze the fitting performance of our competing models since they provide the same performance for each model. Overall, the RealGJR(1,1) model with Student-*t* distribution is the best performing model for the FTSE 100 data for any sample sizes.

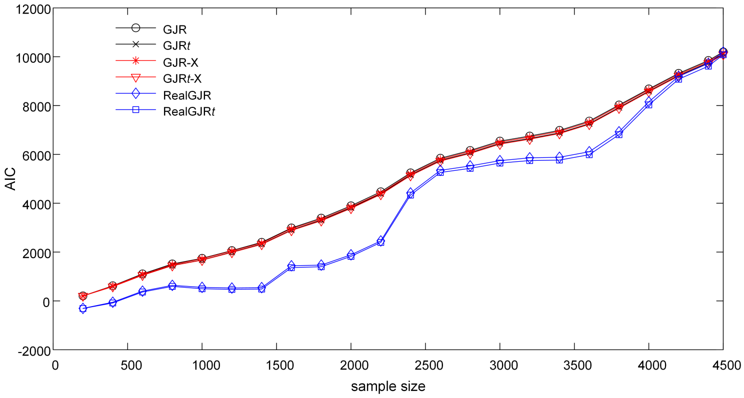

Figure 4 Plots of AIC values at several sample sizes with an interval of 200 and a sample size of 4500.

4. Conclusions

This study examined the fitting performance of the GJR(1,1), GJR-X(1,1) and RealGJR(1,1) models using daily returns and RK series of FTSE100 from January 2000 to December 2017. The models with Normal and Student-*t* distribution for return error were estimated by using the Excel's Solver's GRG Non-Linear and ARWM methods. First, the estimation results obtained from two methods are close to each other in terms of relative error, which concludes that the Excel's Solver's GRG Non-Linear can be considered to be reliable for estimating the proposed model. Second, the results of the comparison of proposed models based on the AIC indicate that the GJR(1,1) models with RK computed from high frequency intraday returns better fit than the plain GJR(1,1) model. In general, the RealGJR(1,1) model with Student-*t* distribution is the preferred model according to the AIC. The empirical merit of this study is, thus, (i) its recommendation on the use of Excel's Solver for financial practioners who need to estimate the GJR models easily, (ii) its demonstration on the potential of the RealGJR(1,1) models to spur the development of several other GARCH-type models and to better model and evaluate an investor's risk of a stock portfolio, and (iii) its recommendation on the use of sample sizes of 400 to 4400 for research analyzis.

Finally, this study would like to mention that the RealGJR(1,1)-type model applied to FTSE200 data can be considered to compete with GARCH(1,1)-type models. This could be an interesting future work when the types of GJR model with RV measure is applied to other RV measures and they can be compared to other asymmetric GARCH-type models, such as Realized Exponential GARCH model proposed by Hansen and Huang (2016). Another promising future work is the use of power transformation functions to the observed data (such as return series or RV) and conditional volatility, see in Nugroho et al. (2021a,b).

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