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A Study on Partially Accelerated Life Test for the Generalized Inverse Lindley Distribution Under Multiple Censored Information

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Abstract

If the objects have high reliability, then checking the life span of components in regular use circumstances takes extra time and expenditure compared to accelerated circumstances. The apparatus put on higher stress than the regular level of stresses to find out premature failures in a short phase to lessen the costs involved in the assessment of apparatus with no change in the worth. This problem is based on constant stress partially accelerated life tests for the generalized inverse Lindley distribution using multiple censoring schemes. The maximum likelihood estimates and asymptotic variance and covariance matrix are achieved. The confidence intervals for parameters are also assembled. Further, a simulation study is used to check and verify the performance of the estimators.

Keywords: Constant stress partially accelerated life tests, generalized inverse Lindley distribution, multiple censoring, Fisher information matrix, simulation study.

1. Introduction

Manufacturing designs are evolving on a daily basis in response to the present market condition and technological advancements. It can be difficult to obtain information regarding the lifetime of an item or product if the item has a high reliability under typical usage conditions at the time of testing. In this case, the accelerated life test (ALT) is the ideal technique to learn more about the objects' existence. ALT is used to obtain knowledge on an item's or a good's life in a short amount of time and at a low cost by testing them under accelerated conditions and then testing them under normal use conditions to trigger early failures. These situations are referred to as stresses, and they can take the form of temperature, voltage, force, and so on.

In general, three types of stresses are used in ALT: constant stress, step-stress, and progressive stress. We are only dealing with constant stress in this paper. During a constant stress ALT, the products or items are subjected to constant levels of stress. From ALT, two types of data are obtained: complete data and censored data. The lifetime of each unit is known in the complete data, but it is unknown in the censored data. In ALT, a mathematical model related to the lifetime of an item or product and stress can be expected. There are numerous situations in which these relationships are either unknown or cannot be summarized. This means that data cannot be hypothesized to use conditions that are obtained from ALT. The partially accelerated life test (PALT) is used in situations where the test objects are subjected to both ordinary and higher-than-ordinary stress conditions. Reliability practitioners use two main methods in PALT: constant stress partially accelerated life test (CSPALT) and step-stress partially accelerated life test (SSPALT). The products or items are tested in either standard or higher-than-normal conditions until the test is completed in CSPALT.

The components of a system may fail in a variety of situations for a variety of reasons, and as a result, the lifetime experiment may become uncontrollable. The test is aborted after a predetermined amount of time in time censoring, also known as type-I censoring. The analysis is terminated after a fixed number of items in item censoring, also known as type II censoring. Because type-I and type-II censorings do not allow the removal of items or components from a test during testing, we move on to other censoring schemes such as progressive type censoring and multiple censoring. Both of the aforementioned schemes permit the removal of items from the test at any time or in any situation. Multiple censoring also occurs when testing of items or components fails for more than one reason. Tobias and Trindada (1995) discovered that the type-I and type-II censoring schemes are a subset of multiple censoring schemes. The following Table 1 summarizes the literature relevant to our study.

Table 1 Review of the literature related to the proposed work

Author(s) Name	Method	Scheme	Failure Model	Strategy
Mohamed et al. (2018)	CSALT	Progressive type-II censoring	Extension of exponential distribution	-
Almarashi (2020)	CSPALT	Progressive type-II censoring scheme	Generalized half-logistic distribution	-
Zhang and Fang (2018)	CSPALT	Type-I censoring	Exponential distribution	-
Alam et al. (2021), Alam and Ahmed (2023)	CSPALT, SSPALT	Progressive censoring, adaptive type-II progressive hybrid censoring	Generalized inverted exponential distribution, Exponentiated Pareto distribution	Maintenance service policy
Ismail and Al Tamimi (2017)	CSPALT	Type-I Censoring	Inverse Weibull distribution	-
Ling et al. (2009)	SSALT	Progressive type-I hybrid censoring scheme	Exponential distribution	-
Xiaolin et al. (2018)	SSPALT	Progressive type-II hybrid censoring	Modified Weibull distribution	-
Abushal and Al-Zaydi (2017)	CSPALT	Progressive type-II censored	Mixture of Pareto distribution	-
Alam et al. (2019)	CSPALT	Multiple censoring	Exponentiated Exponential distribution	-
Mahmoud et al. (2018)	CSPALT	Progressive type-II	Modified Weibull distribution	-

Table 1 (Continued)

Author(s) Name	Method	Scheme	Failure Model	Strategy
Shi and Shi (2016)	CSPALT	Progressive type-II censoring	Complementary exponential distribution	Masked series system
Ismail (2016)	CSPALT	Hybrid censoring	Weibull distribution	Maximum likelihood and percentile bootstrap method
Ullah et al. (2017)	ALT	Complete data	Generalized exponential distribution	Geometric process
Escober and Meeker (1994)	CSPALT	Type-I censoring	Weibull distribution	-
Guan et al. (2014)	CSALT	Complete data	Generalized exponential distribution	-
Nassr and Elharoun (2019)	CSPALT	Multiple censored	Exponentiated Weibull distribution	-
Hassan et al. (2015)	CSPALT	Multiple censoring	Inverted Weibull distribution	-
Cheng and Wang (2012)	CSPALT	Multiple censoring	Burr XII distribution	-
Alam and Ahmed (2022)	SSPALT	Progressive censoring	Generalized inverted exponential distribution	Maintenance service policy
Proposed Work	CSPALT	Multiple censoring	Generalized inverse Lindley distribution	-

The proposed problem is CSPALT under multiple censoring when the examination unit life span follows the generalized inverse Lindley distribution. The novelty of this problem is that CSPALT is designed with multiple censoring for the generalized inverse Lindley distribution, and no previous research on used distribution under the proposed method for multiple censoring schemes is available. The paper is organized as follows. Section 2 contains an explanation of the model as well as the test method. Section 2 also contains the necessary assumptions for CSPALT. Section 3 contains the point Estimation. In this section, the likelihood function of the model is observed under multiple censoring schemes, and the Fisher Information matrix is also investigated. Section 4 develops the simulation study. Section 6 contains the conclusions. Section 6 contains the proposed study's real-world application.

2. Model Description, Test Method and Assumptions

2.1. Model description

In this section, we employ a two-parameter inverse Lindley distribution, also known as the generalized inverse Lindley distribution (GIL) proposed by Sharma et al. (2015). A random variable X follows the GIL distribution if the probability density function (pdf) and cumulative density function (cdf) take the following forms. The pdf of the distribution is given in the following Equation (1).

$$f(x,\alpha,\lambda)=\frac{\lambda\alpha^2}{1+\alpha}\left(\frac{1+x^\lambda}{x^{2\lambda+1}}\right)e^{-\frac{\alpha}{x^\lambda}}\;;\; x,\alpha,\lambda>0 \tag{1}$$

The pdf curve of the distribution is presented in the following Figure 1.

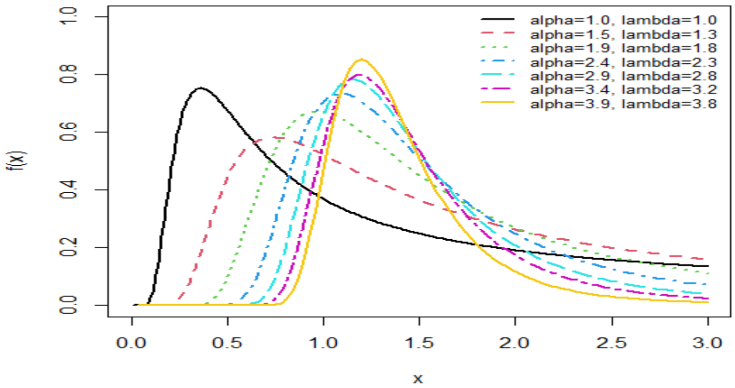

Figure1 pdf curve of the generalized inverse Lindley distribution

The cdf of the distribution is given in the following Equation (2).

$$F(x,\alpha,\lambda)=\left(1+\frac{\alpha}{(1+\alpha)x^{\lambda}}\right)e^{-\frac{\alpha}{x^{\lambda}}} \tag{2}$$

The cdf curve of the distribution is presented in the following Figure 2.

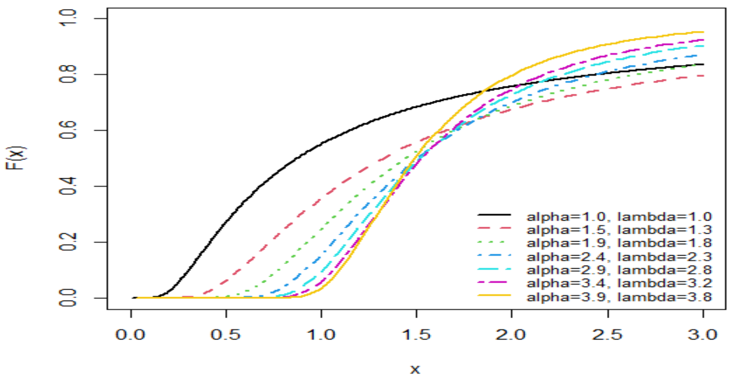

Figure2 cdf curve of the generalized inverse Lindley distribution

The reliability and hazard function are given in Eqn. (3) and Eqn. (4), respectively.

$$S(x,\alpha,\lambda)=1-\left(1+\frac{\alpha}{(1+\alpha)x^{\lambda}}\right)e^{-\frac{\alpha}{x^{\lambda}}} \tag{3}$$

$$h(x,\alpha,\lambda)=\frac{\lambda\alpha^{2}(1+x^{\lambda})}{x^{\lambda+1}\left[(1+\alpha)x^{\lambda}\left(e^{-\frac{\alpha}{x^{\lambda}}}-1\right)-\alpha\right]} \tag{4}$$

where α and λ are scale, shape parameters, respectively. The reliability and hazard functions curves are shown in the following Figures 3 and 4.

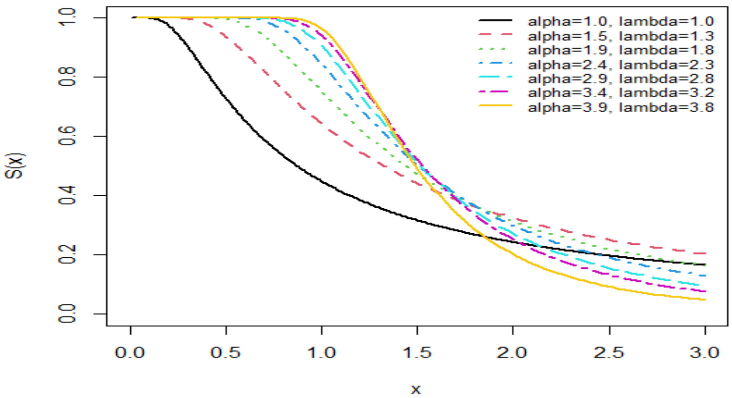

Figure3 Reliability curve of the generalized inverse Lindley distribution

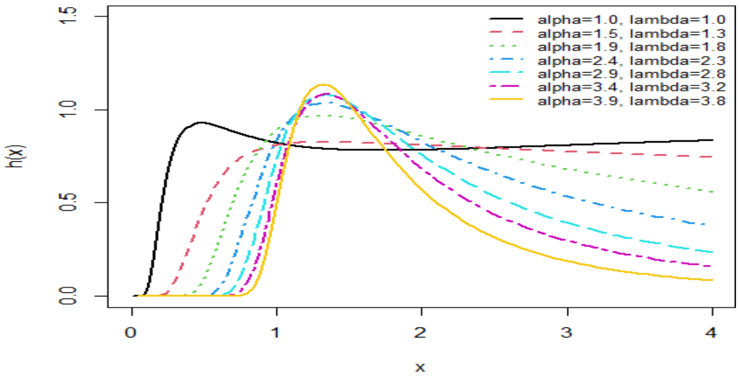

Figure4 Hazard function curve of the generalized inverse Lindley distribution

Sharma et al. (2016) proposed the extended inverse Lindley distribution, which is a subset of the GIL distribution. This distribution is a new statistical inverse model for information about upside-down bathtub survival. Alkarni (2015) introduced the inverse Lindley distribution, which provides more flexibility in modeling upside-down bathtub life span data.

2.2. Test method

If the component's life span follows the GIL distribution, then the CSPALT test procedure under multiple censoring schemes can be defined as follows. The pdf of GIL distribution under normal condition is given in the following Equation (5).

$$f_1(t_i,\alpha,\lambda)=\frac{\lambda\alpha^2}{1+\alpha}\left(\frac{1+t_i^{\lambda}}{t_i^{2\lambda+1}}\right)e^{-\frac{\alpha}{t_i^{\lambda}}};\ t_i,\alpha,\lambda>0,\ i=1,2,\ldots,n_1 \tag{5}$$

The cdf of GIL distribution under normal condition is given in the following Equation (6).

$$F_1(t_i,\alpha,\lambda)=\left(1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}\right)e^{-\frac{\alpha}{t_i^{\lambda}}};\ t_i,\alpha,\lambda>0 \tag{6}$$

Under the accelerated condition, the pdf and cdf of lifetime $X=\beta^{-1}T$ take the following forms, presented in Equations (7) and (8).

$$f_2(x_j,\alpha,\lambda)=\frac{\lambda\beta\alpha^2}{1+\alpha}\left(\frac{1+(\beta x_j)^{\lambda}}{(\beta x_j)^{2\lambda+1}}\right)e^{-\frac{\alpha}{(\beta x_j)^{\lambda}}}\;;x_j,\alpha,\lambda>0\,,\beta>1\,;j=1,2,...,n_2 \tag{7}$$

$$F_2(x_j,\alpha,\lambda)=\left(1+\frac{\alpha}{(1+\alpha)(\beta x_j)^{\lambda}}\right)e^{-\frac{\alpha}{(\beta x_j)^{\lambda}}} \tag{8}$$

where $\beta(>1)$ is the acceleration factor and x_j is j^{th} observed lifetime under the case of the accelerated situation.

2.3. Assumptions

The necessary postulations for CSPALT are given as

(i) The lifetimes of objects T_i, $i=1,2,...,n_1$ are independent and identically distributed random variable with probability pdf presented in (5), which is allocated to normal situation.

(ii) The lifetimes of objects X_j, $j=1,2,...,n_2$ are also independent and identically distributed random variable with pdf presented in (7), which is allocated to accelerated condition.

(iii) T_i and X_j are mutually independent also.

(iv) n_1 and n_2 are the total number of objects at regular and accelerated situations, respectively.

3. Estimation Procedure

3.1. Maximum likelihood estimation

In this section, we estimate parameters using the maximum likelihood estimation (MLE) technique. MLE is the most important and widely used method in statistics. In addition, for large samples, MLEs have the appealing properties of being consistent and asymptotically normal. $t_{(1)}<t_{(2)}<...<t_{(n)}$ are supposed observed values of the total life span T at the usual situation and $t_{(1)}<t_{(2)}<...<t_{(n)}$ are the supposed observed values of the life span X at the accelerated situation. Then the likelihood for GIL distribution for multiple censored data under CSPALT is given in the following Equation (9).

$$L(\alpha,\lambda,\beta)=\prod_{i=1}^{n}\left[f_1(t_i)\right]^{\delta_{i,1,f}}\left[1-F_i(t_i)\right]^{\delta_{i,1,c}}\times\left[f_2(x_i)\right]^{\delta_{i,2,f}}\left[1-F_2(x_i)\right]^{\delta_{i,2,c}}. \tag{9}$$

Then, the log likelihood function takes the following (Equation (10)).

$$\begin{aligned}\ln L=\;&n_f\left(\frac{\lambda\alpha^2}{1+\alpha}\right)+n_{2f}\ln(\beta)+\sum_{i=1}^{n}\delta_{i,1,f}\ln\left(\frac{1+t_i^{\lambda}}{t_i^{2\lambda+1}}\right)-\sum_{i=1}^{n}\delta_{i,1,f}\left(\frac{\alpha}{t_i^{\lambda}}\right)\\&+\sum_{i=1}^{n}\delta_{i,1,c}\ln\left[1-\left(1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}\right)e^{-\alpha/t_i^{\lambda}}\right]\\&+\sum_{i=1}^{n}\delta_{i,2,f}\ln\left(\frac{1+(\beta x_i)^{\lambda}}{(\beta x_i)^{2\lambda+1}}\right)-\sum_{i=1}^{n}\delta_{i,2,f}\left(\frac{\alpha}{(\beta x_i)^{\lambda}}\right)\\&+\sum_{i=1}^{n}\delta_{i,2,c}\ln\left[1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\right],\end{aligned} \tag{10}$$

$\delta_{i,1,f}$, $\delta_{i,1,c}$, $\delta_{i,2,f}$, $\delta_{i,2,c}$ are indicator functions with

$$\delta_{i,1,f}, \delta_{i,2,f} = \begin{cases} 1, & \text{the item failed at stress condition} \\ 0, & \text{otherwise} \end{cases}$$

$$\delta_{i,1,c}, \delta_{i,2,c} = \begin{cases} 1, & \text{the item censored at normal condition} \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{i=1}^{n} \delta_{i,1,f} = n_{1f} = \text{Number of failed items at nomal condition}$$

$$\sum_{i=1}^{n} \delta_{i,2,f} = n_{2f} = \text{Number of failed items at accelerated condition}$$

$$\sum_{i=1}^{n} \delta_{i,1,c} = n_{1c} = \text{Number of censored item at normal condition}$$

$$\sum_{i=1}^{n} \delta_{i,2,c} = n_{2c} = \text{Number of censored item at accelerated condition}$$

$$n_f = n_{1f} + n_{2f}, \ \ln L = \ln L(\alpha, \lambda, \beta).$$

The MLEs of α, λ and β are obtained by differentiating log-likelihood function concerning parameters and equating to zero. Then the expressions are given in the following equations (Equations (11), (12) and (13)).

$$\frac{\partial \ln L}{\partial \alpha} = \lambda n_f \left(\frac{2\alpha - \alpha^2}{(1+\alpha)^2} \right) - \sum_{i=1}^{n} \delta_{i,1,f} \left(\frac{1}{t_i^{\lambda}} \right) + \sum_{i=1}^{n} \delta_{i,1,c} \frac{\left[\frac{\alpha}{1+\alpha} \left(\frac{1}{\alpha} - \frac{1}{1+\alpha} \right) e^{-\alpha / t_i^{\lambda}} \right]}{\left[1 - \left(1 + \frac{\alpha}{(1+\alpha) t_i^{\lambda}} \right) e^{-\alpha / t_i^{\lambda}} \right]}$$
$$- \sum_{i=1}^{n} \delta_{i,2,f} \left(\frac{1}{(\beta x_i)^{\lambda}} \right) + \sum_{i=1}^{n} \delta_{i,2,c} \frac{\left[\frac{\alpha}{1+\alpha} \left(\frac{1}{\alpha} - \frac{1}{1+\alpha} \right) e^{-\alpha / (\beta x_i)^{\lambda}} \right]}{\left[1 - \left(1 + \frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}} \right) e^{-\alpha / (\beta x_i)^{\lambda}} \right]} = 0 \qquad (11)$$

$$\frac{\partial \ln L}{\partial \lambda} = n_f\left(\frac{\alpha^2}{1+\alpha}\right) + \sum_{i=1}^{n}\delta_{i,1,f}\left(\frac{t_i^{\lambda}}{1+t_i^{\lambda}} - 2\right) + \sum_{i=1}^{n}\delta_{i,1,f}\left(\frac{\alpha}{t_i^{\lambda}}\right)\ln(t_i) - \sum_{i=1}^{n}\delta_{i,2,f}\left(\frac{\alpha}{(\beta x_i)^{\lambda}}\right)$$

$$-\sum_{i=1}^{n}\delta_{i,1,c}\frac{\left(1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}\right)e^{-\alpha/t_i^{\lambda}}\left[\frac{\frac{\alpha}{(1+\alpha)}t_i^{\lambda}\ln(t_i)}{1+\frac{\alpha}{(1+\alpha)}t_i^{\lambda}} + t_i^{\lambda}\ln(t_i)\right]}{\left[1-\left(1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}\right)e^{-\alpha/t_i^{\lambda}}\right]} + \sum_{i=1}^{n}\delta_{i,2,f}\left(\frac{(\beta x_i)^{\lambda}}{1+(\beta x_i)^{\lambda}} - 2\right) \quad (12)$$

$$-\sum_{i=1}^{n}\delta_{i,2,c}\frac{\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\left[\frac{\frac{\alpha}{(1+\alpha)}(\beta x_i)^{\lambda}\ln(\beta x_i)}{1+\frac{\alpha}{(1+\alpha)}(\beta x_i)^{\lambda}} + (\beta x_i)^{\lambda}\ln(\beta x_i)\right]}{\left[1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\right]} = 0$$

$$\frac{\partial \ln L}{\partial \beta} = n_{2f}\beta^{-1} + \sum_{i=1}^{n}\delta_{i,2,f}\left(\frac{\lambda x_i^{\lambda}\beta^{\lambda-1}}{1+(\beta x_i)^{\lambda}} - \frac{(2\lambda+1)}{\beta}\right) - \sum_{i=1}^{n}\delta_{i,2,f}\left(\frac{\alpha\lambda x_i}{(\beta x_i)^{\lambda+1}}\right)$$

$$-\sum_{i=1}^{n}\delta_{i,2,c}\frac{\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\left[\frac{\frac{\alpha\lambda x_i}{(1+\alpha)(\beta x_i)^{\lambda+1}}}{1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}} - \alpha x_i(\beta x_i)^{-\lambda-1}\right]}{\left[1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\right]} = 0 \quad (13)$$

The Fisher Information matrix is given in the following Equation (14).

$$I = \begin{bmatrix} -\frac{\partial^2 \ln L}{\partial \alpha^2} & -\frac{\partial^2 \ln L}{\partial \alpha \partial \lambda} & -\frac{\partial^2 \ln L}{\partial \alpha \partial \beta} \\ -\frac{\partial^2 \ln L}{\partial \lambda \partial \alpha} & -\frac{\partial^2 \ln L}{\partial \lambda^2} & -\frac{\partial^2 \ln L}{\partial \lambda \partial \beta} \\ -\frac{\partial^2 \ln L}{\partial \beta \partial \alpha} & -\frac{\partial^2 \ln L}{\partial \beta \partial \lambda} & -\frac{\partial^2 \ln L}{\partial \beta^2} \end{bmatrix} \quad (14)$$

The elements of the matrix are given in the following equations.

$$\frac{\partial^2 \ln L}{\partial\alpha\partial\beta} = -\sum_{i=1}^{n}\delta_{i,2,f}\left(\frac{\lambda x_i}{(\beta x_i)^{\lambda+1}}\right) + \sum_{i=1}^{n}\delta_{i,2,c}\frac{\left[\frac{\alpha}{1+\alpha}\left(\frac{1}{\alpha}-\frac{1}{1+\alpha}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\right]}{\left[1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\right]}\times$$

$$\left[\left(\frac{\alpha\lambda x_i}{(\beta x_i)^{\lambda+1}}\right) + e^{-\alpha/(\beta x_i)^{\lambda}}\frac{\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)\alpha\lambda(\beta x_i)^{-\lambda-1} - \left(\frac{\alpha\lambda x_i}{(1+\alpha)(\beta x_i)^{\lambda+1}}\right)}{1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}}\right]$$

$$\frac{\partial^2 \ln L}{\partial\lambda\partial\beta} = \sum_{i=1}^{n}\delta_{i,2,f}\frac{\alpha\lambda x_i}{(\beta x_i)^{\lambda+1}} + \sum_{i=1}^{n}\delta_{i,2,f}\left[\frac{\lambda(\beta x_i)^{\lambda}}{1+(\beta x_i)^{\lambda}}\left(\frac{1}{\beta}-\frac{x_i(\beta x_i)^{\lambda-1}}{1+(\beta x_i)^{\lambda}}\right)\right]$$

$$-\sum_{i=1}^{n}\delta_{i,2,c}\frac{\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}(\beta x_i)^{\lambda}\ln(\beta x_i)\left[\frac{\frac{\alpha}{(1+\alpha)}}{1+\frac{\alpha}{(1+\alpha)}(\beta x_i)^{\lambda}}+1\right]}{\left[1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\right]}\times$$

$$\left[\frac{1-\frac{\alpha\lambda x_i}{1+\alpha}(\beta x_i)^{-\lambda-1}}{1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}} + \alpha\lambda x_i(\beta x_i)^{-\lambda-1} - \frac{\frac{\alpha}{(1+\alpha)}\left(1+\frac{\alpha}{(1+\alpha)}(\beta x_i)^{\lambda}\right)^{-2}\left(\frac{\alpha\lambda x_i(\beta x_i)^{\lambda-1}}{1+\alpha}\right)}{\left(\frac{\frac{\alpha}{(1+\alpha)}}{1+\frac{\alpha}{(1+\alpha)}(\beta x_i)^{\lambda}}+1\right)} + \right.$$

$$\left. e^{-\alpha/(\beta x_i)^{\lambda}}\frac{\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)\alpha\lambda(\beta x_i)^{-\lambda-1}x_i - \left(\frac{\alpha\lambda x_i}{(1+\alpha)(\beta x_i)^{\lambda+1}}\right)}{1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}} + \frac{\lambda}{\beta} + \frac{1}{\beta\ln(\beta x_i)}\right]$$

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = \lambda n_f \left(\frac{2\alpha-\alpha^2}{(1+\alpha)^2}\right)\left[\frac{2(1-\alpha)}{\alpha(2-\alpha)} - \frac{2}{1+\alpha}\right] + \sum_{i=1}^{n} \delta_{i,1,c} \frac{\left[\frac{\alpha}{1+\alpha}\left(\frac{1}{\alpha}-\frac{1}{1+\alpha}\right)e^{-\alpha/t_i^{\lambda}}\right]}{\left[1-\left(1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}\right)e^{-\alpha/t_i^{\lambda}}\right]} \times$$

$$\left[\left(\alpha^{-1}-(1+\alpha)^{-1}-\frac{\alpha^{-2}-(1+\alpha)^{-2}}{\alpha^{-1}-(1+\alpha)^{-1}} - t_i^{-\lambda}\right) - e^{-\alpha/t_i^{\lambda}} \frac{\left(1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}\right)-\left(\frac{1}{(1+\alpha)^2 t_i^{\lambda}}\right)}{1-\left(1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}\right)e^{-\alpha/t_i^{\lambda}}}\right]$$

$$+\sum_{i=1}^{n} \delta_{i,2,c} \frac{\left[\frac{\alpha}{1+\alpha}\left(\frac{1}{\alpha}-\frac{1}{1+\alpha}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\right]}{\left[1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\right]} \times$$

$$\left[\left(\alpha^{-1}-(1+\alpha)^{-1}-\frac{\alpha^{-2}-(1+\alpha)^{-2}}{\alpha^{-1}-(1+\alpha)^{-1}} - (\beta x_i)^{-\lambda}\right) - e^{-\alpha/(\beta x_i)^{\lambda}} \frac{\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)-\left(\frac{1}{(1+\alpha)^2 (\beta x_i)^{\lambda}}\right)}{1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}}\right]$$

$$\frac{\partial l^2 nL}{\partial \lambda^2} = \sum_{i=1}^{n} \delta_{i,1,f} \left(-t_i^{2\lambda} \ln(t_i)(1+t_i^{\lambda})^{-2} + t_i^{\lambda} \ln(t_i)(1+t_i^{\lambda})^{-1} \right) - \sum_{i=1}^{n} \delta_{i,1,f} \left(\frac{\alpha}{t_i^{\lambda}} \ln(t_i) \right) \ln(t_i)$$

$$+\sum_{i=1}^{n} \delta_{i,2,f} \left[\left\{ -(\beta x_i)^{2\lambda}) \ln(\beta x_i) \left(1+(\beta x_i)^{\lambda}\right)^{-2} + (\beta x_i)^{\lambda} \ln(\beta x_i) \left(1+(\beta x_i)^{\lambda}\right)^{-1} \right\} \right]$$

$$-\sum_{i=1}^{n} \delta_{i,2,f} \left(\frac{\alpha}{(\beta x_i)^{\lambda}} \ln(\beta x_i) \right) - \sum_{i=1}^{n} \delta_{i,1,c} \frac{\left(1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}\right) e^{-\alpha/t_i^{\lambda}} \left[\frac{\frac{\alpha}{(1+\alpha)} t_i^{\lambda} \ln(t_i)}{1+\frac{\alpha}{(1+\alpha)} t_i^{\lambda}} + t_i^{\lambda} \ln(t_i) \right]}{\left[1-\left(1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}\right) e^{-\alpha/t_i^{\lambda}}\right]} \times$$

$$\left[-\frac{\frac{\alpha}{(1+\alpha)t_i^{\lambda}} \ln(t_i)}{1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}} - \alpha t_i^{-\lambda} \ln(t_i) + \frac{\left(1+\frac{\alpha}{(1+\alpha)} t_i^{\lambda}\right)\left(\frac{\alpha}{(1+\alpha)} t_i^{\lambda} \ln^2(t_i)\right) + \left(\frac{\alpha}{(1+\alpha)} t_i^{\lambda} \ln(t_i)\right)\left(\frac{\alpha}{(1+\alpha)} \ln(t_i) t_i^{\lambda}\right)}{\left(\frac{\frac{\alpha}{(1+\alpha)} t_i^{\lambda} \ln(t_i)}{1+\frac{\alpha}{(1+\alpha)} t_i^{\lambda}} + t_i^{\lambda} \ln(t_i)\right)\left(1+\frac{\alpha}{(1+\alpha)} t_i^{\lambda}\right)^2} \right]$$

$$-\sum_{i=1}^{n} \delta_{i,2,c} \frac{\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right) e^{-\alpha/(\beta x_i)^{\lambda}} \left[\frac{\frac{\alpha}{(1+\alpha)} (\beta x_i)^{\lambda} \ln(\beta x_i)}{1+\frac{\alpha}{(1+\alpha)} (\beta x_i)^{\lambda}} + (\beta x_i)^{\lambda} \ln(\beta x_i) \right]}{\left[1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right) e^{-\alpha/(\beta x_i)^{\lambda}}\right]}$$

$$\left[-\frac{\frac{\alpha \ln(\beta x_i)}{(1+\alpha)(\beta x_i)^{\lambda}}}{1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}} - \alpha(\beta x_i)^{-\lambda} \ln(\beta x_i)^{\lambda} + \frac{\left(1+\frac{\alpha}{(1+\alpha)} (\beta x_i)^{\lambda}\right)\left(\frac{\alpha}{(1+\alpha)} (\beta x_i)^{\lambda} \ln^2(\beta x_i)\right) + \left(\frac{\alpha}{(1+\alpha)} (\beta x_i)^{\lambda} \ln(\beta x_i)\right)\left(\frac{\alpha}{(1+\alpha)} \ln(\beta x_i)(\beta x_i)^{\lambda}\right)}{\left(\frac{\frac{\alpha}{(1+\alpha)} (\beta x_i)^{\lambda} \ln(\beta x_i)}{1+\frac{\alpha}{(1+\alpha)} (\beta x_i)^{\lambda}} + (\beta x_i)^{\lambda} \ln(\beta x_i)\right)\left(1+\frac{\alpha}{(1+\alpha)} (\beta x_i)^{\lambda}\right)^2} \right]$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -n_{2f}\beta^{-2} + \sum_{i=1}^{n}\delta_{i,2,f}\left[\frac{\lambda x_i^{\lambda}\beta^{\lambda-1}}{1+(\beta x_i)^{\lambda}}\left(\frac{\lambda-1}{\beta}-\frac{\lambda x_i(\beta x_i)^{\lambda-1}}{1+(\beta x_i)^{\lambda}}\right)\right] + \sum_{i=1}^{n}\delta_{i,2,f}\left(\frac{(\lambda+1)\alpha\lambda x_i^2}{(\beta x_i)^{\lambda+2}}\right)$$

$$-\sum_{i=1}^{n}\delta_{i,2,c}\frac{\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\left[\frac{\frac{\alpha\lambda x_i}{(1+\alpha)(\beta x_i)^{\lambda+1}}}{1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}}-\alpha x_i(\beta x_i)^{-\lambda-1}\right]}{\left[1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\right]}\times(\beta x_i)^{-\lambda-1}$$

$$\left[\begin{array}{c}\frac{\alpha\lambda}{(1+\alpha)\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)}+\alpha\lambda-\frac{\alpha\beta\lambda(1+\alpha)^{-1}e^{-\alpha/(\beta x_i)^{\lambda}}-e^{-\alpha/(\beta x_i)^{\lambda}}\alpha\lambda x_i\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)}{1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}} \\ \frac{\alpha\lambda x_i^{-1}(1+\alpha)^{-1}\left\{\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda+1}}\right)-(\lambda+1)(\beta x_i)^{-1}\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)\right\}-(\beta x_i)^{-1}\alpha x_i(\lambda+1)}{\frac{\frac{\alpha\lambda x_i}{(1+\alpha)(\beta x_i)^{\lambda+1}}}{1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}}-\alpha x_i(\beta x_i)^{-\lambda-1}}\end{array}\right],$$

$$\frac{\partial^2 \ln L}{\partial\alpha\partial\lambda} = n_f\left(\frac{2\alpha-\alpha^2}{(1+\alpha)^2}\right)+\sum_{i=1}^{n}\delta_{i,1,f}\frac{\ln(t_i)}{t_i^{\lambda}}+\sum_{i=1}^{n}\delta_{i,1,c}\frac{\left[\frac{\alpha}{1+\alpha}\left(\frac{1}{\alpha}-\frac{1}{1+\alpha}\right)e^{-\alpha/t_i^{\lambda}}\right]}{\left[1-\left(1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}\right)e^{-\alpha/t_i^{\lambda}}\right]}\times$$

$$\left[\alpha\lambda t_i^{-\lambda}\ln(t_i)+e^{-\alpha/t_i^{\lambda}}\frac{\left(1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}\right)\ln(t_i)\alpha t_i^{-\lambda}+\left(1-\frac{\alpha\ln(t_i)}{(1+\alpha)t_i^{\lambda}}\right)}{1-\left(1+\frac{\alpha}{(1+\alpha)t_i^{\lambda}}\right)e^{-\alpha/t_i^{\lambda}}}\right]$$

$$+\sum_{i=1}^{n}\delta_{i,2,f}\frac{\ln(\beta x_i)}{(\beta x_i)^{\lambda}}+\sum_{i=1}^{n}\delta_{i,2,c}\frac{\left[\frac{\alpha}{1+\alpha}\left(\frac{1}{\alpha}-\frac{1}{1+\alpha}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\right]}{\left[1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}\right]}\times$$

$$\left[\alpha\lambda(\beta x_i)^{-\lambda}\ln(\beta x_i)+e^{-\alpha/(\beta x_i)^{\lambda}}\frac{\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)\ln(\beta x_i)\alpha(\beta x_i)^{-\lambda}+\left(1-\frac{\alpha\ln(\beta x_i)}{(1+\alpha)(\beta x_i)^{\lambda}}\right)}{1-\left(1+\frac{\alpha}{(1+\alpha)(\beta x_i)^{\lambda}}\right)e^{-\alpha/(\beta x_i)^{\lambda}}}\right]$$

The asymptotic variance-covariance of matrix I is given in the following Equation (15).

$$\Sigma = I^{-1} = \begin{bmatrix} -\frac{\partial^2 \ln L}{\partial \alpha^2} & -\frac{\partial^2 \ln L}{\partial \alpha \partial \lambda} & -\frac{\partial^2 \ln L}{\partial \alpha \partial \beta} \\ -\frac{\partial^2 \ln L}{\partial \lambda \partial \alpha} & -\frac{\partial^2 \ln L}{\partial \lambda^2} & -\frac{\partial^2 \ln L}{\partial \lambda \partial \beta} \\ -\frac{\partial^2 \ln L}{\partial \beta \partial \alpha} & -\frac{\partial^2 \ln L}{\partial \beta \partial \lambda} & -\frac{\partial^2 \ln L}{\partial \beta^2} \end{bmatrix}^{-1} = \begin{bmatrix} AVar(\hat{\alpha}) & ACov(\hat{\alpha}\hat{\lambda}) & ACov(\hat{\alpha}\hat{\beta}) \\ ACov(\hat{\lambda}\hat{\alpha}) & AVar(\hat{\lambda}) & ACov(\hat{\lambda}\hat{\beta}) \\ ACov(\hat{\beta}\hat{\alpha}) & ACov(\hat{\beta}\hat{\lambda}) & AVar(\hat{\beta}) \end{bmatrix} \quad (15)$$

where $AVar$ and $ACov$ stand for asymptotic variance, asymptotic covariance, respectively. The two-sided confidence limits can be billed as in Equation (16)

$$p\left[-z \le \frac{\hat{\phi}-\phi}{\sigma(\hat{\phi})} \le z\right] = 1-\kappa. \quad (16)$$

This construction of two-sided confidence limits is for the maximum likelihood estimate $\hat{\phi}$ of a population parameter $\psi = (\alpha, \lambda, \beta)$. In the above (16), z stands for $100(1-\kappa/2)$ the standard normal percentile and κ stands for the significance point. So, for a population parameter ϕ, an appropriate confidence limits can be achieved, such that

$$p\left[\hat{\phi} - z\sigma(\hat{\phi}) \le \phi \le \hat{\phi} + z\sigma(\hat{\phi})\right] = 1-\kappa,$$

where lower confidence limit $L_{\phi} = \hat{\phi} - z\sigma(\hat{\phi})$ and upper confidence limit $U_{\phi} = \hat{\phi} + z\sigma(\hat{\phi})$.

4. Simulation Study

In this section, we conduct a simulation study to ensure that the estimators with GIL distribution are presented correctly under multiple censoring schemes. The study is completed by Monte Carlo Simulation procedure using R-Software. To ensure estimator performance, means square errors (MSEs) and biases are estimated. This task necessitates the following steps.

(i) The total sample n is divided into two parts, n_1 and n_2. $n_1 = n\pi$ and $n_2 = n(1-\pi)$.

(ii) Generate $t_{1,1} < t_{2,2} < ... < t_{n_1,1}$ and $t_{2,1} < t_{2,2} < ... < t_{n_2,2}$ random samples of size n_1 and n_2 in regular and accelerated circumstances, respectively from power Lindley distribution.

(iii) 1000 random of size 30, 60, 90 and 120 are generated and also specified the values of the parameters as

Case (I) $(\alpha = 0.75, \lambda = 0.75, \beta = 1.9)$, Case (II) $(\alpha = 0.75, \lambda = 0.75, \beta = 2.2)$

Case (III) $(\alpha = 0.56, \lambda = 0.95, \beta = 1.9)$, Case (IV) $(\alpha = 0.56, \lambda = 0.75, \beta = 2.2)$.

(iv) The acceleration factor and distribution parameters are calculated for each sample and set of parameters. For each parameter combination, the asymptotic variance and covariance matrix are also obtained.

(v) Finally, for confidence levels $\gamma = 95\%, 99\%$ of acceleration factor, the two sides confidence limits and two parameters are constructed by (Equation (16)) for parameters α, λ and β.

Table 2 The input information of bias and MSE for multiple censored data with different sample sizes

n	Parameters	Case I ($\alpha = 0.75, \lambda = 0.75, \beta = 1.9$)			Case II ($\alpha = 0.75, \lambda = 0.75, \beta = 2.2$)		
		Estimates	Bias	MSE	Estimates	Bias	MSE
30	α	0.863	0.723	0.630	1.532	0.676	0.502
	λ	1.154	0.270	0.221	1.317	0.223	0.198
	β	1.006	0.487	0.392	0.651	0.432	0.328
60	α	0.836	0.610	0.592	1.443	0.539	0.371
	λ	1.154	0.178	0.119	1.832	0.176	0.126
	β	1.625	0.376	0.246	1.254	0.370	0.324
90	α	1.160	0.497	0.376	0.765	0.487	0.287
	λ	0.901	0.115	0.104	0.921	0.184	0.095
	β	1.304	0.221	0.134	1.432	0.287	0.226
120	α	0.602	0.388	0.271	1.754	0.341	0.307
	λ	0.792	0.085	0.021	1.033	0.154	0.088
	β	1.297	0.143	0.098	1.914	0.165	0.158

Table 3 The input information of bias and MSE under the different size of samples for multiple censored data

n	Parameters	Case III ($\alpha = 0.56, \lambda = 0.95, \beta = 1.9$)			Case IV ($\alpha = 0.56, \lambda = 0.75, \beta = 2.2$)		
		Estimates	Bias	MSE	Estimates	Bias	MSE
30	α	1.876	0.478	0.290	2.112	0.548	0.478
	λ	1.254	0.629	0.498	2.023	0.336	0.267
	β	0.978	0.566	0.419	1.976	0.389	0.276
60	α	1.776	0.365	0.248	0.945	0.566	0.465
	λ	1.218	0.530	0.446	1.246	0.289	0.265
	β	1.890	0.486	0.398	1.849	0.281	0.198
90	α	1.486	0.286	0.196	1.196	0.500	0.454
	λ	1.802	0.402	0.335	1.097	0.238	0.158
	β	1.987	0.365	0.289	1.046	0.176	0.111
120	α	1.598	0.254	0.166	0.966	0.456	0.366
	λ	1.843	0.298	0.191	2.086	0.218	0.138
	β	2.543	0.276	0.236	1.725	0.137	0.096

Table 4 The input information of asymptotic variance and covariance matrix of estimators for different size of samples under multiple censored data

n	Parameters	Case I ($\alpha=0.75,\lambda=0.75,\beta=1.9$)			Case II ($\alpha=0.75,\lambda=0.75,\beta=2.2$)		
		α	λ	β	α	λ	β
30	α	0.00684	0.00887	0.00456	0.00776	0.00211	0.00509
	λ	0.00498	–0.00884	0.00387	0.00224	0.00449	0.00277
	β	0.00543	0.00498	0.04541	0.00443	0.00109	0.00118
60	α	0.00576	0.00276	0.00432	0.00654	0.00210	0.00498
	λ	0.00227	–0.00876	0.00267	0.00221	0.00265	0.00176
	β	0.00338	0.00234	0.00453	0.00343	0.00025	0.00101
90	α	0.00465	0.00199	0.00365	0.00554	0.00189	0.00334
	λ	0.00176	–0.00998	0.00223	0.00176	0.00228	0.00116
	β	0.00225	0.00178	0.00116	0.00225	–0.00987	–0.00554
120	α	0.00356	0.00113	0.00294	0.00445	0.00156	0.00223
	λ	0.00114	–0.00887	0.00132	0.00114	0.00189	0.00115
	β	0.00115	0.00117	0.00101	0.00112	–0.00998	–0.00776

Table 5 The input information of asymptotic variance and covariance matrix of estimators for different size of samples under multiple censored data

n	Parameters	Case III ($\alpha=0.56,\lambda=0.95,\beta=1.9$)			Case IV ($\alpha=0.56,\lambda=0.75,\beta=2.2$)		
		α	λ	β	α	λ	β
30	α	0.00332	0.00098	0.00543	0.00376	0.00076	0.00432
	λ	0.00254	0.00221	0.00065	0.00577	0.00981	0.00087
	β	0.00443	0.00545	0.00334	–0.00654	0.00443	–0.00043
60	α	0.00224	0.00065	0.00332	0.00331	0.00054	0.00224
	λ	0.00223	0.00188	0.00045	0.00443	0.00076	0.00066
	β	0.00376	0.00332	0.00224	–0.00765	0.00411	–0.00066
90	α	0.00202	0.00043	0.00223	0.00269	0.00044	0.00187
	λ	0.00123	0.00117	0.00032	0.00332	0.00387	0.00054
	β	0.00321	0.00212	–0.00987	–0.00799	0.00332	–0.00098
120	α	0.00187	0.00011	0.00165	0.00211	0.00012	0.00112
	λ	0.00115	0.00076	0.00011	0.00287	0.00225	0.00043
	β	0.00234	0.00133	–0.00999	–0.00998	0.00225	–0.00076

Table 6 At confidence level $\kappa = 95\%, 99\%,$ the confidence bounds of estimates for different size of samples

n	Parameters	Case I ($\alpha = 0.75, \lambda = 0.75, \beta = 1.9$)					Case II ($\alpha = 0.56, \lambda = 0.75, \beta = 2.2$)				
		95% Confidence Interval		99% Confidence Interval		σ	95% Confidence Interval		99% Confidence Interval		σ
		Lower Bound	Upper Bound	Lower Bound	Upper Bound		Lower Bound	Upper Bound	Lower Bound	Upper Bound	
30	α	0.57	0.73	0.53	0.89	0.08	0.51	0.78	0.61	0.93	0.07
	λ	0.68	0.89	0.57	0.76	0.04	0.55	0.86	0.67	0.83	0.10
	β	0.88	1.32	0.66	0.91	0.38	0.79	1.89	0.87	1.90	0.32
60	α	0.59	0.67	0.55	0.84	0.09	0.57	0.84	0.73	0.99	0.09
	λ	0.61	0.75	0.66	0.80	0.06	0.61	0.82	0.62	0.79	0.06
	β	0.77	1.34	0.73	0.88	0.43	0.98	1.56	0.97	2.11	0.35
90	α	0.64	0.71	0.67	0.81	0.06	0.44	0.60	0.65	0.76	0.05
	λ	0.64	0.76	0.58	0.72	0.09	0.87	0.93	0.56	0.69	0.08
	β	0.79	1.22	0.69	0.81	0.48	0.78	1.23	0.67	1.36	0.42
120	α	0.59	0.67	0.71	0.79	0.08	0.56	0.65	0.54	0.67	0.06
	λ	0.55	0.63	0.69	0.82	0.03	0.74	0.82	0.65	0.76	0.09
	β	0.88	1.01	0.61	0.73	0.35	0.77	1.02	0.68	1.11	0.36

Table 7 At confidence level $\kappa = 95\%, 99\%,$ the confidence bounds of estimates for different size of samples

n	Parameters	Case III ($\alpha = 0.56, \lambda = 0.95, \beta = 1.9$)					Case IV ($\theta = 0.5, \lambda = 0.8, \beta = 1.9$)				
		95% Confidence Interval		99% Confidence Interval		σ	95% Confidence Interval		99% Confidence Interval		σ
		Lower Bound	Upper Bound	Lower Bound	Upper Bound		Lower Bound	Upper Bound	Lower Bound	Upper Bound	
30	α	0.67	0.89	0.64	1.07	0.09	0.62	0.81	0.45	0.79	0.38
	λ	0.80	0.96	0.70	0.90	0.43	0.59	0.73	0.72	0.99	0.16
	β	0.81	1.32	0.69	0.88	0.64	0.79	1.36	0.81	1.40	0.33
60	α	0.55	0.69	0.61	0.79	0.08	0.68	0.94	0.64	0.79	0.08
	λ	0.56	0.71	0.77	0.89	0.07	0.68	0.87	0.62	0.79	0.10
	β	0.66	1.23	0.63	0.78	0.37	0.78	1.36	0.97	2.11	0.42
90	α	0.61	0.72	0.63	0.71	0.05	0.55	0.68	0.69	0.81	0.03
	λ	0.54	0.65	0.61	0.69	0.08	0.78	0.85	0.56	0.69	0.13
	β	0.68	1.10	0.79	0.85	0.42	0.67	1.11	0.67	1.36	0.47
120	α	0.55	0.62	0.65	0.72	0.02	0.64	0.71	0.65	0.69	0.07
	λ	0.59	0.66	0.65	0.76	0.05	0.64	0.69	0.68	0.79	0.02
	β	0.72	0.81	0.69	0.76	0.37	0.86	1.02	0.79	1.19	0.39

5. Conclusions

This paper's study presents a CSPALT inference for the GIL distribution with multiple censoring schemes. Based on the simulation study, the following declaration has been made: In the Tables 2 and 3, the MSE and bias of estimators are obtained in four cases. We can observe that the sample size increases as the biases and MSEs decrease. The maximum likelihood estimates have good statistical properties for set of parameters $(\alpha = 0.75, \lambda = 0.75, \beta = 1.9)$ because this set have the smallest biases all sample sizes in comparison of all sets of parameters.In the Tables 4and 5, the asymptotic variance and covariance matrix are obtained. We can observe that the asymptotic variance-covariance of estimators decreases as sample size increases for the all sets of parameters. In the Tables 6 and 7, the confidence limits of the intervals for the parameters and the acceleration factor at 95% and 99% are obtained. The standard deviation (σ) of estimators is also obtained. We can observe that the width of the interval decreases as sample size increases for all sets of parameters.

6. Industrial Applications

The industrial applications of this work are as follows: the product has a longer life and is more reliable as a result of the rapid development of high technology in the industrial area. It may take several years for an object to fail, making it difficult, if not impossible, to obtain the stoppage information in a traditional situation for such high-reliability items. CSPALT is used in industrialized industries to assess or demonstrate the trustworthiness of parts and subsystems, to verify parts, to spot failure approaches so they can be accurate, to evaluate different manufacturers, and so on. In general, every manufacturer wants to test and improve the reliability of their products before releasing them to the market. As a result, the proposed work becomes extremely useful in this situation.

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