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# Estimation of Maintenance Costs Under Partially Accelerated Life Tests with Multiply Censoring for Reduced Kies Distribution

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**Abstract**

In this paper, we explain how to analyze and design the accelerated life test plans to improve the quality and reliability of the product. This study's key feature is to estimate the costs of maintenance service policy because it plays a significant part in assisting any manufacturing organization for sale and available its equipment and maintenance cost-effective. The constant-stress partially accelerated life test is assumed when the lifetime of test units follows two-parameter Reduced Kies (RK) distribution under a multiply censoring scheme. The maximum likelihood estimates, Fisher Information matrix, and the asymptotic variance and covariance matrix are obtained. The confidence intervals of the estimators are also obtained. In addition, a simulation study is conducted to ensure the accuracy of the results.

**Keywords**: Life testing, partially accelerated life test, expected cost rate, total cost, minimal repair time, maintenance service policy, simulation.

## 1. Introduction

At the present time, in the market, industrialized designs are upgrading day by day because of the changing tools and equipment. When high-reliability manufactured goods are tested, it is too complex to get information regarding the life span of items under ordinary usage circumstances at the time of testing. Some frequently used life experiments give no or very tiny quantity of failures by the ending of the assessment. In such a circumstance, a result is the accelerated life test (ALT), in which the manufactured goods or material is experienced under higher than usual used circumstances to attain the information rapidly on the life distribution or product performance. These conditions are referred to as stresses, and stresses are temperature, voltage, force, etc.

Generally, ALT methods occur in three forms in reliability theory, and the methods are constant stress ALT, step-stress ALT, and Progressive stress ALT. Here we are focusing only on constant stress. In constant stress ALT, the experimenter may have preset stress stages applied for different tested items. So, each component is subjected to only one stress stage until the component is unsuccessful or the investigation is closed for several other causes.

In ALT, the statistical representation involving the life span of a component and stress is well-known or can be assumed. Conversely, in various situations, these relationships are not known and cannot be assumed, i.e. the information achieved from ALT cannot be extrapolated to use circumstance. So, a partially accelerated life test (PALT) can be applied in such situations in which the assessment objects are run at both normal and higher than normal stress environments. Constant stress partially accelerated life test (CSPALT) and step stress partially accelerated life test (SSPALT) are two important and well-known ways in PALT. In CSPALT, the components are experienced at also normal or advanced than normal circumstances until the examination is done. Another approach to step up failures is the SSPALT which increases the load function to the components in a specific discrete progression. An investigation item is first to go at the use circumstance. If it does not be unsuccessful for a particular moment, then it is run at accelerated circumstances until a prespecified amount of failures have occurred or a prespecified moment has achieved.

When life information is examined, an experiment can be out of control for many reasons, like components of a system may break accidentally, and all the units in the trial may not fail. Such information is called censored or incomplete data. Due to several forms of censoring, censored data can be divided into Type I censored (or time censored) data and Type II censored (or failure-censored) data. Type I censoring takes place if a test has a set number of subjects or items and stops the test at a predetermined time, at which point any subjects remaining are right-censored. Type II censoring takes place if a test has a set number of subjects or items and stops the test when a predetermined number are observed to have failed; the remaining subjects are then right-censored. Removing units from the test at points is unacceptable in these two censoring schemes while the removal is acceptable only on termination. Although, the removal of items or components from the test during testing is possible in the progressive type censoring scheme. In such situations, the multiple censoring schemes are the best choice for an engineer or reliability practitioner because multiple censoring schemes allow removing items from the test during the testing at any situation or any time. Tobias and Trindada (1995) showed that the Type-I and Type-II censoring schemes are a particular case of multiple censoring schemes. The literature related to our study is presented in the following Table 1.

**Table 1** Review of the literature related to the proposed work

| Author(s) Name | Method | Scheme | Failure Model | Strategy |
|---|---|---|---|---|
| Elsayed and Zhang (2007) | multiple-stress-type ALT | - | - | - |
| Xin et al. (2020) | CSALT | - | Doubly truncated three-parameter BurrXII distribution | - |
| Abushal and Al-Zaydi (2017) | CSPALT | Progressive censoring | Mixture of Pareto distributions | - |
| Zhang and Fang (2017) | CSPALT | Type-I censoring | Exponential distribution | - |
| Abdel-Ghaly et al. (2016) | CSALT | - | Family of Exponential distributions | - |
| Shi and Shi (2016) | CSPALT | Progressive Type-II | Complementary Exponential distribution | - |
| Ismail (2016, 2014) | CSPALT, SSPALT | Hybrid censoring, Type-I progressively hybrid censoring | Weibull distribution | - |
| Nassar and Elharoun (2019) | CSPALT | Multiply censoring | Exponentiated Weibull distribution | - |
| Hassan et al. (2015) | CSPALT | Multiply censoring | Inverted Weibull distribution | - |

**Table 1** (Continued)

| Author(s) Name | Method | Scheme | Failure Model | Strategy |
|---|---|---|---|---|
| Cheng and Wang (2012) | CSPALT | Multiply censoring | Burr XII distribution | - |
| Alam et al. (2019) | ALT | Type-II Censoring Schemes | Burr Type-X | - |
| Kumar et al. (2021) | Progressive-stress ALT | Progressively type-II | Burr X distribution | - |
| El-Sagheer and Mohamed (2018) | CSALT | Progressively type-II | Modified Weibull distribution | - |
| Abushal and Soliman (2015) | CSPALT | Progressive censoring | Pareto distribution | - |
| Abdel-Hamid and Al-Hussaini (2008) | SPALT | Type-I censoring | Finite mixture of distributions | - |
| Ling et al. (2011) | SSALT | Type-I hybrid censoring scheme | Exponential distribution | - |
| Chang, and Tse (2013) | ALT | Progressive Type-II interval censoring | Weibull distribution | - |
| Balakrishnan and Ling (2014) | CSALT for one-shot device testing | - | Weibull distribution | - |
| Alam et al. (2020) | SSPALT | Progressive Censoring Schemes | Power function distribution | Maintenance service policy |
| El-Dessouky (2017) | SSPALT | Type-II | Extension of Exponential distribution | Maintenance service policy |
| Alam et al. (2021) | CSPALT | Progressive Censoring Schemes | Generalized inverted Exponential distribution | Maintenance service policy |
| Proposed Work | CSPALT | Multiply Censoring | Reduced Kies distribution | Maintenance service policy |

The authors presented their studies on ALT, CSALT, CSPALT, SSPALT, and progressive-stress ALT with different censoring schemes under different failure models in the above literature but no one presented a study on CSPALT for two parameters RK distribution under Multiply censoring scheme. So, in this work, we design CSPALT under a Multiply censoring scheme while the lifetime of test items follows two parameters, RK distribution. Additionally, we present the application of the designed test in the field of maintenance service policy.

The main contribution of this paper is two-fold: first, we design the CSPALT under multiply censoring when the lifetime of test units follows the RK distribution. Second, we provide the application of the designed test in the field of maintenance service policy. Under this policy, we estimate the expected cost rate, total cost, minimal repair time, and confidence level.

The rest of the paper is prepared as follows. The model description, test procedure, and basic assumptions are given in Section 2. The point estimation, interval estimation, Fisher information matrix, and confidence intervals are developed in Section 3. The estimating costs of maintenance service policy under two parameters, RK distribution, are presented in Section 4. A simulation study is proposed in Section 5. The industrial use of the proposed work is presented in Section 6. Finally, the conclusions are made in Section 7.

## 2. Model Description and Test Procedure

### 2.1. Model description

Kumar and Dharmaja (2013) have presented a new one-parameter distribution that becomes a better fit for representing information and a generalization of the Kies failure model called one parameter RK failure model. The two-parameter RK distribution is a suitable failure model with left-skewed, right-skewed, symmetrical, and reversed-J Shaped densities. The hazard of this distribution is increasing, decreasing, bathtub, upside-down bathtub, and reversed-J Shaped. Here, we choose this distribution because it has many exciting applications in reliability, survival analysis, design of accelerated life test plans, engineering, hydrology, medication, psychology, and pharmacy, etc. (See Murthy et al. (2004), Rinne (2009) and references therein for brief and detailed literature).

Let $X_1,...,X_n$ be a random variable (r.v.) that appear from a two-parameter RK distribution, then its probability density function (pdf) and cumulative distribution function (cdf) take the following forms. The pdf of the distribution is given as

$$f(x,\theta,\alpha)=\frac{\theta(x-\alpha)^{\theta-1}}{\left(1-(x-\alpha)\right)^{\theta+1}}e^{-\left(\frac{x-\alpha}{1-(x-\alpha)}\right)^{\theta}},x>\alpha>0,\theta>0, \tag{1}$$

where $\theta$ and $\alpha$ are shape and location parameters, respectively. The curve of the above pdf is shown in the following Figure 1.

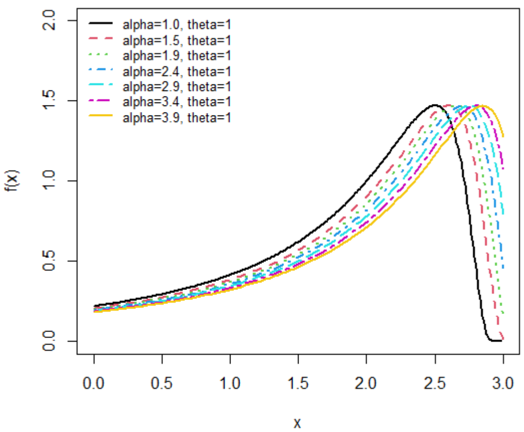


**Figure 1** pdf curve of RK distribution

The cdf of the distribution is given as

$$F(x,\theta,\alpha)=1-e^{-\left(\frac{x-\alpha}{1-(x-\alpha)}\right)^{\theta}},x>\alpha>0,\theta>0. \tag{2}$$

The curve of the above cdf is shown in the following Figure 2.

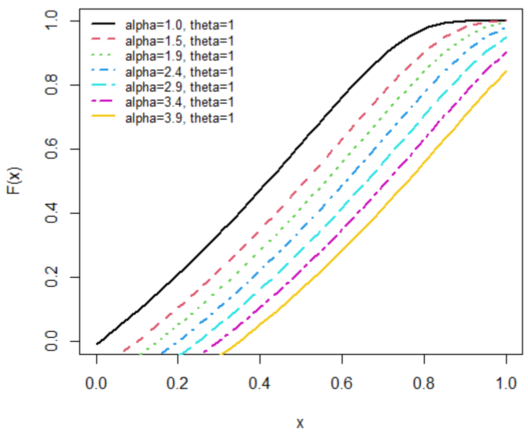


**Figure 2** Cdf curve of RK distribution

The survival function is given as

$$R(x,\theta,\alpha)=e^{-\left(\frac{x-\alpha}{1-(x-\alpha)}\right)^{\theta}}.$$

The curve of the above survival function is shown in Figure 3.

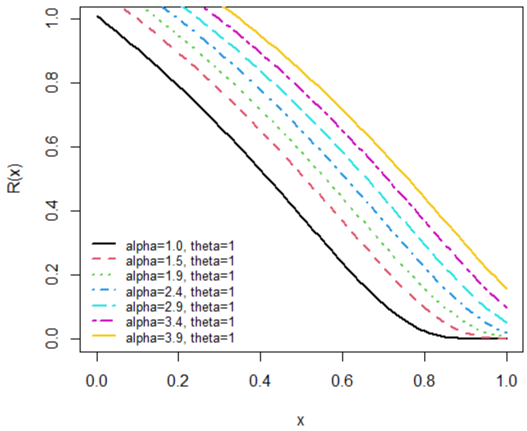


**Figure 3** Survival function curve of RK distribution

The hazard function is given as

$$h(x,\theta,\alpha)=\frac{\theta(x-\alpha)^{\theta-1}}{\left(1-(x-\alpha)\right)^{\theta+1}}.$$

The curve of the above hazard function is shown in the following Figure 4.

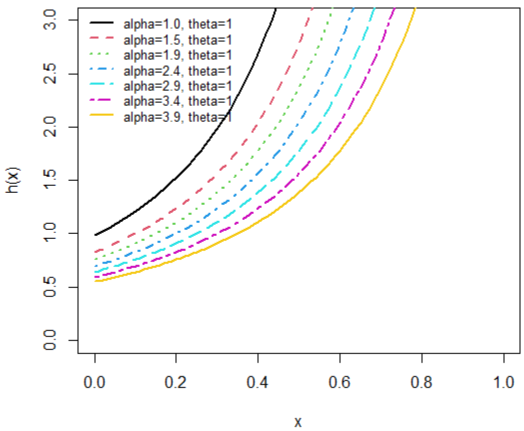


**Figure 4** Hazard function curve of RK distribution

Some interesting properties of RK distribution are proposed by Kumar and Dharmaja (2013). They also presented the estimation of RK distribution parameters by the ML estimation process. Dey et al. (2019) tackled the estimation of the RK parameter based on progressive Type-II censoring.

The pdf of the RK distribution is uni-modal, positively skewed, and approximately symmetric, and the hazard rate function is very bendable. It can have increasing, decreasing, and upside-down bathtub or bathtub failure rates.

**2.2. Test procedure**

The examination procedure of CSPALT based on multiply censoring assuming the lifetime item has two parameters RK distribution is described as follows. The pdf under normal conditions takes the following form

$$f_1(t_i)=\frac{\theta(t_i-\alpha)^{\theta-1}}{\left(1-(t_i-\alpha)\right)^{\theta+1}}e^{-\left(\frac{t_i-\alpha}{1-(t_i-\alpha)}\right)^{\theta}},t_i>\alpha>0,\theta>0,i=1,2,...,m_1. \quad (3)$$

The cdf under normal condition is given as follows

$$F_1(t_i)=1-e^{-\left(\frac{t_i-\alpha}{1-(t_i-\alpha)}\right)^{\theta}}, \quad (4)$$

where $t_i$ is the experimental lifetime of an item $i$, that is tested at the ordinary circumstance. The pdf and cdf of the lifetime $Y=\beta^{-1}T,$ under accelerated condition, obtain the following form given in (5) and (6)

$$f_2(y_j,\theta,\beta)=\frac{\beta\theta(\beta y_j-\alpha)^{\theta-1}}{\left(1-(\beta y_j-\alpha)\right)^{\theta+1}}e^{-\left(\frac{\beta y_j-\alpha}{1-(\beta y_j-\alpha)}\right)^{\theta}},\ y_j>\alpha>0,\theta>0,\beta>0,\ j=1,2,...,m_2, \quad (5)$$

$$F_2(y_j,\theta,\beta)=1-e^{-\left(\frac{\beta y_j-\alpha}{1-(\beta y_j-\alpha)}\right)^{\theta}}, \quad (6)$$

where $y_j$ is the observed lifetime of an item $j$, that is tested at the accelerated condition.

**2.3. Basic assumptions**

The necessary assumptions for CSPALT are given as:

(i) The lifetimes of items $T_i,\ i=1,2,...,m_1$ are independent and identically distributed (i.i.d.) random variable with pdf provided in (3), which is allocated to normal condition.

(ii) The lifetimes of items $Y_j,\ j=1,2,...,m_2$ are also independent and identically distributed (i.i.d.) random variable with pdf given in (5), which is allocated to accelerated condition.

(iii) $T_i$ and $Y_j$ are mutually independent also.

(iv) $m_1$ and $m_2$ are the total number of objects at ordinary and accelerated circumstances, respectively and $m=m_1+m_2$ is the total number of items.

## 3. Estimation of Parameters

**3.1. Point estimation**

In this sub-section, we apply the maximum likelihood (ML) procedure for estimating parameters. ML method is the most significant technique for fitting the statistical model; it has many motivating and engaging properties like asymptotic unbiased, asymptotic efficiency and asymptotic normality, etc.

$t_{(1)}<t_{(2)}<...<t_{(m_1)}$ are supposed observed values of the total lifetime $T$ at the ordinary situation and $t_{(1)}<t_{(2)}<...<t_{(m_2)}$ are the supposed observed values of the lifetime $Y$ at the accelerated condition. Then the likelihood of RK distribution under proposed censoring is given as

$$L=\prod_{i=1}^{m}\left[f_1(t_i)\right]^{\delta_i,1,f}\left[1-F_i(t_i)\right]^{\delta_i,1,c}\times\left[f_2(y_i)\right]^{\delta_i,2,f}\left[1-F_2(y_i)\right]^{\delta_i,2,c}$$

$$= \prod_{i=1}^{m}\left[\frac{\theta(t_i-\alpha)^{\theta-1}}{\left(1-(t_i-\alpha)\right)^{\theta+1}}e^{-\left(\frac{t_i-\alpha}{1-(t_i-\alpha)}\right)^{\theta}}\right]^{\delta_i,1,f}\left[e^{-\left(\frac{\beta y_j-\alpha}{1-(\beta y_j-\alpha)}\right)^{\theta}}\right]^{\delta_i,1,c}\times$$
$$\left[\frac{\beta\theta(\beta y_i-\alpha)^{\theta-1}}{\left(1-(\beta y_i-\alpha)\right)^{\theta+1}}e^{-\left(\frac{\beta y_i-\alpha}{1-(\beta y_i-\alpha)}\right)^{\theta}}\right]^{\delta_i,2,f}\left[e^{-\left(\frac{\beta y_i-\alpha}{1-(\beta y_i-\alpha)}\right)^{\theta}}\right]^{\delta_i,2,c} \tag{6}$$

The indicator functions are denoted by $\delta_{i,1,f}$, $\delta_{i,1,c}$, $\delta_{i,2,f}$ and the values of functions are given as

$$\delta_{i,1,f},\delta_{i,2,f}=\begin{cases}1, & \text{the item failed at stress condition,}\\ 0, & \text{otherwise,}\end{cases}$$

$$\delta_{i,1,c},\delta_{i,2,c}=\begin{cases}1, & \text{the item censored at normal condition,}\\ 0, & \text{otherwise,}\end{cases}$$

$$\sum_{i=1}^{n}\delta_{i,1,f}=m_{1f}=\text{number of failed items at nomal condition,}$$

$$\sum_{i=1}^{n}\delta_{i,2,f}=m_{2f}=\text{number of failed items at accelerated condition,}$$

$$\sum_{i=1}^{n}\delta_{i,1,c}=m_{1c}=\text{number of censored item at normal condition,}$$

$$\sum_{i=1}^{n}\delta_{i,2,c}=m_{2c}=\text{number of censored item at accelerated condition.}$$

Then, the log-likelihood is given as

$$\ln L=\sum_{i=1}^{m}\delta_{i,1,f}\ln\left(\theta(t_i-\alpha)^{(\theta-1)}\right)-(\theta+1)\sum_{i=1}^{m}\delta_{i,1,f}\ln\left(1-(t_i-\alpha)\right)-\sum_{i=1}^{m}\delta_{i,1,c}{}^{\left(\frac{t_i-\alpha}{1-(t_i-\alpha)}\right)^{\theta}}$$
$$+\sum_{i=1}^{m}\delta_{i,2,f}\ln\left(\beta\theta(\beta y_i-\alpha)^{\theta-1}\right)-(\theta+1)\sum_{i=1}^{m}\delta_{i,2,f}\ln\left(1-(\beta y_i-\alpha)\right)-\sum_{i=1}^{m}\delta_{i,2,c}{}^{\left(\frac{\beta y_i-\alpha}{1-(\beta y_i-\alpha)}\right)^{\theta}}. \tag{7}$$

The ML estimates of $\theta,\alpha$ and $\beta$ are obtained by differentiating (7) concerning $\theta,\alpha$ and $\beta$ respectively and equating to zero,

$$\frac{\partial\ln L}{\partial\theta}=\sum_{i=1}^{m}\delta_{i,1,f}\left(\frac{\theta(\theta-1)(t_i-\alpha)^{-1}-1}{\theta}\right)+\sum_{i=1}^{m}\delta_{i,2,f}\left(\frac{\theta(\theta-1)(\beta y_i-\alpha)^{-1}-1}{\theta}\right)$$
$$-\left[\sum_{i=1}^{m}\delta_{i,1,f}\ln\left(1-(t_i-\alpha)\right)+\sum_{i=1}^{m}\delta_{i,2,f}\ln\left(1-(\beta y_i-\alpha)\right)\right] \tag{8}$$
$$-\left[\theta\sum_{i=1}^{m}\delta_{i,1,c}\left(\frac{t_i-\alpha}{1-(t_i-\alpha)}\right)^{\theta-1}+\theta\sum_{i=1}^{m}\delta_{i,2,c}\left(\frac{\beta y_i-\alpha}{1-(\beta y_i-\alpha)}\right)^{\theta-1}\right]=0,$$

$$\frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^{m} \delta_{i,1,f} \left( \frac{\theta(\theta-1)(t_i-\alpha)^{\theta-2}}{\theta(t_i-\alpha)^{\theta-1}} \right) + \sum_{i=1}^{m} \delta_{i,2,f} \left( \frac{\theta(\theta-1)(\beta y_i-\alpha)^{\theta-2}}{\theta(\beta y_i-\alpha)^{\theta-1}} \right)$$
$$-(\theta+1)\left[ \sum_{i=1}^{m} \delta_{i,1,f} \left(1-(t_i-\alpha)\right)^{-1} + \sum_{i=1}^{m} \delta_{i,2,f} \left(1-(\beta y_i-\alpha)\right)^{-1} \right] \tag{9}$$
$$+\theta \left[ \begin{array}{l} \sum_{i=1}^{m} \delta_{i,1,c} \left( \frac{t_i-\alpha}{1-(t_i-\alpha)} \right)^{\theta} \left( (t_i-\alpha)^{-1} + (1-(t_i-\alpha))^{-1} \right) + \\ \sum_{i=1}^{m} \delta_{i,2,c} \left( \frac{\beta y_i-\alpha}{1-(\beta y_i-\alpha)} \right)^{\theta} \left( (\beta y_i-\alpha)^{-1} + (1-(\beta y_i-\alpha))^{-1} \right) \end{array} \right] = 0,$$

$$\frac{\partial \ln L}{\partial \beta} = \sum_{i=1}^{m} \delta_{i,2,f} \left[ \frac{\theta(\theta-1)\beta y_i(\beta y_i-\alpha)^{\theta-2} - (\beta y_i-\alpha)^{\theta-1}}{\beta\theta(\beta y_i-\alpha)^{\theta-1}} \right] + (\theta+1)\sum_{i=1}^{m} \delta_{i,2,f}\, y_i \left(1-(\beta y_i-\alpha\right)^{-1}$$
$$-\left[ \theta \sum_{i=1}^{m} \delta_{i,2,c} \left( \frac{\beta y_i-\alpha}{1-(\beta y_i-\alpha)} \right)^{\theta} y_i \left( (\beta y_i-\alpha)^{-1} + (1-(\beta y_i-\alpha))^{-1} \right) \right] = 0. \tag{10}$$

This is a very tough or impossible task to solve the above three equations manually. So the Newton-Raphson technique can be used for solving these equations.

Another approach can be used to solve the above three equations to get ML estimates of the parameters, and it is preferable to solve the following optimization problem

$$\max \ln L$$
$$\text{s.t. } \alpha > 0,\ \theta > 0,\ \beta > 0.$$

The $\ln L$ is the log-likelihood function given in (7). The following optimization problem has linear inequality constraints and can be solved by both the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm and the Nelder-Mead algorithm.

### 3.2. Interval estimation

The Fisher information matrix under multiply censored data is

$$I = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}. \tag{11}$$

The elements of the matrix are

$$I_{11} = \frac{\partial^2 \ln L}{\partial \alpha^2} = -\sum_{i=1}^{m} \delta_{i,1,f} \frac{(\theta-1)}{(t_i-\alpha)} - \sum_{i=1}^{m} \delta_{i,2,f} \frac{(\theta-1)}{(\beta y_i-\alpha)} - (\theta+1) \left[ \begin{array}{l} \sum_{i=1}^{m} \delta_{i,1,f} \left(1-(t_i-\alpha)\right)^{-1} \\ +\sum_{i=1}^{m} \delta_{i,2,f} \left(1-(\beta y_i-\alpha)\right)^{-1} \end{array} \right]$$
$$-\theta \left[ \begin{array}{l} \sum_{i=1}^{m} \delta_{i,1,c} \left( \frac{t_i-\alpha}{1-(t_i-\alpha)} \right)^{\theta} \left( (1-(t_i-\alpha))^{-1} - (t_i-\alpha)^{-1} \right) + \\ \sum_{i=1}^{m} \delta_{i,2,c} \left( \frac{\beta y_i-\alpha}{1-(\beta y_i-\alpha)} \right)^{\theta} \left( (1-(\beta y_i-\alpha))^{-1} - (\beta y_i-\alpha)^{-1} \right) \end{array} \right],$$

$$I_{33}=\frac{\partial^2 \ln L}{\partial \beta^2}=\sum_{i=1}^{m}\delta_{i,2,f}(\beta y_i-\alpha)^{\theta-1}\frac{\left(\beta-(\theta-1)(\beta y_i-\alpha)^{-1}\right)}{\beta(\beta y_i-\alpha)^{\theta-1}}+(\theta+1)\left[\sum_{i=1}^{m}\delta_{i,2,f}y_i\left(1-(\beta y_i-\alpha)\right)^{-1}\right]$$

$$+\sum_{i=1}^{m}\delta_{i,2,c}\theta y_i(\beta y_i-\alpha)^{\theta}\left(1-(\beta y_i-\alpha)\right)^{-\theta}\left[(\beta y_i-\alpha)^{-1}+(1-(\beta y_i-\alpha))^{-1}\right],$$

$$I_{22}=\frac{\partial^2 \ln L}{\partial \theta^2}=\sum_{i=1}^{m}\delta_{i,1,f}\left(\frac{\theta(\theta-1)(t_i-\alpha)^{-1}-1}{\theta}\right)\times$$

$$\left[\begin{array}{l}\theta(\theta-1)(t_i-\alpha)^{\theta-2}\left(\dfrac{\theta^{-1}+(\theta-1)^{-1}+(\theta-2)(t_i-\alpha)^{\theta-1}-(\theta-1)(t_i-\alpha)^{\theta-2}}{\theta(\theta-1)(t_i-\alpha)^{\theta-2}}\right)-\\ \left(\dfrac{\theta(\theta-1)(t_i-\alpha)^{\theta-2}-(t_i-\alpha)^{\theta-1}}{\theta(t_i-\alpha)^{\theta-1}}\right)\end{array}\right]+$$

$$\sum_{i=1}^{m}\delta_{i,2,f}\left(\frac{\theta(\theta-1)(\beta y_i-\alpha)^{-1}-1}{\theta}\right)\times$$

$$\left[\begin{array}{l}\theta(\theta-1)(\beta y_i-\alpha)^{\theta-2}\left(\dfrac{\theta^{-1}+(\theta-1)^{-1}+(\theta-2)(\beta y_i-\alpha)^{\theta-1}-(\theta-1)(\beta y_i-\alpha)^{\theta-2}}{\theta(\theta-1)(\beta y_i-\alpha)^{\theta-2}}\right)\\ -\left(\dfrac{\theta(\theta-1)(\beta y_i-\alpha)^{\theta-2}-(\beta y_i-\alpha)^{\theta-1}}{\theta(\beta y_i-\alpha)^{\theta-1}}\right)\end{array}\right]$$

$$-\sum_{i=1}^{m}\delta_{i,1,c}\left[\theta(\theta-1)\left(\frac{t_i-\alpha}{1-(t_i-\alpha)}\right)^{\theta-2}+\left(\frac{t_i-\alpha}{1-(t_i-\alpha)}\right)^{\theta-1}\right]$$

$$+\sum_{i=1}^{m}\delta_{i,2,c}\left[\theta(\theta-1)\left(\frac{\beta y_i-\alpha}{1-(\beta y_i-\alpha)}\right)^{\theta-2}+\left(\frac{\beta y_i-\alpha}{1-(\beta y_i-\alpha)}\right)^{\theta-1}\right],$$

$$I_{23}=I_{32}=\frac{\partial^2 \ln L}{\partial \beta \partial \alpha}=\sum_{i=1}^{m}\delta_{i,2,f}\left[\frac{\theta(\theta-1)\beta y_i(\beta y_i-\alpha)^{-1}-1}{\beta\theta \backslash}\left(\frac{\beta\theta y_i(\beta y_i-\alpha)^{-2}}{\beta\theta(\theta-1)y_i(\beta y_i-\alpha)^{-1}-1}\right)\right]-$$

$$(\theta+1)\sum_{i=1}^{m}\delta_{i,2,f}y_i\left(1-(\beta y_i-\alpha\right)^{-2}$$

$$-\theta\sum_{i=1}^{m}\delta_{i,2,c}\left(\frac{\beta y_i-\alpha}{1-(\beta y_i-\alpha)}\right)^{\theta}y_i\left((\beta y_i-\alpha)^{-1}+(1-(\beta y_i-\alpha))^{-1}\right)\times$$

$$\left[-\theta\left((\beta y_i-\alpha)^{-1}+(1-(\beta y_i-\alpha))^{-1}\right)+\left(\frac{(\beta y_i-\alpha)^{-2}+(1-(\beta y_i-\alpha))^{-2}}{(\beta y_i-\alpha)^{-1}+(1-(\beta y_i-\alpha))^{-1}}\right)\right],$$

$$I_{13}=I_{31}=\frac{\partial^2 \ln L}{\partial \theta \partial \beta}$$

$$=\sum_{i=1}^{m}\delta_{i,2,f}\left(\frac{\theta(\theta-1)(\beta y_i-\alpha)^{-1}-1}{\theta}\right)\left[\beta^{-1}+\frac{\theta(\theta-1)y_i(\theta-2)(\beta y_i-\alpha)^{-2}-(\theta-1)(\beta y_i-\alpha)^{-1}y_i}{(\theta-1)(\beta y_i-\alpha)^{-1}-1}\right]$$

$$+\sum_{i=1}^{m}\delta_{i,2,f}y_i\left(1-(\beta y_i-\alpha)\right)^{-1}-\left[\theta(\theta-1)\sum_{i=1}^{m}\delta_{i,2,c}y_i\left(\frac{\beta y_i-\alpha}{1-(\beta y_i-\alpha)}\right)^{\theta-1}\left((\beta y_i-\alpha)^{-1}+(1-(\beta y_i-\alpha))^{-1}\right)\right],$$

$$I_{12}=I_{21}=\frac{\partial^2 \ln L}{\partial\alpha\partial\theta}=\sum_{i=1}^{m}\delta_{i,1,f}\left(\frac{\theta(\theta-1)(t_i-\alpha)^{-1}-1}{\theta}\right)\left[\frac{\theta(t_i-\alpha)^{-1}+1}{\theta}\right]-\sum_{i=1}^{m}\delta_{i,2,f}\left(\frac{(\theta-1)(\beta y_i-\alpha)^{\theta-2}}{(\beta y_i-\alpha)^{\theta-1}}\right)$$
$$-(\theta+1)\left[\sum_{i=1}^{m}\delta_{i,1,f}\left(1-(t_i-\alpha)\right)^{-1}+\sum_{i=1}^{m}\delta_{i,2,f}\left(1-(\beta y_i-\alpha)\right)^{-1}\right]$$
$$-\begin{bmatrix}\theta\sum_{i=1}^{m}\delta_{i,1,c}\left(\frac{t_i-\alpha}{1-(t_i-\alpha)}\right)^{\theta}\left((1-(t_i-\alpha))^{-1}-(t_i-\alpha)^{-1}+\right)+\\ \theta\sum_{i=1}^{m}\delta_{i,2,c}\left(\frac{\beta y_i-\alpha}{1-(\beta y_i-\alpha)}\right)^{\theta}\left((1-(\beta y_i-\alpha))^{-1}(\beta y_i-\alpha)^{-1}+\right)\end{bmatrix}.$$

The asymptotic variance-covariance is given as

$$\Sigma=I^{-1}=\begin{bmatrix}I_{11}&I_{12}&I_{13}\\I_{21}&I_{22}&I_{23}\\I_{31}&I_{32}&I_{33}\end{bmatrix}^{-1}=\begin{bmatrix}AVar(\hat{\theta})&ACov(\hat{\theta}\hat{\alpha})&ACov(\hat{\theta}\hat{\beta})\\ACov(\hat{\alpha}\hat{\theta})&AVar(\hat{\alpha})&ACov(\hat{\alpha}\hat{\beta})\\ACov(\hat{\beta}\hat{\theta})&ACov(\hat{\beta}\hat{\alpha})&AVar(\hat{\beta})\end{bmatrix}. \quad (12)$$

A confidence interval (CI) for distribution parameters is a form of interval estimate calculated from the information of the observed statistics that consist of the true value of an unknown population parameter. In other words, a confidence interval is a probability. A CI is a probability that the value of a parameter will fall among the lower and upper bounds of a probability distribution. Generally, 90%, 95%, and 99% confidence levels are applied. The two-sided confidence limits can be denoted as

$$P\left[-z\le\frac{\hat{\rho}-\rho}{\sigma(\hat{\rho})}\le z\right]=1-\gamma. \quad (13)$$

This formation of two- sided confidence limits is for the ML estimate $\hat{\rho}$ of a population parameter $\psi=(\alpha,\theta,\beta)$. $z$ stands for $100(1-\gamma/2)$ the standard normal percentile and $\gamma$ stands for the significance level. So, for a population parameter $\rho$, appropriate confidence limits can be achieved, such that

$$P\left[\hat{\rho}-z\sigma(\hat{\rho})\le\rho\le\hat{\rho}+z\sigma(\hat{\rho})\right]=1-\gamma, \quad (14)$$

where lower confidence limit $L_\rho=\hat{\rho}-z\sigma(\hat{\rho})$ and upper confidence limit $U_\rho=\hat{\rho}+z\sigma(\hat{\rho})$.

## 4. Maintenance Service Policy

Many types of research investigate the topic of maintenance service policy. The problem of cost-driven predictive safeguarding strategy for structural airframe repairs is tackled by Yiwei et al. (2017). This plan is suitably resulting in the trade-off among prospected of unplanned and planned safeguarding. Wang et al. (2018) studied extrapolative airframe preservation strategies based on model-based prognostics. In this study, the authors proposed two foretelling maintenance strategies dependent on the developed prognostic form. These strategies are helpful to fatigue spoil extend in fuselage panels. For the one-unit arrangement failures which have sudden socks and inner deterioration, a preventive maintenance strategy is also discussed by Yang et al. (2018). The major desire of this study is to minimize the expected cost per unit time, defining the most favourable preventive alternate interval, check interval, and the number of assessments. For manipulative and optimizing the maintenance service strategy, the study is presented by Hong et al. (2016). Gonçalves and Trabasso (2018) tackled with a method to explicit safeguarding intermissions to those of related arrangements under improvement. The method has been helpful in an industrialized aircraft corporation with a modern operation database. Bozoudis et al. (2018) tackled an aircraft maintenance grounding optimization problem and its applications to an aircraft part. Shey-Huei et al. (2015) examined the most favourable defensive protection strategy for multi-state arrangements. Alam et al. (2020) tackled the proposed policy under the SSPALT for Power function failure model under the progressive Type-II censoring scheme. Mudibbo et al. (2021) handled a problem that would be very helpful for decision-makers in the complex system arrangements to the estimation of reliability of arrangement parts, optimize the allocation and producer assortment problems. Currently, Alam and Aquil (2021) proposed a study on SSPALT for generalized inverted exponential distribution under progressive censoring and provide its application in maintenance policy.

This strategy ends when the arrangement phase reaches time, i.e., the usage level $(L)$. The system's regeneration is not included, and the preventive and corrective maintenances are included in the proposed strategy. The arrangement should go for periodic precautionary protection at an interval of time $(\kappa)$ in the proposed plan while time is constant. The arrangement should go for only be fixed at every failure within succeeding precautionary maintenances. An intricate repairable arrangement with a long life is utterly appropriate for this kind of service arrangement. The key points which cover this policy are

(i) The succeeding failures are equally independent random actions and known on parameters of distributions.

(ii) Whether the repairs were completed in preservation, only minimal repairs are done.

(iii) The Servicing action restores life a bit.

(iv) The repair period is slight compared to the item's age.

(v) After every preventive maintenance, the age renewal is stable.

(vi) The unit quantity of minimum repairs between the unit quantity of preventive maintenances and preventive maintenances has a constant average.

According to Anisur (2007), the expected cost of maintenance service policy can be explained in the following steps:

(i) Taking the same length of the preventive maintenance period $(\kappa)$, the expected cost of minimal repairs among preventive maintenance for two parameters RK distribution is provided as

$$E(C_{mr}) = C_{mr}\left[\sum_{p=0}^{N-1}\int_{p\kappa}^{(p+1)\kappa} h(t-p\kappa)dt\right] = \theta C_{mr}\left[\sum_{p=0}^{N-1}\left(\sum_{i=0}^{\theta-1}(-1)^i\binom{\theta-1}{i}\frac{\kappa^{i-\theta}}{i-\theta}\right)\right]. \quad (15)$$

(ii) The expected cost of preventive maintenance is provided as

$$E(C_{pm}) = NC_{pm}. \quad (16)$$

Here, the arrangement is periodically maintained at $N^{\text{th}}$ preventive maintenance. The total expected cost per unit time $C(\kappa, N)$ for two parameters RK distribution is provided as

$$E(C(\kappa,N)) = \frac{E(C_{mr})+E(C_{pm})}{L} = \frac{C_{mr}\left[\sum_{p=0}^{N-1}\sum_{i=0}^{\theta-1}(-1)^i\binom{\theta-1}{i}\frac{\kappa^{i-\theta}}{i-\theta}\right]+E(C_{pm})}{L}, \quad (17)$$

where $L = N\times\kappa$.

## 5. Simulation Study and Results

Now, we carry out a simulation study to check the presentation of the estimators having RK distribution using multiply censored data. This study is done by the Monte Carlo simulation technique using R-Software. To check the estimator's performance, the means square error (MSEs) and absolute relative bias (RAB) are estimated. The steps for the study are

(i) The total sample $m$ is divided into two parts $m_1$ and $m_2$, where $m_1 = m\pi$ and $m_2 = m(1-\pi)$.

(ii) Generate random samples of size $m_1$ $(t_{1,1} < t_{2,2} < \ldots < t_{m_1,1})$ and $m_2$ $(t_{2,1} < t_{2,2} < \ldots < t_{m_2,2})$ under normal and accelerated conditions, respectively, from RK distribution using the inverse CDF method.

(iii) Generate 1,000 random samples of sizes 40, 90, and 140 and specify the following values,
Case (I) $(\alpha = 0.7, \theta = 0.7, \beta = 1.7)$, Case (II) $(\alpha = 0.7, \theta = 0.7, \beta = 1.9)$
Case (III) $(\alpha = 0.5, \theta = 0.9, \beta = 1.7)$, Case (IV) $(\alpha = 0.5, \theta = 0.9, \beta = 1.9)$.

(iv) The model parameters and acceleration factors are obtained for each sample and each set of parameters. Also, the asymptotic variance and covariance matrix are obtained.

(v) By (13), for confidence levels $\gamma = 95\%$, 99% of acceleration factor $\beta$, the two-sided confidence limits and two parameters are obtained for parameters $\alpha, \theta$ and $\beta$.

(vi) All nonlinear equations are solved by the Newton Raphson method.

(vii) The above steps are replicated 1000 times with different values of parameters.

(viii) From (15)-(17), the expected cost of maintenance service strategy is estimated for preventive maintenance, total costs, minimal repairs, and expected cost rate, and the duration of the maintenance service policy $(L)$ is chosen as three years.

(ix) At the usual cost $(C_{pm} = 1100)$, preventive maintenance is every four months $(\kappa = 0.30)$. If there are failures among two succeeding preventive maintenance, the minimal repairs will be completed at an average cost $(C_{mr} = 700)$. Finally, the expected cost of preventive maintenance is, $E(C_{pm}) = 18800$.

**Table 2** The biases and MSEs with different sizes of samples for multiply censoring for Cases I and II

| $m$ | Parameters | Case I $(\alpha = 0.7, \theta = 0.7, \beta = 1.7)$ | | | Case II $(\alpha = 0.7, \theta = 0.7, \beta = 1.9)$ | | |
|---|---|---|---|---|---|---|---|
| | | Estimates | Bias | MSE | Estimates | Bias | MSE |
| 40 | $\alpha$ | 0.809 | 0.559 | 0.092 | 0.618 | 0.417 | 0.071 |
| | $\theta$ | 0.602 | 0.105 | 0.069 | 0.801 | 0.100 | 0.055 |
| | $\beta$ | 1.668 | 0.094 | 0.531 | 1.303 | 0.094 | 0.260 |
| 90 | $\alpha$ | 0.739 | 0.504 | 0.060 | 0.503 | 0.318 | 0.059 |
| | $\theta$ | 0.654 | 0.098 | 0.044 | 0.703 | 0.073 | 0.024 |
| | $\beta$ | 1.509 | 0.076 | 0.202 | 1.104 | 0.070 | 0.124 |
| 140 | $\alpha$ | 0.762 | 0.499 | 0.033 | 0.505 | 0.203 | 0.033 |
| | $\theta$ | 0.583 | 0.055 | 0.014 | 0.518 | 0.044 | 0.015 |
| | $\beta$ | 1.431 | 0.021 | 0.104 | 0.942 | 0.037 | 0.106 |

**Table 3** The biases and MSEs with different sizes of samples for multiply censoring for Cases III and IV

| $m$ | Parameters | Case III ($\alpha=0.5,\theta=0.9,\beta=1.7$) | | | Case IV ($\alpha=0.5,\theta=0.9,\beta=1.9$) | | |
|---|---|---|---|---|---|---|---|
| | | Estimates | Bias | MSE | Estimates | Bias | MSE |
| 40 | $\alpha$ | 0.611 | 0.705 | 0.092 | 0.708 | 0.663 | 0.104 |
| | $\theta$ | 0.498 | 0.491 | 0.069 | 0.550 | 0.300 | 0.087 |
| | $\beta$ | 1.245 | 0.051 | 0.531 | 1.090 | 0.082 | 0.400 |
| 90 | $\alpha$ | 0.671 | 0.629 | 0.060 | 0.813 | 0.508 | 0.110 |
| | $\theta$ | 0.512 | 0.362 | 0.044 | 0.610 | 0.210 | 0.042 |
| | $\beta$ | 1.001 | 0.033 | 0.202 | 1.310 | 0.063 | 0.204 |
| 140 | $\alpha$ | 0.602 | 0.550 | 0.033 | 0.905 | 0.444 | 0.072 |
| | $\theta$ | 0.443 | 0.290 | 0.014 | 0.709 | 0.142 | 0.024 |
| | $\beta$ | 1.109 | 0.025 | 0.104 | 0.809 | 0.071 | 0.171 |

**Table 4** The 95% and 99% confidence limits of estimates at various size of samples for Cases I and II

| $m$ | Parameters | Case I ($\alpha=0.7,\theta=0.7,\beta=1.7$) | | | | $\sigma$ | Case II ($\alpha=0.7,\theta=0.7,\beta=1.9$) | | | | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 95% CI | | 99% CI | | | 95% CI | | 99% CI | | |
| | | Lower Bound | Upper Bound | Lower Bound | Upper Bound | | Lower Bound | Upper Bound | Lower Bound | Upper Bound | |
| 40 | $\alpha$ | 0.78 | 1.50 | 1.03 | 1.91 | 0.06 | 0.88 | 2.06 | 0.90 | 1.71 | 0.02 |
| | $\theta$ | 0.42 | 1.01 | 0.87 | 1.70 | 0.11 | 1.15 | 1.91 | 1.07 | 1.60 | 0.18 |
| | $\beta$ | 0.60 | 1.11 | 0.92 | 1.61 | 0.48 | 1.29 | 1.78 | 0.71 | 1.43 | 0.66 |
| 90 | $\alpha$ | 0.70 | 1.19 | 0.85 | 1.44 | 0.08 | 1.37 | 1.64 | 1.11 | 1.59 | 0.09 |
| | $\theta$ | 0.50 | 0.95 | 0.60 | 1.20 | 0.26 | 0.91 | 1.42 | 0.82 | 1.49 | 0.21 |
| | $\beta$ | 0.52 | 0.84 | 0.93 | 1.39 | 0.66 | 0.78 | 1.26 | 0.57 | 0.80 | 0.50 |
| 140 | $\alpha$ | 0.64 | 0.91 | 0.79 | 1.08 | 0.03 | 0.44 | 0.75 | 0.50 | 0.86 | 0.09 |
| | $\theta$ | 0.58 | 1.06 | 0.58 | 0.72 | 0.19 | 0.68 | 0.93 | 0.42 | 0.71 | 0.40 |
| | $\beta$ | 0.66 | 0.91 | 0.88 | 1.13 | 0.31 | 0.71 | 0.90 | 0.77 | 1.06 | 0.91 |

**Table 5** The 95% and 99% confidence limits of estimates at various size of samples for Cases III and IV

| $m$ | Parameters | Case III $(\alpha = 0.5, \theta = 0.9, \beta = 1.7)$ | | | | $\sigma$ | Case IV $(\alpha = 0.5, \theta = 0.9, \beta = 1.9)$ | | | | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 95% CI | | 99% CI | | | 95% CI | | 99% CI | | |
| | | Lower Bound | Upper Bound | Lower Bound | Upper Bound | | Lower Bound | Upper Bound | Lower Bound | Upper Bound | |
| 40 | $\alpha$ | 1.05 | 1.80 | 0.55 | 2.22 | 0.08 | 1.11 | 1.87 | 0.50 | 1.29 | 0.07 |
| | $\theta$ | 0.77 | 1.61 | 1.32 | 2.19 | 0.37 | 0.30 | 1.17 | 0.68 | 1.35 | 0.12 |
| | $\beta$ | 1.20 | 1.82 | 1.07 | 1.99 | 0.10 | 1.37 | 1.80 | 0.60 | 1.05 | 0.26 |
| 90 | $\alpha$ | 0.94 | 1.65 | 0.74 | 1.80 | 0.14 | 0.91 | 1.29 | 0.89 | 1.20 | 0.17 |
| | $\theta$ | 0.69 | 1.74 | 0.77 | 150 | 0.29 | 0.78 | 1.16 | 1.00 | 1.47 | 0.26 |
| | $\beta$ | 1.14 | 1.51 | 0.60 | 1.22 | 0.09 | 0.44 | 0.69 | 0.74 | 1.19 | 0.25 |
| 140 | $\alpha$ | 1.28 | 1.49 | 0.48 | 0.80 | 0.20 | 0.68 | 0.95 | 0.88 | 1.31 | 0.22 |
| | $\theta$ | 0.97 | 1.55 | 0.74 | 1.19 | 0.40 | 0.71 | 0.85 | 0.60 | 0.82 | 0.13 |
| | $\beta$ | 1.05 | 1.80 | 0.55 | 2.22 | 0.08 | 1.11 | 1.87 | 0.50 | 1.29 | 0.07 |

**Table 6** Expected cost rate, total cost, minimal repair time, and its confidence level in the maintenance service policy

| $m$ | Minimal repair cost | | | Total cost | | | Cost rate | | |
|---|---|---|---|---|---|---|---|---|---|
| | $E(C_{mr})$ | Lower Bound | Upper Bound | $E(C_{total})$ | Lower Bound | Upper Bound | $E(C(\tau, N))$ | Lower Bound | Upper Bound |
| Case I $(\alpha = 0.7, \theta = 0.7, \beta = 1.7)$ | | | | | | | | | |
| 40 | 174450.2 | 68732.8 | 80654.4 | 73092.3 | 37912.4 | 64562.3 | 66532.2 | 23421.6 | 59821.5 |
| 90 | 161152.9 | 60732.1 | 90823.7 | 69032.3 | 54132.5 | 73099.4 | 61093.2 | 65320.2 | 102313.4 |
| 140 | 15287.6 | 75432.4 | 99193.5 | 64901.4 | 32987.8 | 66983.3 | 55421.4 | 4654295 | 129874.3 |
| Case II $(\alpha = 0.7, \theta = 0.7, \beta = 1.9)$ | | | | | | | | | |
| 40 | 74098.4 | 55020.7 | 81345.8 | 51876.9 | 22970.9 | 29876.9 | 27654.6 | 21076.9 | 45786.9 |
| 90 | 69012.8 | 83912.4 | 85935.0 | 56830.6 | 44765.8 | 74876.8 | 21598.4 | 44765.2 | 72176.2 |
| 140 | 65413.4 | 49023.5 | 69839.5 | 49763.8 | 49762.3 | 37965.3 | 17954.2 | 49859.5 | 137654.5 |
| Case III $(\alpha = 0.5, \theta = 0.9, \beta = 1.7)$ | | | | | | | | | |
| 40 | 65409.6 | 69754.7 | 74321.9 | 47654.7 | 87432.8 | 99876.8 | 26543.6 | 76543.9 | 10654.9 |
| 90 | 59876.9 | 55876.6 | 107654.4 | 41987.5 | 64325.8 | 72354.9 | 20987.8 | 61543.5 | 226543.9 |
| 140 | 50987.4 | 91565.5 | 146549.4 | 37654.9 | 65435.0 | 66543.9 | 86543.6 | 34876.3 | 88765.8 |

## 6. Results and Discussion

Tables 2 and 3 show the biases and MSEs with different samples sizes for a multiply censoring scheme at different values of parameters. Tables 4 and 5 show the confidence limits of estimates at the various sizes of samples for multiply censoring schemes for different parameter values. While Table 5 shows the Expected cost rate, total cost, minimal repair time, and its confidence level in the maintenance service policy.

## 7. Industrial Applications

Due to the speedy development of high machinery, the products become more consistent in an industrial area, and the product's life gets longer and longer. It might take a lengthy occasion, such as several years, for a product to be unsuccessful, making it complicated or even impracticable to obtain the failure information under ordinary conditions for such high dependable products. In such difficulties, CSPALT is used in mechanized industries to assess or demonstrate part and subsystem consistency, certified components, detect failure modes to be corrected, compare different manufacturers, etc. Generally, before letting them a product in the market, every manufacturing company wants to check and improve its reliability. Therefore, this proposed work becomes instrumental for this task.

## 8. Conclusion

The paper has presented CSPALT and estimating costs of maintenance service policy using the multiply censoring scheme for the two-parameters RK distribution. The following conclusions are made based on this study:

(i) The values of MSEs and biases reduce as the sample size increases and confidence intervals become narrower, or the confidence interval size decreases. Thus, the MLEs have encouraging statistical results. We can also observe that the numerical outcomes and theoretical conclusions support each other, and our suppositions are also satisfied. (see Table 2,3,4 and 5)

(ii) There is a direct relationship between model parameters and costs of maintenance service policy for two parameters RK distribution. While Costs of maintenance service and sample size are in an inverse relationship. (see Table 6)

For future research, this work can be extended with several other censoring schemes for different failure models using SSPALT or CSPALT.

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