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A New Exponential Ratio-Type Estimator of Population Mean Using Mean and Median with Two Auxiliary Variables under Double Sampling

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Abstract

Using auxiliary information for estimating the study variable is a popular and well established technique to enhance the efficiency of the estimator. In this paper, we have considered an exponential ratio-type estimator under double sampling scheme involving two auxiliary variables. Both cases of independence and dependence of auxiliary variables have been considered. Here, we make use of the sample mean and sample median of the auxiliary variables of both phases with the intention to improve upon the efficiency of the said estimator. It is found that the proposed estimator is more efficient than the regression estimator proposed by Vadlamudi et al. (2017) with one auxiliary variable and also over other classical estimators. The study was tested empirically using simulated data.

Keywords: Mean squared error, bias, ratio estimator, regression estimator, study variable, median estimation.

1. Introduction

In the pursuit of finding a good estimator of the population mean of a certain population, authors and researchers have often resorted to using the sample mean in the structural form of the estimator as have been done by classical and standard estimators such as ratio estimators, regression estimators, etc. In particular, the standard ratio and regression estimators also makes use of the information of a known mean of an auxiliary variable, which is highly correlated to the study variable, alongwith the sample means of the auxiliary and study variables (Cochran, 1977). In the case where the population mean of the auxiliary variable is not known, then double sampling is implemented and the sample means of the two phases are used in the form of the estimators of the population mean. However, there is a gain in efficiency or precision in the estimation of the population mean if the estimator involves the use of a known median or sample median of the variables along side the means of the auxiliary and study variables. Several authors have made a significant contribution to the study of median estimation, however, Gross (1980), Kuk and Mak (1989) and others have been the main pioneers and contributors to this particular field of study. Furthermore, the inclusion of both mean and median of the auxiliary variable in a regression estimator of the population mean was introduced by Lamichhane et al. (2017) and this same estimator was introduced under double sampling scheme by Vadlamudi et al. (2017). The most recent study on ratio type estimators of population mean using the known median of the study variable was given by Yadav et al. (2021). Their estimator uses the known value of the population median of the study variable under simple random sampling without replacement (SRSWOR) in order to estimate the population mean.

In this paper, a new exponential ratio-type estimator of the population mean has been put forward under double sampling scheme and it involves the use of two auxiliary variables which also makes use of the sample mean and sample median of the auxiliary variables in both the phases. The motivation behind this work is the goal of achieving increased precision and efficiency in mean estimation through modification of the previous estimator proposed by Vadlamudi et al. (2017) by involving an additional auxiliary variable and changing the structural form of the estimator to an exponential ratio-type estimator. This estimator differs slightly from the estimator given by Yadav (2021) in the sense that it makes use of the sample medians of the auxiliary variables of both the phases, whereas, Yadav (2021) utilizes the known population median of the study variable in his estimator.

Let us consider a population of size N. Let the variable under study be Y having population values $Y_1, Y_2, ..., Y_N$. Similarly, let the two auxiliary variables under consideration be X and Z with population values $X_1, X_2, ..., X_N$ and $Z_1, Z_2, ..., Z_N$, respectively. The study variable Y is taken to be highly correlated with auxiliary variable X as well as auxiliary variable Z.

Now, as double sampling is to be implemented, firstly, a large preliminary sample (or first phase sample) of size m is drawn from the population of size N using simple random sampling without replacement (SRSWOR) measuring the values of X and Z, respectively according to the population unit numbers selected viz., $x_{r'}$ and $z_{r'}$ $(r' = 1, 2, ..., m)$. Based on this first phase sample, we define

$$\bar{x}_m = \frac{1}{m}\sum_{r'=1}^{m} x_{r'}, \quad \bar{z}_m = \frac{1}{m}\sum_{r'=1}^{m} z_{r'}. \tag{1}$$

Next, a smaller sample (or second phase sample) of size n is drawn from the previous first phase sample of size m using SRSWOR measuring the values of Y, X and Z, respectively according to the population unit numbers selected viz., y_r, x_r, and z_r $(r = 1, 2, ..., n)$. Similarly, we define the following based on the second phase sample:

$$\bar{y}_n = \frac{1}{n}\sum_{r=1}^{n} y_r, \quad \bar{x}_n = \frac{1}{n}\sum_{r=1}^{n} x_r, \quad \bar{z}_n = \frac{1}{n}\sum_{r=1}^{n} z_r. \tag{2}$$

Let M_x and M_z be the population medians of variables X and Z, respectively. As stated by Vadlamudi et al. (2017), "the pioneer contributors, to the problem of estimating median, are Kuk and Mak (1989) by proposing very clear estimators of median in the presence of auxiliary information". Therefore, in our approach of implementing the use of the various sample medians in our estimator, we make use of the same assumptions put forward by Kuk and Mak (1989) for a finite population which utilizes the asymptotic properties of the sample median and also defines its asymptotic variance (Gross 1980).

We further define $\hat{M}_x^*$ and $\hat{M}_z^*$ as the sample medians of X and Z, respectively based on the first phase samples of size m. Similarly, we define $\hat{M}_x$ and $\hat{M}_z$ as the sample medians of X and Z respectively based on the second phase sample of size n.

The estimator proposed by Lamichhane et al. (2017) which makes use of known population mean and known population median of the single auxiliary variable X is given by

$$\bar{y}_c = \bar{y} + \beta_1\left(\bar{X} - \bar{x}\right) + \beta_2\left(M_x - \hat{M}_x\right), \tag{3}$$

where β_1 and β_2 are to be estimated such that the variance of $\bar{y}_c$ is minimum.

The minimum variance of this estimator is given as follows

$$Min.Var(\bar{y}_c) = Var(\bar{y})\left[1 - R^2_{\bar{y}\bar{x}\hat{M}_x}\right], \tag{4}$$

where $R^2_{\bar{y}\bar{x}\hat{M}_x} = \dfrac{\rho^2_{\bar{y}\hat{M}_x} - 2\rho_{\bar{y}\bar{x}}\rho_{\bar{y}\hat{M}_x} + \rho^2_{\bar{y}\bar{x}}}{1 - \rho^2_{\bar{y}\hat{M}_x}}$ is the multiple R^2 such that $0 \le R^2 \le 1$. Next, Vadlamudi et al. (2017) modified the above estimator by introducing it under double sampling and proposed the following estimator given by

$$\bar{y}_{kal} = \bar{y} + \beta^*_1(\bar{x}_m - \bar{x}_n) + \beta_2\left(\hat{M}_x^* - \hat{M}_x\right), \tag{5}$$

where β_1^* and β_2^* are unknown partial regression coefficients to be determined such that the variance of the estimator is minimum. The minimum variance of this estimator is given as follows

$$Min.Var(\bar{y}_{kal}) = \left(\frac{1}{m} - \frac{1}{N}\right)S_y^2 + \left(\frac{1}{n} - \frac{1}{m}\right)S_y^2\left\{1 - \rho_{yx}^2 - \frac{(S_x^2 S_{YM_x} - S_{xy}S_{XM_x})^2}{S_y^2 S_x^2(0.25S_x^2 - S^2_{XM_x})}\right\}, \tag{6}$$

where

$$S_y^2 = \left(1/(N-1)\right)\sum_{i=1}^{N}(Y_i - \bar{Y})^2,\ S_x^2 = \left(1/(N-1)\right)\sum_{i=1}^{N}(X_i - \bar{X})^2,\ S_{yx} = \left(1/(N-1)\right)\sum_{i=1}^{N}(Y_i - \bar{Y})(X_i - \bar{X}),$$

$$S_{YM_x} = \left(1/(N-1)\right)\sum_{i=1}^{N}(Y_i - \bar{Y})(I_{x_i} - 0.5),\ S_{XM_x} = \left(1/(N-1)\right)\sum_{i=1}^{N}(X_i - \bar{X})(I_{x_i} - 0.5),$$

$$I_{x_i} = \begin{cases}1, \text{if } X_i \le M_x\\ 0, \text{otherwise}\end{cases},\ I_{z_i} = \begin{cases}1, \text{if } Z_i \le M_z\\ 0, \text{otherwise.}\end{cases}$$

2. Proposed Estimator

The proposed exponential ratio-type estimator of the population mean $\bar{Y}$ under double sampling with two auxiliary variables is defined by

$$\bar{y}_{proposed} = \bar{y}_n \exp\left(\beta_1\frac{\bar{x}_m - \bar{x}_n}{\bar{x}_m + \bar{x}_n}\right)\exp\left(\beta_2\frac{\hat{M}_x^* - \hat{M}_x}{\hat{M}_x^* + \hat{M}_x}\right)\exp\left(\beta_3\frac{\bar{z}_m - \bar{z}_n}{\bar{z}_m + \bar{z}_n}\right)\exp\left(\beta_4\frac{\hat{M}_z^* - \hat{M}_z}{\hat{M}_z^* + \hat{M}_z}\right), \tag{7}$$

where $\beta_1, \beta_2, \beta_3$ and β_4 are unknown constants to be determined such that the MSE of the estimator $\bar{y}_{proposed}$ is minimum. Since this estimator makes use of the medians of the two auxiliary variables, then, the same assumption as given by Lamichhane et al. (2017) that the distributions of Y, X and Z are non-normal and follow a skewed distribution is also adopted here so that their corresponding means and medians do not coincide.

Now, in order to show that the proposed estimator is unbiased and also to determine its minimum variance we define the following error functions:

$$e_0 = \left(\frac{\bar{y}_n - \bar{Y}}{\bar{Y}}\right),\ e_1 = \left(\frac{\bar{x}_n - \bar{X}}{\bar{X}}\right),\ e_2 = \left(\frac{\hat{M}_x - M_x}{M_x}\right),\ \text{(Based on 2}^{\text{nd}}\text{ phase sample)}$$

$$e_3 = \left(\frac{\bar{x}_m - \bar{X}}{\bar{X}}\right),\ e_4 = \left(\frac{\hat{M}_x^* - M_x}{M_x}\right),\ \text{(Based on 1}^{\text{st}}\text{ phase sample)}$$

$$e_5 = \left(\frac{\bar{z}_n - \bar{Z}}{\bar{Z}}\right),\ e_6 = \left(\frac{\hat{M}_z - M_z}{M_z}\right),\ \text{(Based on 2}^{\text{nd}}\text{ phase sample)}$$

$$e_7 = \left(\frac{\bar{z}_m - \bar{Z}}{\bar{Z}}\right),\ e_8 = \left(\frac{\hat{M}_z^* - M_z}{M_z}\right).\ \text{(Based on 1}^{\text{st}}\text{ phase sample)}$$

Let us define the following,

$$\left.\begin{aligned}
&\lambda_1 = \left(\frac{1}{m} - \frac{1}{N}\right),\ \lambda_2 = \left(\frac{1}{n} - \frac{1}{N}\right),\ \lambda_3 = \left(\frac{1}{n} - \frac{1}{m}\right),\ \lambda_{xy} = \rho_{xy} C_x C_y,\ \lambda_{xz} = \rho_{xz} C_x C_z, \\
&\lambda_{yz} = \rho_{yz} C_y C_z,\ \lambda_{M_x} = \frac{1}{4\left[f_x(M_x)\right]^2 M_x^2},\ \lambda_{M_z} = \frac{1}{4\left[f_z(M_z)\right]^2 M_z^2},\ \lambda_{YM_x} = \frac{S_{YM_x}\left[f_x(M_x)\right]^{-1}}{\bar{Y} M_x}, \\
&\lambda_{YM_z} = \frac{S_{YM_z}\left[f_z(M_z)\right]^{-1}}{\bar{Y} M_z},\quad \lambda_{XM_x} = \frac{S_{XM_x}\left[f_x(M_x)\right]^{-1}}{\bar{X} M_x},\quad \lambda_{ZM_z} = \frac{S_{ZM_z}\left[f_z(M_z)\right]^{-1}}{\bar{Z} M_z}, \\
&\lambda_{XM_z} = \frac{S_{XM_z}\left[f_z(M_z)\right]^{-1}}{\bar{X} M_z},\quad \lambda_{ZM_x} = \frac{S_{ZM_x}\left[f_x(M_x)\right]^{-1}}{\bar{Z} M_x}, \\
&\lambda_{M_x M_z} = \frac{\left[4P_{11}(x,z) - 1\right]}{4 M_x M_z}\left[f_x(M_x)\right]^{-1}\left[f_z(M_z)\right]^{-1},
\end{aligned}\right\} \tag{8}$$

where $C_y = S_y / \bar{Y}$, $C_x = S_x / \bar{X}$, $C_z = S_z / \bar{Z}$. Following the footsteps of Kuk and Mak (1989) for defining a matrix of proportions P_{ij}, we consider the following:

	$Z \le M_z$	$Z > M_z$	Total
$X \le M_x$	$P_{11}(x,z)$	$P_{21}(x,z)$	$P_{\bullet 1}(x,z)$
$X > M_x$	$P_{12}(x,z)$	$P_{22}(x,z)$	$P_{\bullet 2}(x,z)$
Total	$P_{1\bullet}(x,z)$	$P_{2\bullet}(x,z)$	1

Here, the expectations are given in the following table as follows, noting that the symbols have their usual meaning,

$$E(e_0) = E(e_1) = E(e_2) = E(e_5) = E(e_6) = 0,$$
$$E(e_3) = E(e_4) = E(e_7) = E(e_8) \approx 0.$$

Table 1 Table of expectations of the product of the error functions

Error Functions	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
e_0	$E(e_0^2)=\lambda_2 C_y^2$	$E(e_0e_1)=\lambda_2\lambda_{xy}$	$E(e_0e_2)=-\lambda_2\lambda_{YM_x}$	$E(e_0e_3)=\lambda_1\lambda_{xy}$	$E(e_0e_4)=-\lambda_1\lambda_{YM_x}$	$E(e_0e_5)=\lambda_2\lambda_{yz}$	$E(e_0e_6)=-\lambda_2\lambda_{YM_z}$	$E(e_0e_7)=\lambda_1\lambda_{yz}$	$E(e_0e_8)=-\lambda_1\lambda_{YM_z}$
e_1	-	$E(e_1^2)=\lambda_2 C_x^2$	$E(e_1e_2)=-\lambda_2\lambda_{XM_x}$	$E(e_1e_3)=\lambda_1 C_x^2$	$E(e_1e_4)=-\lambda_1\lambda_{XM_x}$	$E(e_1e_5)=\lambda_2\lambda_{xz}$	$E(e_1e_6)=-\lambda_2\lambda_{XM_z}$	$E(e_1e_7)=\lambda_1\lambda_{xz}$	$E(e_1e_8)=-\lambda_1\lambda_{XM_z}$
e_2	-	-	$E(e_2^2)=\lambda_2\lambda_{M_x}$	$E(e_2e_3)=-\lambda_1\lambda_{XM_x}$	$E(e_2e_4)=\lambda_1\lambda_{M_x}$	$E(e_2e_5)=-\lambda_2\lambda_{ZM_x}$	$E(e_2e_6)=\lambda_2\lambda_{M_xM_z}$	$E(e_2e_7)=-\lambda_1\lambda_{ZM_x}$	$E(e_2e_8)=\lambda_1\lambda_{M_xM_z}$
e_3	-	-	-	$E(e_3^2)=\lambda_1 C_x^2$	$E(e_3e_4)=-\lambda_1\lambda_{XM_x}$	$E(e_3e_5)=\lambda_1\lambda_{xz}$	$E(e_3e_6)=-\lambda_1\lambda_{XM_z}$	$E(e_3e_7)=\lambda_1\lambda_{xz}$	$E(e_3e_8)=-\lambda_1\lambda_{XM_z}$
e_4	-	-	-	-	$E(e_4^2)=\lambda_1\lambda_{M_x}$	$E(e_4e_5)=-\lambda_1\lambda_{ZM_x}$	$E(e_4e_6)=\lambda_1\lambda_{M_xM_z}$	$E(e_4e_7)=-\lambda_1\lambda_{ZM_x}$	$E(e_4e_8)=\lambda_1\lambda_{M_xM_z}$
e_5	-	-	-	-	-	$E(e_5^2)=\lambda_2 C_z^2$	$E(e_5e_6)=-\lambda_2\lambda_{ZM_z}$	$E(e_5e_7)=\lambda_1 C_z^2$	$E(e_5e_8)=-\lambda_1\lambda_{ZM_z}$
e_6	-	-	-	-	-	-	$E(e_6^2)=\lambda_2\lambda_{M_z}$	$E(e_6e_7)=-\lambda_1\lambda_{ZM_z}$	$E(e_6e_8)=\lambda_1\lambda_{M_z}$
e_7	-	-	-	-	-	-	-	$E(e_7^2)=\lambda_1 C_z^2$	$E(e_7e_8)=-\lambda_1\lambda_{ZM_z}$
e_8	-	-	-	-	-	-	-	-	$E(e_8^2)=\lambda_1\lambda_{M_z}$

Lamichhane et al. (2015) referred the work of Sedory and Singh (2013), who have formulated the following results

$$E[(\bar{y}_n-\bar{Y})(\hat{M}_x-M_x)]=-\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_{YM_x}}{[f_x(M_x)]},$$

$$E[(\bar{x}_n-\bar{X})(\hat{M}_x-M_x)]=-\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_{XM_x}}{[f_x(M_x)]},\quad E[(\bar{y}_n-\bar{Y})(\bar{x}_n-\bar{X})]=\left(\frac{1}{n}-\frac{1}{N}\right)S_{xy}. \tag{9}$$

Further, Sedory and Singh (2013) referred to Gross (1980), who have formulated the following

$$Var(\hat{M}_x)=\left(\frac{1}{n}-\frac{1}{N}\right)\frac{1}{4[f_x(M_x)]^2}. \tag{10}$$

The proposed estimator $\bar{y}_{proposed}$ in terms of $e_i's\,(i=0,1,2,3,4,5,6,7,8)$ reduces to the following form,

$$\bar{y}_{proposed} = \bar{Y}(1+e_0)\exp\left[\beta_1(e_3-e_1)(2+e_3+e_1)^{-1}\right]\exp\left[\beta_2(e_4-e_2)(2+e_4+e_2)^{-1}\right]$$
$$\exp\left[\beta_3(e_7-e_5)(2+e_7+e_5)^{-1}\right]\exp\left[\beta_4(e_8-e_6)(2+e_8+e_6)^{-1}\right]. \quad (11)$$

On expanding the terms and neglecting terms of $e_i's$ of order 3 and higher we deduce the expression of the proposed estimator as follows,

$$\bar{y}_{proposed} \approx \bar{Y} + \bar{Y}e_0 + \bar{Y}\{\beta_1 E_1 + \beta_2 E_2 + \beta_3 E_3 + \beta_4 E_4 + \beta_1^2 E_5 + \beta_2^2 E_6 + \beta_3^2 E_7 + \beta_4^2 E_8$$
$$+ \beta_1\beta_2 E_9 + \beta_1\beta_3 E_{10} + \beta_1\beta_4 E_{11} + \beta_2\beta_3 E_{12} + \beta_2\beta_4 E_{13} + \beta_3\beta_4 E_{14}\}, \quad (12)$$

where $E_1 = \frac{e_3}{2} - \frac{e_1}{2} - \frac{e_3^2}{4} + \frac{e_1^2}{4} + \frac{e_0e_3}{2} - \frac{e_0e_1}{2}$, $E_2 = \frac{e_4}{2} - \frac{e_2}{2} - \frac{e_4^2}{4} + \frac{e_2^2}{4} + \frac{e_0e_4}{2} - \frac{e_0e_2}{2}$,

$$E_3 = \frac{e_7}{2} - \frac{e_5}{2} - \frac{e_7^2}{4} + \frac{e_5^2}{4} + \frac{e_0e_7}{2} - \frac{e_0e_5}{2},\; E_4 = \frac{e_8}{2} - \frac{e_6}{2} - \frac{e_8^2}{4} + \frac{e_6^2}{4} + \frac{e_0e_8}{2} - \frac{e_0e_6}{2},$$

$$E_5 = \frac{1}{2}\left(\frac{e_3^2}{4} + \frac{e_1^2}{4} - \frac{e_1e_3}{2}\right),\; E_6 = \frac{1}{2}\left(\frac{e_4^2}{4} + \frac{e_2^2}{4} - \frac{e_2e_4}{2}\right),\; E_7 = \frac{1}{2}\left(\frac{e_7^2}{4} + \frac{e_5^2}{4} - \frac{e_5e_7}{2}\right),$$

$$E_8 = \frac{1}{2}\left(\frac{e_8^2}{4} + \frac{e_6^2}{4} - \frac{e_6e_8}{2}\right),\; E_9 = \frac{1}{4}(e_3e_4 - e_2e_3 - e_1e_4 + e_1e_2),$$

$$E_{10} = \frac{1}{4}(e_3e_7 - e_3e_5 - e_1e_7 + e_1e_5),\; E_{11} = \frac{1}{4}(e_3e_8 - e_3e_6 - e_1e_8 + e_1e_6),$$

$$E_{12} = \frac{1}{4}(e_4e_7 - e_4e_5 - e_2e_7 + e_2e_5),\; E_{13} = \frac{1}{4}(e_4e_8 - e_4e_6 - e_2e_8 + e_2e_6),$$

$$E_{14} = \frac{1}{4}(e_7e_8 - e_7e_6 - e_5e_8 + e_5e_6). \quad (13)$$

Clearly, the proposed estimator is a biased estimator of population mean, however, it will be unbiased up to the first order of approximation.

3. Approximation to the Bias and Mean Square Error of the Proposed Estimator

3.1. Case-I : Auxiliary variables X and Z are independent of each other

In the case of independence of the auxiliary variables, note that

$$E(e_1e_5) = E(e_1e_6) = E(e_1e_7) = E(e_1e_8) = E(e_3e_5) = E(e_3e_6) = E(e_3e_7) = E(e_3e_8) = 0,$$
$$E(e_2e_5) = E(e_2e_6) = E(e_2e_7) = E(e_2e_8) = E(e_4e_5) = E(e_4e_6) = E(e_4e_7) = E(e_4e_8) = 0.$$

The expression of the approximated bias of the proposed estimator is given as follows

$$Bias(\bar{y}_{proposed}) = E(\bar{y}_{proposed}) - \bar{Y}$$
$$\approx \bar{Y}\lambda_3\left\{\beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3 + \beta_4 I_4 + \beta_1^2 I_5 + \beta_2^2 I_6 + \beta_3^2 I_7 + \beta_4^2 I_8 + \beta_1\beta_2 I_9 + \beta_3\beta_4 I_{10}\right\} \quad (14)$$

where

$$I_1 = \frac{C_x^2}{4} - \frac{\lambda_{xy}}{2},\; I_2 = \frac{\lambda_{M_x}}{4} + \frac{\lambda_{YM_x}}{2},\; I_3 = \frac{C_z^2}{4} - \frac{\lambda_{yz}}{2},\; I_4 = \frac{\lambda_{M_z}}{4} + \frac{\lambda_{YM_z}}{2},\; I_5 = \frac{C_x^2}{8},$$
$$I_6 = \frac{\lambda_{M_x}}{8},\; I_7 = \frac{C_z^2}{8},\; I_8 = \frac{\lambda_{M_z}}{8},\; I_9 = -\frac{\lambda_{XM_x}}{4},\; I_{10} = -\frac{\lambda_{ZM_z}}{4}. \quad (15)$$

Neglecting the terms of $e_i's$ of order 3 and higher, the MSE of the proposed estimator is approximated as follows:

$$MSE\left(\bar{y}_{proposed}\right)=E\left(\bar{y}_{proposed}-\bar{Y}\right)^2$$
$$\approx \lambda_2 S_y^2+\lambda_3\left\{\beta_1 I_{11}+\beta_2 I_{12}+\beta_3 I_{13}+\beta_4 I_{14}+\beta_1^2 I_{15}+\beta_2^2 I_{16}+\beta_3^2 I_{17}+\beta_4^2 I_{18}+\beta_1\beta_2 I_{19}+\beta_3\beta_4 I_{20}\right\}, \quad (16)$$

where

$$I_{11}=-\frac{\bar{Y}S_{xy}}{\bar{X}},\ I_{12}=\bar{Y}^2\lambda_{YM_x},\ I_{13}=-\frac{\bar{Y}S_{yz}}{\bar{Z}},\ I_{14}=\bar{Y}^2\lambda_{YM_z},\ I_{15}=-\frac{\bar{Y}^2S_x^2}{4\bar{X}^2},$$
$$I_{16}=\frac{\bar{Y}^2\lambda_{M_x}}{4},\ I_{17}=-\frac{\bar{Y}^2S_z^2}{4\bar{Z}^2},\ I_{18}=\frac{\bar{Y}^2\lambda_{M_z}}{4},\ I_{19}=-\frac{\bar{Y}^2\lambda_{XM_x}}{2},\ I_{20}=-\frac{\bar{Y}^2\lambda_{ZM_z}}{2}. \quad (17)$$

On differentiating the above MSE w.r.t. $\beta_1,\beta_2,\beta_3,\beta_4$ and equating each to zero we obtain their least squares estimates respectively as follows:

$$\hat{\beta}_1=\frac{2\bar{X}S_{xy}}{\bar{Y}S_x^2}+\frac{\beta_2\bar{X}S_{XM_x}[f_x(M_x)]^{-1}}{S_x^2M_x},\ \hat{\beta}_2=\frac{-2M_x\left(S_x^2S_{YM_x}-S_{xy}S_{XM_x}\right)}{[f_x(M_x)]^{-1}\bar{Y}(0.25S_x^2-S_{XM_x}^2)},$$
$$\hat{\beta}_3=\frac{2\bar{Z}S_{yz}}{\bar{Y}S_z^2}+\frac{\beta_4\bar{Z}S_{ZM_z}[f_z(M_z)]^{-1}}{S_z^2M_z},\ \hat{\beta}_4=\frac{-2M_z\left(S_z^2S_{YM_z}-S_{yz}S_{ZM_z}\right)}{[f_z(M_z)]^{-1}\bar{Y}(0.25S_z^2-S_{ZM_z}^2)}. \quad (18)$$

Substituting the values of these estimates in the expression of the MSE, we obtain the minimum MSE of the proposed estimator given as follows:

$$\min MSE(\bar{y}_{proposed})=\left(\frac{1}{m}-\frac{1}{N}\right)S_y^2$$
$$+\left(\frac{1}{n}-\frac{1}{m}\right)S_y^2\left\{1-\rho_{xy}^2-\rho_{yz}^2-\frac{(S_x^2S_{YM_x}-S_{xy}S_{XM_x})^2}{S_y^2S_x^2(0.25S_x^2-S_{XM_x}^2)}-\frac{(S_z^2S_{YM_z}-S_{yz}S_{ZM_z})^2}{S_y^2S_z^2(0.25S_z^2-S_{ZM_z}^2)}\right\}. \quad (19)$$

3.2. Case-II : Auxiliary variables X and Z are not independent of each other

The expression of the approximated bias of the proposed estimator is given as follows

$$Bias\left(\bar{y}_{proposed}\right)=E\left(\bar{y}_{proposed}\right)-\bar{Y}$$
$$\approx\bar{Y}\lambda_3\{\beta_1J_1+\beta_2J_2+\beta_3J_3+\beta_4J_4+\beta_1^2J_5+\beta_2^2J_6+\beta_3^2J_7+\beta_4^2J_8$$
$$+\beta_1\beta_2J_9+\beta_1\beta_3J_{10}+\beta_1\beta_4J_{11}+\beta_2\beta_3J_{12}+\beta_2\beta_4J_{13}+\beta_3\beta_4J_{14}\}, \quad (20)$$

where

$$J_1=\frac{C_x^2}{4}-\frac{\lambda_{xy}}{2},\ J_2=\frac{\lambda_{M_x}}{4}+\frac{\lambda_{YM_x}}{2},\ J_3=\frac{C_z^2}{4}-\frac{\lambda_{yz}}{2},\ J_4=\frac{\lambda_{M_z}}{4}+\frac{\lambda_{YM_z}}{2},\ J_5=\frac{C_x^2}{8},$$
$$J_6=\frac{\lambda_{M_x}}{8},\ J_7=\frac{C_z^2}{8},\ J_8=\frac{\lambda_{M_z}}{8},\ J_9=-\frac{\lambda_{XM_x}}{4},\ J_{10}=\frac{\lambda_{xz}}{4}, \quad (21)$$
$$J_{11}=-\frac{\lambda_{XM_z}}{4},\ J_{12}=-\frac{\lambda_{ZM_x}}{4},\ J_{13}=\frac{\lambda_{M_xM_z}}{4},\ J_{14}=-\frac{\lambda_{ZM_z}}{4}.$$

Neglecting the terms of e_i's of order 3 and higher, the MSE of the proposed estimator is approximated as follows:

$$MSE\left(\bar{y}_{proposed}\right)=E\left(\bar{y}_{proposed}-\bar{Y}\right)^2$$
$$\approx \lambda_2 S_y^2+\lambda_3\{\beta_1 J_{15}+\beta_2 J_{16}+\beta_3 J_{17}+\beta_4 J_{18}+\beta_1^2 J_{19}+\beta_2^2 J_{20}+\beta_3^2 J_{21}+\beta_4^2 J_{22}$$
$$+\beta_1\beta_2 J_{23}+\beta_1\beta_3 J_{24}+\beta_1\beta_4 J_{25}+\beta_2\beta_3 J_{26}+\beta_2\beta_4 J_{27}+\beta_3\beta_4 J_{28}\}, \quad (22)$$

where

$$J_{15}=-\frac{\bar{Y}S_{xy}}{\bar{X}},\ J_{16}=\bar{Y}^2\lambda_{YM_x},\ J_{17}=-\frac{\bar{Y}S_{yz}}{\bar{Z}},\ J_{18}=\bar{Y}^2\lambda_{YM_z},\ J_{19}=-\frac{\bar{Y}^2S_x^2}{4\bar{X}^2},$$
$$J_{20}=\frac{\bar{Y}^2\lambda_{M_x}}{4},\ J_{21}=-\frac{\bar{Y}^2S_z^2}{4\bar{Z}^2},\ J_{22}=\frac{\bar{Y}^2\lambda_{M_z}}{4},\ J_{23}=-\frac{\bar{Y}^2\lambda_{XM_x}}{2},\ J_{24}=\frac{\bar{Y}^2S_{xz}}{2\bar{X}\bar{Z}}, \quad (23)$$
$$J_{25}=-\frac{\bar{Y}^2\lambda_{XM_z}}{2},\ J_{26}=-\frac{\bar{Y}^2\lambda_{ZM_x}}{2},\ J_{27}=\frac{\bar{Y}^2\lambda_{M_xM_z}}{2},\ J_{28}=-\frac{\bar{Y}^2\lambda_{ZM_z}}{2}.$$

On differentiating the above MSE w.r.t $\beta_1,\beta_2,\beta_3,\beta_4$ and equating each to zero we obtain the normal equations as follows:

$$\beta_1A_1+\beta_2A_2+\beta_3A_3+\beta_4A_4=A_0, \quad (24)$$
$$\beta_1B_1+\beta_2B_2+\beta_3B_3+\beta_4B_4=B_0, \quad (25)$$
$$\beta_1C_1+\beta_2C_2+\beta_3C_3+\beta_4C_4=C_0 \quad (26)$$
$$\beta_1D_1+\beta_2D_2+\beta_3D_3+\beta_4D_4=D_0, \quad (27)$$

where

$$A_1=\frac{S_x^2}{\bar{X}},\ A_2=-\frac{S_{XM_x}\left[f_x\left(M_x\right)\right]^{-1}}{M_x},\ A_3=\frac{S_{xz}}{\bar{Z}},\ A_4=-\frac{S_{XM_z}\left[f_z\left(M_z\right)\right]^{-1}}{M_z},\ A_0=\frac{2S_{xy}}{\bar{Y}},$$
$$B_1=-\frac{S_{XM_x}}{\bar{X}},\ B_2=\frac{(0.25)\left[f_x\left(M_x\right)\right]^{-1}}{M_x},\ B_3=-\frac{S_{ZM_x}}{\bar{Z}},\ B_0=-\frac{2S_{YM_x}}{\bar{Y}},$$
$$B_4=\frac{(0.25)\left[4P_{11}(x,z)-1\right]\left[f_z\left(M_z\right)\right]^{-1}}{M_z},\ C_1=\frac{S_{xz}}{\bar{X}},\ C_2=-\frac{S_{ZM_x}\left[f_x\left(M_x\right)\right]^{-1}}{M_x},\ C_3=\frac{S_z^2}{\bar{Z}},$$
$$C_4=-\frac{S_{ZM_z}\left[f_z\left(M_z\right)\right]^{-1}}{M_z},\quad C_0=\frac{2S_{yz}}{\bar{Y}},\quad D_1=-\frac{S_{XM_z}}{\bar{X}},\quad D_2=\frac{(0.25)\left[4P_{11}(x,z)-1\right]\left[f_x\left(M_x\right)\right]^{-1}}{M_x},$$
$$D_3=-\frac{S_{ZM_z}}{\bar{Z}},\ D_2=\frac{(0.25)\left[4P_{11}(x,z)-1\right]\left[f_x\left(M_x\right)\right]^{-1}}{M_x},\ D_3=-\frac{S_{ZM_z}}{\bar{Z}},\ D_4=\frac{(0.25)\left[f_z\left(M_z\right)\right]^{-1}}{M_z},$$
$$D_0=-\frac{2S_{YM_z}}{\bar{Y}}. \quad (28)$$

Using Cramer's rule to solve the above normal equations, we obtain the least squares estimates of $\beta_1,\beta_2,\beta_3,\beta_4$, respectively as follows:

$$\hat{\beta}_1=\frac{\Delta_1}{\Delta},\ \hat{\beta}_2=\frac{\Delta_2}{\Delta},\ \hat{\beta}_3=\frac{\Delta_3}{\Delta},\ \hat{\beta}_4=\frac{\Delta_4}{\Delta}, \quad (29)$$

where

$$\Delta = \begin{vmatrix} A_1 & A_2 & A_3 & A_4 \\ B_1 & B_2 & B_3 & B_4 \\ C_1 & C_2 & C_3 & C_4 \\ D_1 & D_2 & D_3 & D_4 \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} A_0 & A_2 & A_3 & A_4 \\ B_0 & B_2 & B_3 & B_4 \\ C_0 & C_2 & C_3 & C_4 \\ D_0 & D_2 & D_3 & D_4 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} A_1 & A_0 & A_3 & A_4 \\ B_1 & B_0 & B_3 & B_4 \\ C_1 & C_0 & C_3 & C_4 \\ D_1 & D_0 & D_3 & D_4 \end{vmatrix},$$

$$\Delta_3 = \begin{vmatrix} A_1 & A_2 & A_0 & A_4 \\ B_1 & B_2 & B_0 & B_4 \\ C_1 & C_2 & C_0 & C_4 \\ D_1 & D_2 & D_0 & D_4 \end{vmatrix}, \quad \Delta_4 = \begin{vmatrix} A_1 & A_2 & A_3 & A_0 \\ B_1 & B_2 & B_3 & B_0 \\ C_1 & C_2 & C_3 & C_0 \\ D_1 & D_2 & D_3 & D_0 \end{vmatrix}.$$

4. Efficiency Comparison

A theoretical comparison of the proposed estimator with other estimators is done only for the case of independent auxiliary variables as a comparable expression of the minimum MSE can be obtained.

In the case of dependent auxiliary variables, only an empirical comparative study is carried out as will be shown in the later section. The proposed estimator is compared with the usual mean estimators and the most recently developed estimators as follows

4.1. Comparison with sample mean $(\bar{y})$:

The sample mean is given by $\bar{y} = \frac{1}{n}\sum_{r=1}^{n} y_r$. Under SRSWOR, the variance of the sample mean is given by

$$Var(\bar{y}) = \left(\frac{1}{n} - \frac{1}{N}\right) S_y^2, \tag{31}$$

$$\min MSE(\bar{y}_{proposed}) - Var(\bar{y})$$
$$= -\left(\frac{1}{n} - \frac{1}{m}\right)\left\{\frac{S_{xy}^2}{S_x^2} + \frac{S_{yz}^2}{S_z^2} + \frac{(S_x^2 S_{YM_x} - S_{xy} S_{XM_x})^2}{S_x^2(0.25 S_x^2 - S_{XM_x}^2)} + \frac{(S_z^2 S_{YM_z} - S_{yz} S_{ZM_z})^2}{S_z^2(0.25 S_z^2 - S_{ZM_z}^2)}\right\} < 0. \tag{32}$$

Hence, the proposed estimator $\bar{y}_{proposed}$ is always more efficient than the sample mean.

4.2. Comparison with the ratio estimator under double-sampling $\bar{y}_{rat(d)}$:

The ratio estimator under double-sampling scheme is given by $\bar{y}_{rat(d)} = \bar{y}_n \frac{\bar{x}_m}{\bar{x}_n}$. Its MSE is given as follows:

$$MSE(\bar{y}_{rat(d)}) = \left(\frac{1}{m} - \frac{1}{N}\right) S_y^2 + \left(\frac{1}{n} - \frac{1}{m}\right)\left(S_y^2 - 2RS_{xy} + R^2 S_x^2\right). \tag{33}$$

Thus,

$$\min MSE(\bar{y}_{proposed}) - MSE(\bar{y}_{rat(d)})$$
$$= -\left(\frac{1}{n} - \frac{1}{m}\right)\left\{\frac{S_{xy}^2}{S_x^2} + \frac{S_{yz}^2}{S_z^2} + \frac{(S_x^2 S_{YM_x} - S_{xy} S_{XM_x})^2}{S_x^2(0.25 S_x^2 - S_{XM_x}^2)} + \frac{(S_z^2 S_{YM_z} - S_{yz} S_{ZM_z})^2}{S_z^2(0.25 S_z^2 - S_{ZM_z}^2)} + R^2 S_x^2 - 2RS_{xy}\right\} < 0. \tag{34}$$

Hence, the proposed estimator $\bar{y}_{proposed}$ is always more efficient than $\bar{y}_{rat(d)}$.

4.3. Comparison with the regression estimator under double-sampling $\bar{y}_{reg(d)}$:

The regression estimator under double-sampling scheme is given by $\bar{y}_{reg(d)} = \bar{y}_n + b\left(\bar{x}_m - \bar{x}_n\right)$ whose MSE is given by

$$MSE\left(\bar{y}_{reg(d)}\right) = \left(\frac{1}{m} - \frac{1}{N}\right) S_y^2 + \left(\frac{1}{n} - \frac{1}{m}\right)\left(S_y^2 - S_y^2 \rho_{xy}^2\right). \quad (35)$$

Therefore,

$$\min MSE\left(\bar{y}_{proposed}\right) - MSE\left(\bar{y}_{reg(d)}\right)$$

$$= -\left(\frac{1}{n} - \frac{1}{m}\right)\left\{\frac{S_{xy}^2}{S_x^2} + \frac{S_{yz}^2}{S_z^2} + \frac{(S_x^2 S_{YM_x} - S_{xy} S_{XM_x})^2}{S_x^2(0.25S_x^2 - S_{XM_x}^2)} + \frac{(S_z^2 S_{YM_z} - S_{yz} S_{ZM_z})^2}{S_z^2(0.25S_z^2 - S_{ZM_z}^2)} - S_y^2 \rho_{xy}^2\right\} < 0. \quad (36)$$

Hence, the proposed estimator $\bar{y}_{proposed}$ is always more efficient than $\bar{y}_{reg(d)}$.

4.4. Comparison with Vadlamudi et al. (2017) estimator $\bar{y}_{kal}$:

The unbiased estimator proposed by Vadlamudi et al. (2017) is given by

$$\bar{y}_{kal} = \bar{y}_n + \beta_1\left(\bar{x}_m - \bar{x}_n\right) + \beta_2\left(\hat{M}_x^* - \hat{M}_x\right), \quad (37)$$

whose minimum variance is given by

$$\min Var(\bar{y}_{kal}) = \left(\frac{1}{m} - \frac{1}{N}\right) S_y^2 + \left(\frac{1}{n} - \frac{1}{m}\right)\left\{S_y^2 - \frac{S_{xy}^2}{S_x^2} - \frac{(S_x^2 S_{YM_x} - S_{xy} S_{XM_x})^2}{S_x^2(0.25S_x^2 - S_{XM_x}^2)}\right\}, \quad (38)$$

$$\min MSE\left(\bar{y}_{proposed}\right) - \min Var\left(\bar{y}_{kal}\right) = -\left(\frac{1}{n} - \frac{1}{m}\right)\left\{\frac{S_{yz}^2}{S_z^2} + \frac{(S_z^2 S_{YM_z} - S_{yz} S_{ZM_z})^2}{S_z^2(0.25S_z^2 - S_{ZM_z}^2)}\right\} < 0. \quad (39)$$

Hence, the proposed estimator $\bar{y}_{proposed}$ is always more efficient than $\bar{y}_{reg(d)}$. Note that

$$\left(0.25S_x^2 - S_{XM_x}^2\right) > 0 \text{ and } \left(0.25S_z^2 - S_{ZM_z}^2\right) > 0. \quad (40)$$

5. Empirical Study (Data Generated by Simulation)

For a simulation study we considered five scenarios. In each scenario, three sets of data of variables Y, X and Z have been generated using R-software from Poisson distribution for various values of parameter λ until particular values of the different correlation coefficients have been obtained in each case. Double sampling is employed in each scenario for various values of the sample sizes m and n. The MSE/Variances of each estimator under consideration have been computed and comparison is made amongst them for different values of the sample sizes of both the first phase and the second phase sample.

Scenario-I: $N = 500$, $\lambda = 140$, $\rho_{xy} = 0.7175$, $\rho_{yz} = 0.6097$, $\rho_{xz} = -0.0453$

Scenario-II: $N = 250$, $\lambda = 84$, $\rho_{xy} = 0.6857$, $\rho_{yz} = 0.7088$, $\rho_{xz} = -0.0274$

Scenario-III: $N = 250$, $\lambda = 84$, $\rho_{xy} = 0.9774$, $\rho_{yz} = 0.1843$, $\rho_{xz} = -0.0274$

Scenario-IV: $N = 250$, $\lambda = 24$, $\rho_{xy} = 0.3187$, $\rho_{yz} = 0.9223$, $\rho_{xz} = -0.0724$

Scenario-V: $N = 250$, $\lambda = 24$, $\rho_{xy} = 0.8820$, $\rho_{yz} = 0.4942$, $\rho_{xz} = 0.0263$.

Therefore, the MSE/variances of the various estimators are given in Table 2 for comparison.

Table 2 Comparison of the MSE of the proposed estimator with the other estimators under consideration

Scenario-I ($N = 500$)	MSE / Variances	Sample Sizes ($m = 220$, $n = 80$)	($m = 231$, $n = 150$)
$\rho_{xy} = 0.7175$	MSE($\hat{\bar{y}}_{\text{proposed}}$)(X &ZInd) =	0.8455	0.6364
$\rho_{yz} = 0.6097$	MSE($\hat{\bar{y}}_{\text{proposed}}$)(X&Z Dep) =	0.8398	0.6347
$\rho_{xz} = -0.0453$	V($\bar{y}_{\text{kal}}$) =	1.5712	0.8497
	V($\bar{y}_{\text{SRSWOR}}$) =	2.5766	1.1451
	MSE($\bar{y}_{\text{rat(d)}}$) =	3.4332	3.3801
	MSE($\bar{y}_{\text{reg(d)}}$) =	1.5718	0.8499
Scenario-II ($N = 250$)	**MSE / Variances**	**Sample Sizes ($m = 137$, $n = 95$)**	**($m = 158$, $n = 116$)**
$\rho_{xy} = 0.6857$	MSE($\hat{\bar{y}}_{\text{proposed}}$)(X &ZInd) =	0.2642	0.1865
$\rho_{yz} = 0.7088$	MSE($\hat{\bar{y}}_{\text{proposed}}$)(X&Z Dep) =	0.2587	0.1826
$\rho_{xz} = -0.0274$	V($\bar{y}_{\text{kal}}$) =	0.3917	0.2771
	V($\bar{y}_{\text{SRSWOR}}$) =	0.5117	0.3623
	MSE($\bar{y}_{\text{rat(d)}}$) =	0.5337	0.3779
	MSE($\bar{y}_{\text{reg(d)}}$) =	0.3928	0.2778
Scenario-III ($N = 250$)	**MSE / Variances**	**Sample Sizes ($m = 137$, $n = 95$)**	**($m = 158$, $n = 116$)**
$\rho_{xy} = 0.9774$	MSE($\hat{\bar{y}}_{\text{proposed}}$)(X &ZInd) =	0.2526	0.1783
$\rho_{yz} = 0.1843$	MSE($\hat{\bar{y}}_{\text{proposed}}$)(X&Z Dep) =	0.2508	0.1771
$\rho_{xz} = -0.0274$	V($\bar{y}_{\text{kal}}$) =	0.2617	0.1848
	V($\bar{y}_{\text{SRSWOR}}$) =	0.4962	0.3513
	MSE($\bar{y}_{\text{rat(d)}}$) =	0.2733	0.1930
	MSE($\bar{y}_{\text{reg(d)}}$) =	0.2618	0.1849
Scenario-IV ($N = 250$)	**MSE / Variances**	**Sample Sizes ($m = 137$, $n = 95$)**	**($m = 158$, $n = 116$)**
$\rho_{xy} = 0.3187$	MSE($\hat{\bar{y}}_{\text{proposed}}$)(X &ZInd) =	0.0879	0.0621
$\rho_{yz} = 0.9223$	MSE($\hat{\bar{y}}_{\text{proposed}}$)(X&Z Dep) =	0.0841	0.0593
$\rho_{xz} = -0.0274$	V($\bar{y}_{\text{kal}}$) =	0.1579	0.1118
	V($\bar{y}_{\text{SRSWOR}}$) =	0.1663	0.1177
	MSE($\bar{y}_{\text{rat(d)}}$) =	0.2281	0.1616
	MSE($\bar{y}_{\text{reg(d)}}$) =	0.1579	0.1118

Table 2 (Continued)

Scenario-V (N = 250)	MSE / Variances	Sample Sizes (m = 137, n = 95)	(m = 158, n = 116)
ρ_{xy} = 0.8820	MSE($\hat{\bar{y}}_{proposed}$)(X &ZInd) =	0.0933	0.0659
ρ_{yz} = 0.4942	MSE($\hat{\bar{y}}_{proposed}$)(X&Z Dep) =	0.0910	0.0642
ρ_{xz} = 0.0263	V($\bar{y}_{kal}$) =	0.1135	0.0802
	V($\bar{y}_{SRSWOR}$) =	0.1845	0.1306
	MSE($\bar{y}_{rat(d)}$) =	0.1421	0.1006
	MSE($\bar{y}_{reg(d)}$) =	0.1135	0.0802

5.1. Remarks

From Table 2 we see that the MSE of the proposed estimator (independent and dependent case) possesses the minimum value amongst the other estimators under consideration based on the generated data in each scenario. As sample sizes are increased, the efficiency of each estimator also increases, which is to be expected. It may be pointed out that in each scenario the proposed estimator is always slightly more efficient (smallest MSE) when the auxiliary variables X and Z are not independent as compared to the case when they are independent. This may be attributed to the fact that the correlation between the auxiliary variables X and Z has not been neglected and taken into account for the computation of the MSE.

Hence, we can conclude that the proposed estimator is always more efficient than the previous estimator and the other standard estimators under consideration.

6. Conclusions

The motivation behind the present study is to examine whether the inclusion of information on both mean and median of two auxiliary variables would contribute to the efficiency of the proposed estimator. The proposed exponential ratio-type estimator $\bar{y}_{proposed}$ was found to be more efficient than the regression estimator which makes use of only one auxiliary variable and the other classical estimators of mean. The result of the empirical study also confirms these findings. Hence, we conclude that as more auxiliary information is included in the estimation process, the more precise the estimator will be for estimating the population mean.

The advantage behind the proposed estimator is that it can be used for estimating the population mean of a certain study variable if information on not one but two auxiliary variables is available provided that they have some level of linear dependence and correlation amongst them which is mostly true in cases of real life data.

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